I/O-Efficient Algorithms for Computing Contours on a Terrain

Bardia Sadri
Duke University

joint work with:

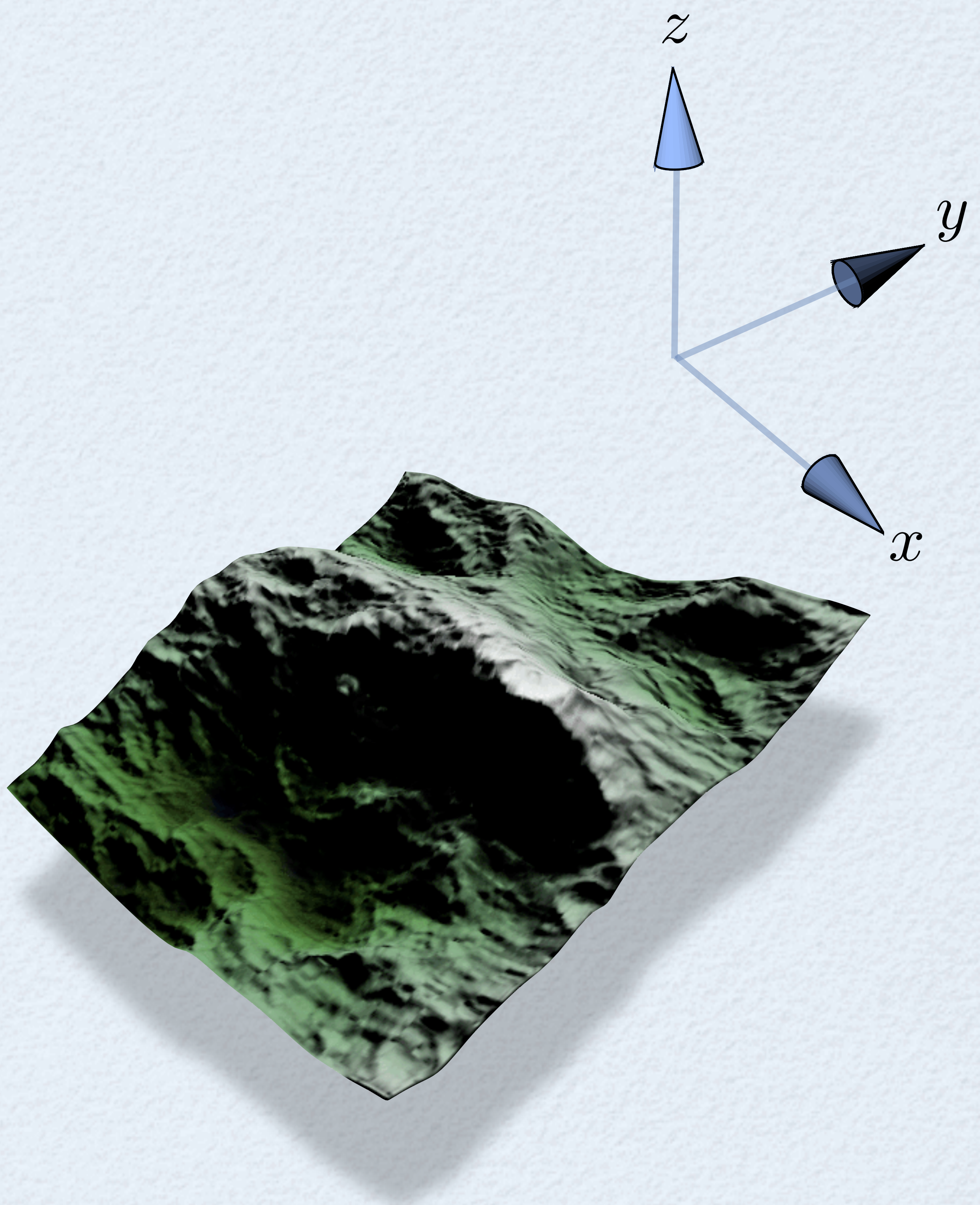
Pankaj K. Agarwal
Duke University

Lars Arge
MADALGO

Thomas Mølhave
MADALGO

Terrains

A terrain is the graph of a continuous bivariate function.



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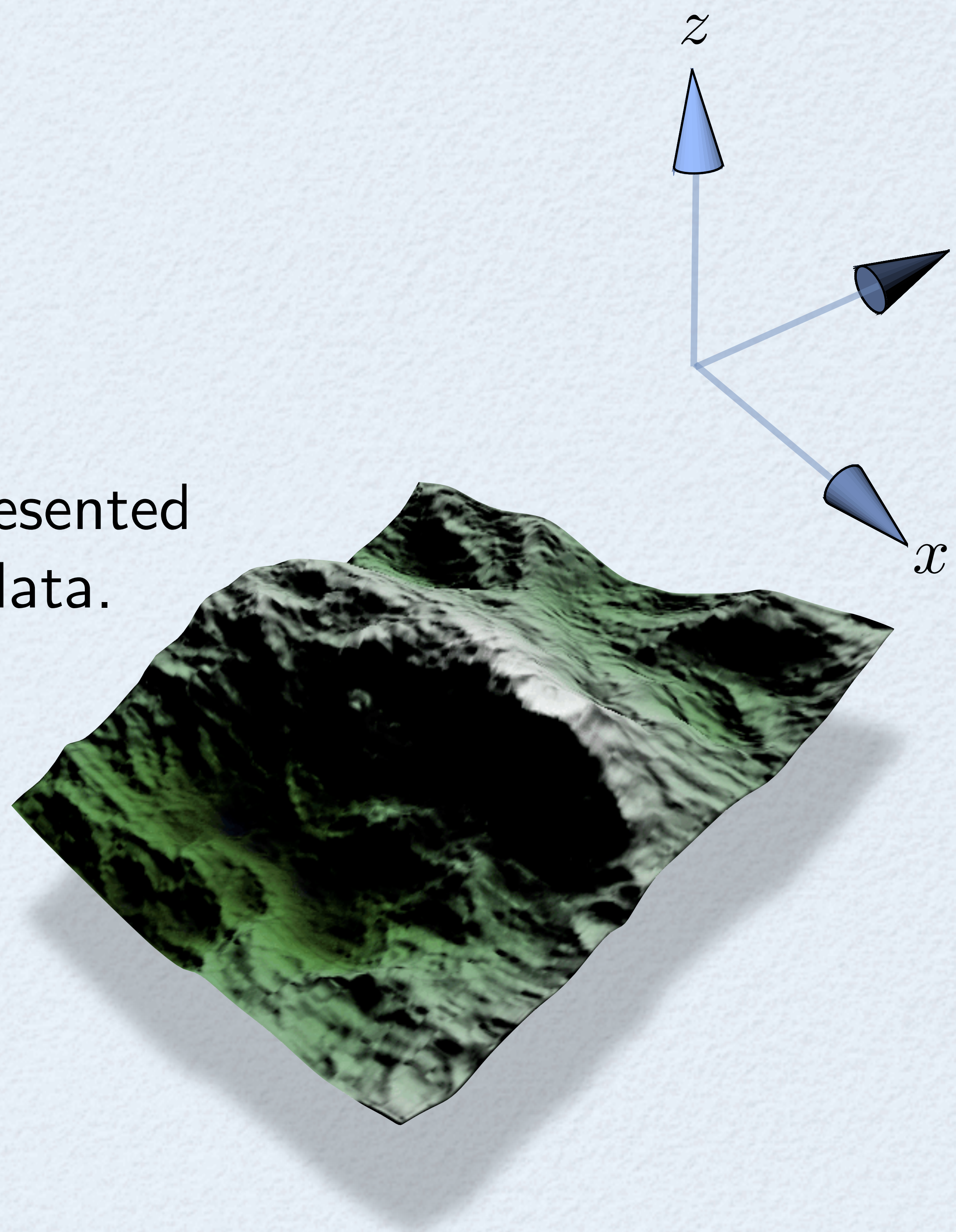
In GIS the surface of earth is often represented as a terrain that interpolates collected data.



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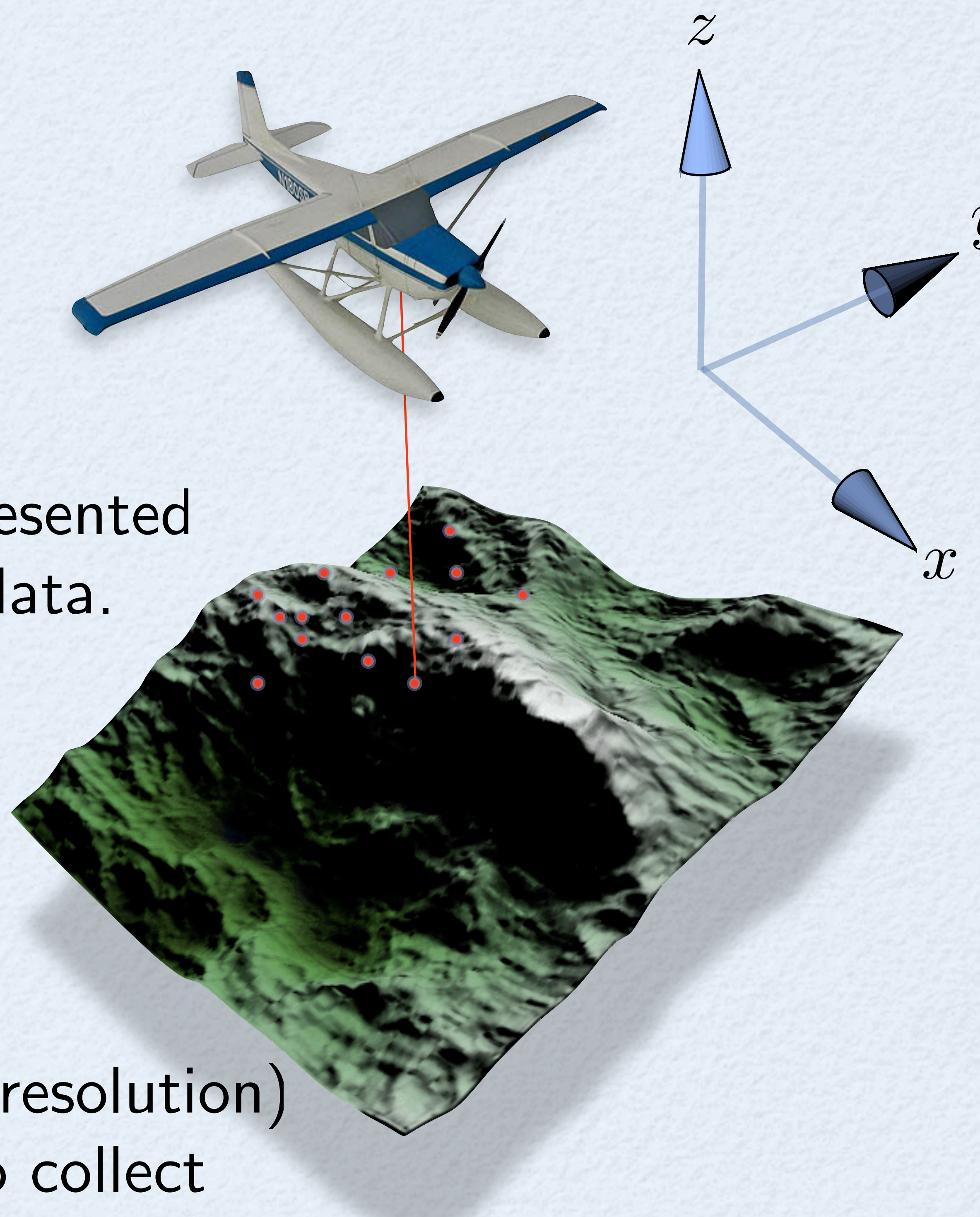

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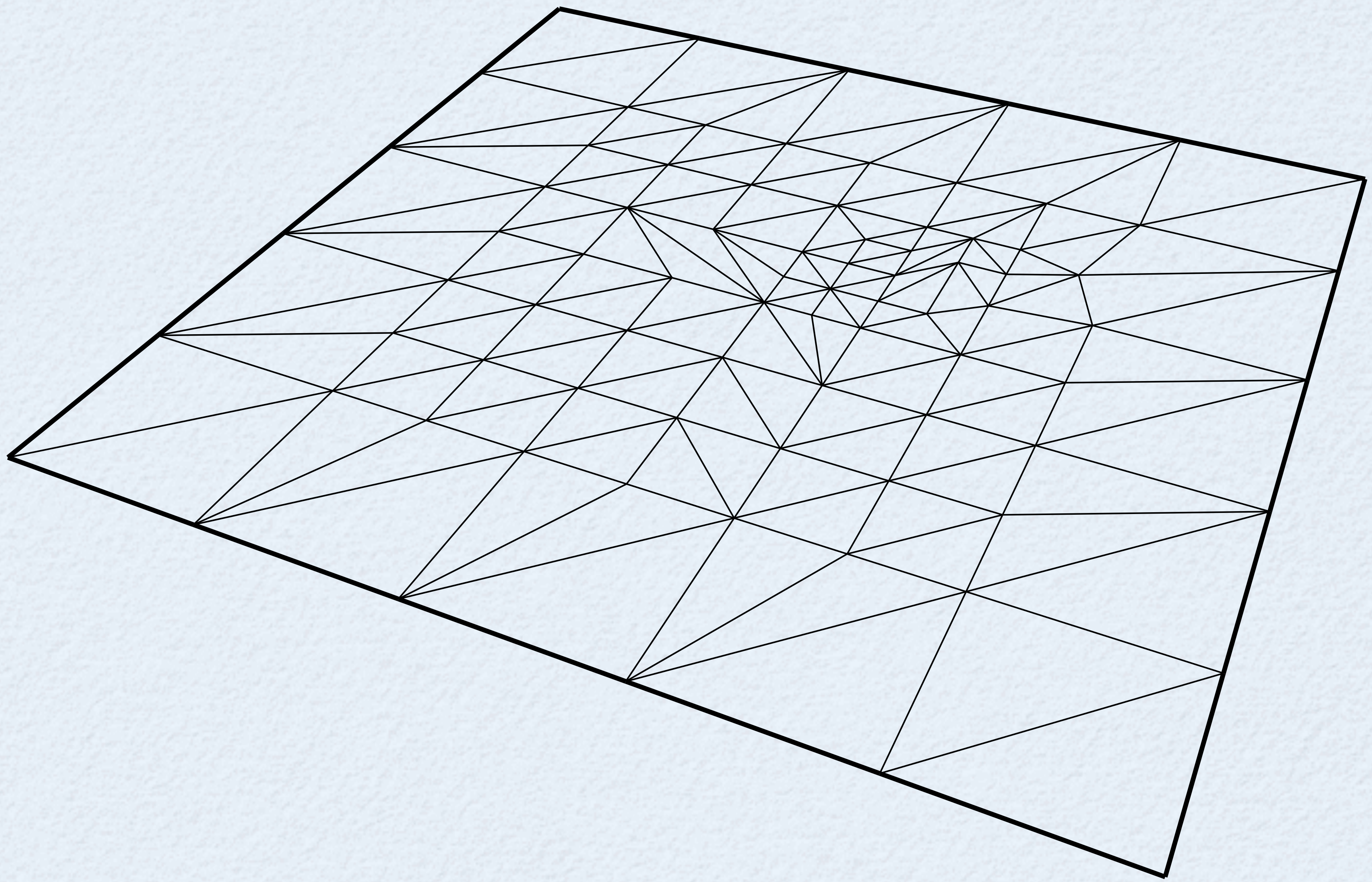
LIDAR (Light Detection and Ranging)

- Massive (irregular) point sets (1-10m resolution)
- Becoming relatively cheap and easy to collect
- Appalachian mountains between 50GB to 5TB

Representation: Triangulated Irregular Network (TIN)

Given a plane triangulation $\mathbb{M}$ with a height $h(v)$ for each vertex v, one can linearly interpolate h in the interior of every face of $\mathbb{M}$.



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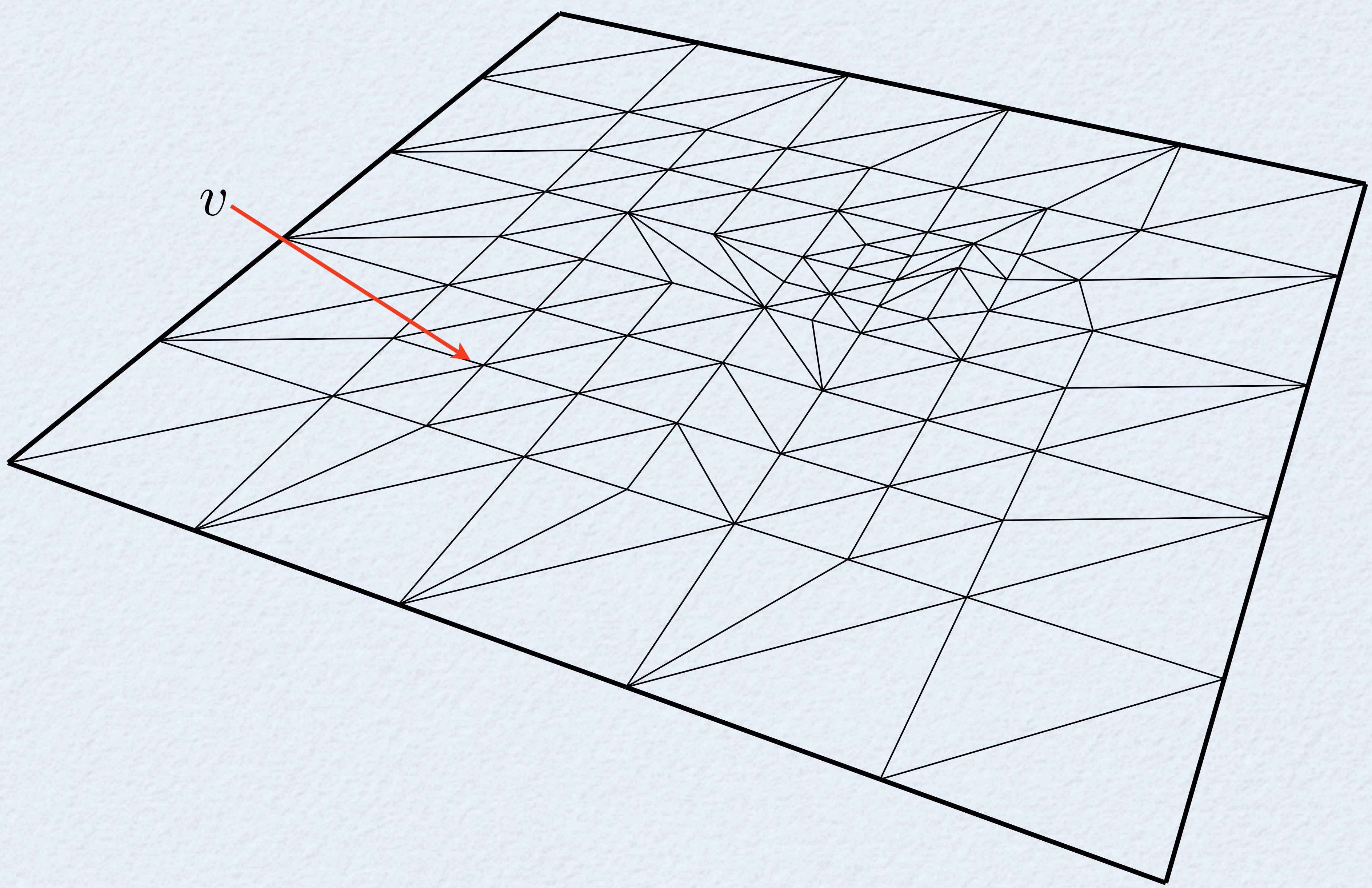

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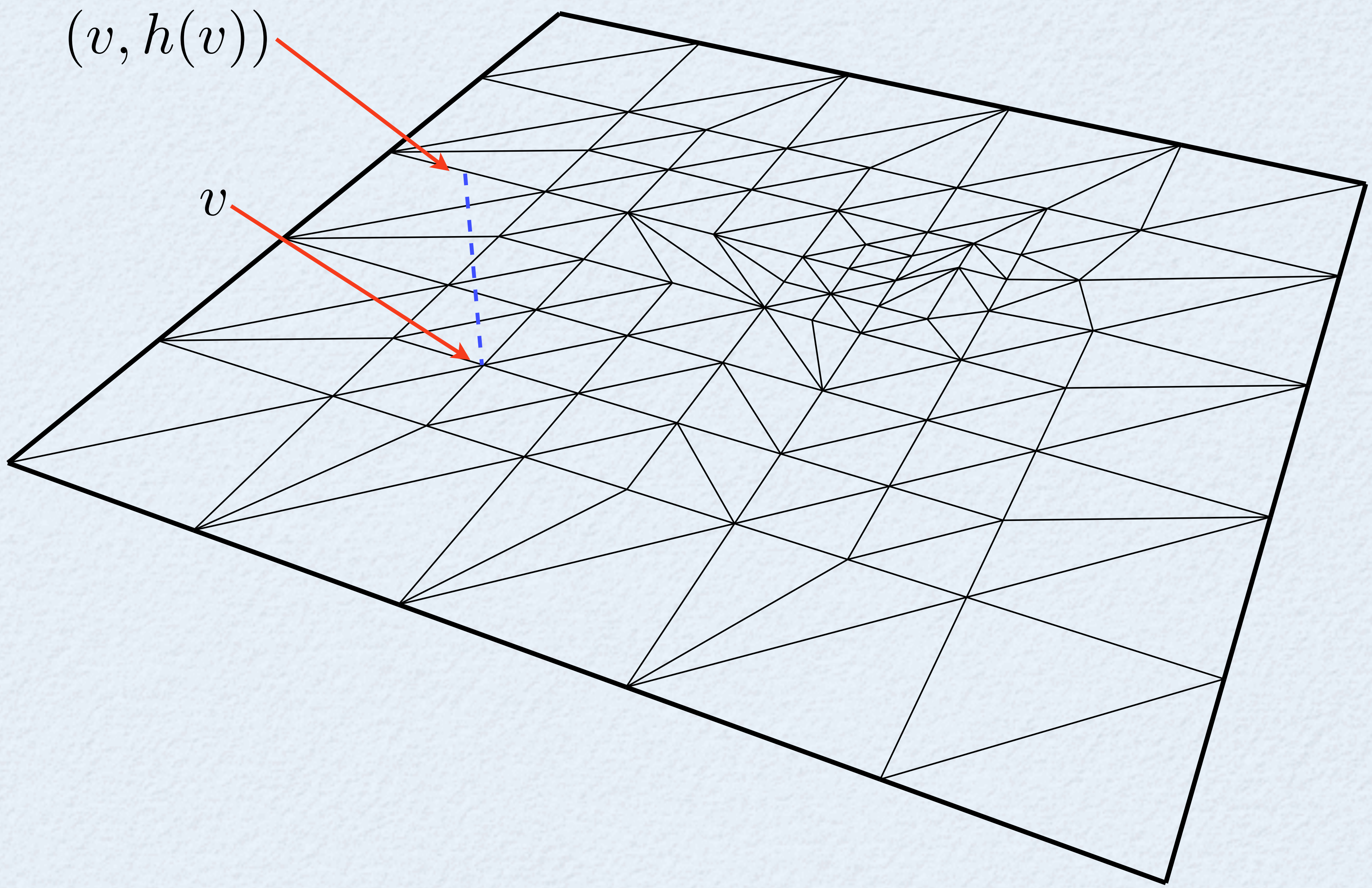

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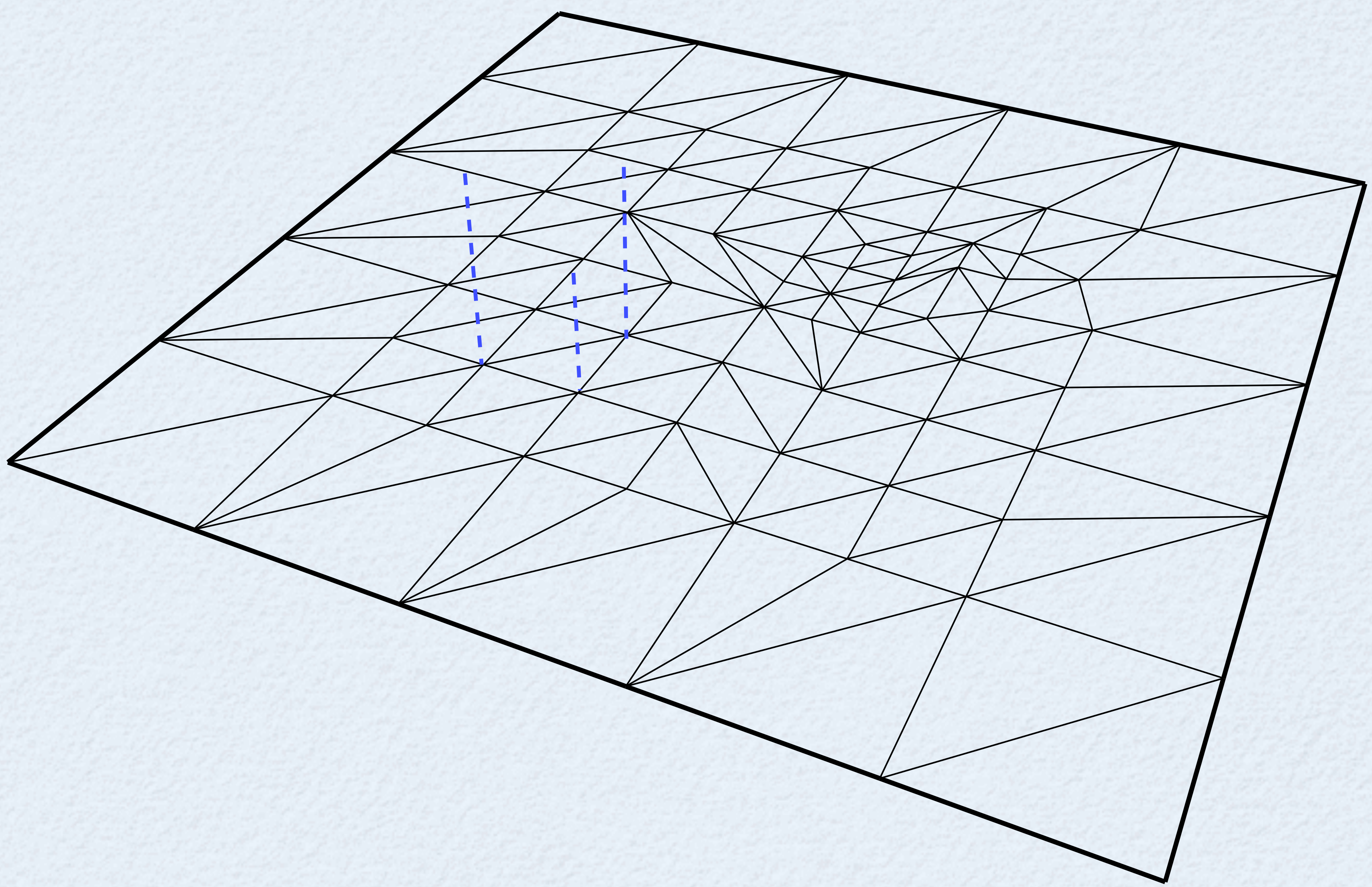

