Solutions to Assignment 3

1. The form of default reasoning proposed in this question corresponds very closely to the CWA. Indeed, lets consider the example in page 217 in the book:

$$\mathrm{KB} = \{\forall x.\, Bird(x) \wedge \neg Ab(x) \supset Flies(x), Bird(c), Bird(d), \neg Flies(c) \vee \neg Flies(d)\}$$

This KB has two minimal models. The first, say $\mathfrak{F} = \langle \mathcal{D}, \mathcal{I} \rangle$, is such that $\mathcal{I}[Ab] = \{\mathcal{I}[c]\}$. The second, say $\mathfrak{F}' = \langle \mathcal{D}, \mathcal{I}' \rangle$, is such that $\mathcal{I}'[Ab] = \{\mathcal{I}'[d]\}$. Therefore, $\mathrm{KB} \models_{\leq} Flies(c) \vee Flies(d)$.

On the other hand since $KB \not\models Flies(c)$ and $KB \not\models Ab(d)$,

$$\mathrm{KB}' = \mathrm{KB} \cup \{\neg Ab(t) \mid \mathrm{KB} \not\models Ab(t)\} = KB \cup \{\neg Ab(c), \neg Ab(d)\}$$

$KB' \models Flies(c) \wedge Flies(d)$ and therefore is inconsitent.

There are more discrepancies. For example, the question's notion of default reasoning will not entail conclusions about unnamed individuals. For example, when

$$\mathrm{KB} = \{\forall x.\, Bird(x) \wedge \neg Ab(x) \supset Flies(x), \exists x\, Bird(x)\},$$

$\mathrm{KB}' = \mathrm{KB}$, since there are no ground terms in the language. However, in minimal models, the extension of Ab is empty and we can conclude $\exists x\, Flies(x)$.

A sufficient condition that make these notions equivalent is restricting the KB to contain only sentences of the form: $P(c)$, $\forall x\, P(x) \wedge \neg Ab(x) \supset Q(x)$, where c is a constant, and P and Q are predicates. In this case, we avoid refering to unnamed individuals (all assertions are about named individuals).