

Heuristic Search for Planning

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Acknowledgements

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Outline

- 1 How to obtain a heuristic
 - The STRIPS heuristic
 - Relaxation and abstraction
- 2 Towards relaxations for planning: Positive normal form
 - Motivation
 - Definition & algorithm
 - Example
- 3 Relaxed planning tasks
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A simple heuristic for deterministic planning

STRIPS (Fikes & Nilsson, 1971) used the number of state variables that differ in current state s and a STRIPS goal $I_1 \wedge \dots \wedge I_n$:

$$h(s) := |\{i \in \{1, \dots, n\} \mid s(a) \neq I_i\}|.$$

Intuition: more true goal literals \rightsquigarrow closer to the goal

\rightsquigarrow **STRIPS heuristic** (properties?)

Note: From now on, for convenience we usually write heuristics as functions of states (as above), not nodes.

Node heuristic h' is defined from state heuristic h as $h'(\sigma) := h(\text{state}(\sigma))$.

What is wrong with the STRIPS heuristic?

- quite **uninformative**:
the range of heuristic values in a given task is small;
typically, most successors have the same estimate
 - very sensitive to **reformulation**:
can easily transform any planning task into an equivalent one
where $h(s) = 1$ for all non-goal states
 - ignores almost all **problem structure**:
heuristic value does not depend on the set of operators!
- need a better, principled way of coming up with heuristics

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Coming up with heuristics in a principled way

General procedure for obtaining a heuristic

Solve an easier version of the problem.

Two common methods:

- **relaxation**: consider **less constrained** version of the problem
- **abstraction**: consider **smaller** version of real problem

Both have been very successfully applied in planning.

We consider both in this course, beginning with **relaxation**.

Relaxing a problem

How do we relax a problem?

Example (Route planning for a road network)

The road network is formalized as a weighted graph over points in the Euclidean plane. The weight of an edge is the **road distance** between two locations.

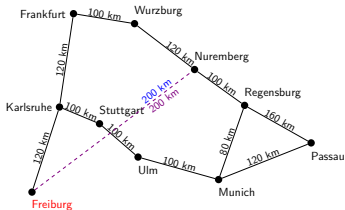
A relaxation **drops constraints** of the original problem.

Example (Relaxation for route planning)

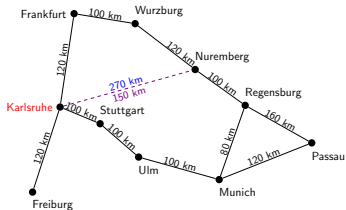
Use the **Euclidean distance** $\sqrt{|x_1 - y_1|^2 + |x_2 - y_2|^2}$ as a heuristic for the road distance between (x_1, x_2) and (y_1, y_2) .
This is a **lower bound** on the road distance (→ admissible).

→ We drop the constraint of having to travel on roads.

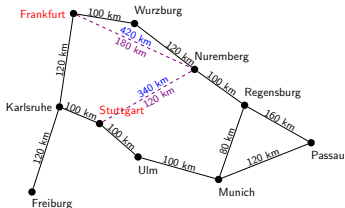
A* using the Euclidean distance heuristic



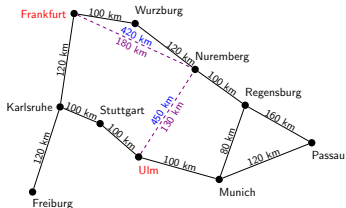
A* using the Euclidean distance heuristic



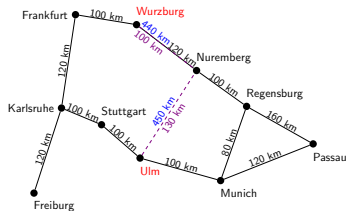
A* using the Euclidean distance heuristic