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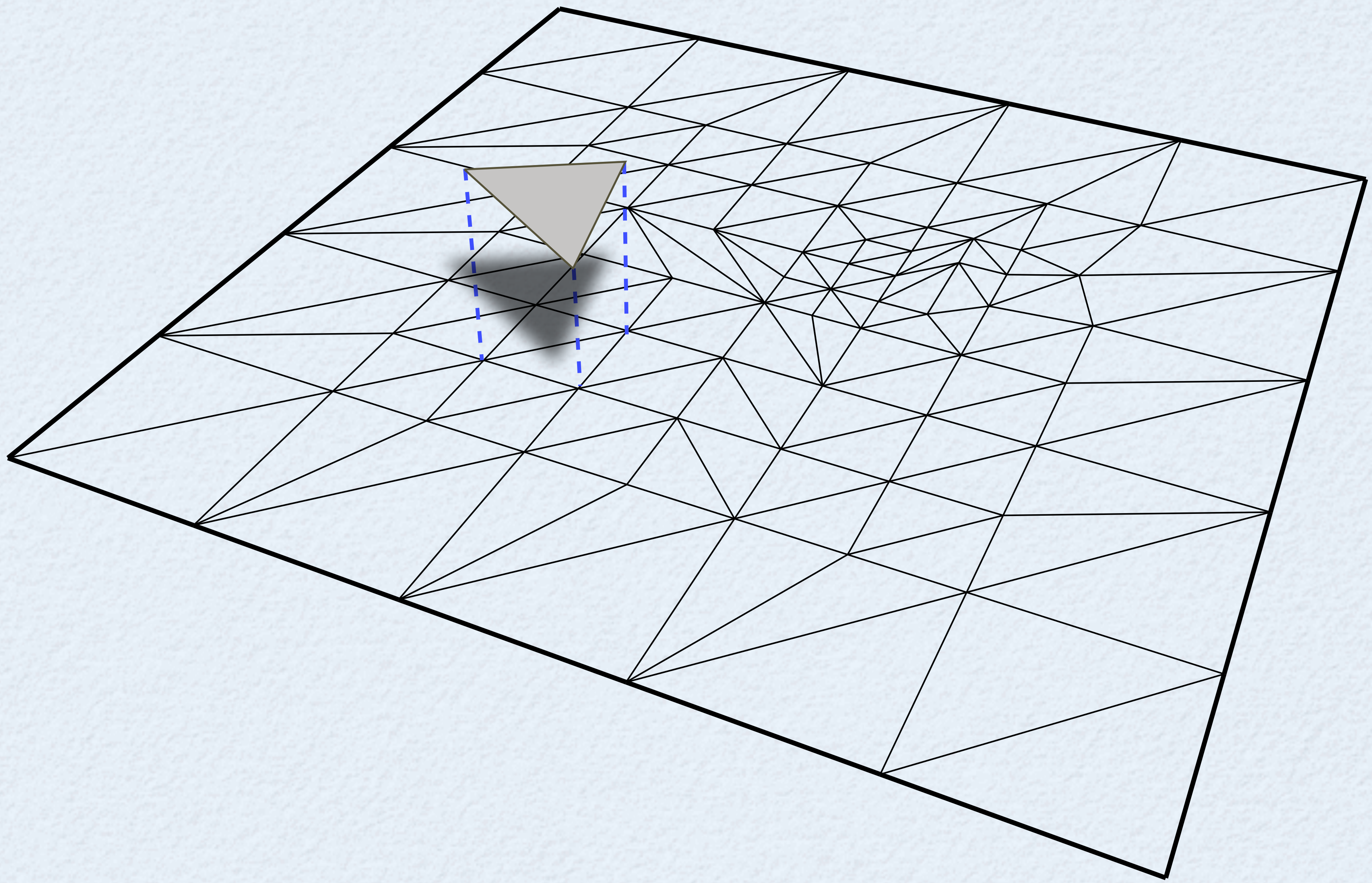

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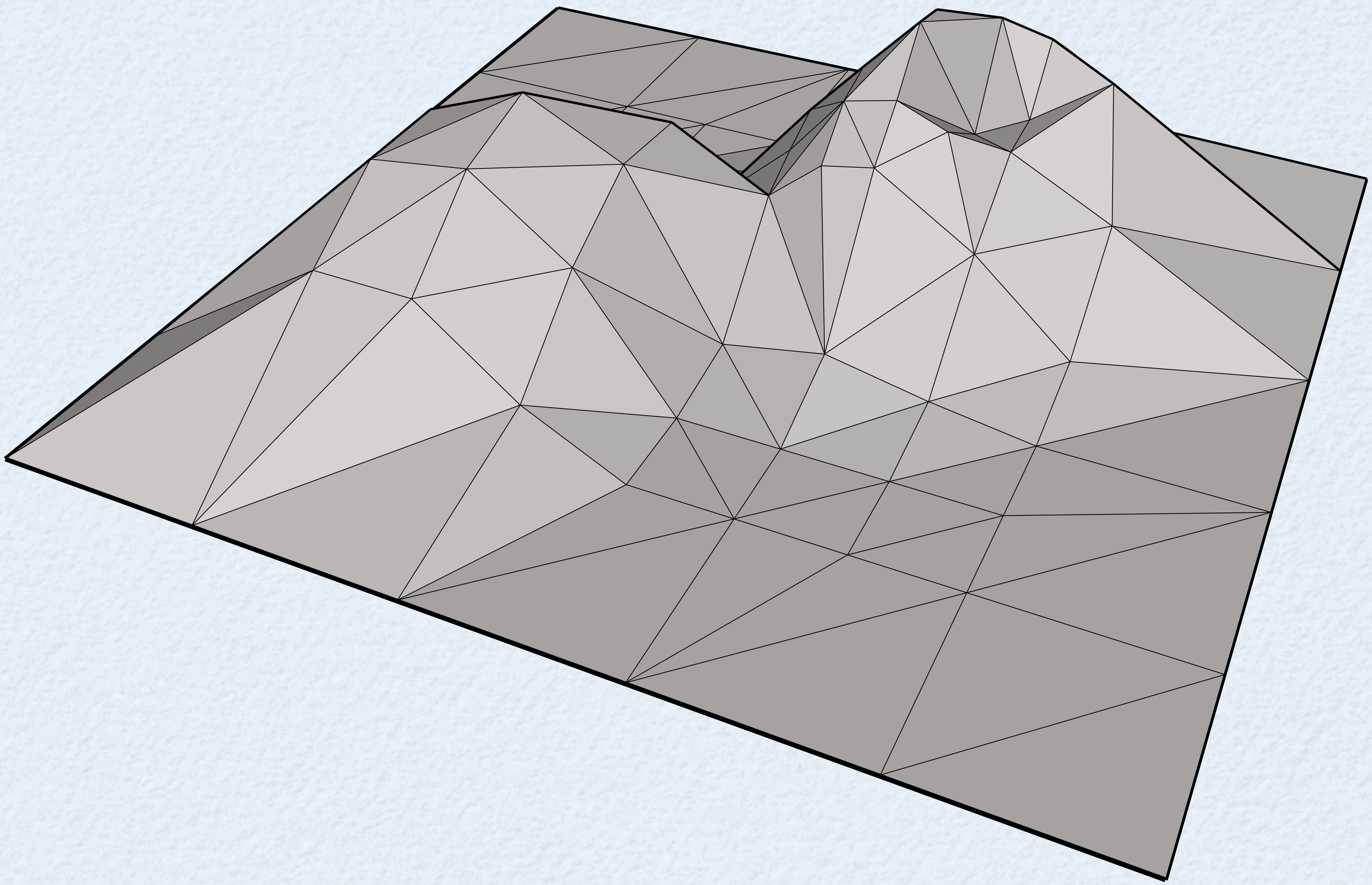

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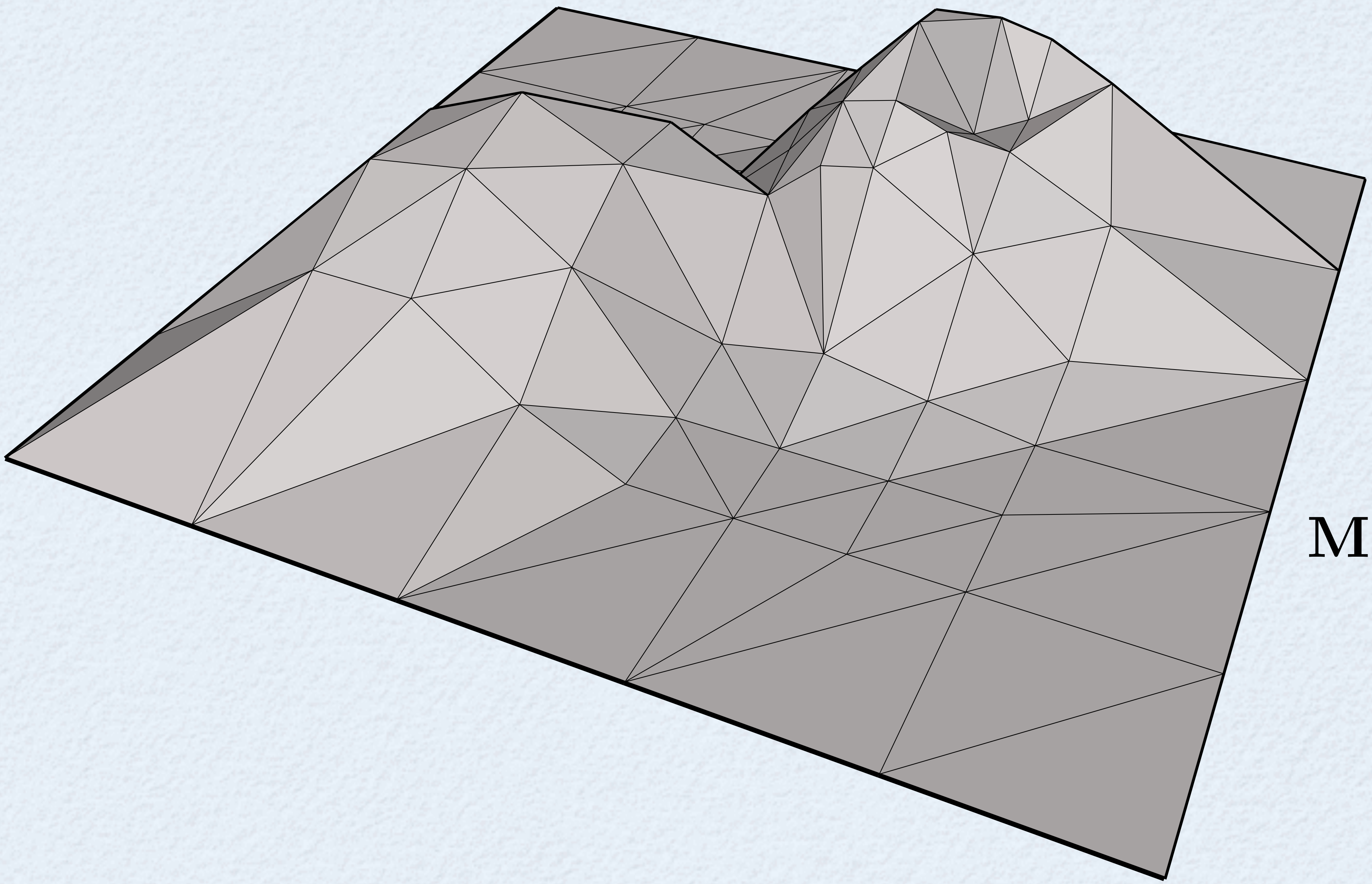

$\mathbf{M} = (\mathbb{M}, h)$

Level Sets, Contours, and Contour Maps

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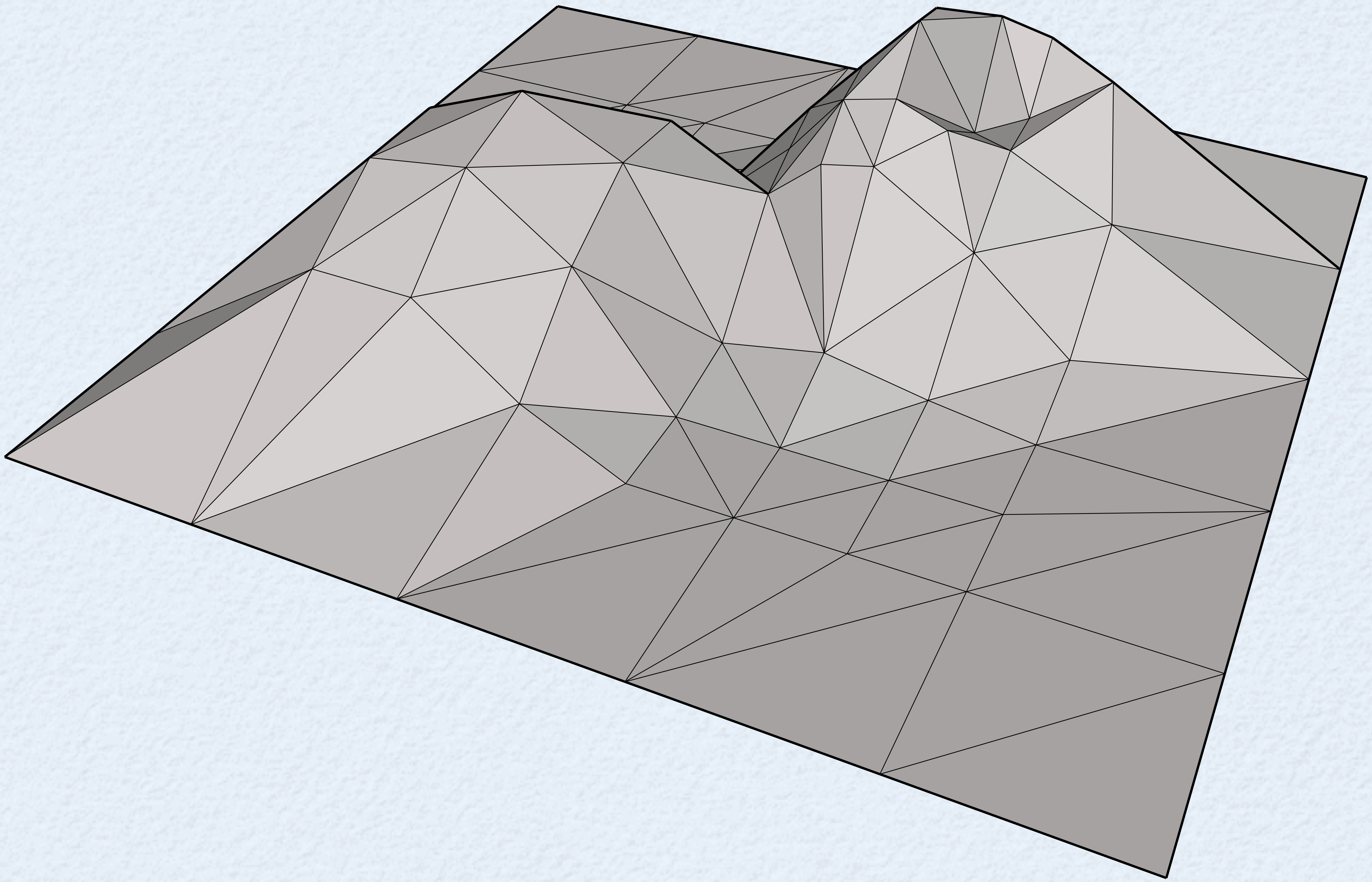

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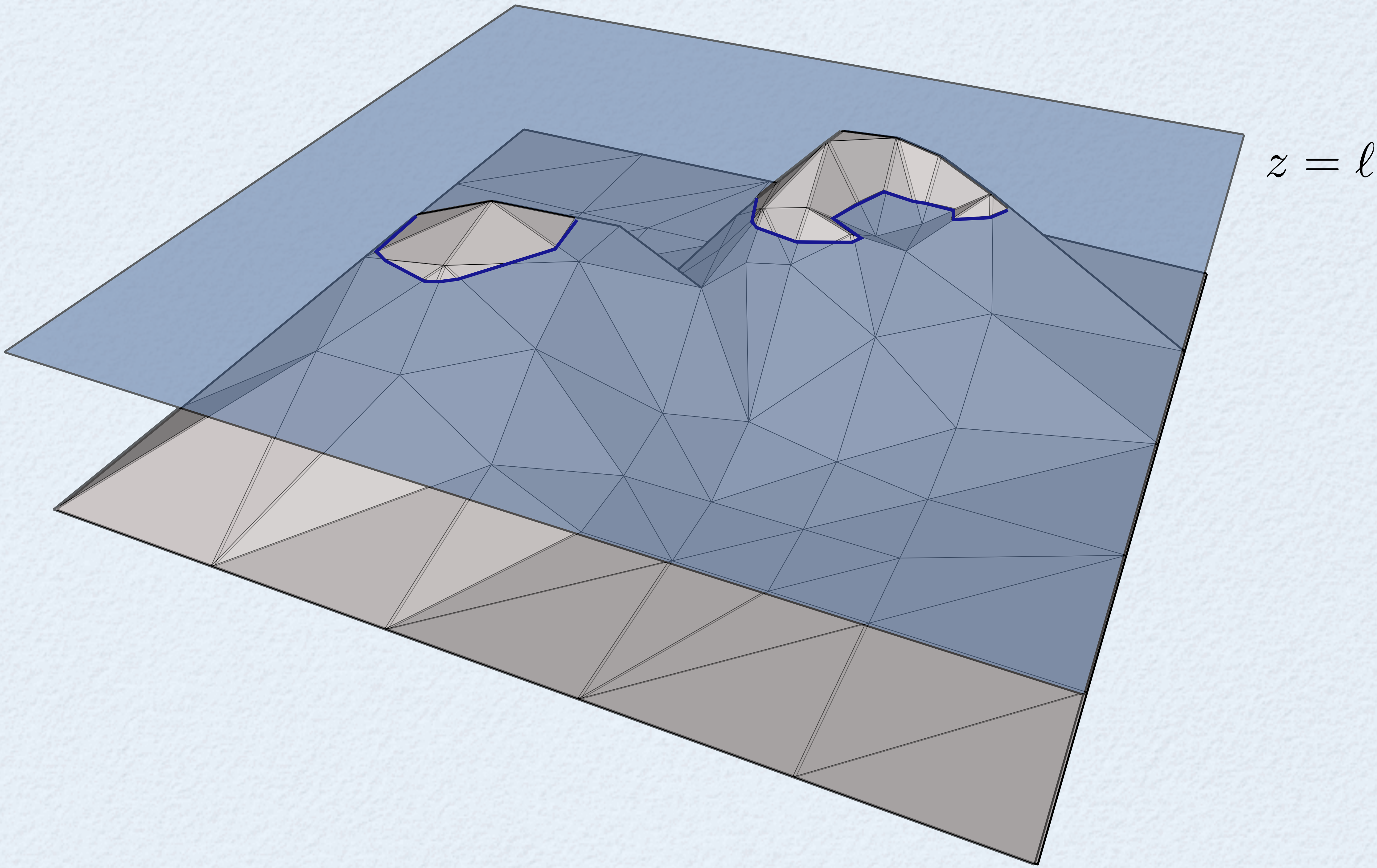

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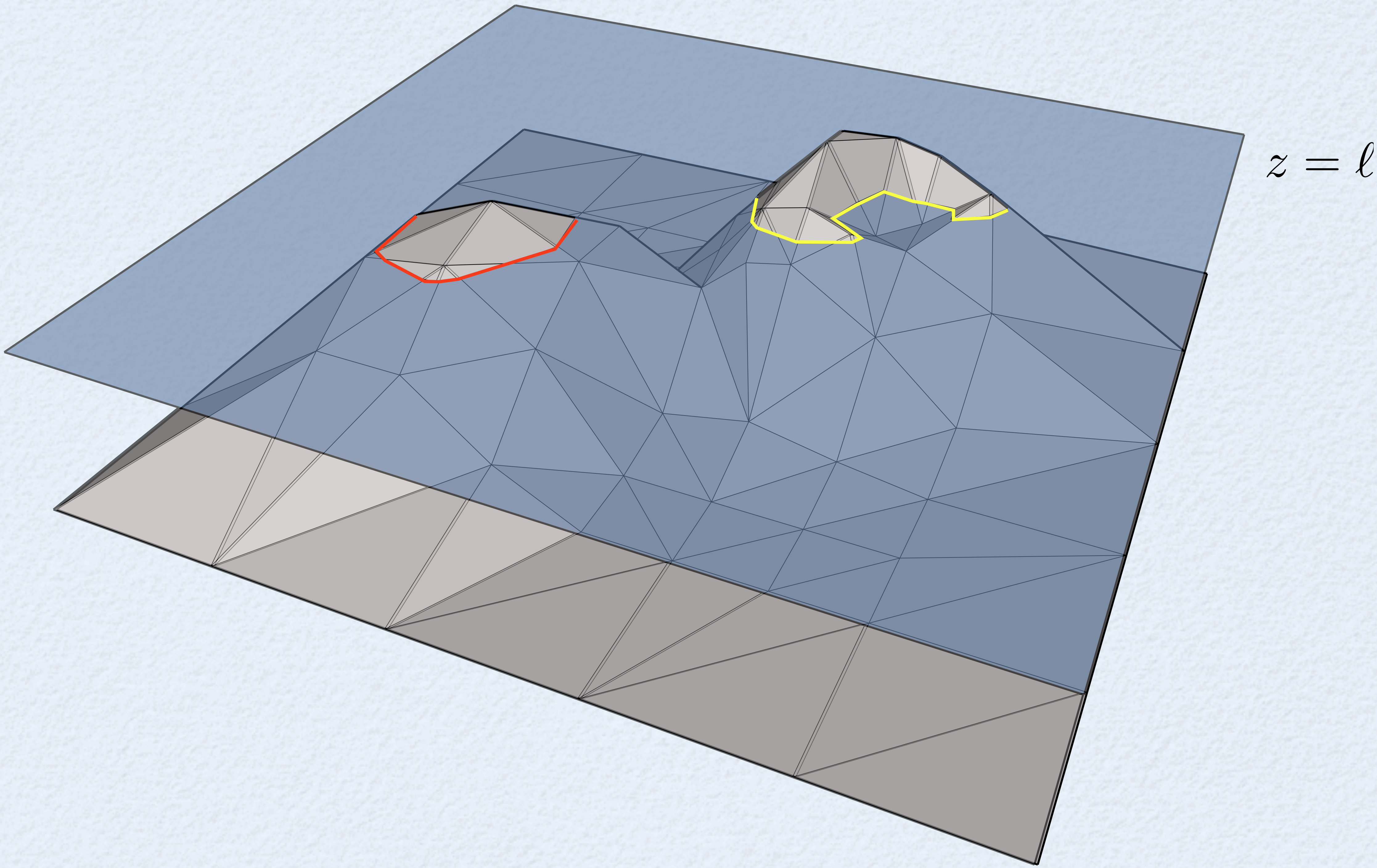

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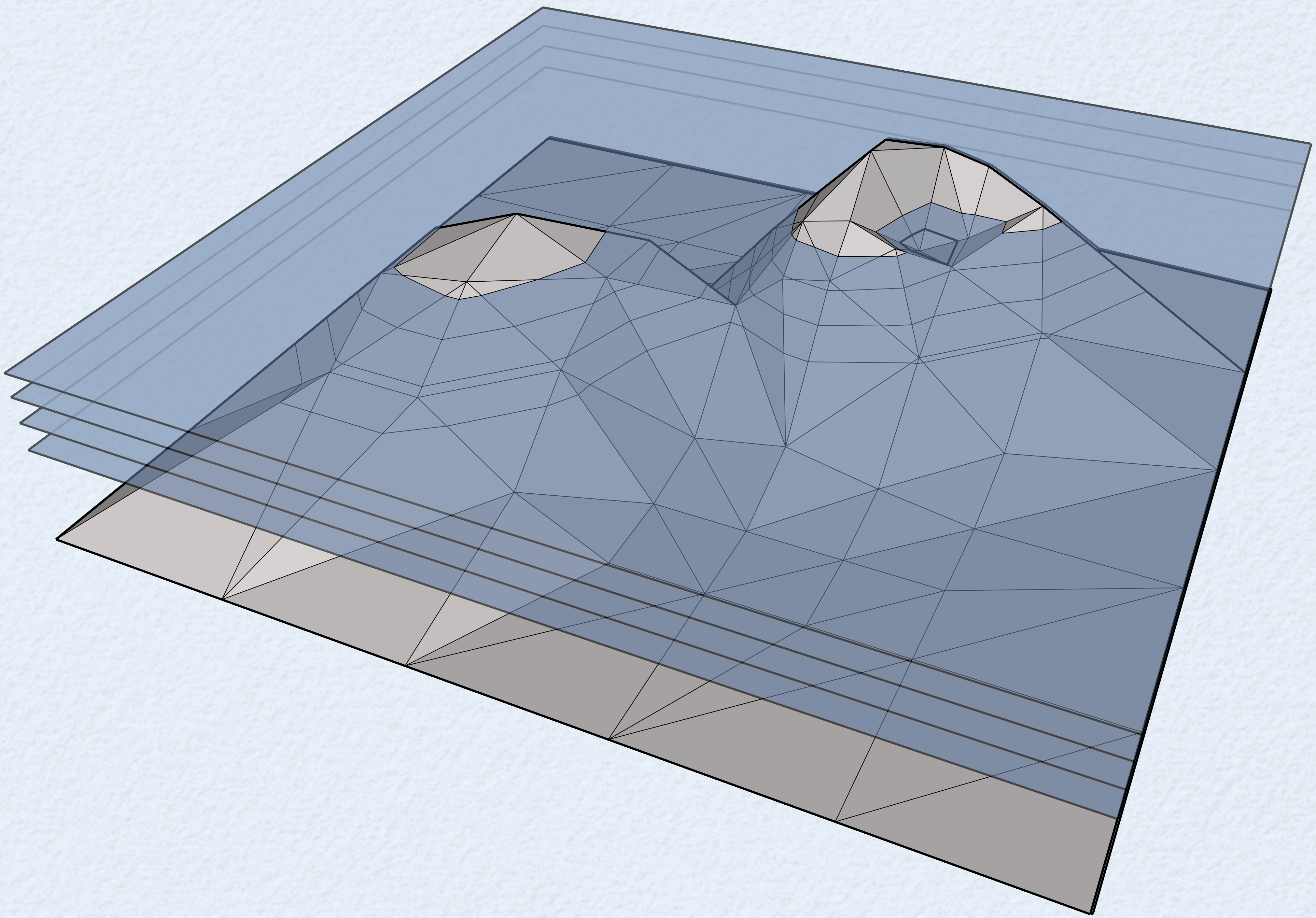

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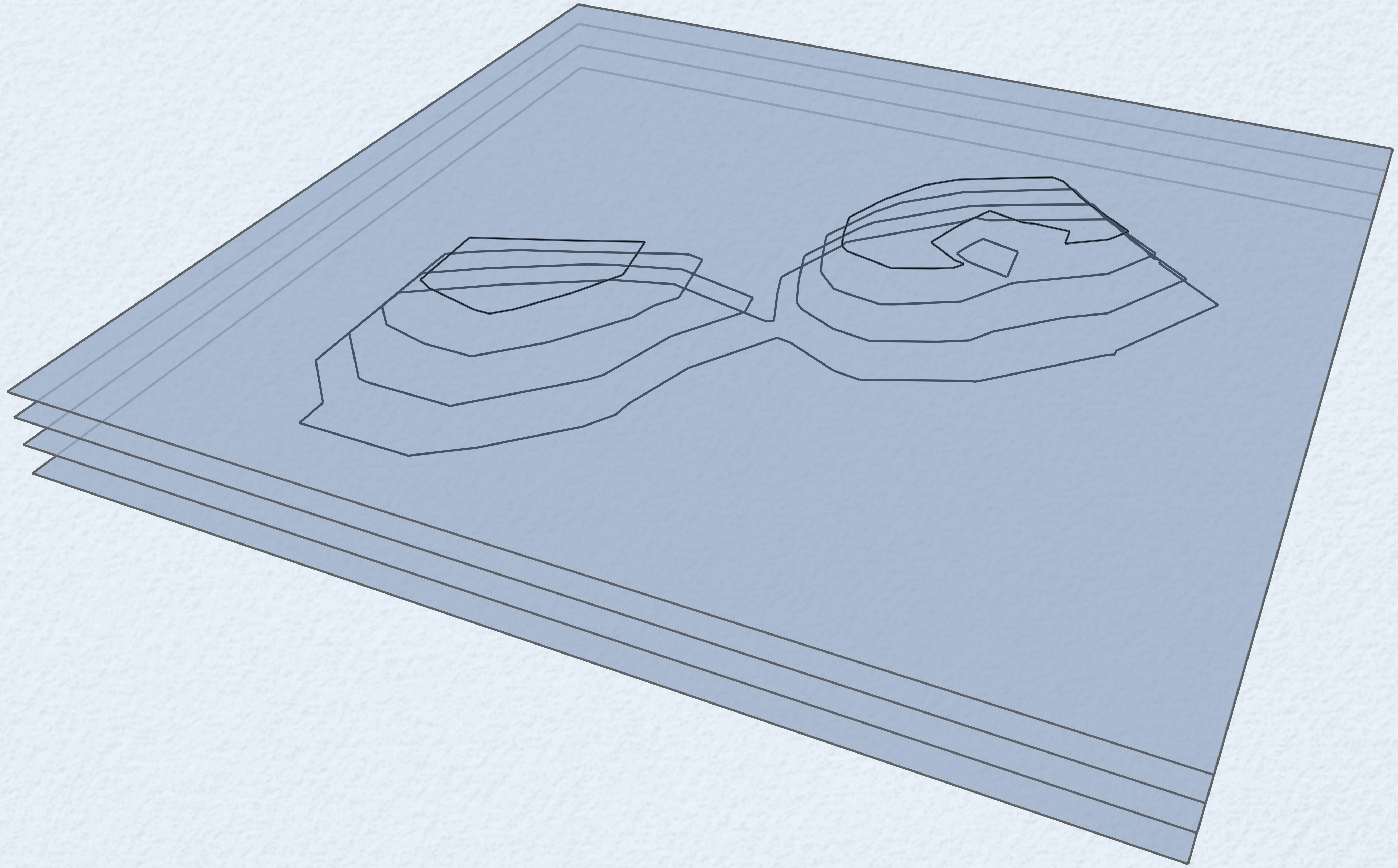

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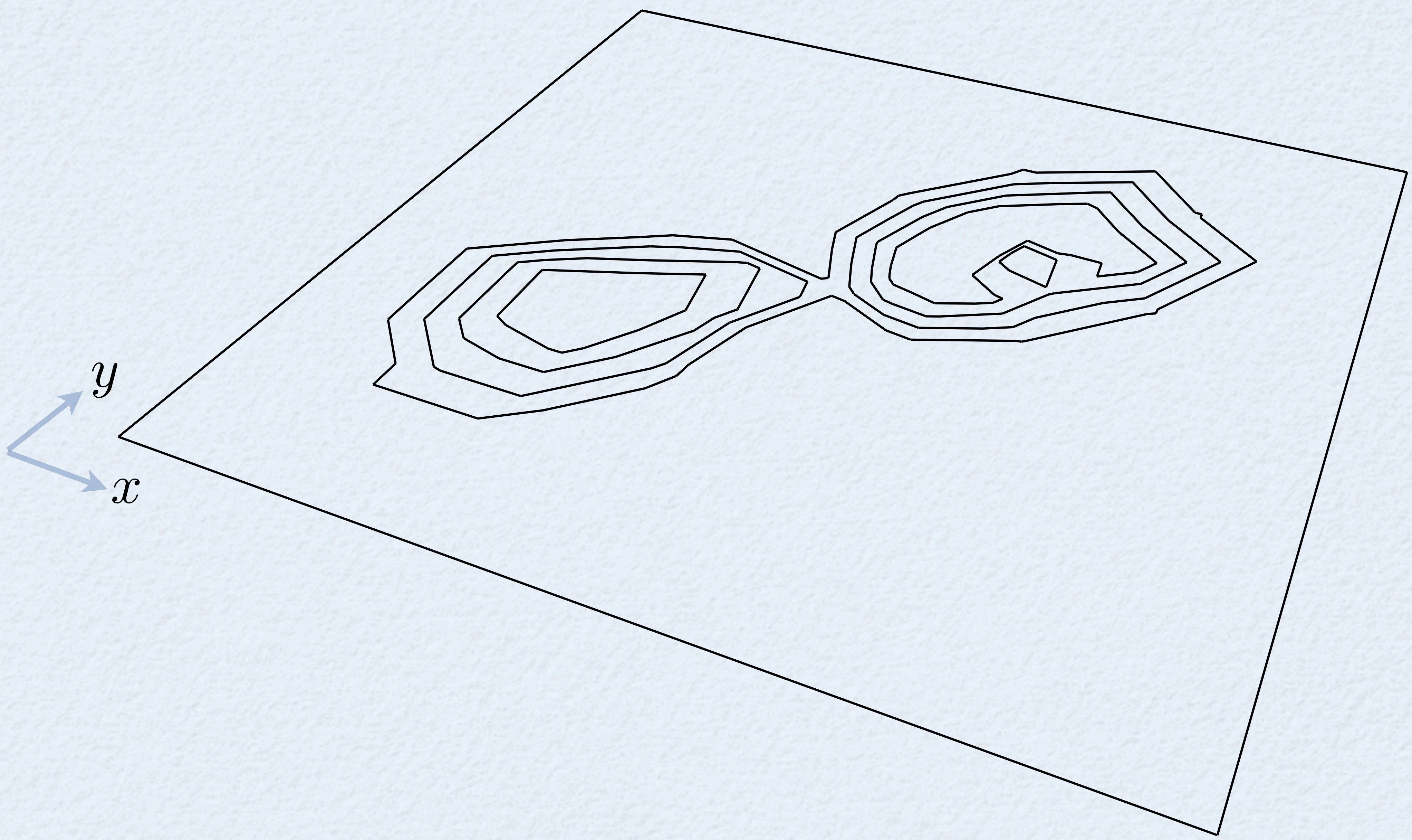

Pretty Old Stuff!

Applications at least as early as the 18^{th} century.

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XXXIII. *An Account of the Calculations made from the Survey and Meaſures taken at* Schehallien, *in order to aſcertain the mean Denſity of the Earth. By* Charles Hutton, *Eſq. F. R. S.*

This circumſtance at firſt gave me much trouble and diſſatisfaction, till I fell upon the following method by which the defect was in a great meaſure ſupplied, and by which I was enabled to proceed in the eſtimation of the altitudes both with much expedition and a conſiderable degree of accuracy. This method was the connecting together by a faint line all the points which were of the ſame relative altitude: by ſo doing, I obtained a great number of irregular polygons lying within, and at ſome diſtance from, one another, and bearing a conſiderable degree of reſemblance to each other: theſe polygons were the figures of ſo many level or horizontal ſections of the hills, the relative altitudes of all the parts of them being known; and as every baſe or little ſpace had

Philosophical Transactions of Royal Society of London, 1779

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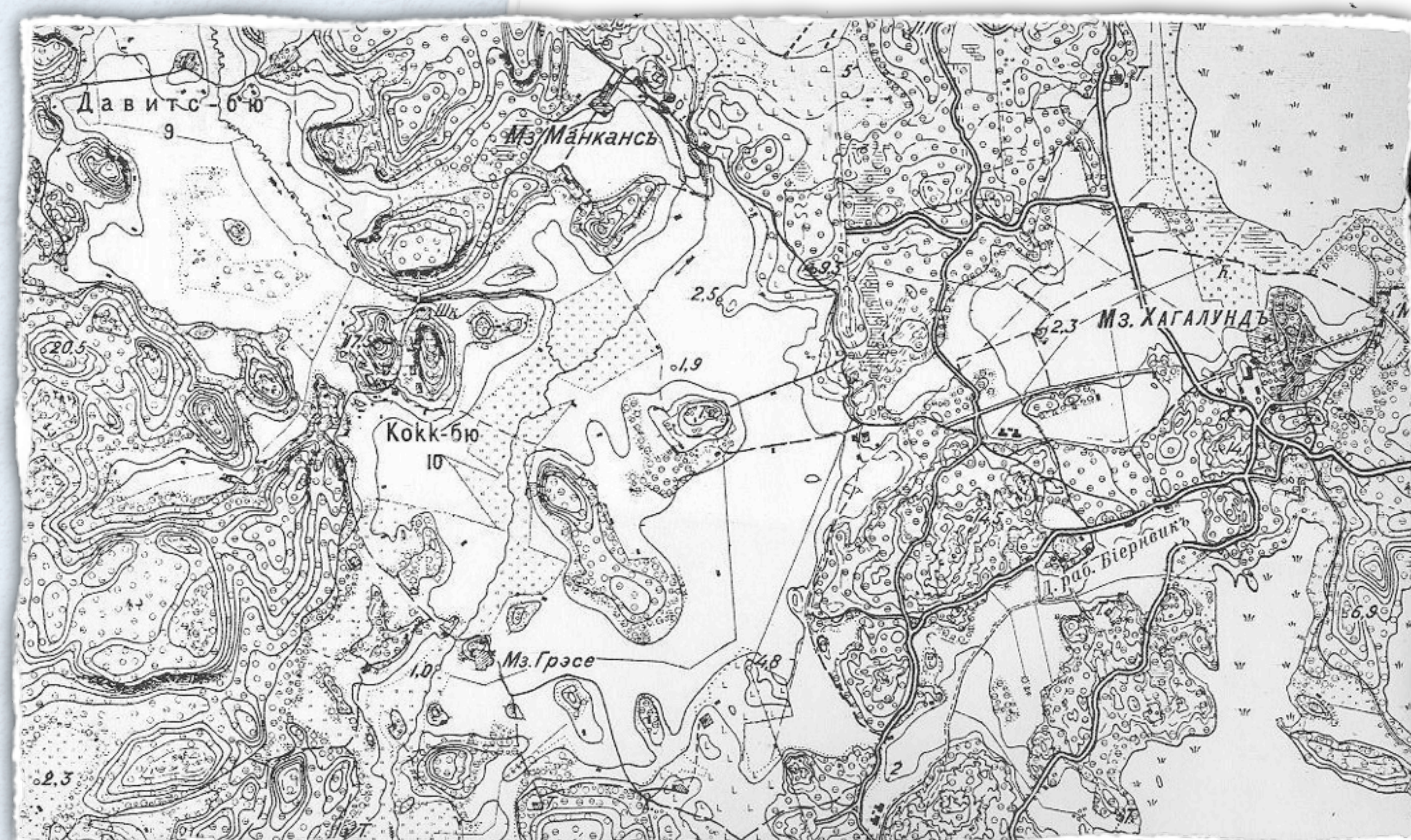

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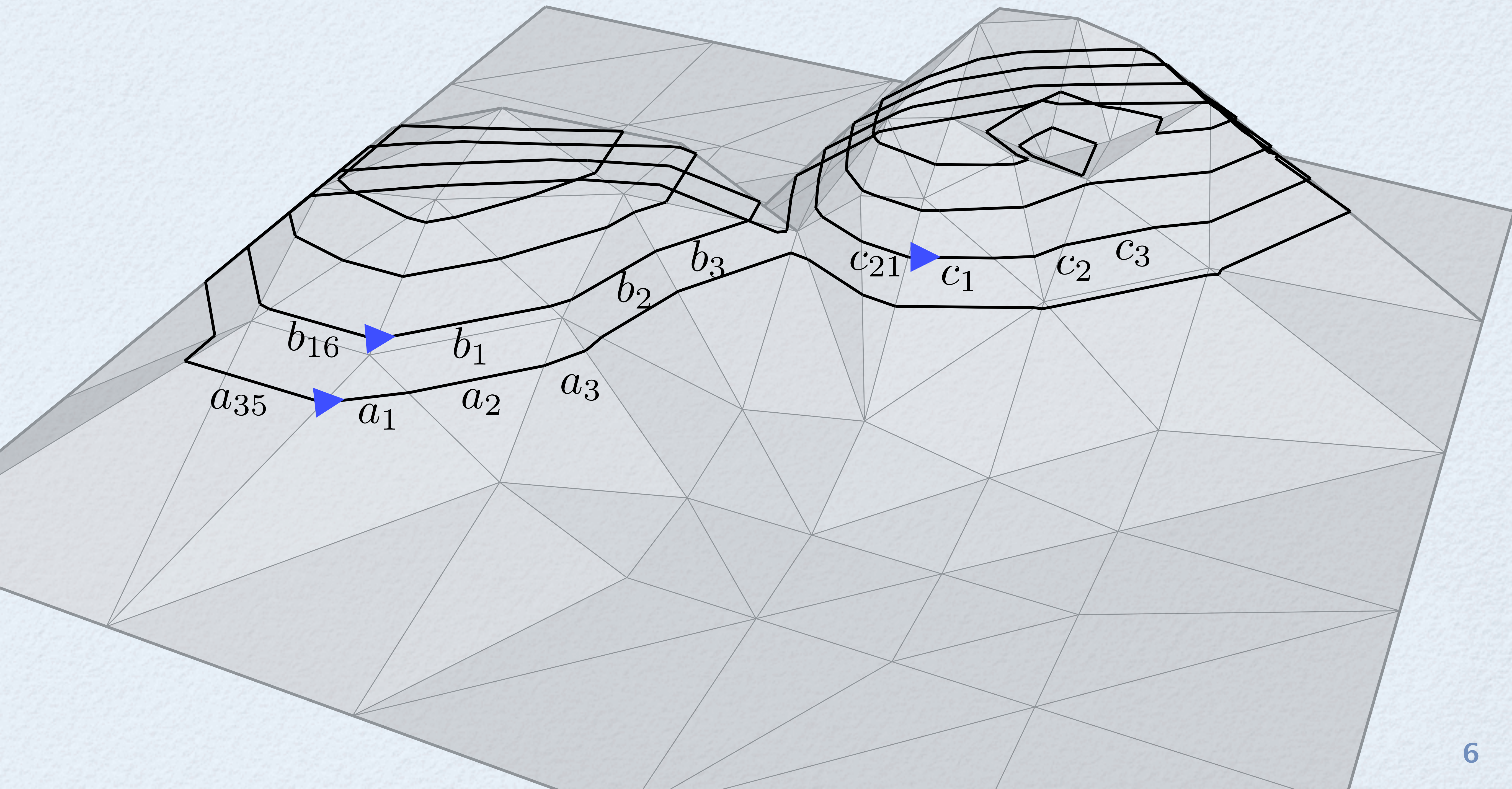
Computing Contour Maps

Given a set of levels $L = \{\ell_1, \ldots, \ell_k\}$, compute the contour map $h^{-1}(L)$ such that each contour is reported separately and in sorted (circular) order.



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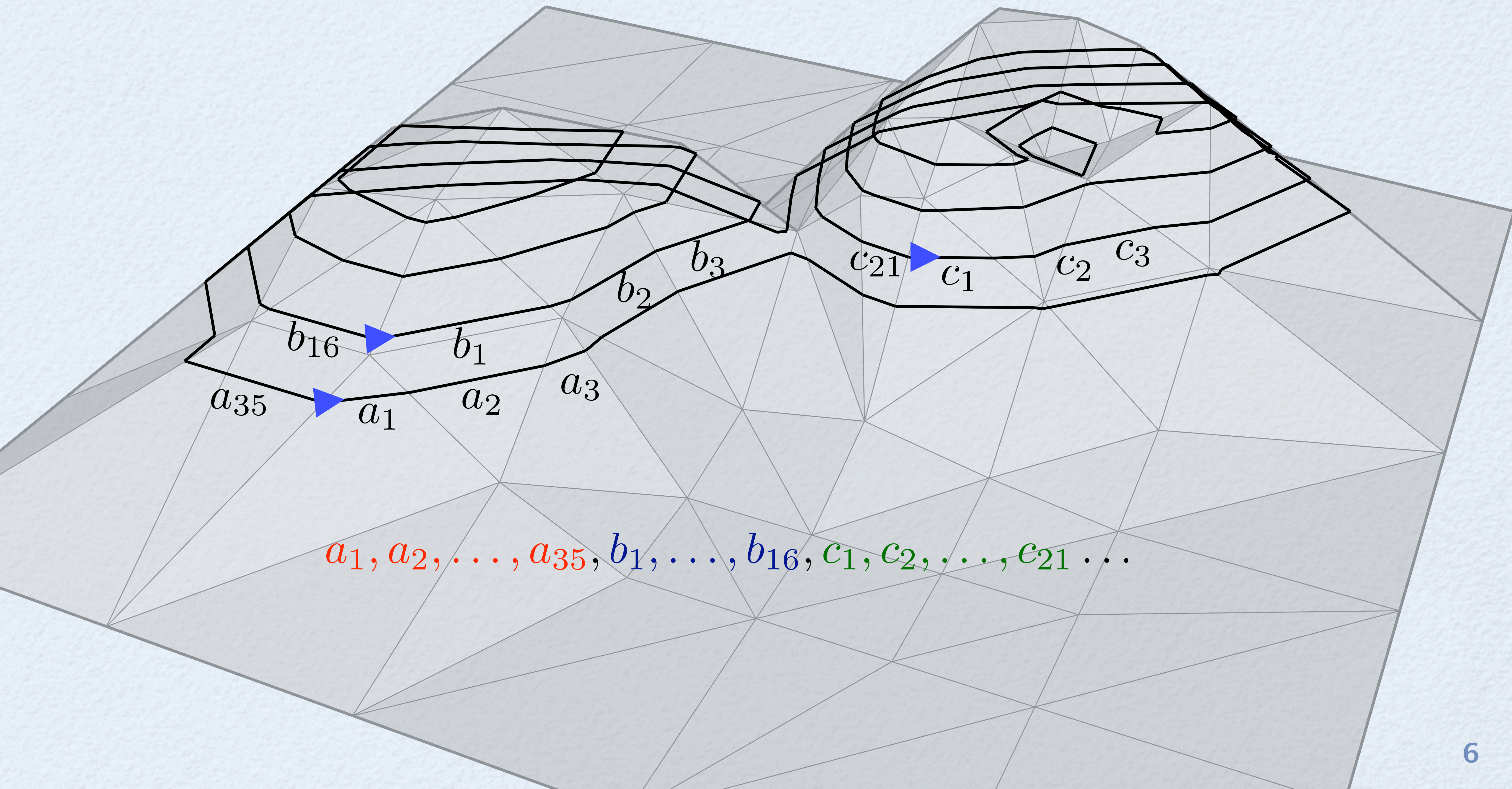

$a_1, a_2, \ldots, a_{35}, b_1, \ldots, b_{16}, c_1, c_2, \ldots, c_{21} \ldots$

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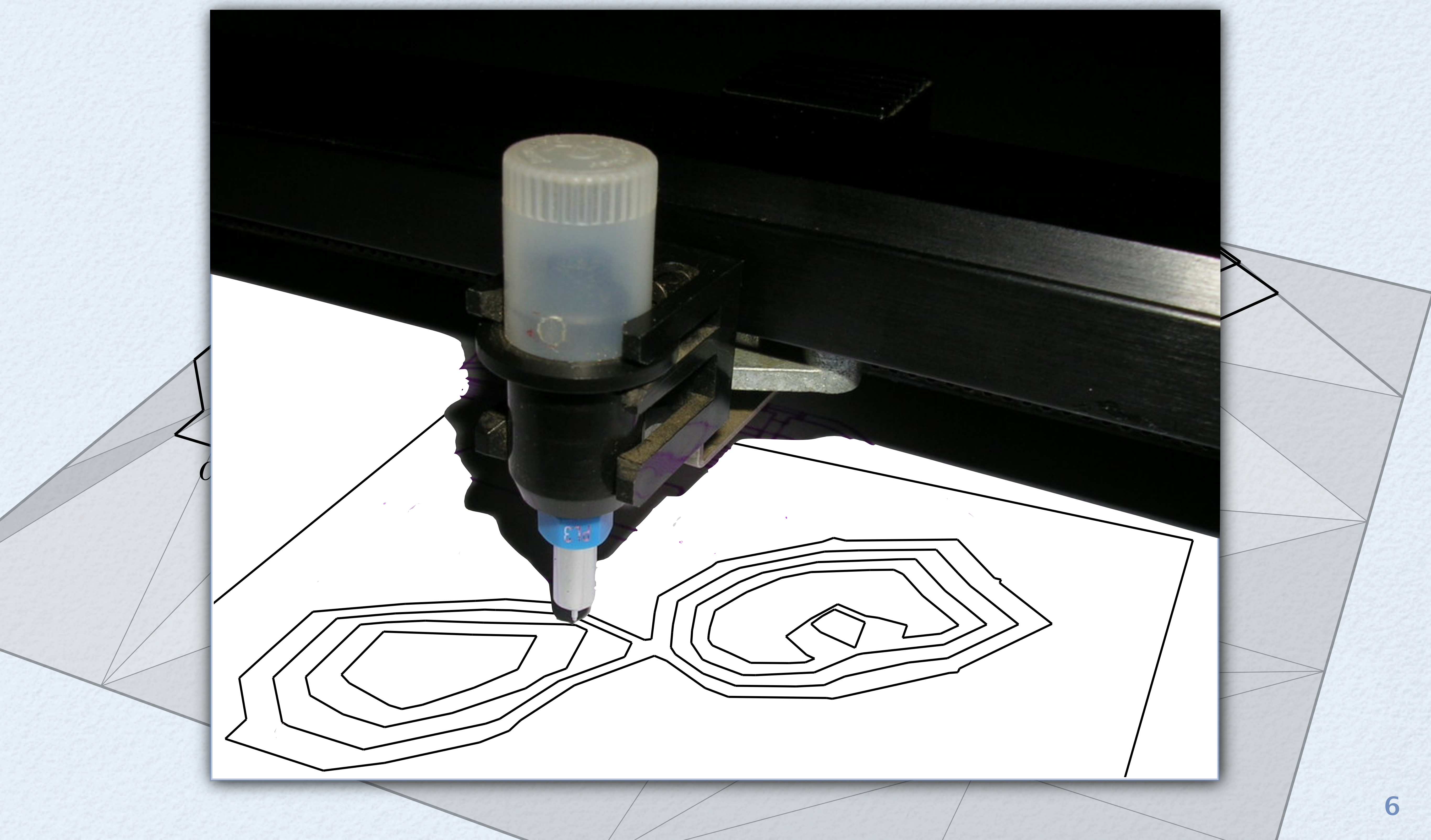

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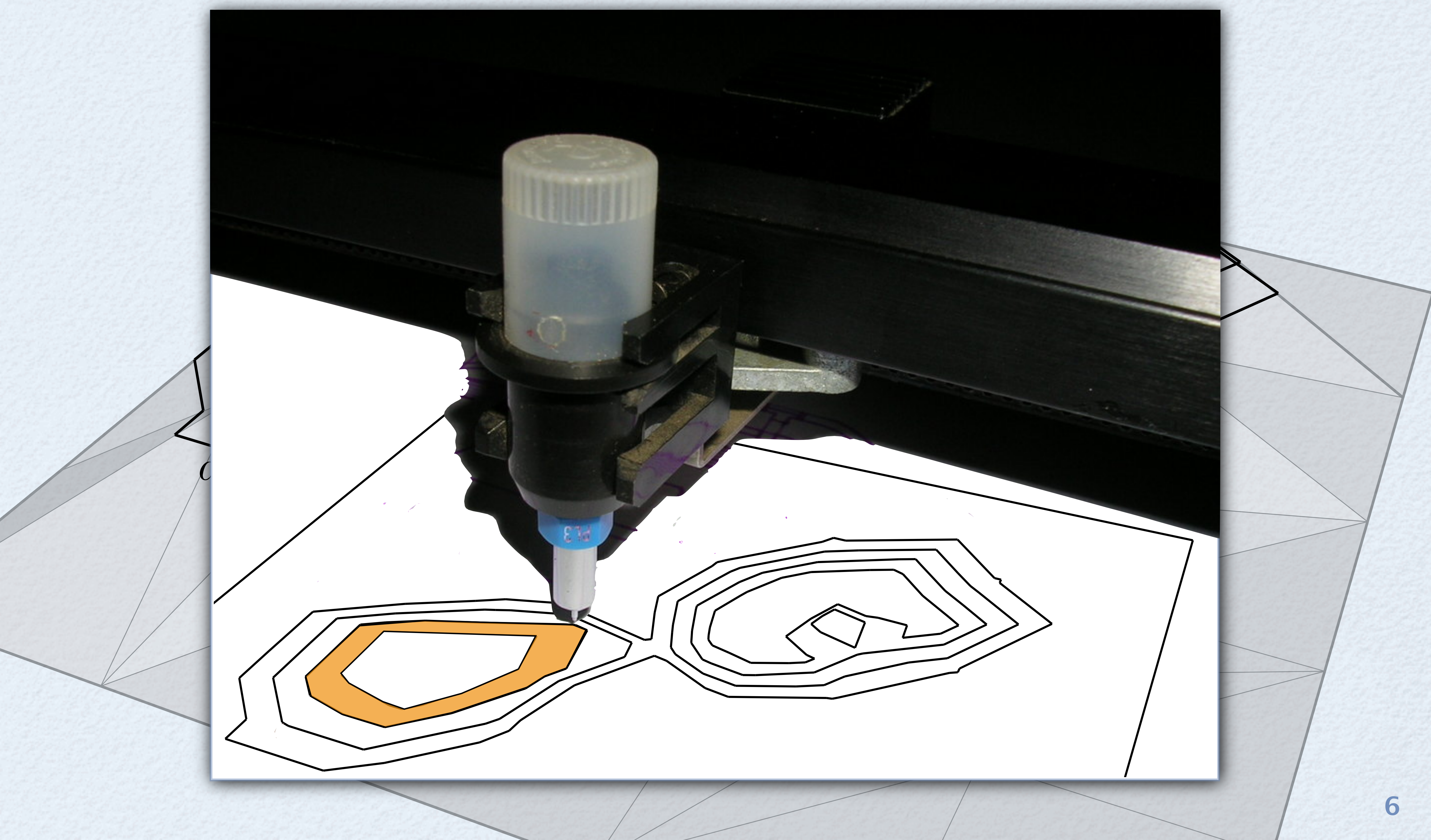
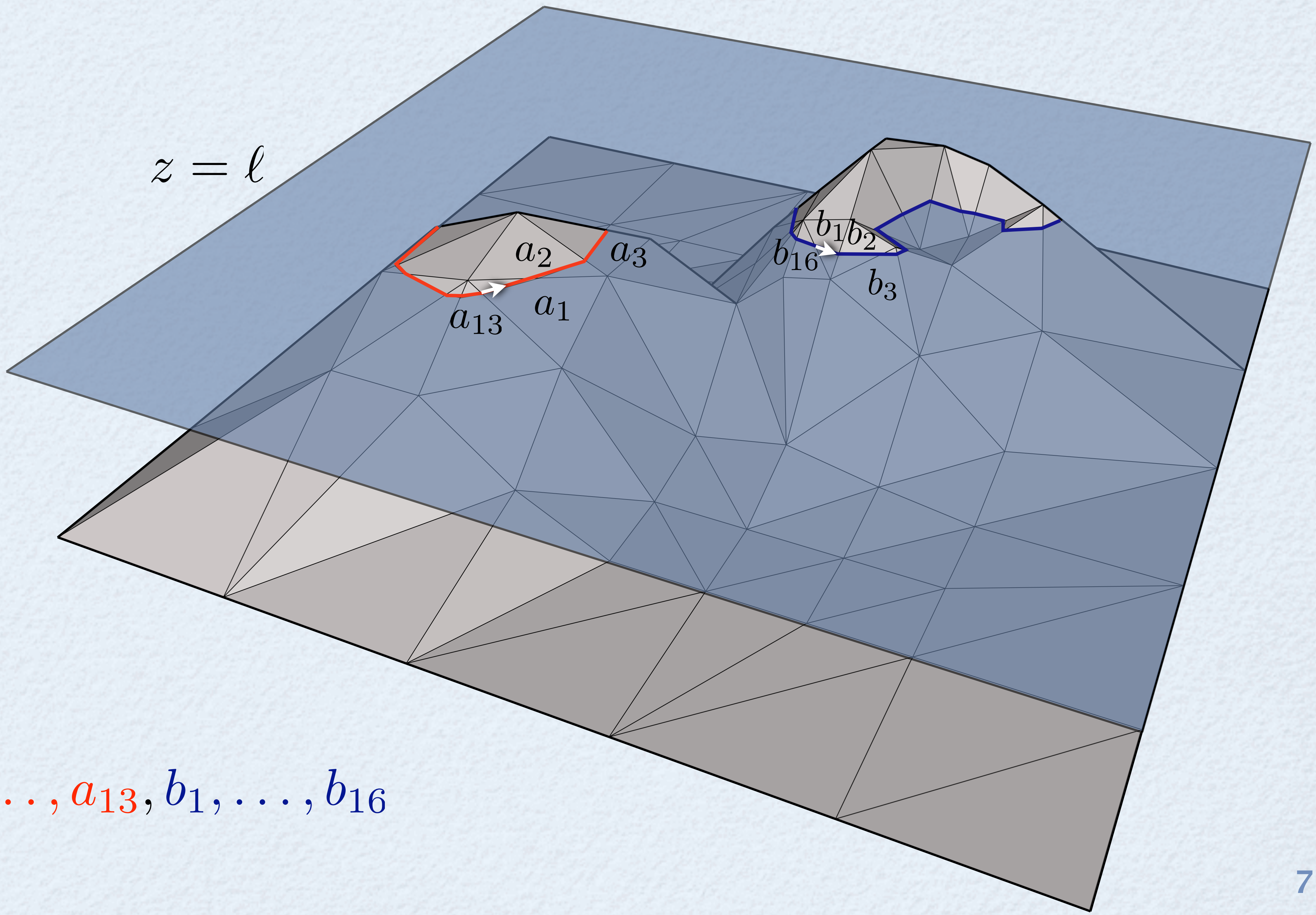


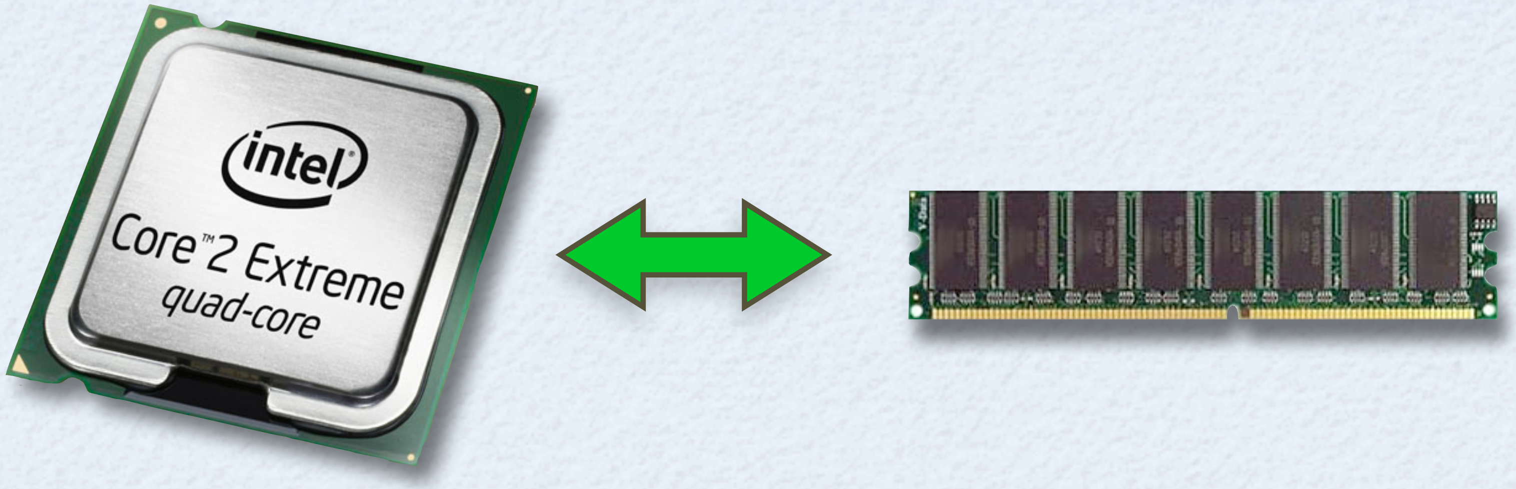
Answering Contour Queries

Preprocess the terrain to answer contour queries efficiently:
Given a level $\ell \in \mathbb{R}$, return the level set $h^{-1}(\ell)$ such that each contour is reported separately and in sorted (circular) order.