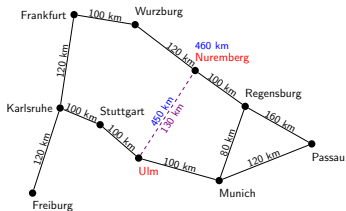
A* using the Euclidean distance heuristic



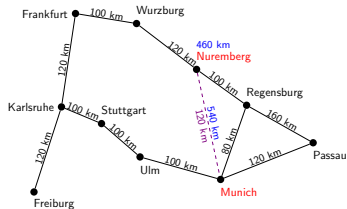
A* using the Euclidean distance heuristic



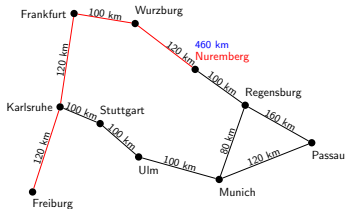
A* using the Euclidean distance heuristic



A* using the Euclidean distance heuristic



A* using the Euclidean distance heuristic



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- Relaxation is a general technique for heuristic design:
 - **Straight-line heuristic** (route planning): Ignore the fact that one must stay on roads.
 - **Manhattan heuristic** (15-puzzle): Ignore the fact that one cannot move through occupied tiles.
- We want to apply the idea of relaxations to planning.
- Informally, we want to ignore **bad side effects** of applying operators.

What is a good or bad effect?

Question: Which operator effects are good, and which are bad?

Difficult to answer in general, because it depends on context:

- Locking the entrance door is **good** if we want to keep burglars out.
- Locking the entrance door is **bad** if we want to enter.

We will now consider a reformulation of planning tasks that makes the distinction between good and bad effects obvious.

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The notation we use here is a generalization of the notation used in previous introductory lectures, which was based on the GNT textbook. Recall:

Definition

An operator $\langle c, e \rangle$ is a **STRIPS operator** if

- 1 **precondition** c is a conjunction* of literals, and
- 2 **effect** e is a conjunction of atomic effects.

*We previously used "set" rather than "conjunction".

Here we extend the expressiveness of our operator definition as follows:

- **precondition** c is an arbitrary propositional formula.
- (Deterministic) **effect** e is defined recursively as follows:
 - 1 If $a \in A$ is a state variable, then a and $\neg a$ are effects (**atomic effects**).
 - 2 If e_1, \dots, e_n are effects, then $e_1 \wedge \dots \wedge e_n$ is an effect (**conjunctive effects**). The special case with $n = 0$ is the empty conjunction \top .
 - 3 If c is a propositional formula and e is an effect, then $c \triangleright e$ is an effect (**conditional effects**).

Atomic effects a and $\neg a$ are best understood as assignments $a := 1$ and $a := 0$, respectively.

Positive normal form

Definition (operators in positive normal form)

An operator $o = \langle c, e \rangle$ is in **positive normal form** if it is in normal form, no negation symbols appear in c , and no negation symbols appear in any effect condition in e .

Definition (planning tasks in positive normal form)

A planning task $\langle A, I, O, G \rangle$ is in **positive normal form** if all operators in O are in positive normal form and no negation symbols occur in the goal G .

Positive normal form: existence

Theorem (positive normal form)

Every planning task Π has an equivalent planning task Π' in positive normal form.

Moreover, Π' can be computed from Π in polynomial time.

Note: Equivalence here means that the represented transition systems of Π and Π' , limited to the states that can be reached from the initial state, are isomorphic.

We prove the theorem by describing a suitable algorithm. (However, we do not prove its correctness or complexity.)

Transformation of $\langle A, I, O, G \rangle$ to positive normal form

Convert all operators $o \in O$ to normal form.

Convert all conditions* to negation normal form (NNF).

while any condition contains a negative literal $\neg a$:

Let a be a variable which occurs negatively in a condition.

$A := A \cup \{\hat{a}\}$ for some new state variable \hat{a}

$I(\hat{a}) := 1 - I(a)$

Replace the effect a by $(a \wedge \neg \hat{a})$ in all operators $o \in O$.