Output: $a_1, a_2, \ldots, a_{13}, b_1, \ldots, b_{16}$

The I/O Model



Classical Complexity: Number of basic operations as a function of N.

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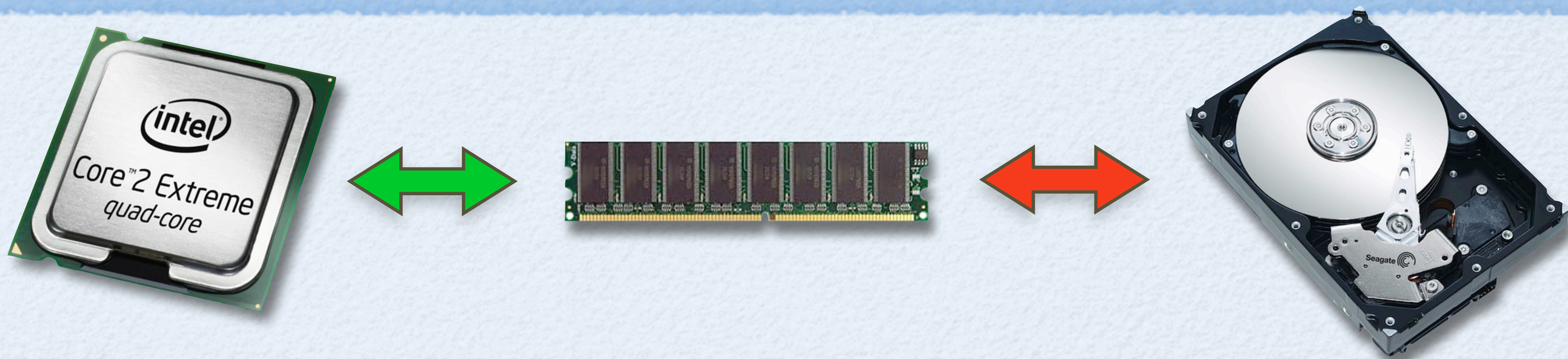

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Disk access is about 10^6 times slower than main memory access.

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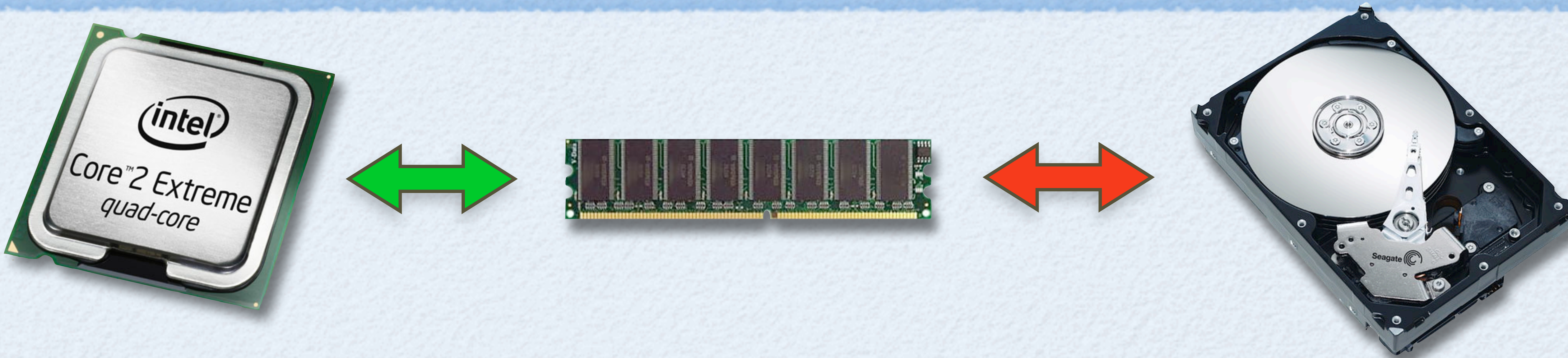

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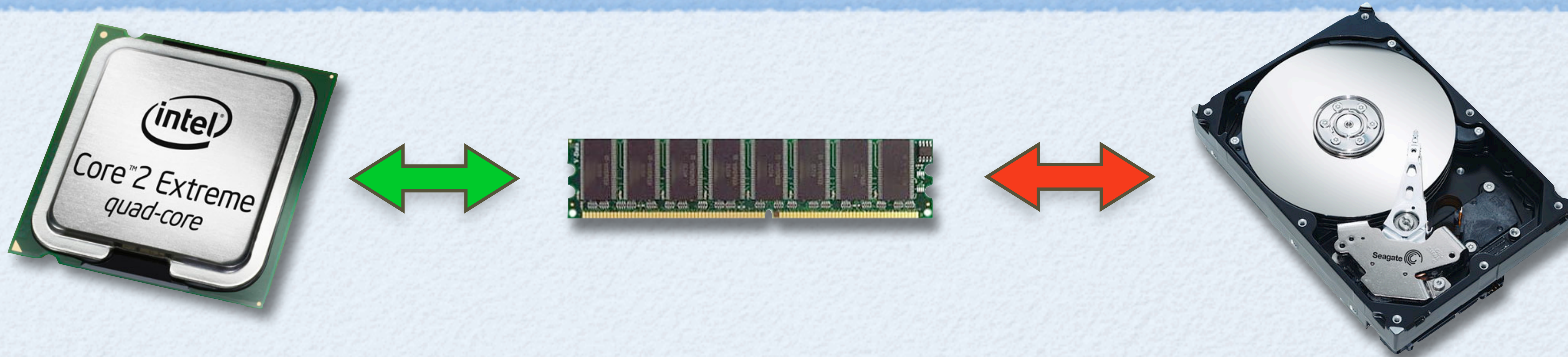

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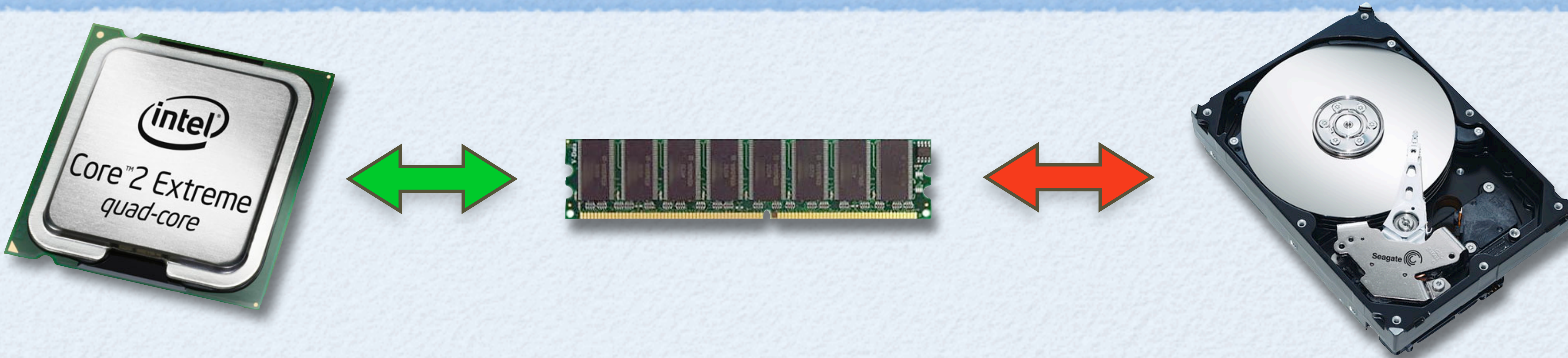

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I/O-Complexity: Number of I/Os as a function of N, B, M, and T.

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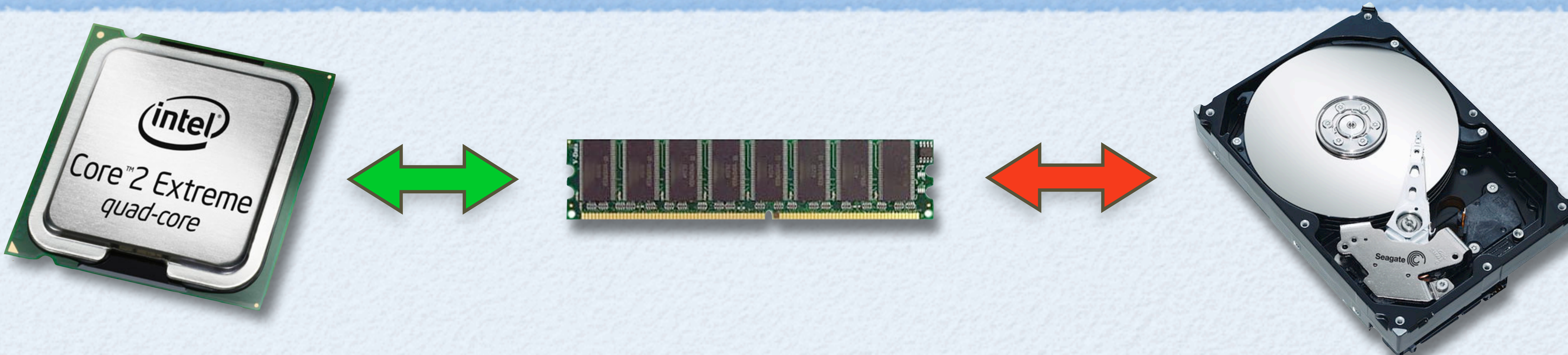

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Important to store/access data to take advantage of locality.

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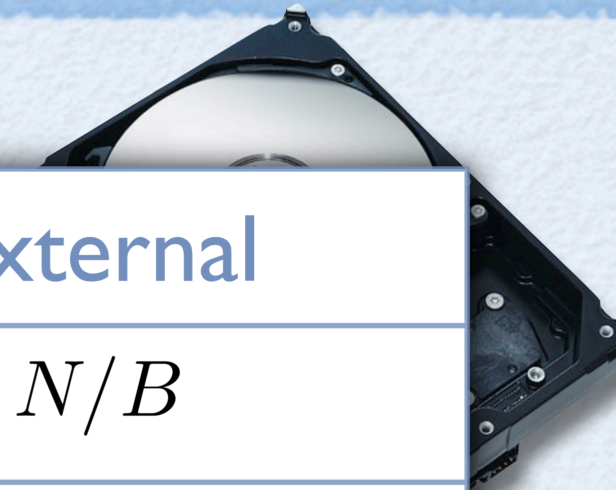

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Scanning	N	N/B
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To a .

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Previous Work

Answering contour queries I/O-efficiently:

	Preprocessing I/Os	Structure Size	Query I/Os
Chiang, Silva'97	$O(\text{Sort}(N))$	$O(N)$	$O(\log_B N + T/B)$
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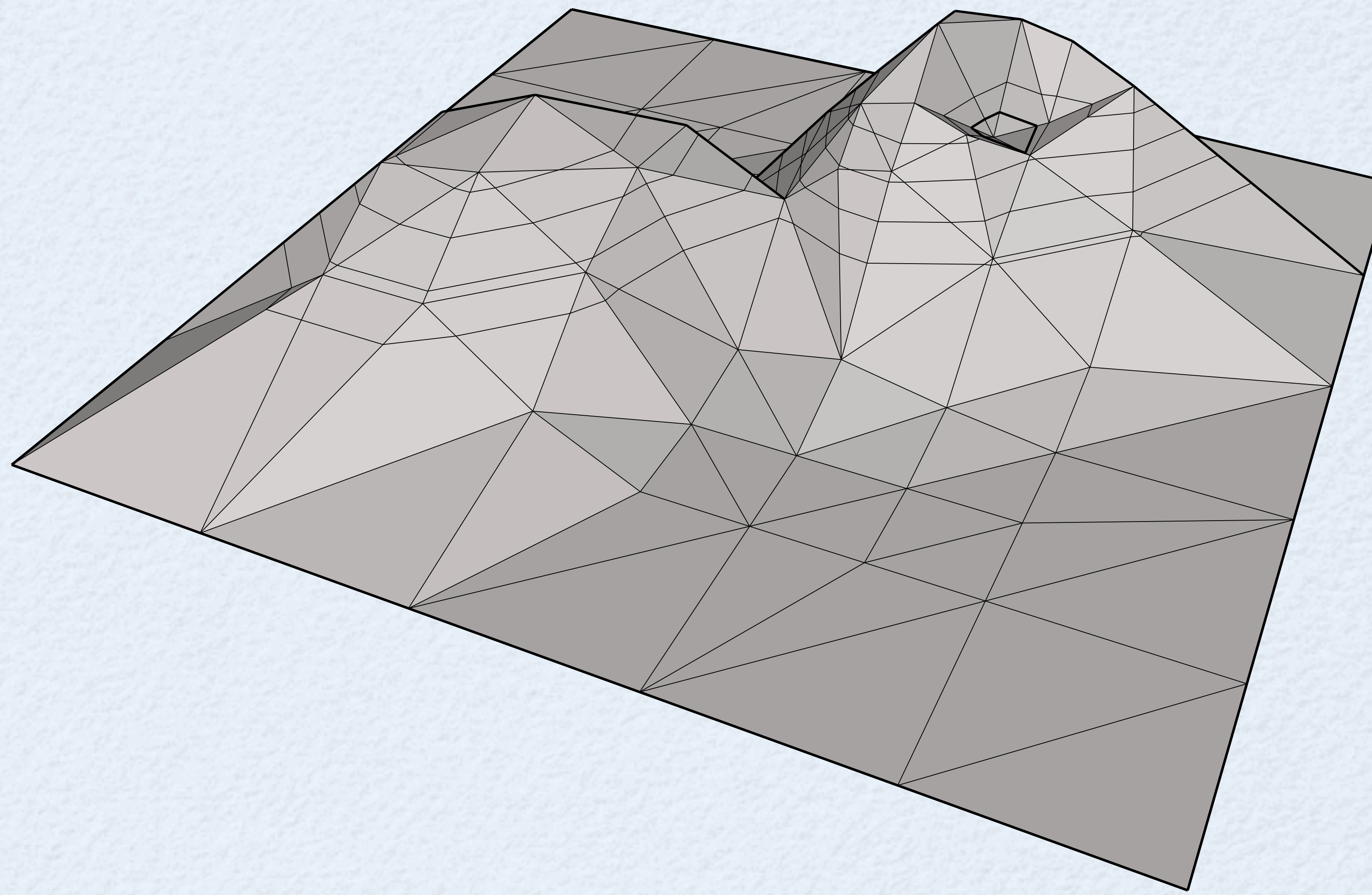
Unsorted Contours!

“Very naïve” Algorithm: Generate one contour at a time

Find one “seed” triangle intersecting each contour and trace out the contour sequentially.



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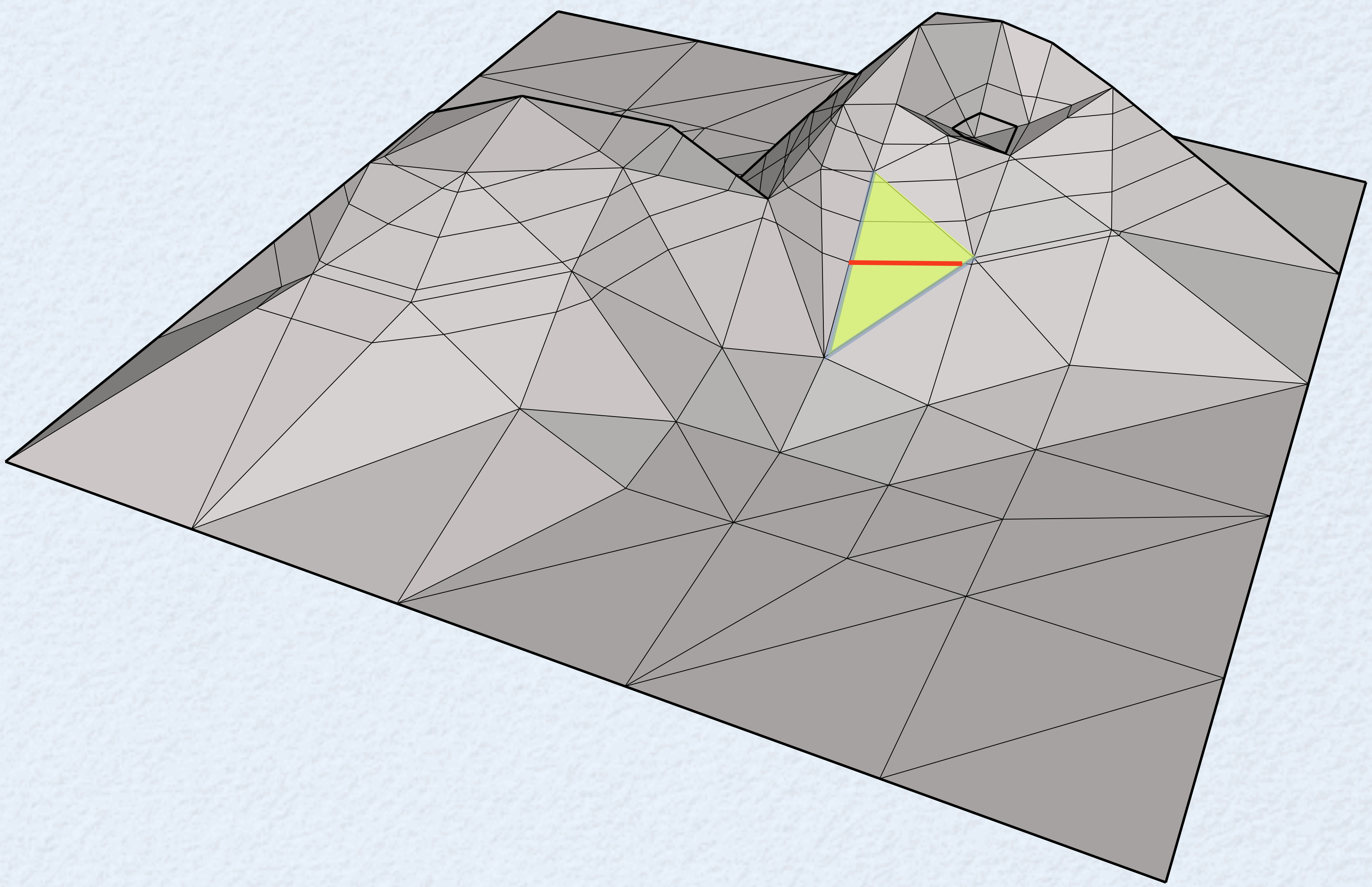

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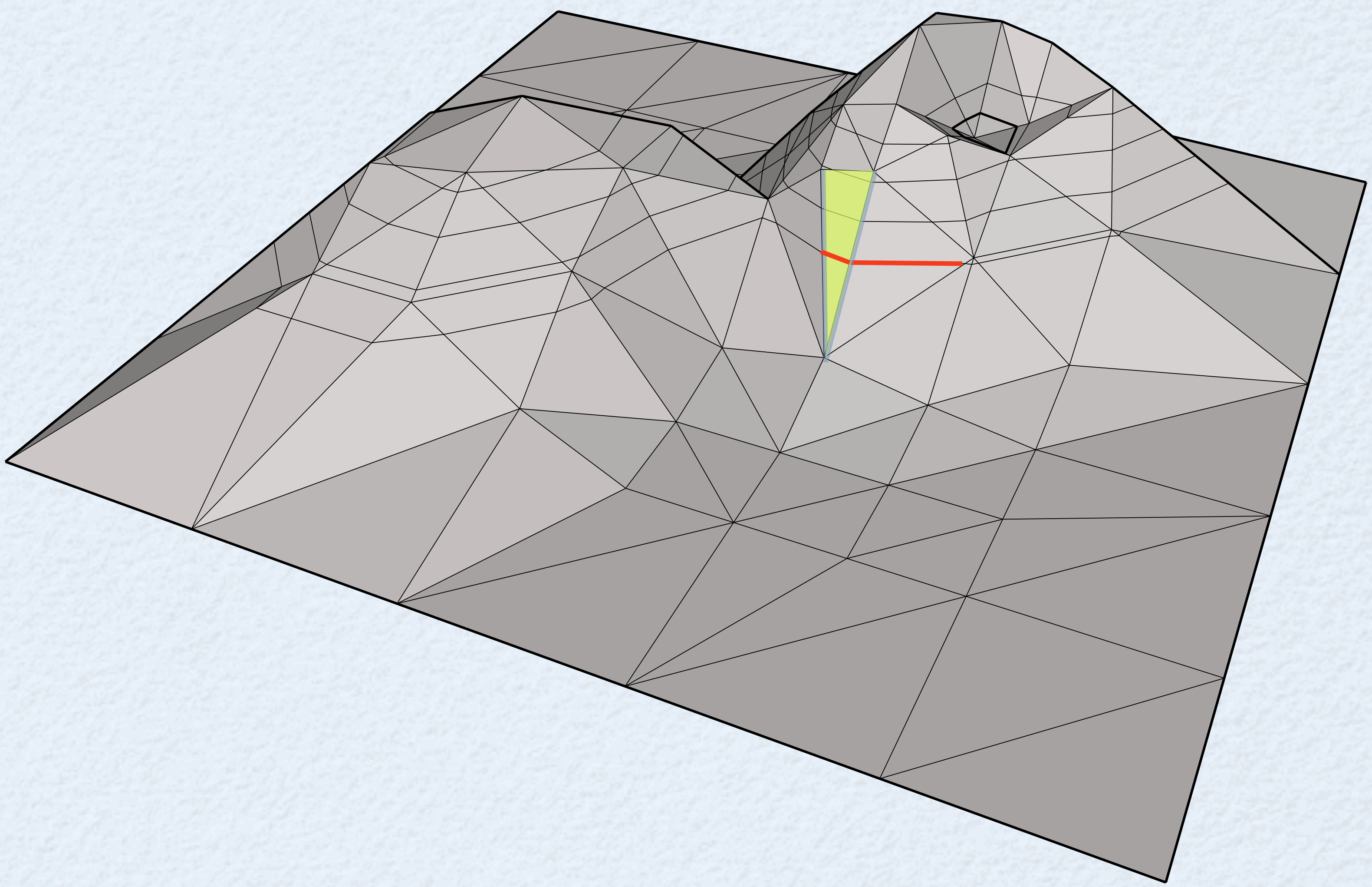

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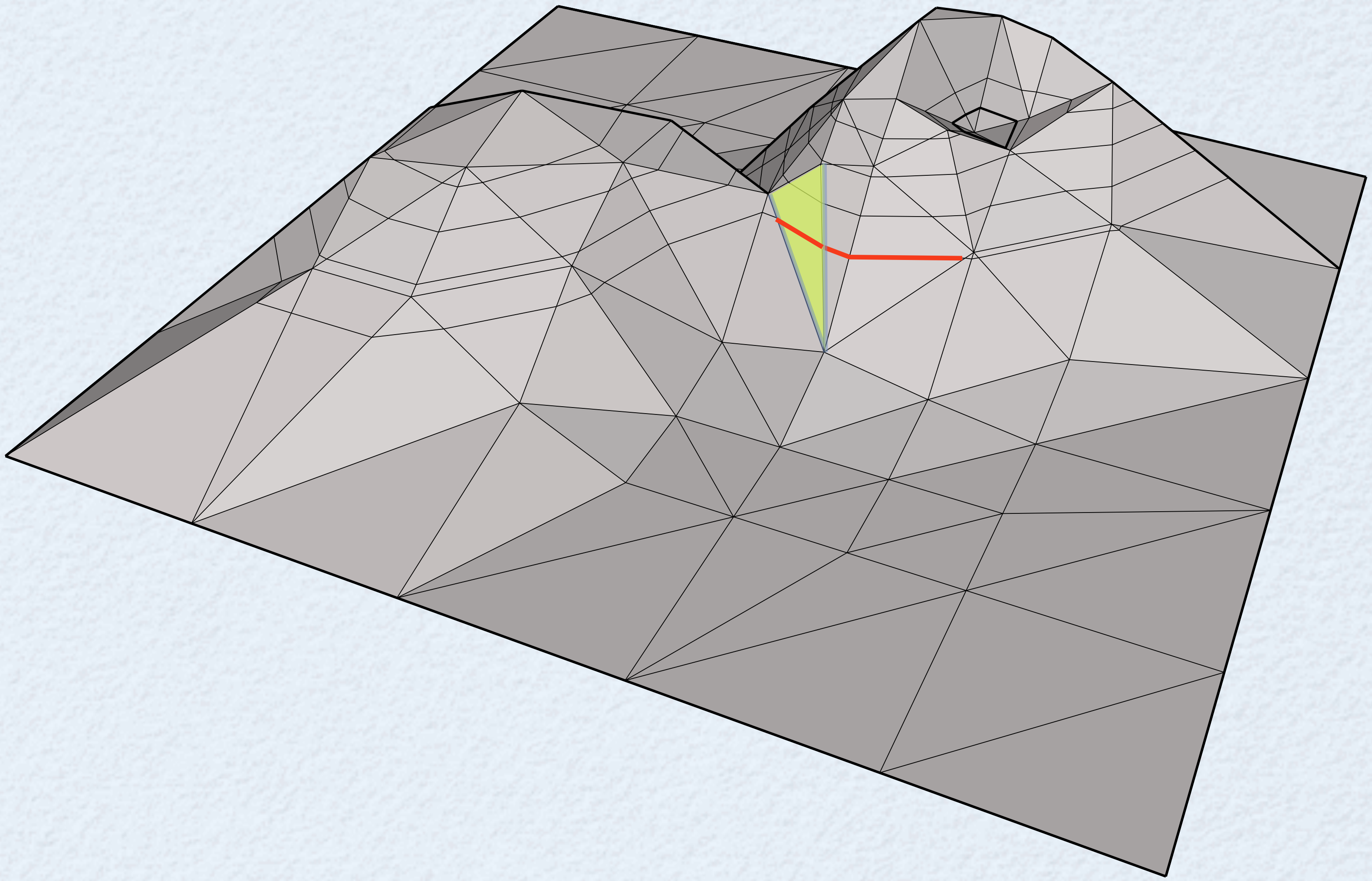