Replace the effect $\neg a$ by $(\neg a \wedge \hat{a})$ in all operators $o \in O$.

Replace $\neg a$ by \hat{a} in all conditions.

Convert all operators $o \in O$ to normal form (again).

* Here, *all conditions* refers to all operator preconditions, operator effect conditions and the goal.

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Example (transformation to positive normal form)

$A = \{home, uni, lecture, bike, bike-locked\}$

$I = \{home \mapsto 1, bike \mapsto 1, bike-locked \mapsto 1, uni \mapsto 0, lecture \mapsto 0\}$

$O = \{\langle home \wedge bike \wedge \neg bike-locked, \neg home \wedge uni \rangle, \langle bike \wedge bike-locked, \neg bike-locked \rangle, \langle bike \wedge \neg bike-locked, bike-locked \rangle, \langle uni, lecture \wedge ((bike \wedge \neg bike-locked) \triangleright \neg bike) \rangle\}$

$G = lecture \wedge bike$

Example (transformation to positive normal form)

$A = \{home, uni, lecture, bike, bike-locked\}$

$I = \{home \mapsto 1, bike \mapsto 1, bike-locked \mapsto 1, uni \mapsto 0, lecture \mapsto 0\}$

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$G = lecture \wedge bike$

Identify state variable a occurring negatively in conditions.

Positive normal form: example

Example (transformation to positive normal form)

$$\begin{aligned}A &= \{home, uni, lecture, bike, bike-locked, \mathbf{bike-unlocked}\} \\I &= \{home \mapsto 1, bike \mapsto 1, bike-locked \mapsto 1, \\&\quad uni \mapsto 0, lecture \mapsto 0, \mathbf{bike-unlocked \mapsto 0}\} \\O &= \{\langle home \wedge bike \wedge \neg bike-locked, \neg home \wedge uni \rangle, \\&\quad \langle bike \wedge bike-locked, \neg bike-locked \rangle, \\&\quad \langle bike \wedge \neg bike-locked, bike-locked \rangle, \\&\quad \langle uni, lecture \wedge ((bike \wedge \neg bike-locked) \triangleright \neg bike) \rangle\} \\G &= lecture \wedge bike\end{aligned}$$

Introduce new variable \hat{a} with complementary initial value.

Positive normal form: example

Example (transformation to positive normal form)

$$\begin{aligned}A &= \{home, uni, lecture, bike, bike-locked, bike-unlocked\} \\I &= \{home \mapsto 1, bike \mapsto 1, bike-locked \mapsto 1, \\&\quad uni \mapsto 0, lecture \mapsto 0, bike-unlocked \mapsto 0\} \\O &= \{\langle home \wedge bike \wedge \neg bike-locked, \neg home \wedge uni \rangle, \\&\quad \langle bike \wedge bike-locked, \mathbf{\neg bike-locked} \rangle, \\&\quad \langle bike \wedge \neg bike-locked, \mathbf{bike-locked} \rangle, \\&\quad \langle uni, lecture \wedge ((bike \wedge \neg bike-locked) \triangleright \neg bike) \rangle\} \\G &= lecture \wedge bike\end{aligned}$$

Identify effects on variable a .

Positive normal form: example

Example (transformation to positive normal form)

$$\begin{aligned}A &= \{home, uni, lecture, bike, bike-locked, bike-unlocked\} \\I &= \{home \mapsto 1, bike \mapsto 1, bike-locked \mapsto 1, \\&\quad uni \mapsto 0, lecture \mapsto 0, bike-unlocked \mapsto 0\} \\O &= \{\langle home \wedge bike \wedge \neg bike-locked, \neg home \wedge uni \rangle, \\&\quad \langle bike \wedge bike-locked, \mathbf{\neg bike-locked \wedge bike-unlocked} \rangle, \\&\quad \langle bike \wedge \neg bike-locked, \mathbf{bike-locked \wedge \neg bike-unlocked} \rangle, \\&\quad \langle uni, lecture \wedge ((bike \wedge \neg bike-locked) \triangleright \neg bike) \rangle\} \\G &= lecture \wedge bike\end{aligned}$$

Introduce complementary effects for \hat{a} .