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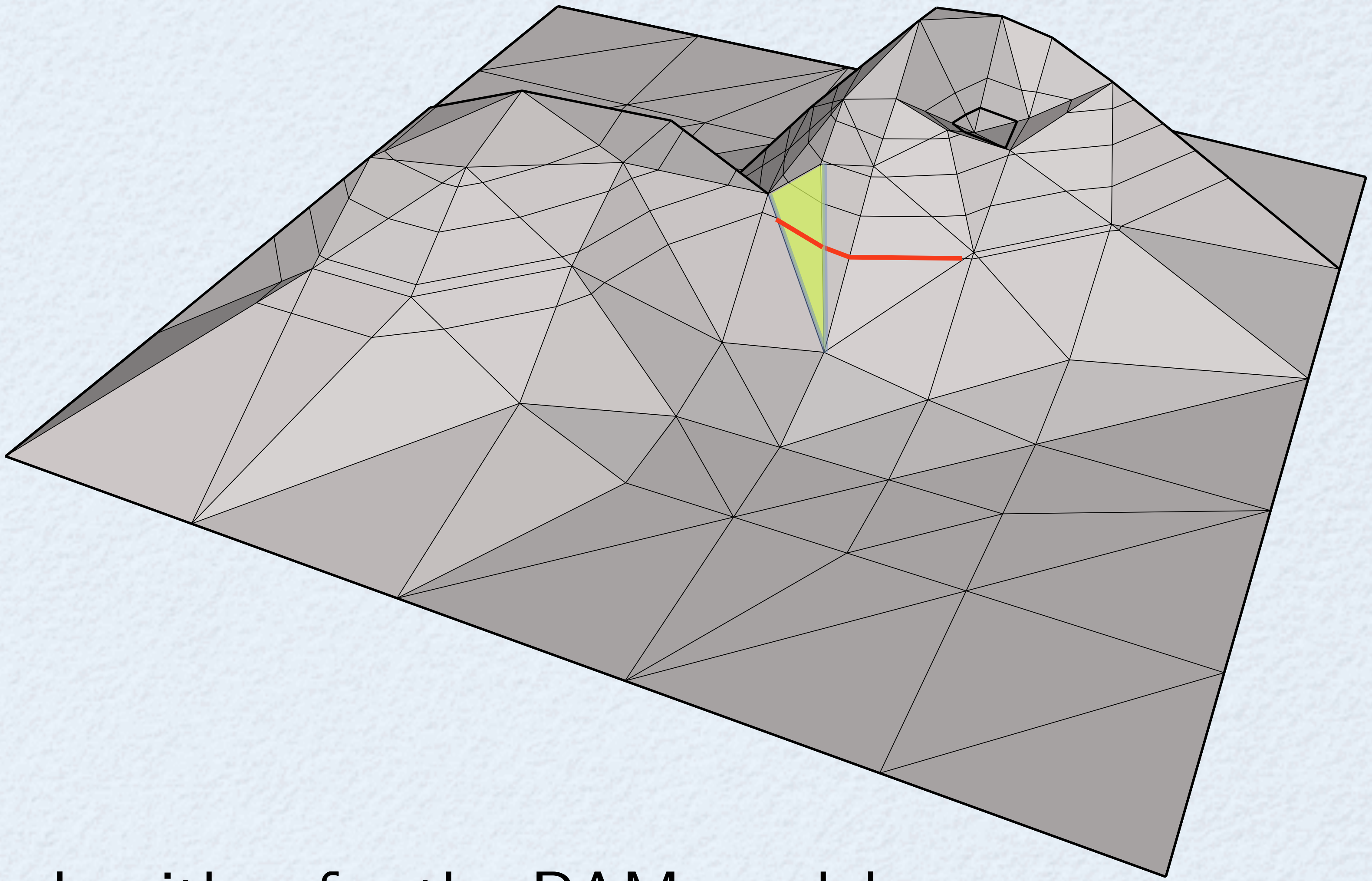

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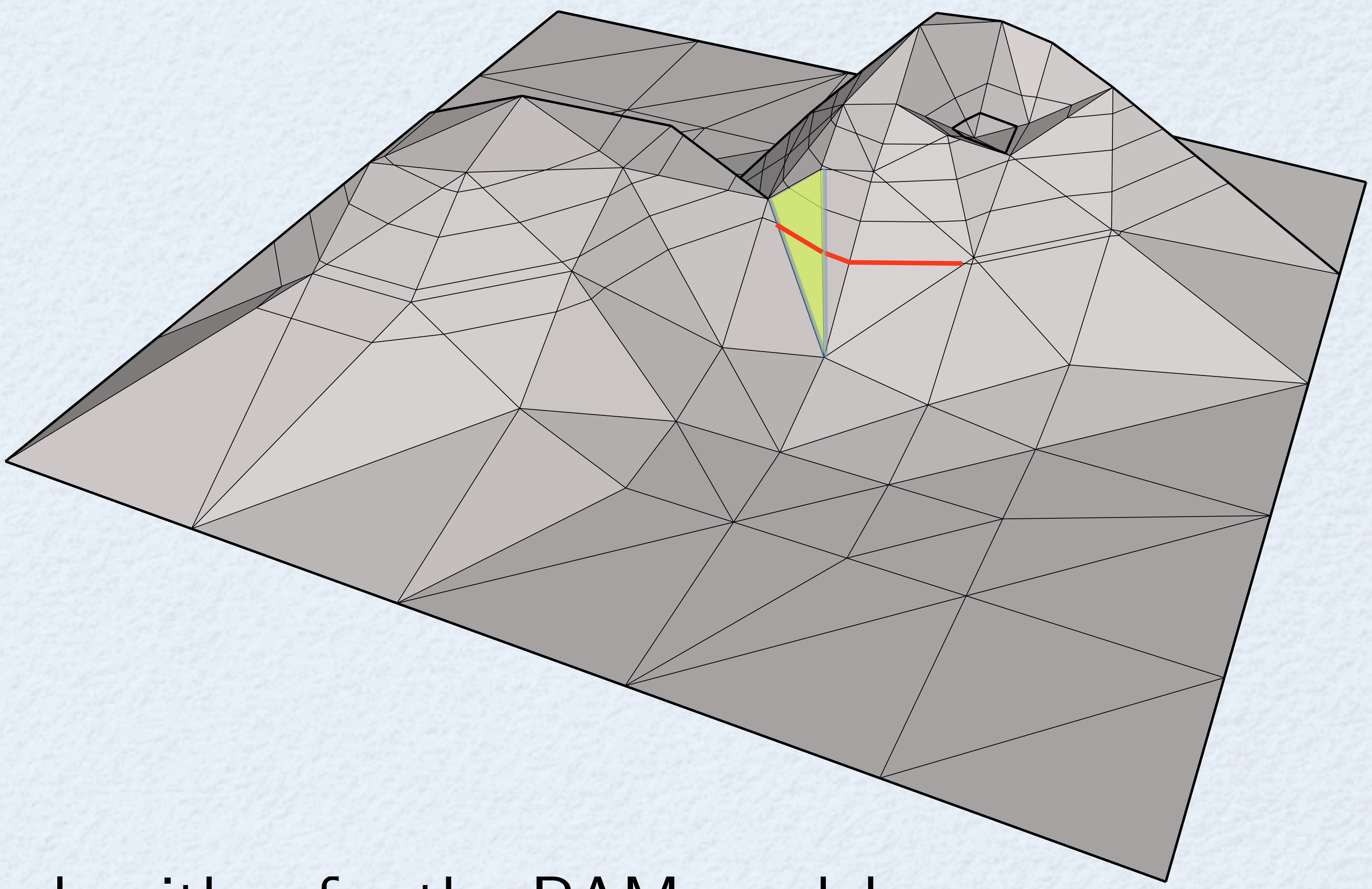

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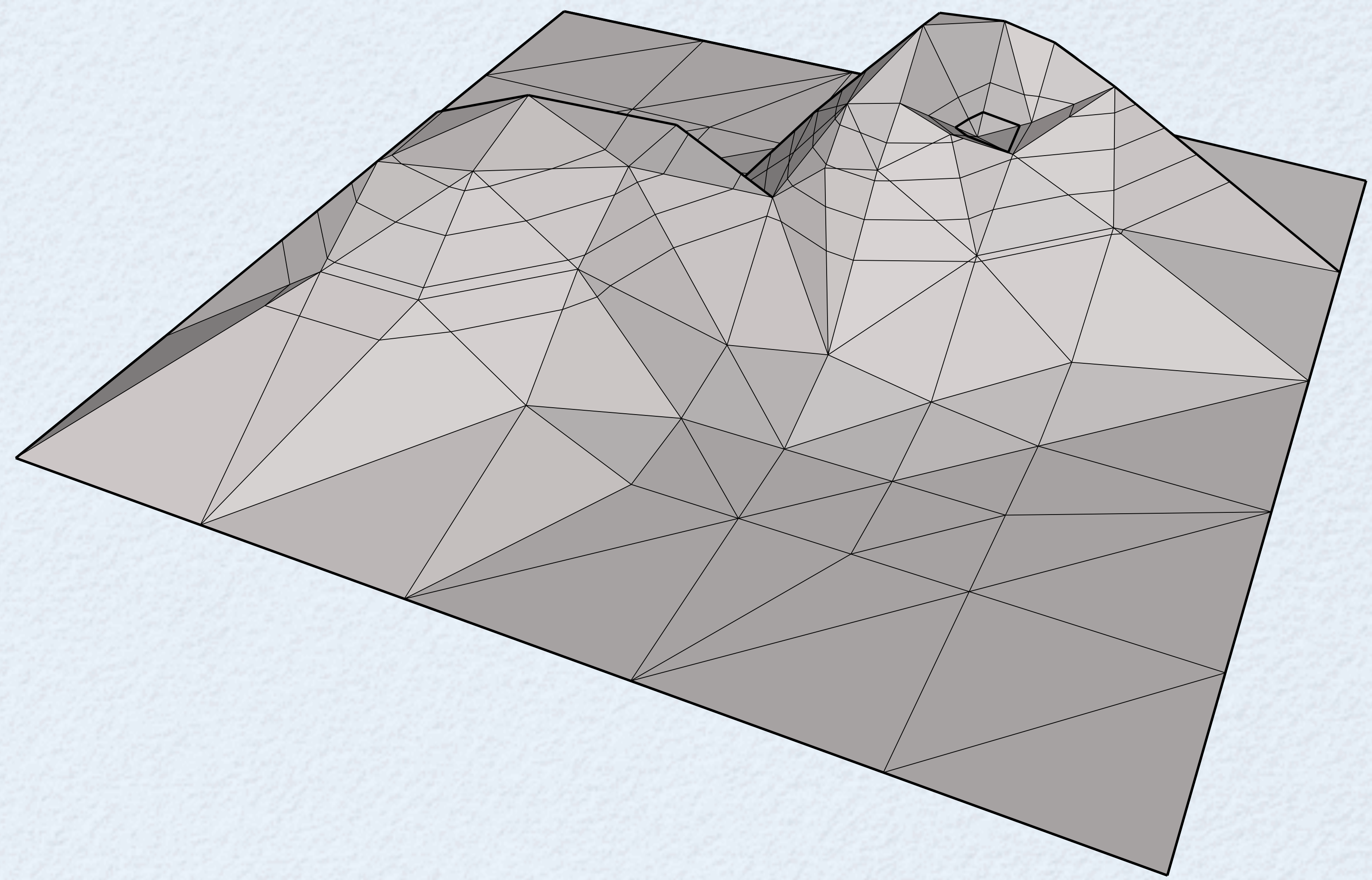
I/O Complexity:

$$O(N/B + T).$$

\# segments in the output

"Less naïve" Algorithm: Generate pieces, sort later

Scan the triangles (in the order laid out on the disk) and generate all segments. Then sort the output.



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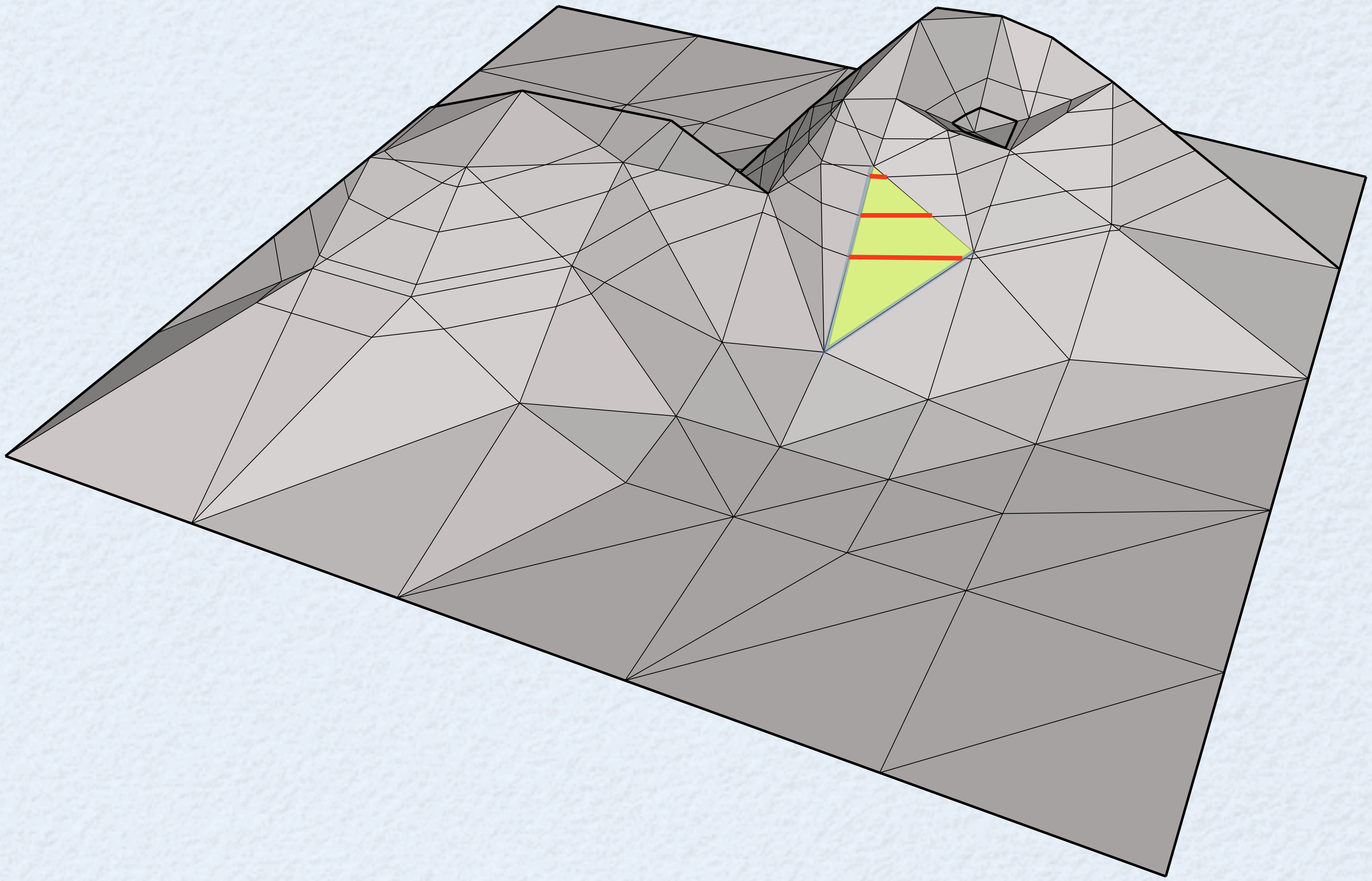

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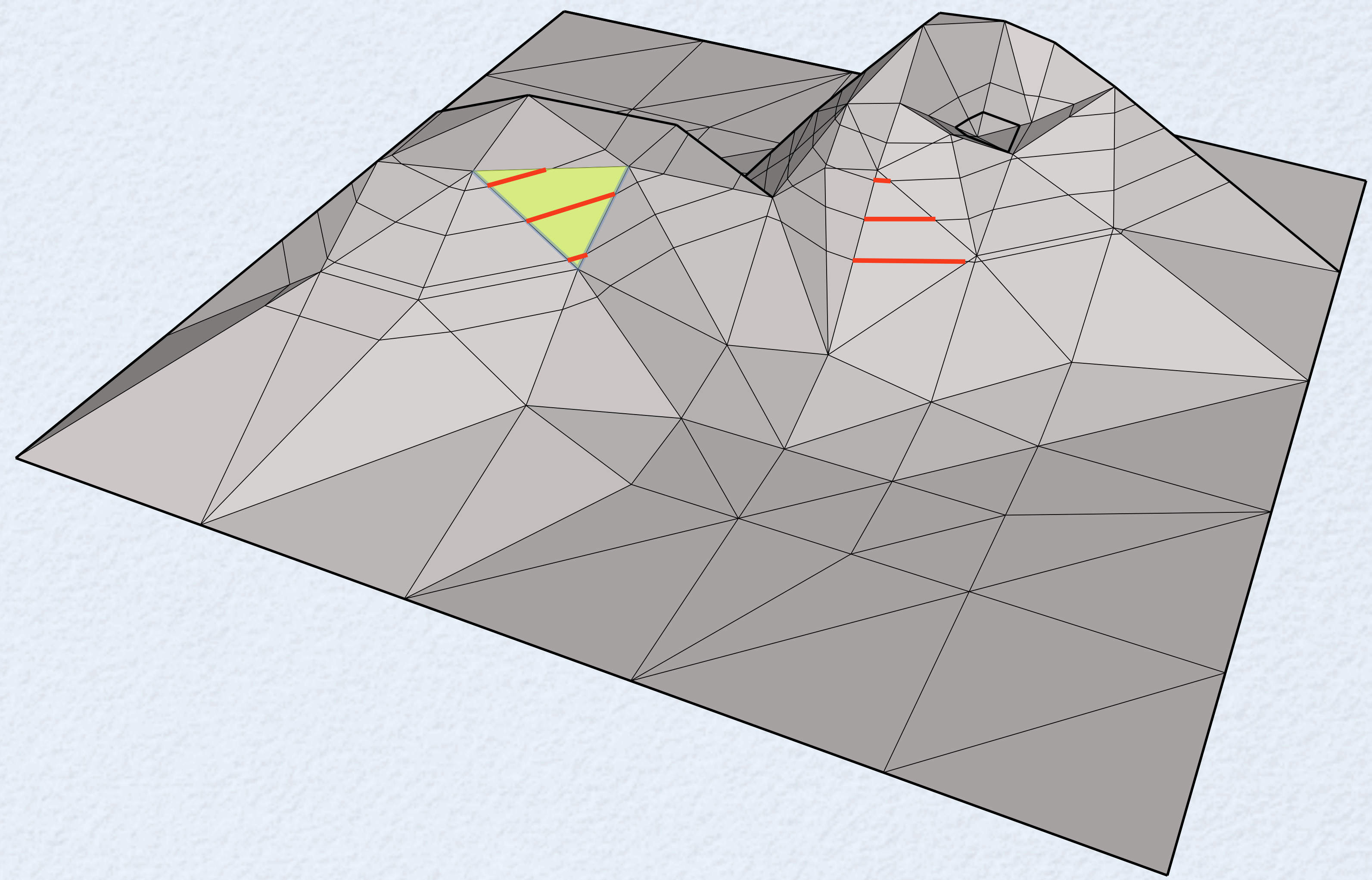

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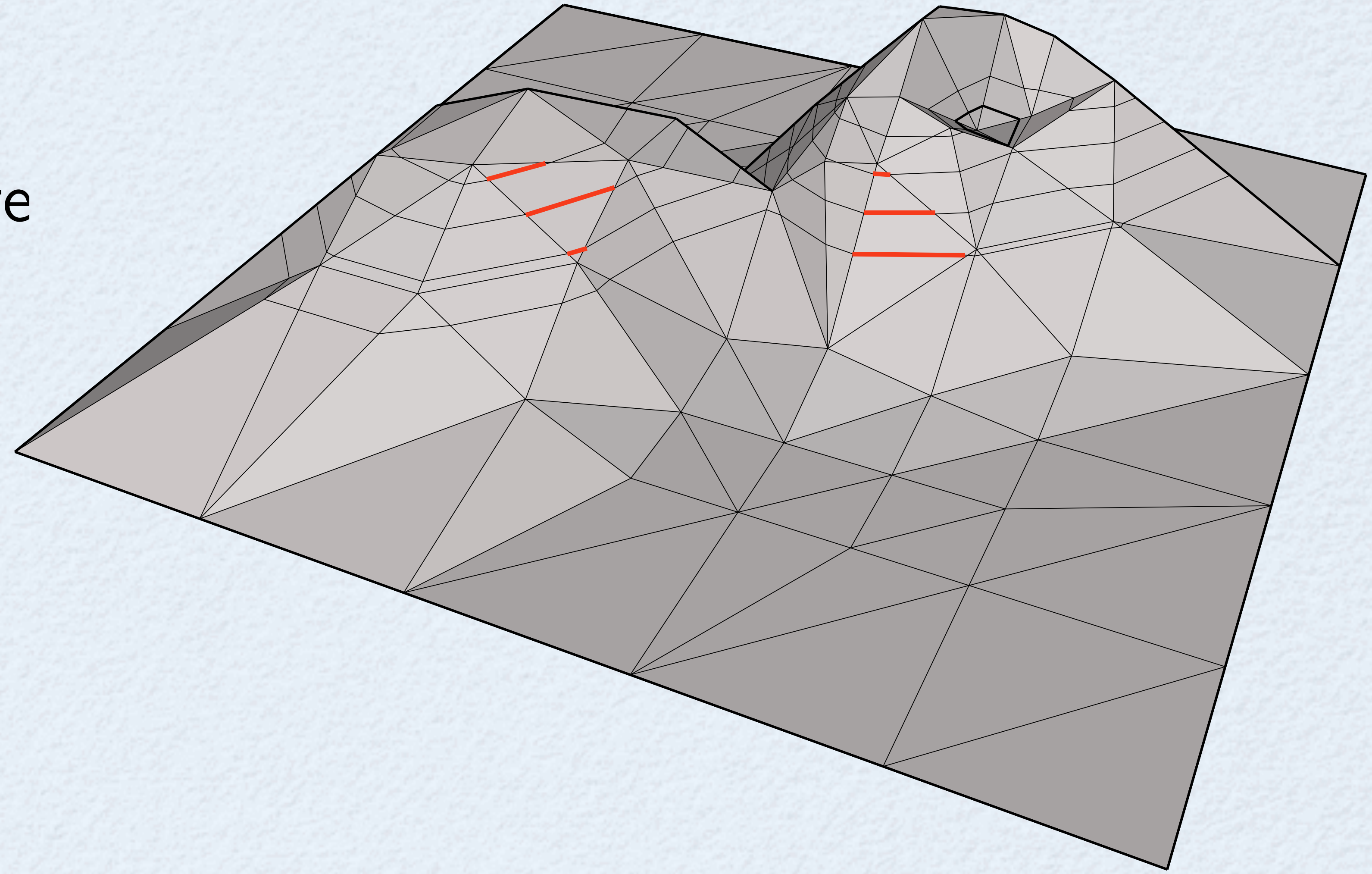

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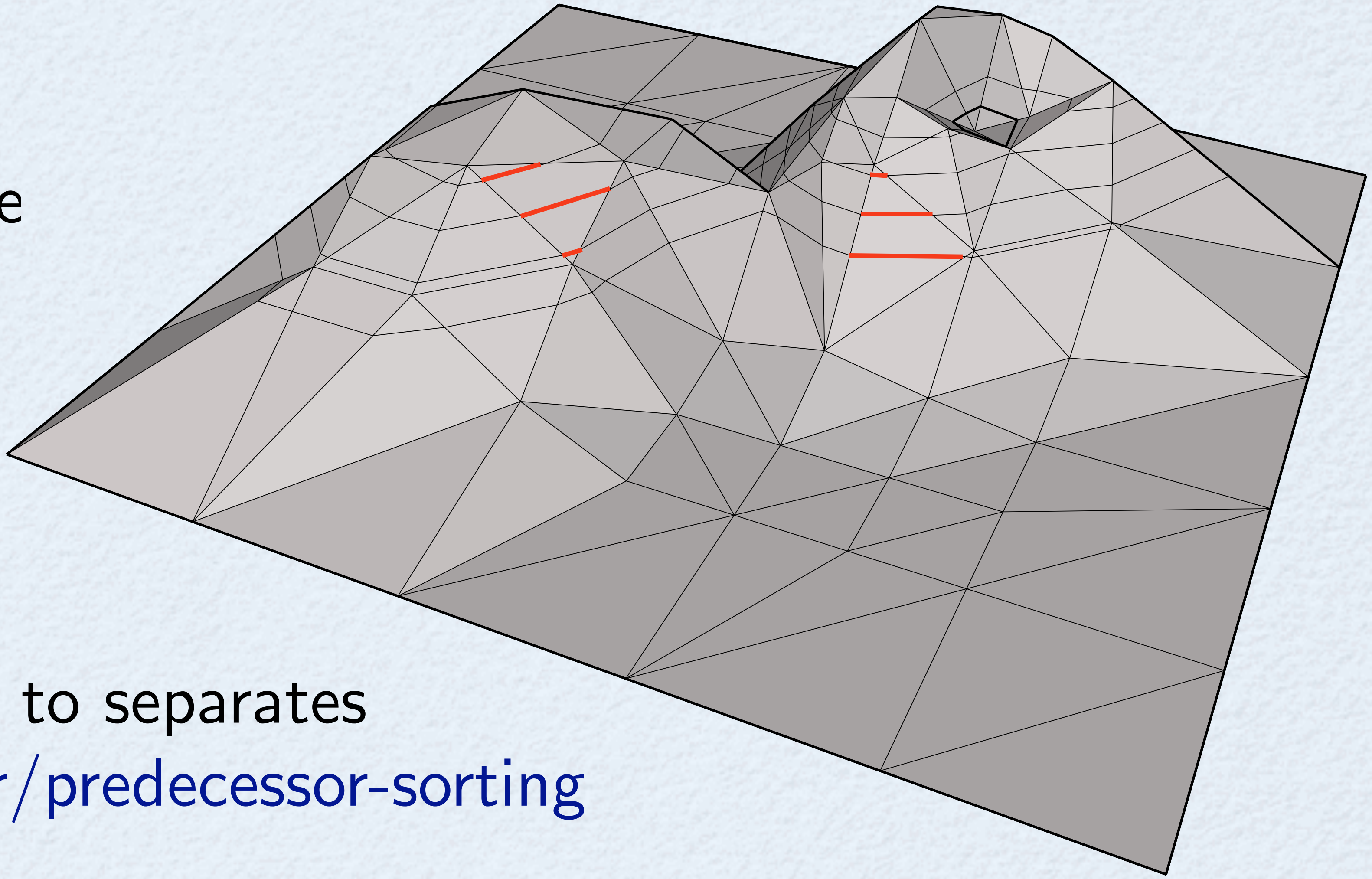

Sort pairs on first component to separates level sets. Then use successor/predecessor-sorting to put contours in order.

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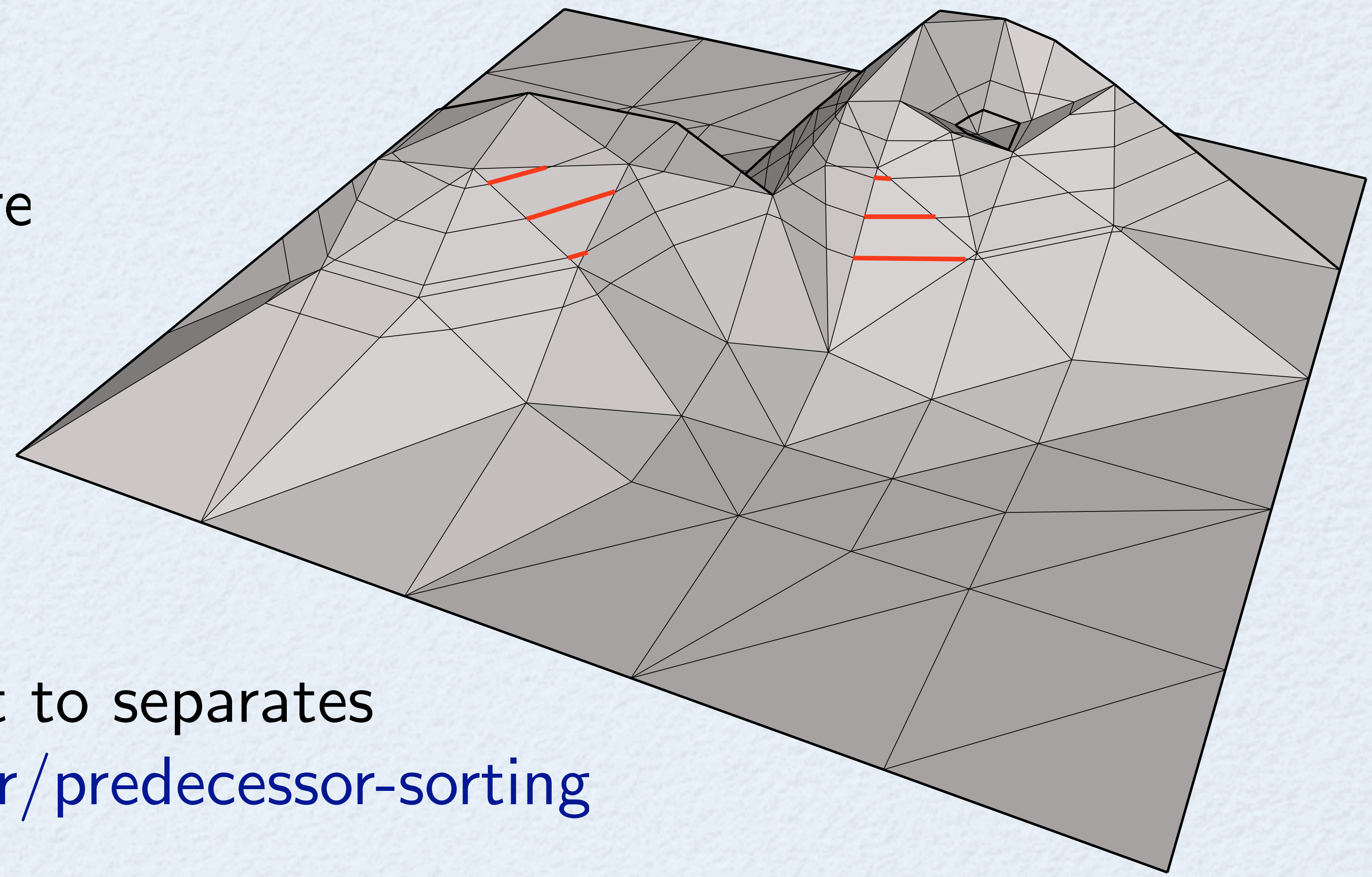

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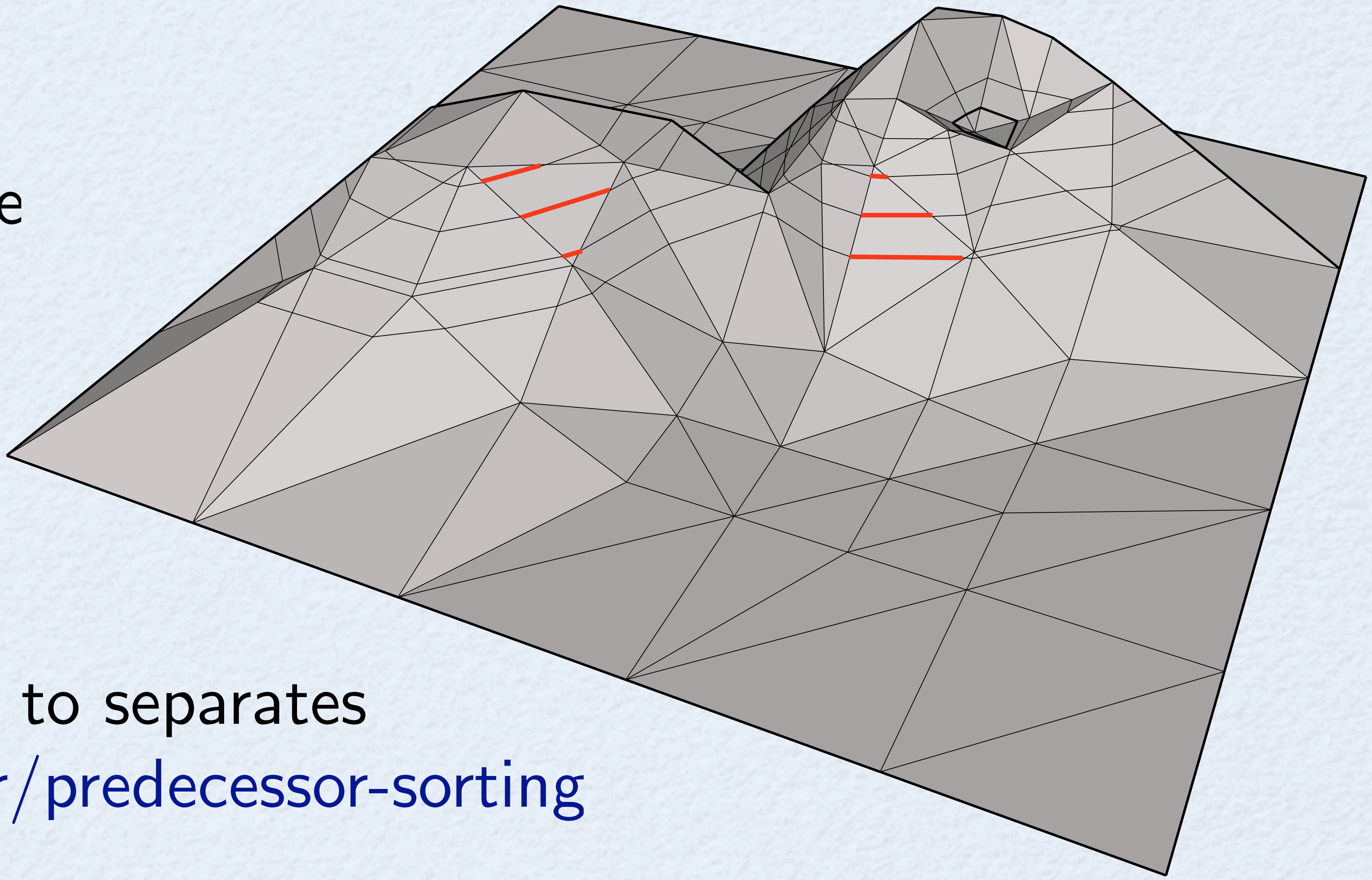

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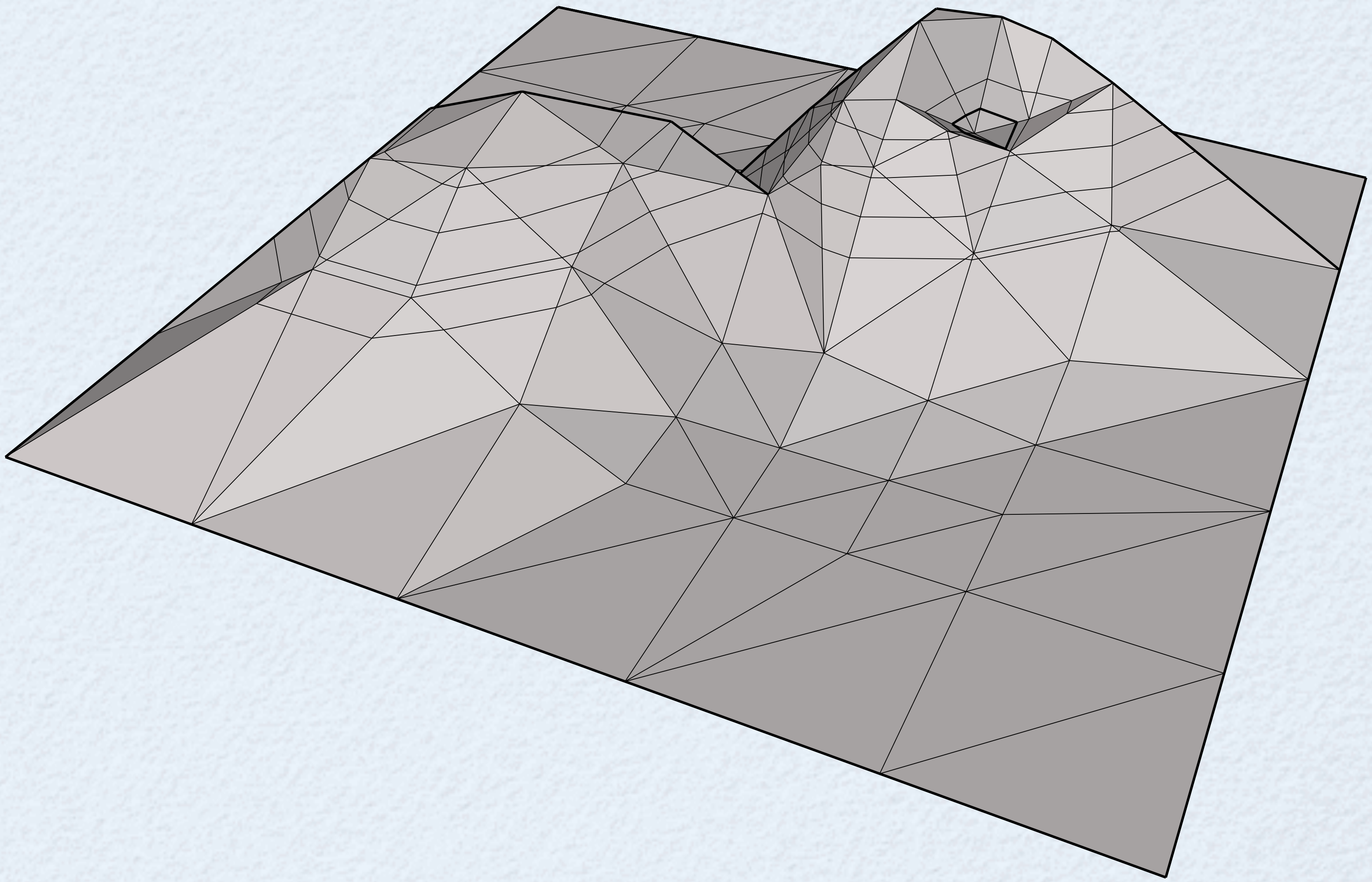

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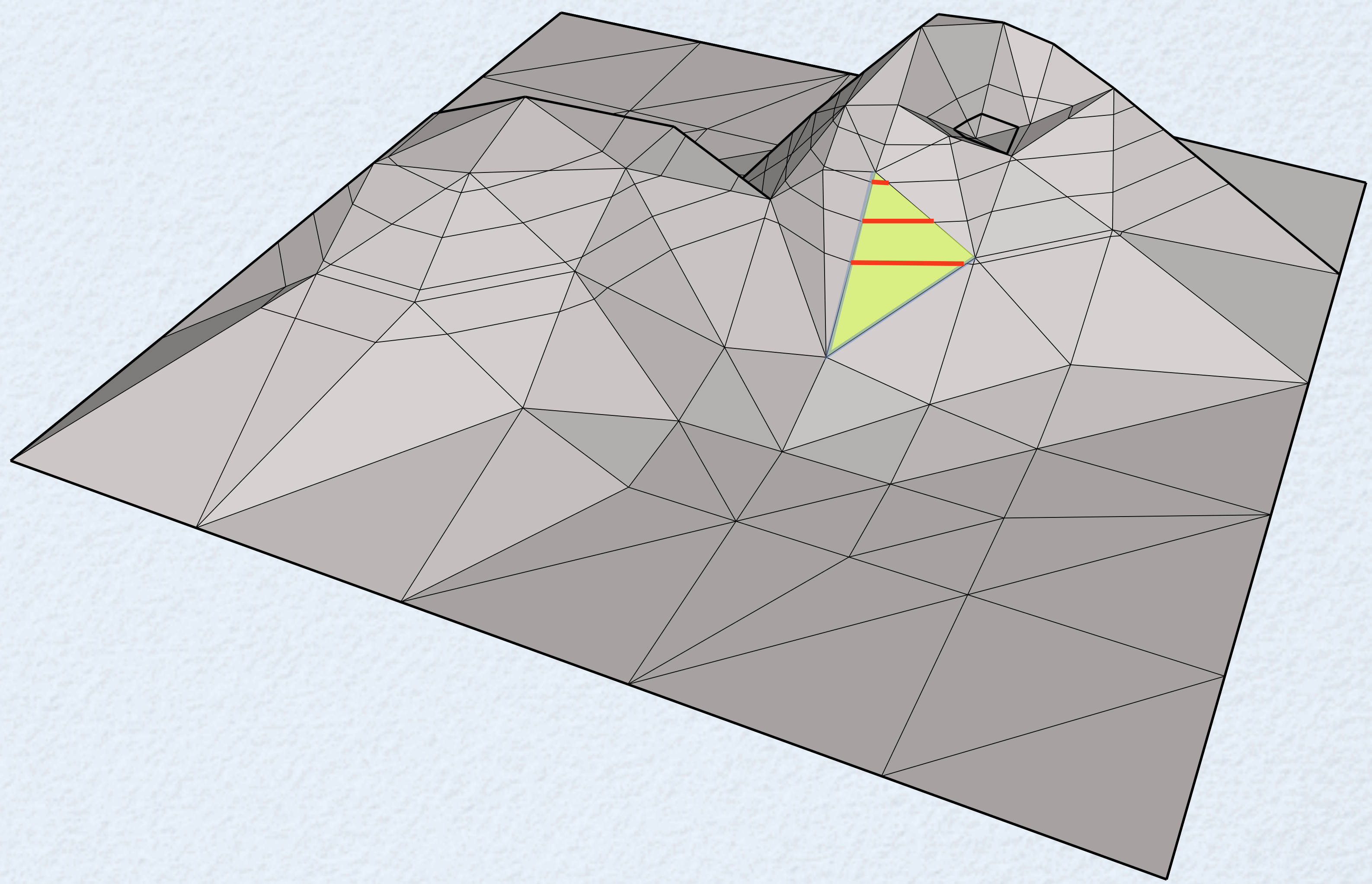

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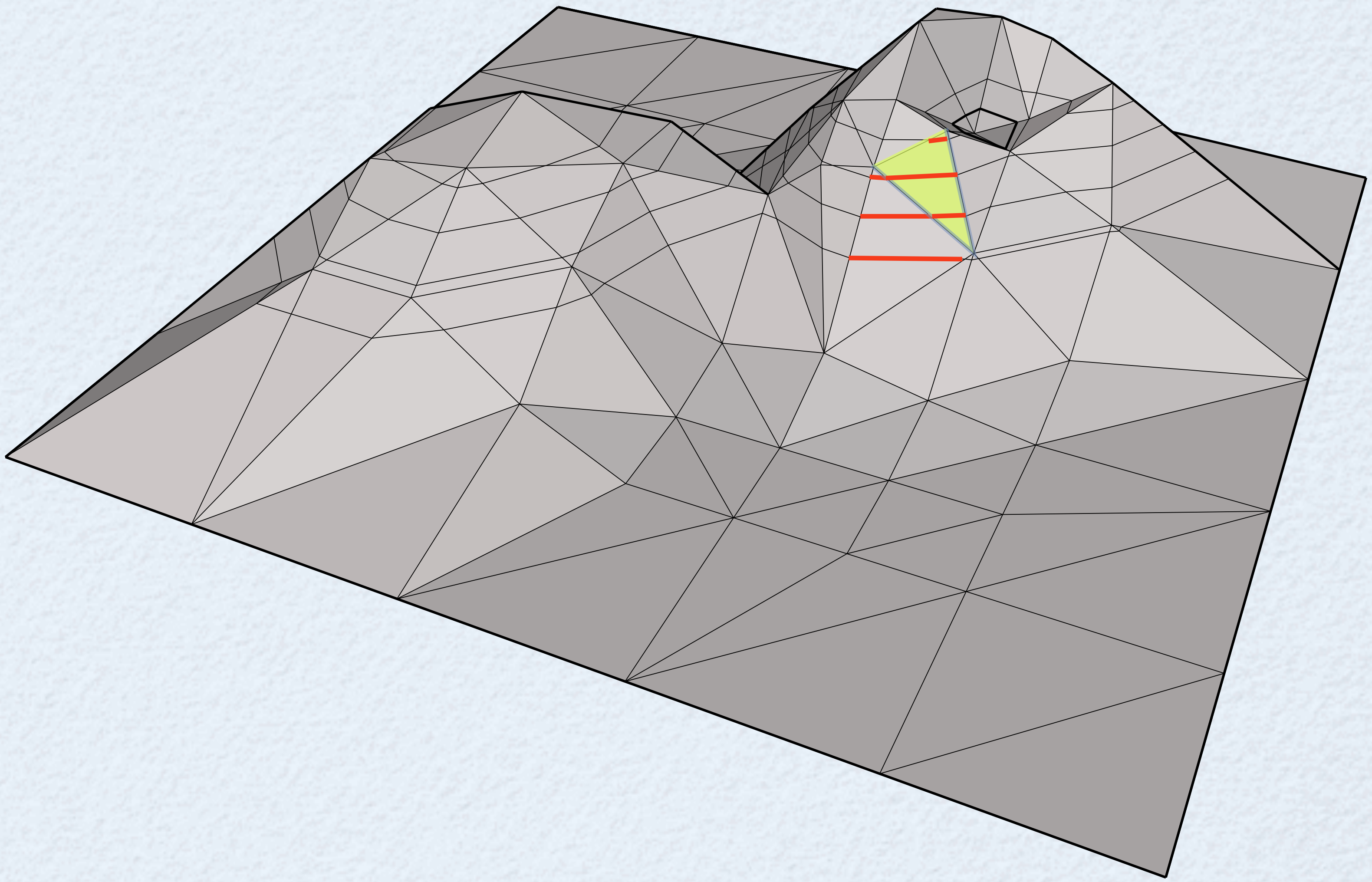

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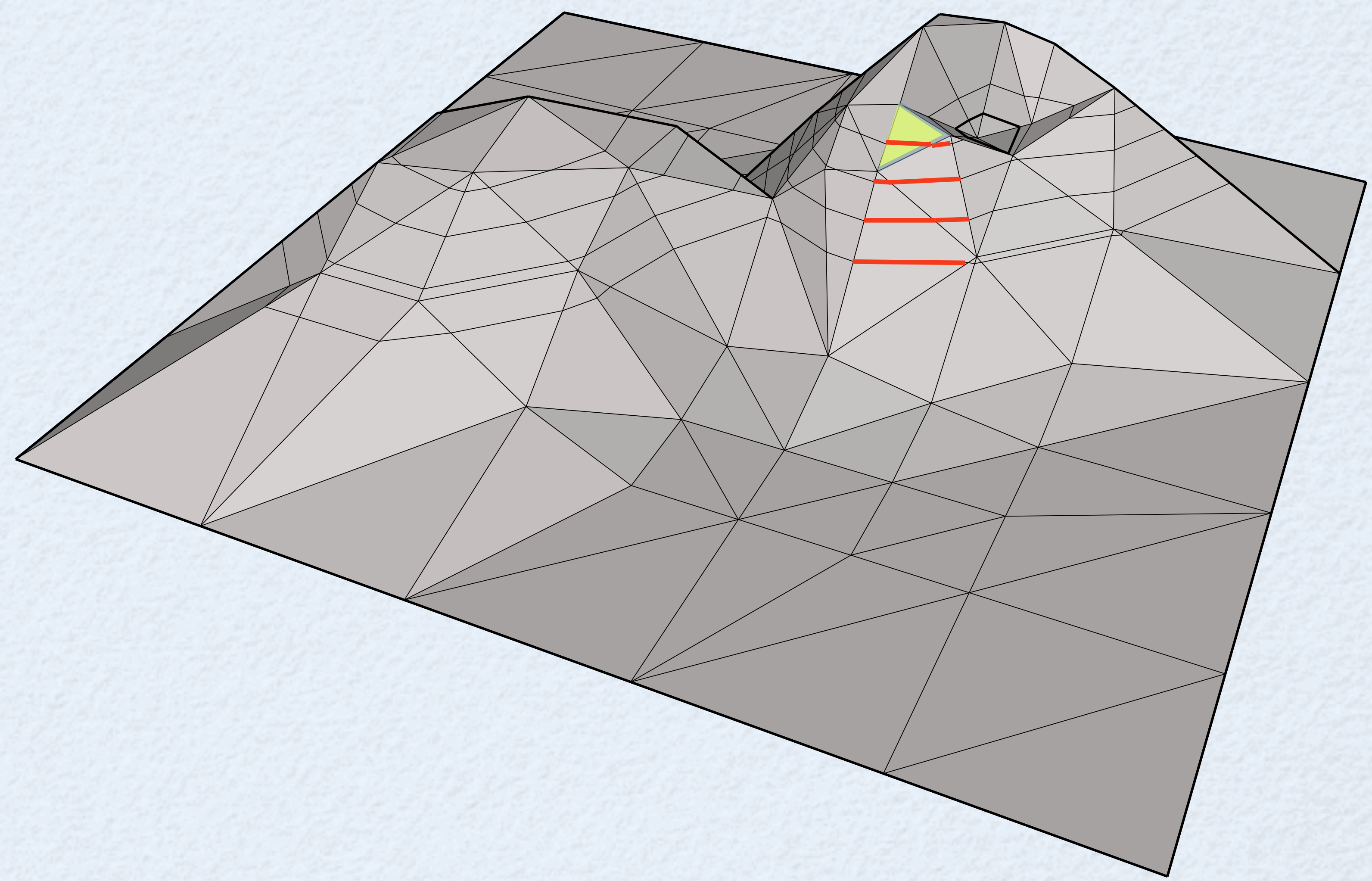

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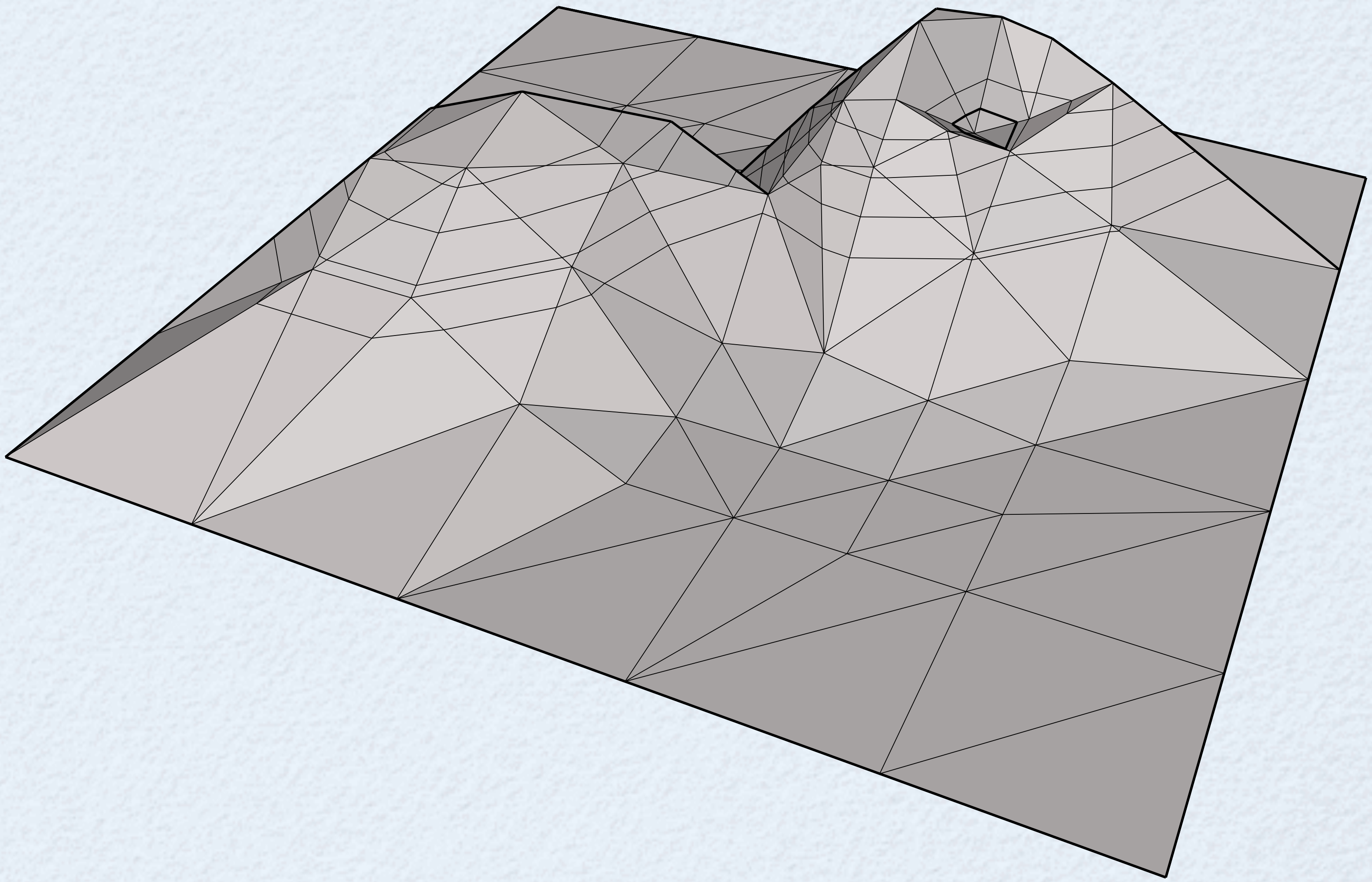

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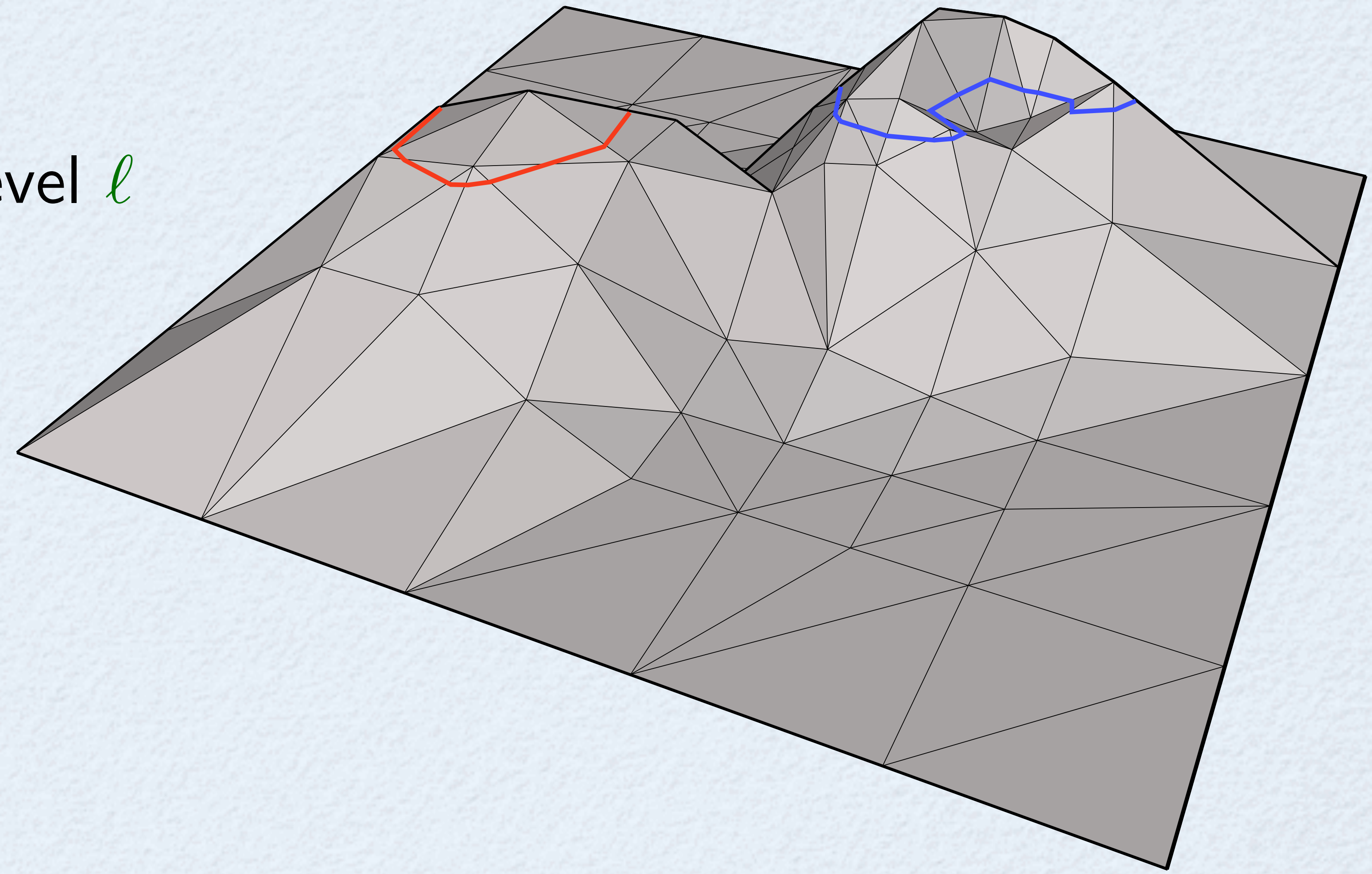