Positive normal form: example

Example (transformation to positive normal form)

$$\begin{aligned}A &= \{home, uni, lecture, bike, bike-locked, bike-unlocked\} \\I &= \{home \mapsto 1, bike \mapsto 1, bike-locked \mapsto 1, \\&\quad uni \mapsto 0, lecture \mapsto 0, bike-unlocked \mapsto 0\} \\O &= \{\langle home \wedge bike \wedge \neg bike-locked, \neg home \wedge uni \rangle, \\&\quad \langle bike \wedge bike-locked, \neg bike-locked \wedge bike-unlocked \rangle, \\&\quad \langle bike \wedge \mathbf{\neg bike-locked}, bike-locked \wedge \neg bike-unlocked \rangle, \\&\quad \langle uni, lecture \wedge ((bike \wedge \mathbf{\neg bike-locked}) \triangleright \neg bike) \rangle\} \\G &= lecture \wedge bike\end{aligned}$$

Identify negative conditions for \hat{a} .

Example (transformation to positive normal form)

$$\begin{aligned}
 A &= \{home, uni, lecture, bike, bike-locked, bike-unlocked\} \\
 I &= \{home \mapsto 1, bike \mapsto 1, bike-locked \mapsto 1, \\
 &\quad uni \mapsto 0, lecture \mapsto 0, bike-unlocked \mapsto 0\} \\
 O &= \{(home \wedge bike \wedge \mathbf{bike-unlocked}, \neg home \wedge uni), \\
 &\quad (bike \wedge bike-locked, \neg bike-locked \wedge bike-unlocked), \\
 &\quad (bike \wedge \mathbf{bike-unlocked}, bike-locked \wedge \neg bike-unlocked), \\
 &\quad (uni, lecture \wedge ((bike \wedge \mathbf{bike-unlocked}) \supset \neg bike))\} \\
 G &= lecture \wedge bike
 \end{aligned}$$

Replace by positive condition \hat{a} .

Example (transformation to positive normal form)

$$\begin{aligned}
 A &= \{home, uni, lecture, bike, bike-locked, bike-unlocked\} \\
 I &= \{home \mapsto 1, bike \mapsto 1, bike-locked \mapsto 1, \\
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 &\quad (bike \wedge bike-unlocked, bike-locked \wedge \neg bike-unlocked), \\
 &\quad (uni, lecture \wedge ((bike \wedge bike-unlocked) \supset \neg bike))\} \\
 G &= lecture \wedge bike
 \end{aligned}$$

What does this transformation achieve?

We have expanded the size of our domain by introducing new propositions to ensure that all the conditions that affect planning:

- preconditions
- conditions of conditional effects
- goals

are expressed in terms of positive literals, and we've adjusted the effects of operators to ensure that they are consistent with the introduction of these new propositions.

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Relaxed planning tasks: idea

In positive normal form, good and bad effects are easy to distinguish:

- Effects that make state variables true are good (add effects).
- Effects that make state variables false are bad (delete effects).

*** Idea for the heuristic: Ignore all delete effects. **

Relaxed planning tasks

Definition (relaxation of operators)

The **relaxation** o^+ of an operator $o = \langle c, e \rangle$ in positive normal form is the operator which is obtained by replacing all negative effects $\neg a$ within e by the do-nothing effect \top .

Definition (relaxation of planning tasks)

The **relaxation** Π^+ of a planning task $\Pi = \langle A, I, O, G \rangle$ in positive normal form is the planning task $\Pi^+ := \langle A, I, \{o^+ \mid o \in O\}, G \rangle$.