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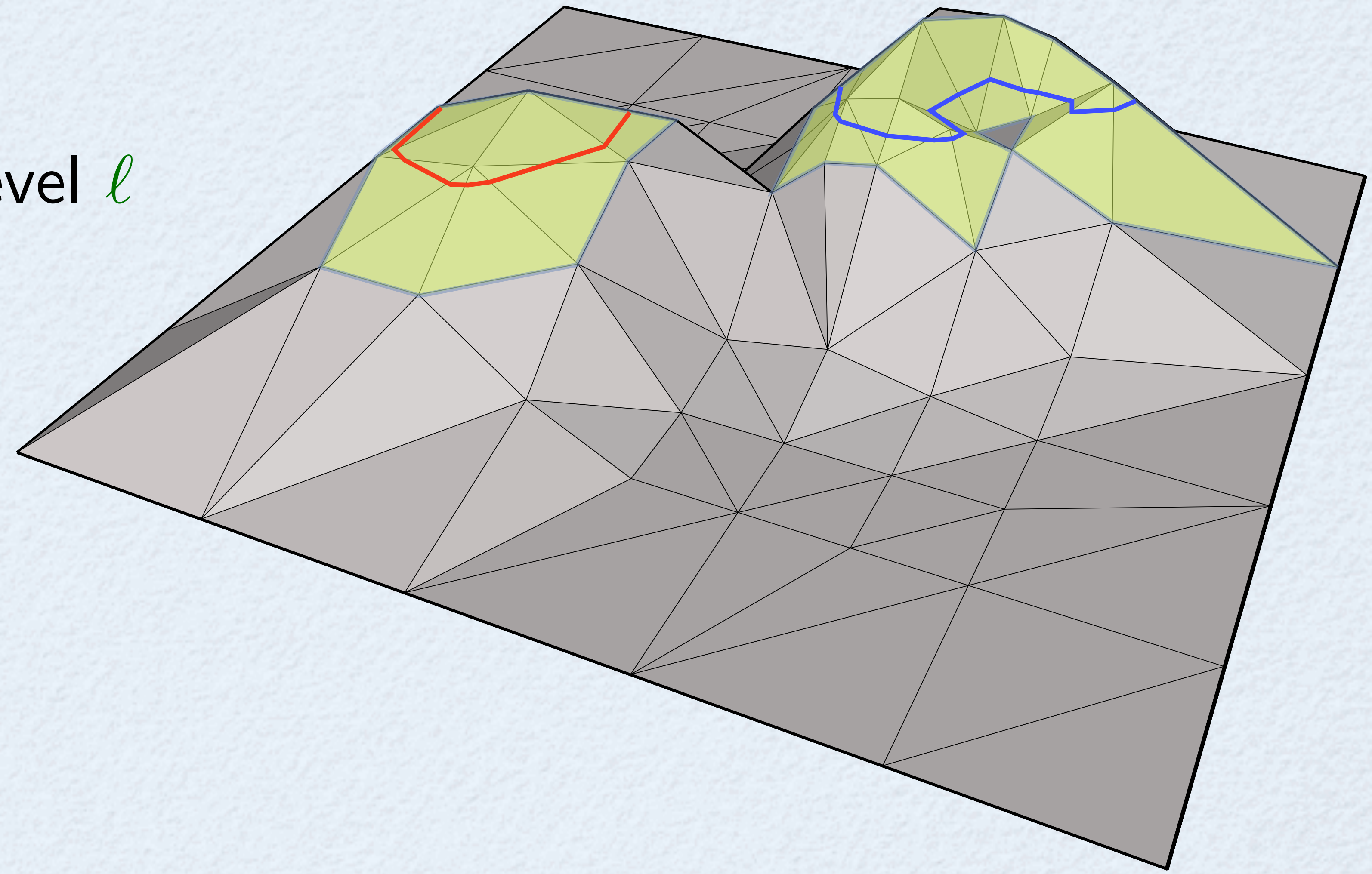

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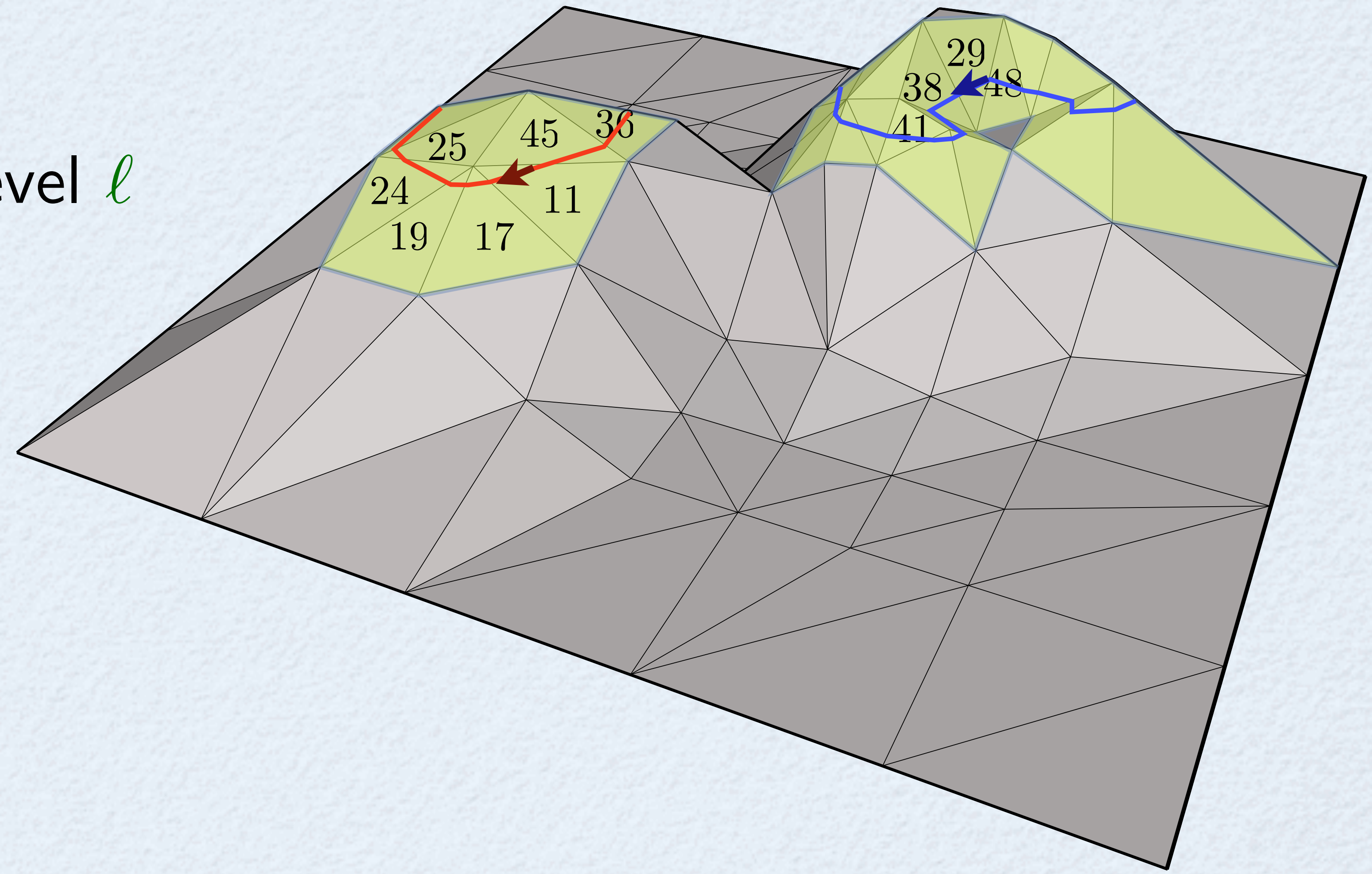
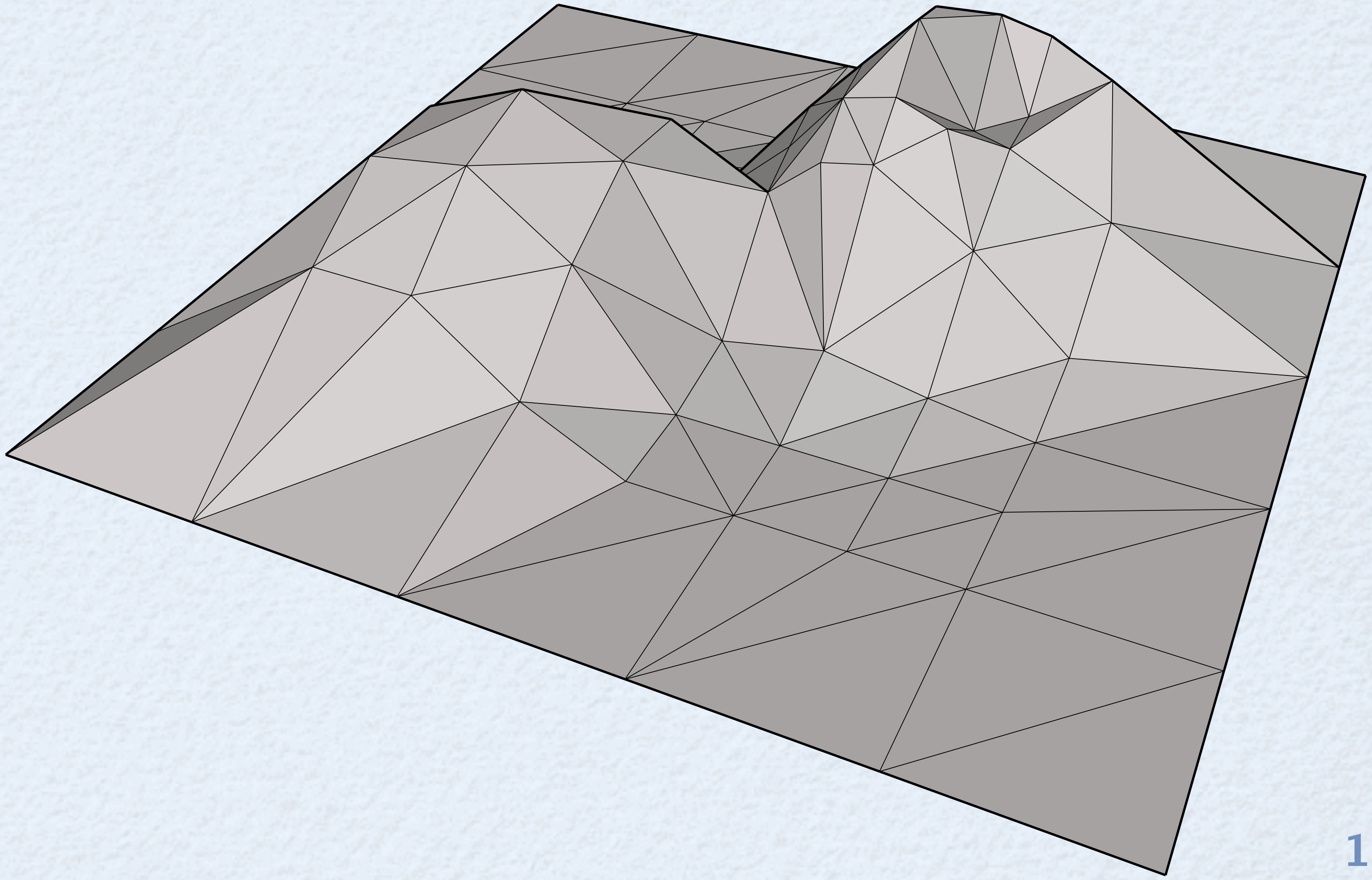


The restriction of $\prec$ to Δ_ℓ traverses each contour of $\mathbb{M}$ in circular order.

The Algorithm

1. Sweep the terrain by a horizontal plane in the $+z$ direction.



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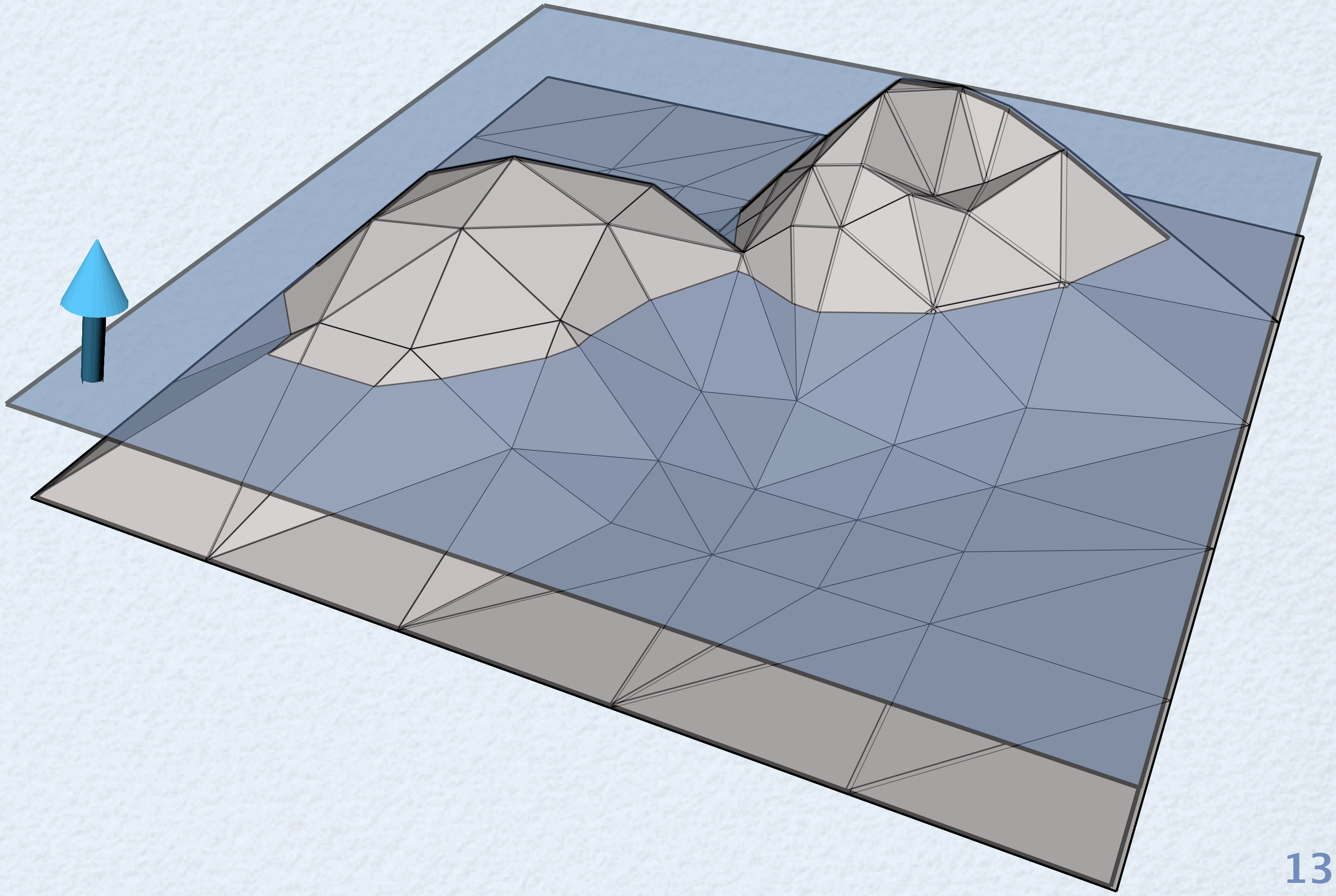

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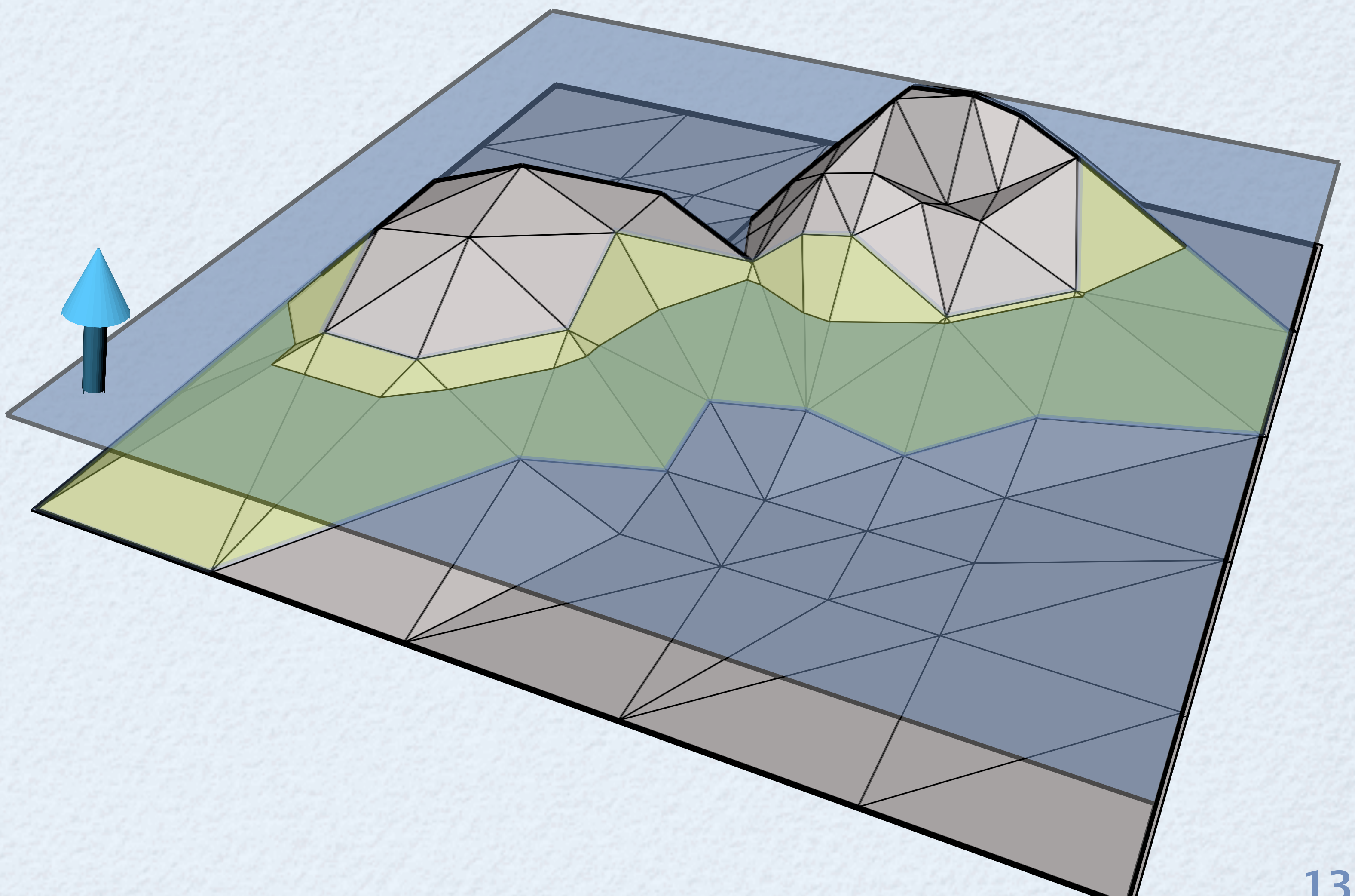

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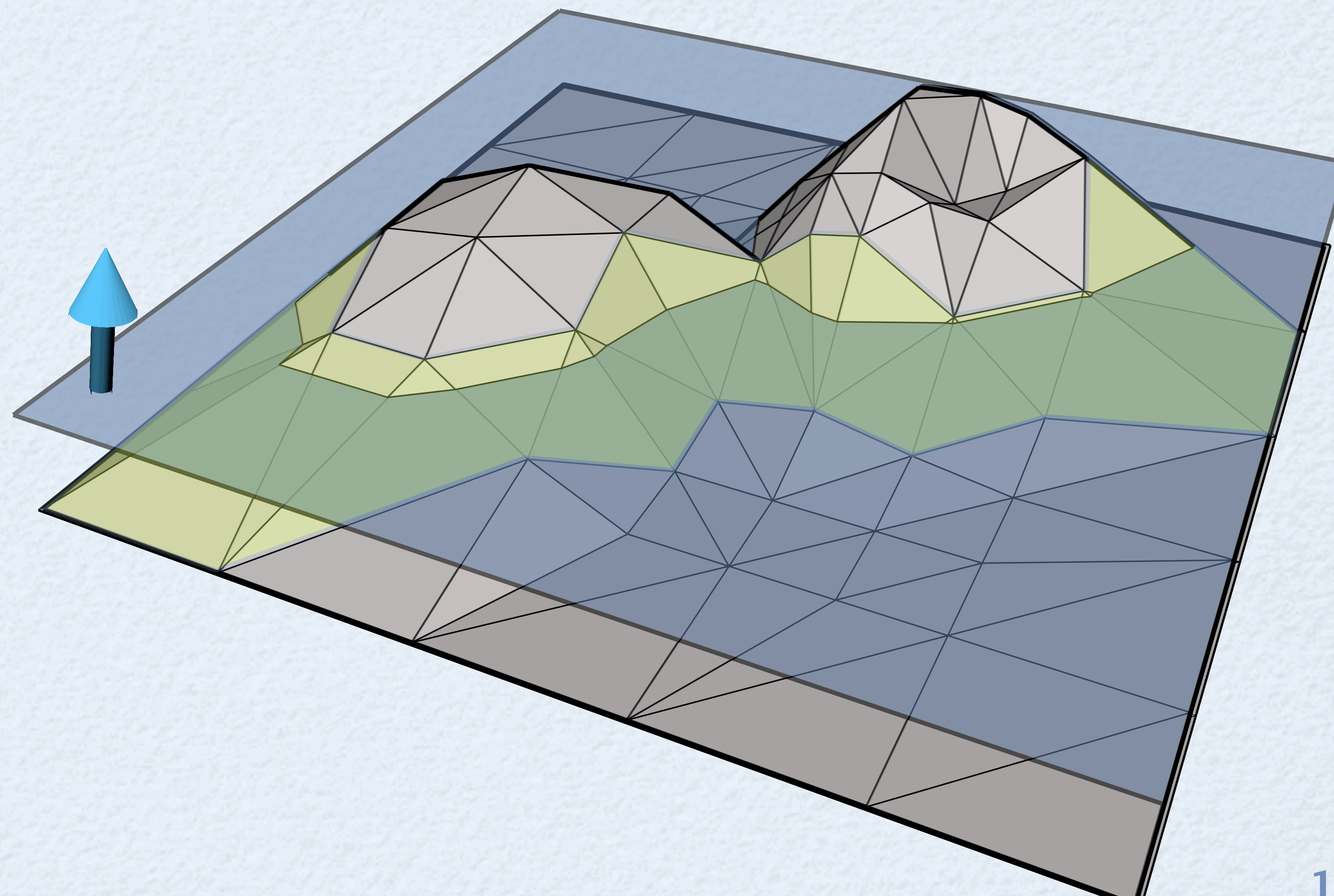
3. When passing target level $\ell_i \in L$, dump contents of tree to disk.



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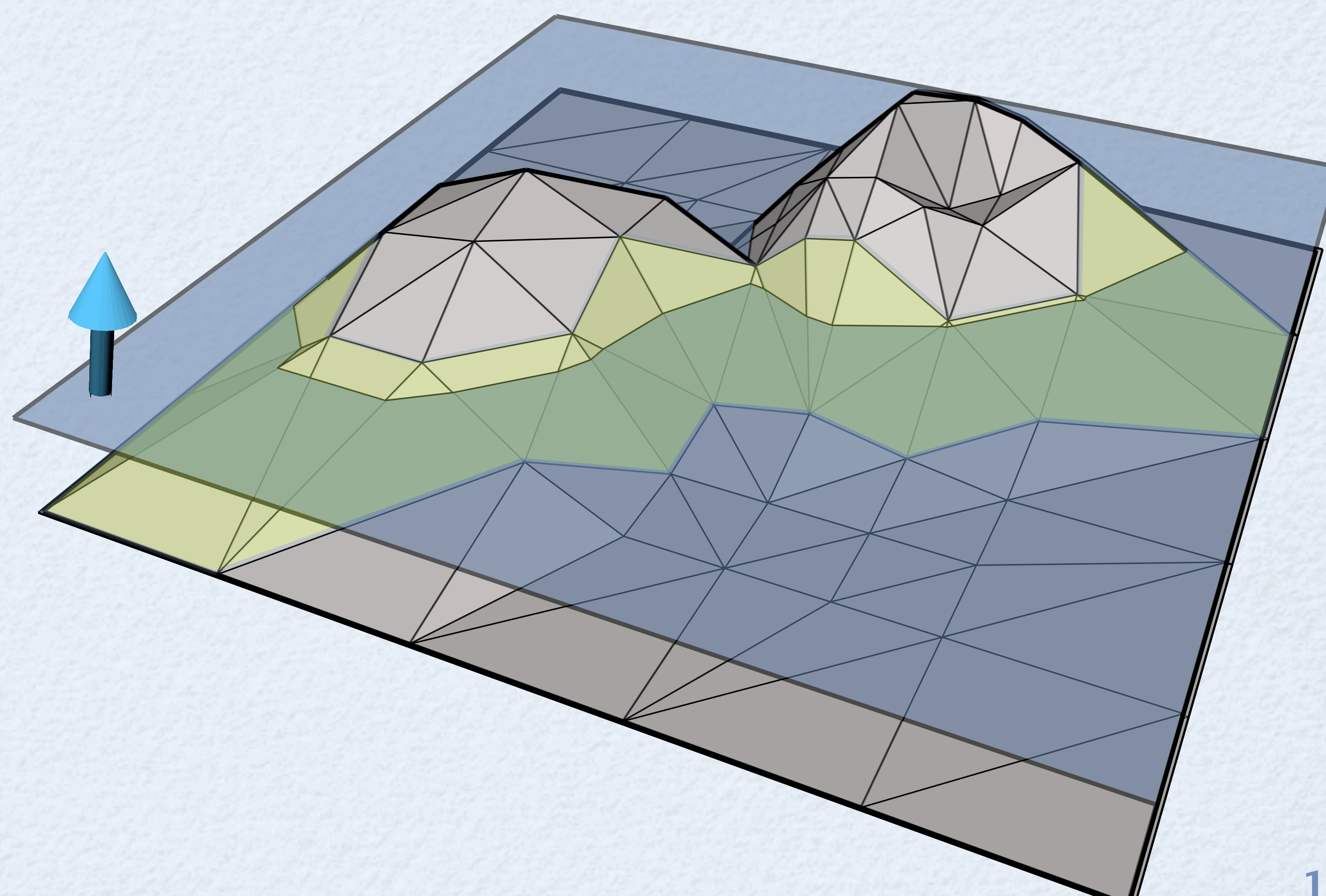

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Buffer Tree [Arge'95]

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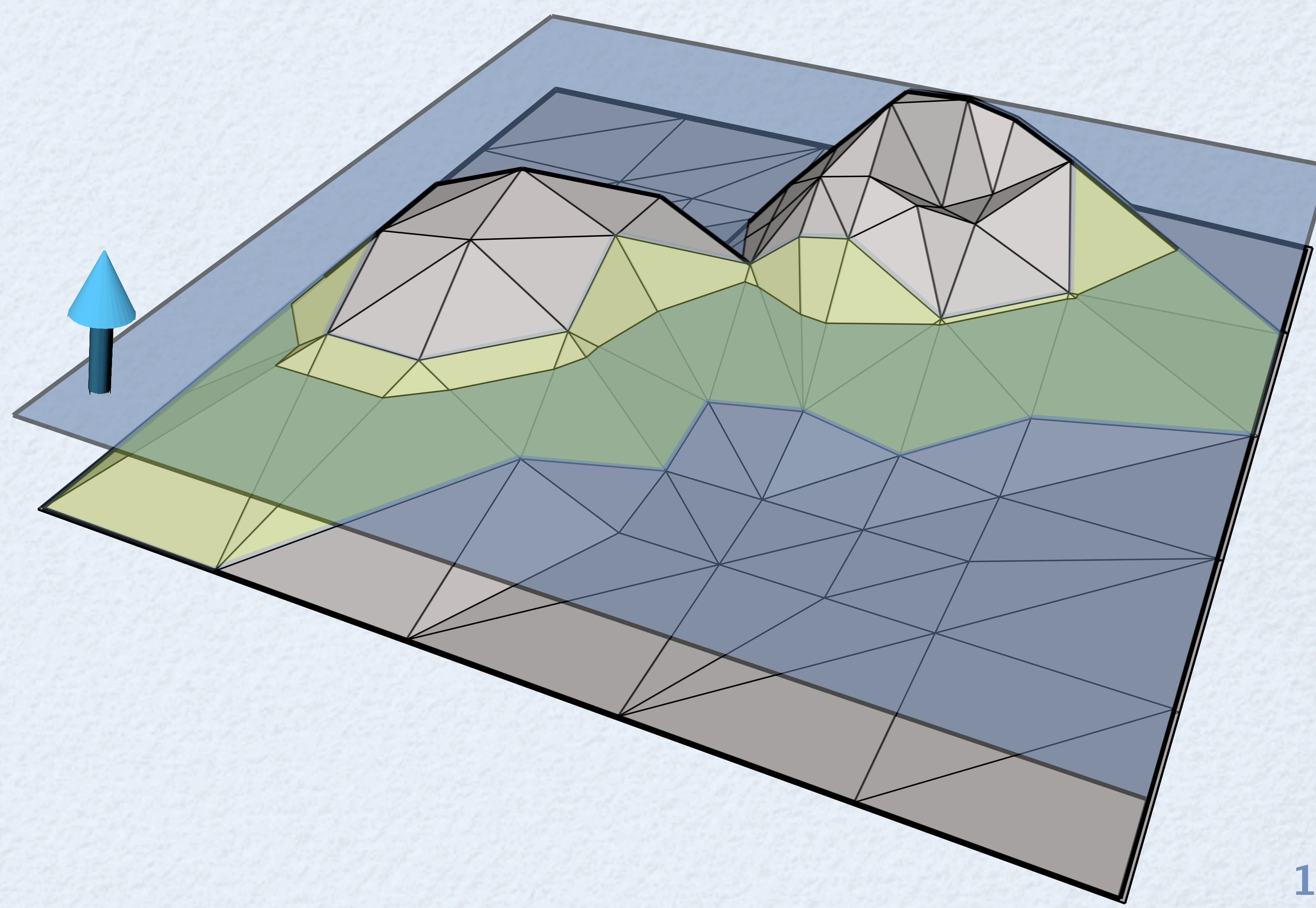