Definition (relaxation of operator sequences)

The **relaxation** of an operator sequence $\pi = o_1 \dots o_n$ is the operator sequence $\pi^+ := o_1^+ \dots o_n^+$.

Relaxed planning tasks: terminology

- Planning tasks in positive normal form without delete effects are called **relaxed planning tasks**.
- Plans for relaxed planning tasks are called **relaxed plans**.
- If Π is a planning task in positive normal form and π^+ is a plan for Π^+ , then π^+ is called a **relaxed plan for Π** .

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Greedy algorithm for relaxed planning tasks

The relaxed planning task can be solved in polynomial time using a simple greedy algorithm:

Greedy planning algorithm for $\langle A, I, O^+, G \rangle$

```
 $s := I$ 
 $\pi^+ := \epsilon$ 
forever:
  if  $s \models G$ :
    return  $\pi^+$ 
  else if there is an operator  $o^+ \in O^+$  applicable in  $s$ 
    with  $app_{o^+}(s) \neq s$ :
    Append such an operator  $o^+$  to  $\pi^+$ .
     $s := app_{o^+}(s)$ 
  else:
    return unsolvable
```

Correctness of the greedy algorithm

The algorithm is **sound**:

- If it returns a plan, this is indeed a correct solution.
- If it returns "unsolvable", the task is indeed unsolvable

What about **completeness** (termination) and **runtime**?

- Each iteration of the loop adds at least one atom to the set of true state variables in s .
- This guarantees termination after at most $|A|$ iterations.
- Thus, the algorithm can clearly be implemented to run in polynomial time.

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Using the greedy algorithm as a heuristic

We can apply the greedy algorithm within heuristic search:

- In a search node σ , solve the relaxation of the planning task with $state(\sigma)$ as the initial state.
- Set $h(\sigma)$ to the length of the generated relaxed plan.

Is this an **admissible** heuristic?

- Yes if the relaxed plans are **optimal** (due to the plan preservation corollary).
- However, usually they are not, because our greedy planning algorithm is very poor.

Generating an admissible heuristic is NP-hard

- To obtain an *admissible* heuristic, we need to generate an *optimal* relaxed plan.
- The problem of deciding whether a given relaxed planning task has a length at most K is NP-complete (through a reduction of part of the problem to the set cover problem).
- Thus, generating an optimal relaxed plan for the purposes of generating a heuristic (not even solving the problem!) is not a good strategy.

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Using relaxations in practice

How can we use relaxations for heuristic planning in practice?

Different possibilities:

- Implement an **optimal planner** for relaxed planning tasks and use its solution lengths as an estimate, even though it is NP-hard.
→ h^+ heuristic
- Do not actually solve the relaxed planning task, but compute an estimate of its difficulty in a different way.
→ h_{\max} heuristic, h_{add} heuristic
- Compute a solution for relaxed planning tasks which is not necessarily optimal, but "reasonable".
→ h_{FF} heuristic

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Why does the greedy algorithm compute low-quality plans?

- It may apply many operators which are not **goal-directed**.

How can this problem be fixed?

- **Reaching the goal** of a relaxed planning task is most easily achieved with **forward search**.
- Analyzing **relevance** of an operator for achieving a goal (or subgoal) is most easily achieved with **backward search**.

Idea: Use a **forward-backward** algorithm that first finds a path to the goal greedily, then prunes it to a relevant subplan. *Does this sound similar to an algorithm we've seen before?*

In the tutorial today you will learn about the Relaxed Plan Graph (RPG) heuristic and how it is used in one particular planner, Fast-Forward (FF) (Hoffmann & Nebel, JAIR-01).

- **Heuristic:** Solve the relaxed planning problem using a planning graph approach.
- **Search:** Hill-climbing extended by breadth-first search on plateaus and with pruning
- **Pruning:** Only those successors are considered that are part of a relaxed solution – i.e., the result of so-called *helpful actions*
- **Fall-back strategy:** Complete best-first search