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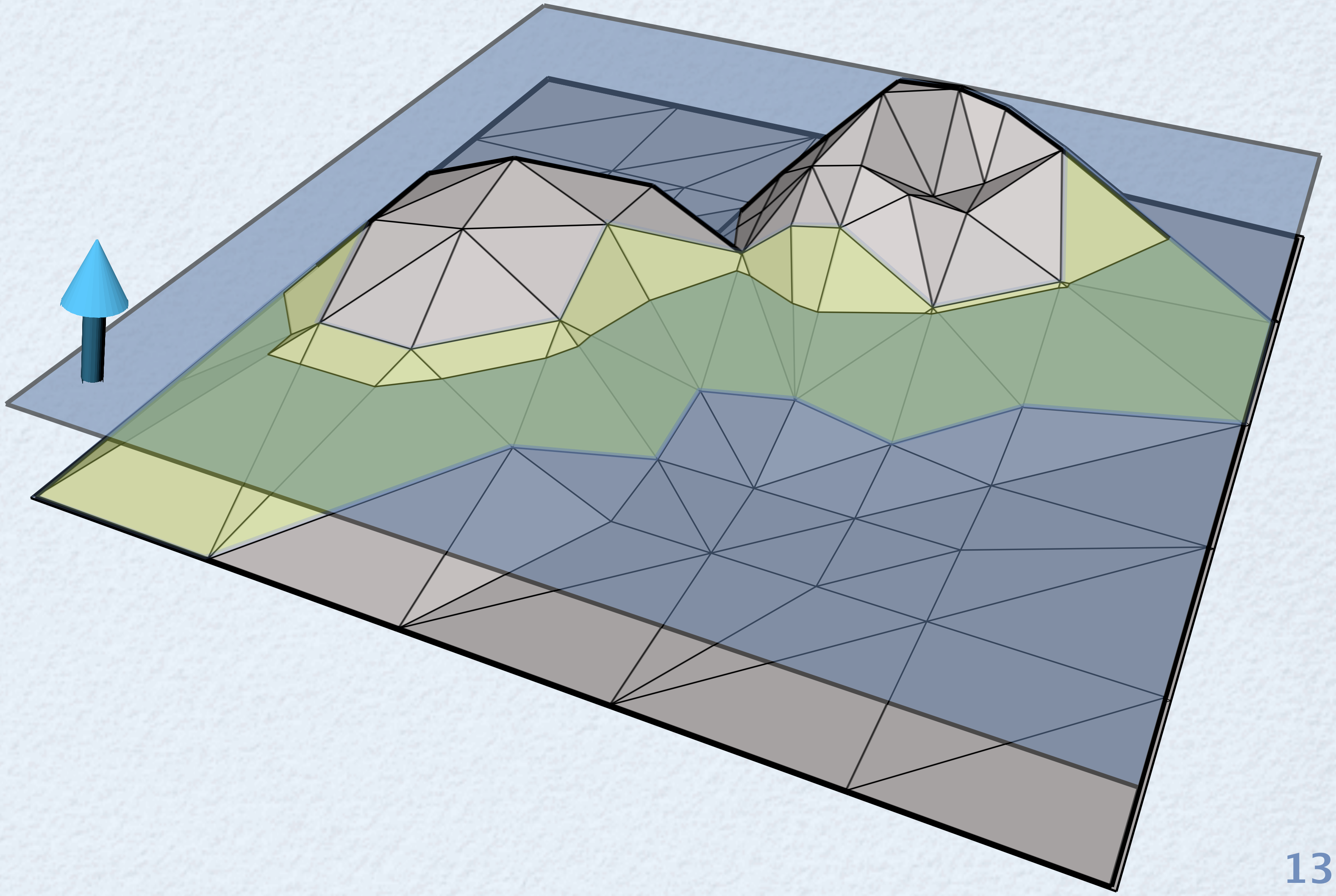

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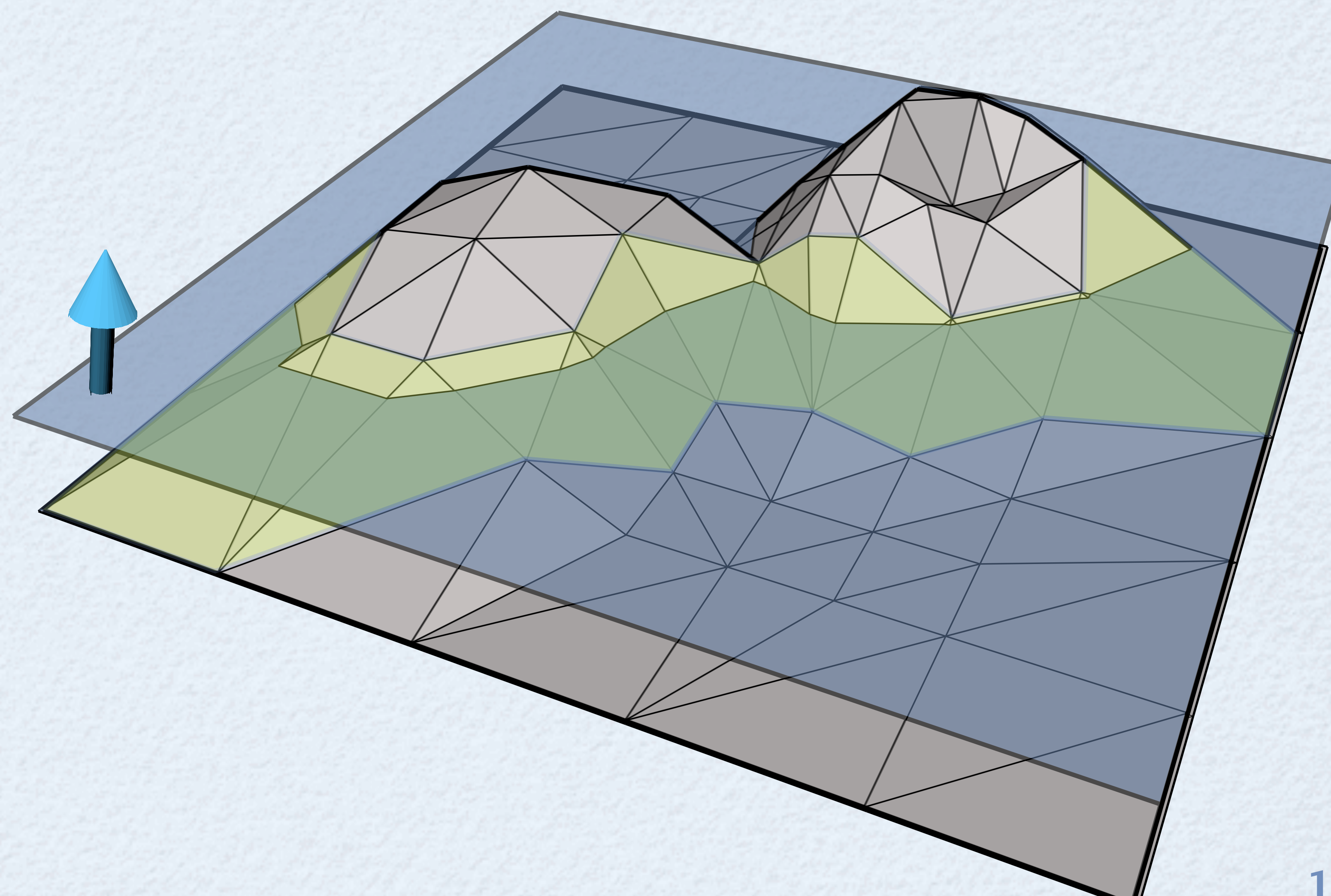

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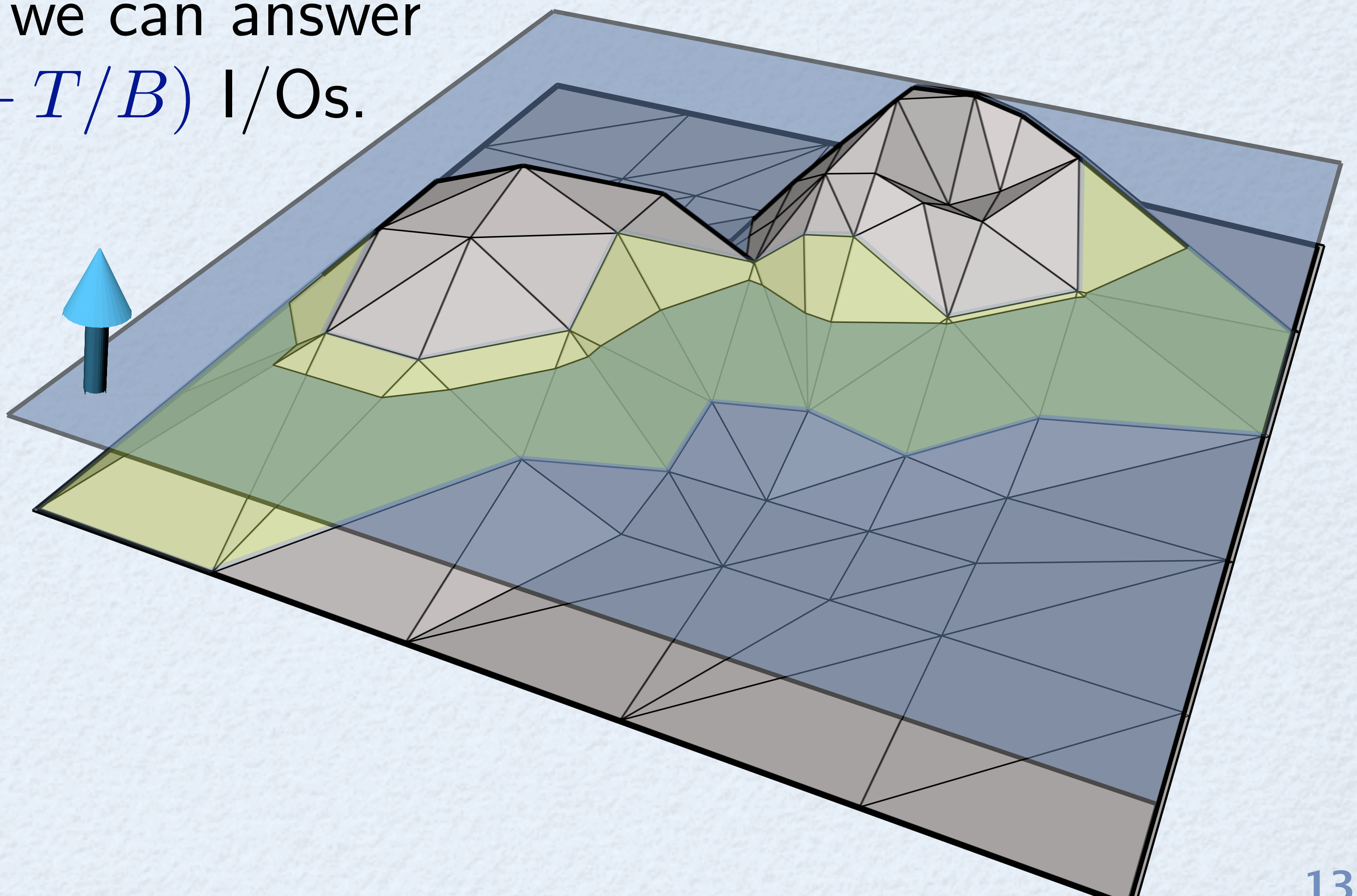

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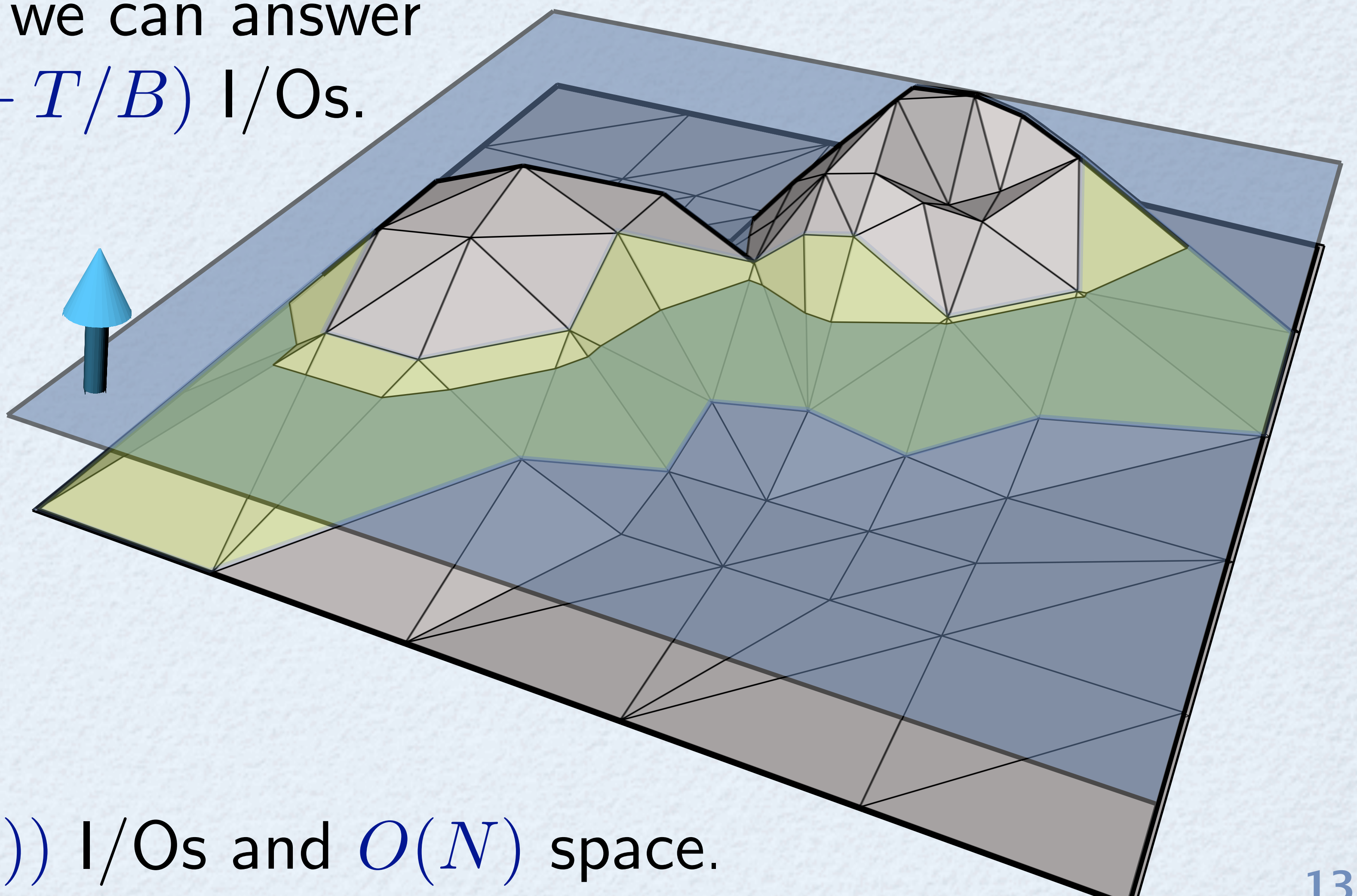
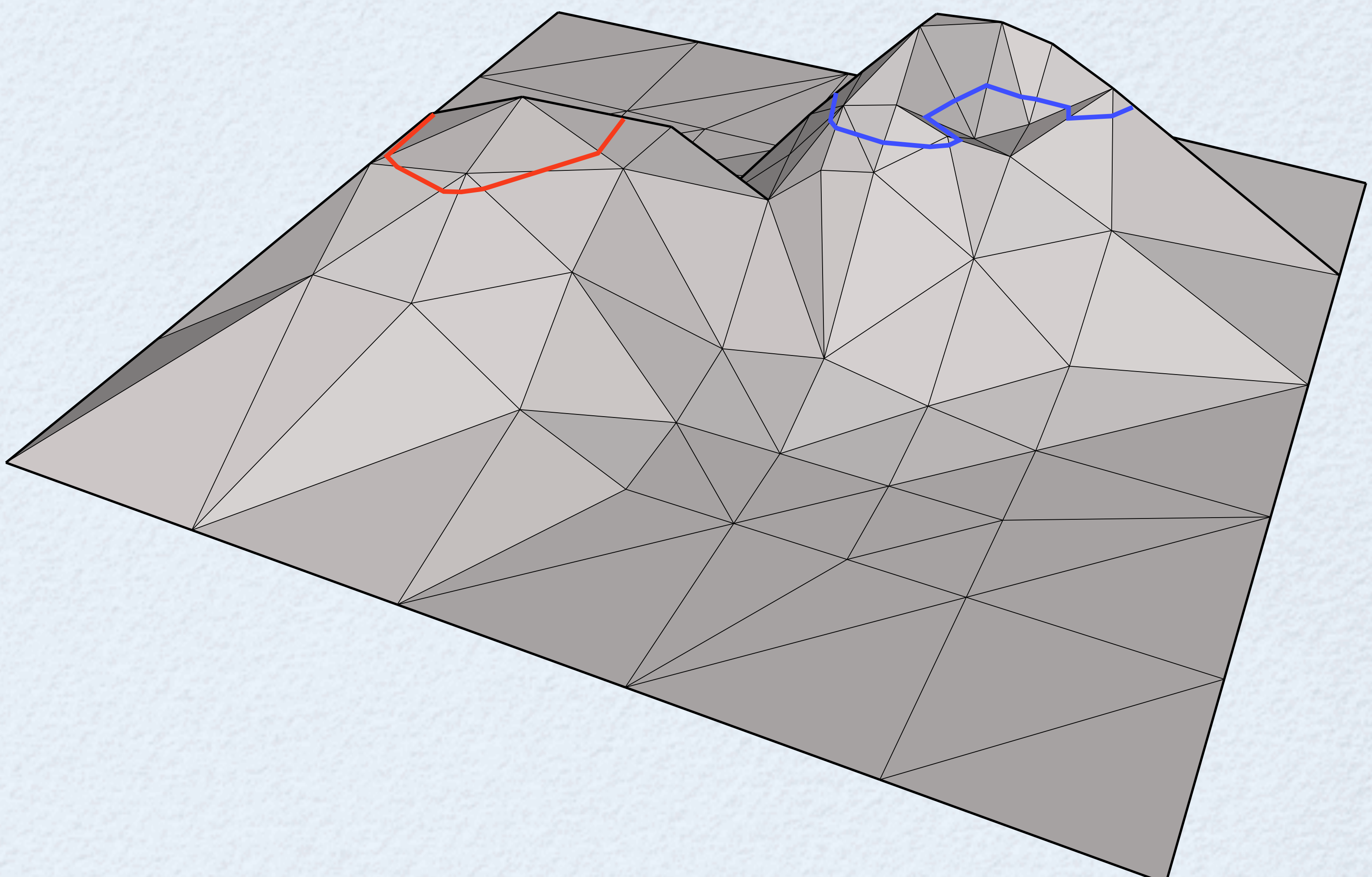


Preprocessing needs $O(\text{Sort}(N))$ I/Os and $O(N)$ space.

Level Ordering Theorem

Theorem. One can find in $O(\mathrm{Sort}(N))$ I/Os a total ordering "$\prec$" of triangles of $\mathbb{M}$, s.t. for any ℓ:

1. Triangles in Δ_ℓ are $\prec$-sorted in cw or ccw order.



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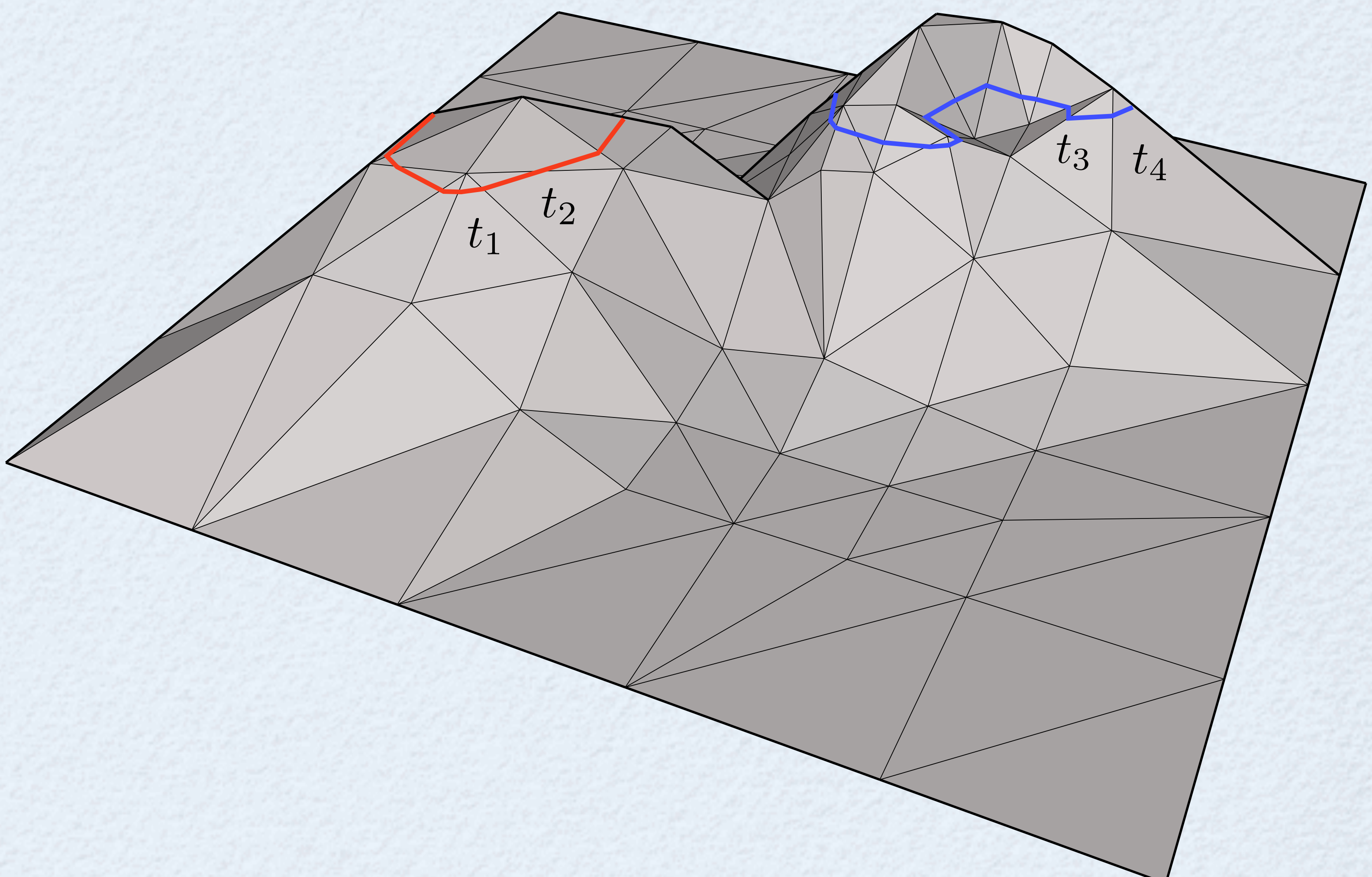
$$t_1 \prec t_3 \prec t_2 \quad \Longrightarrow \quad t_1 \prec t_4 \prec t_2$$



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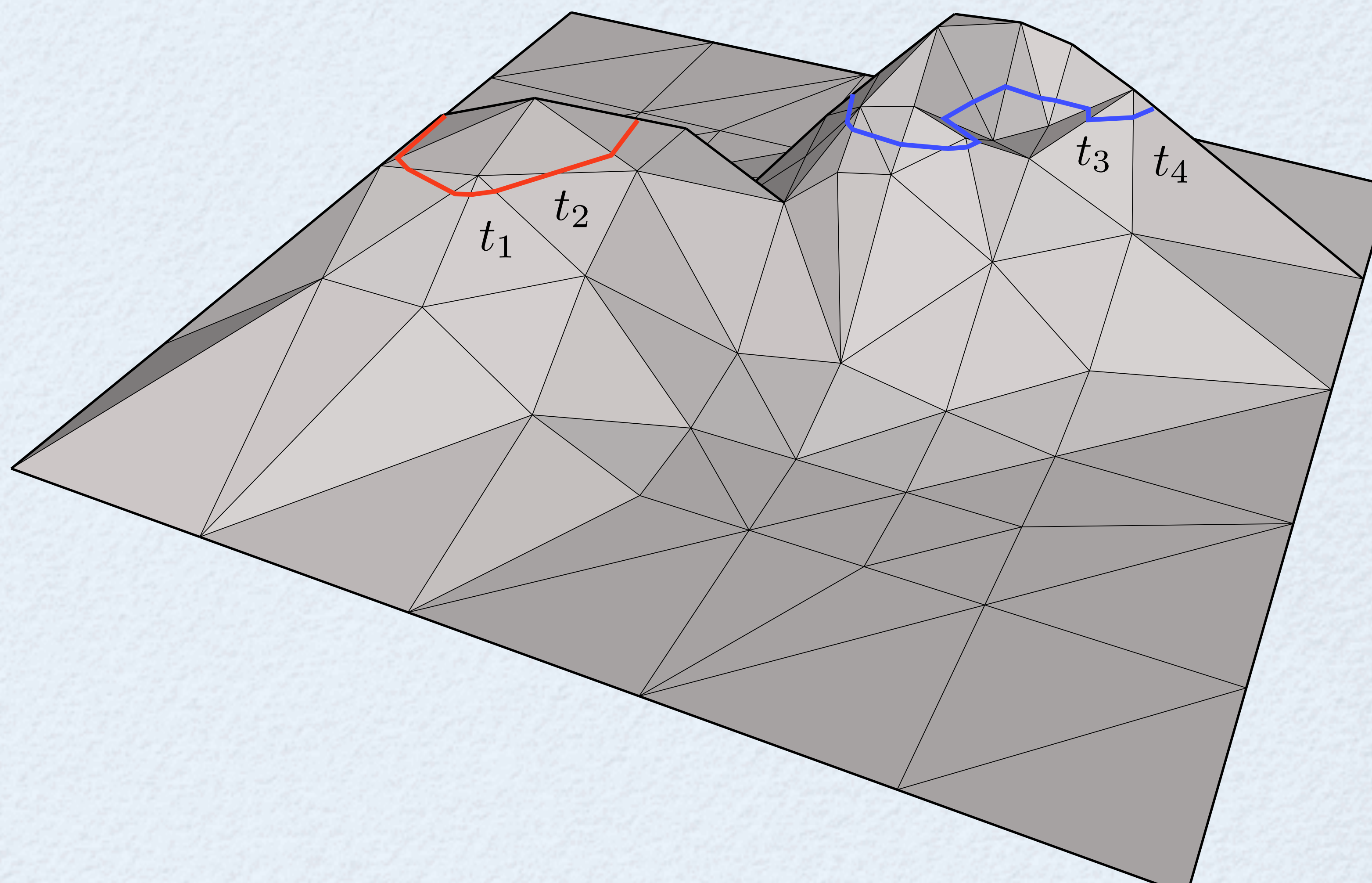

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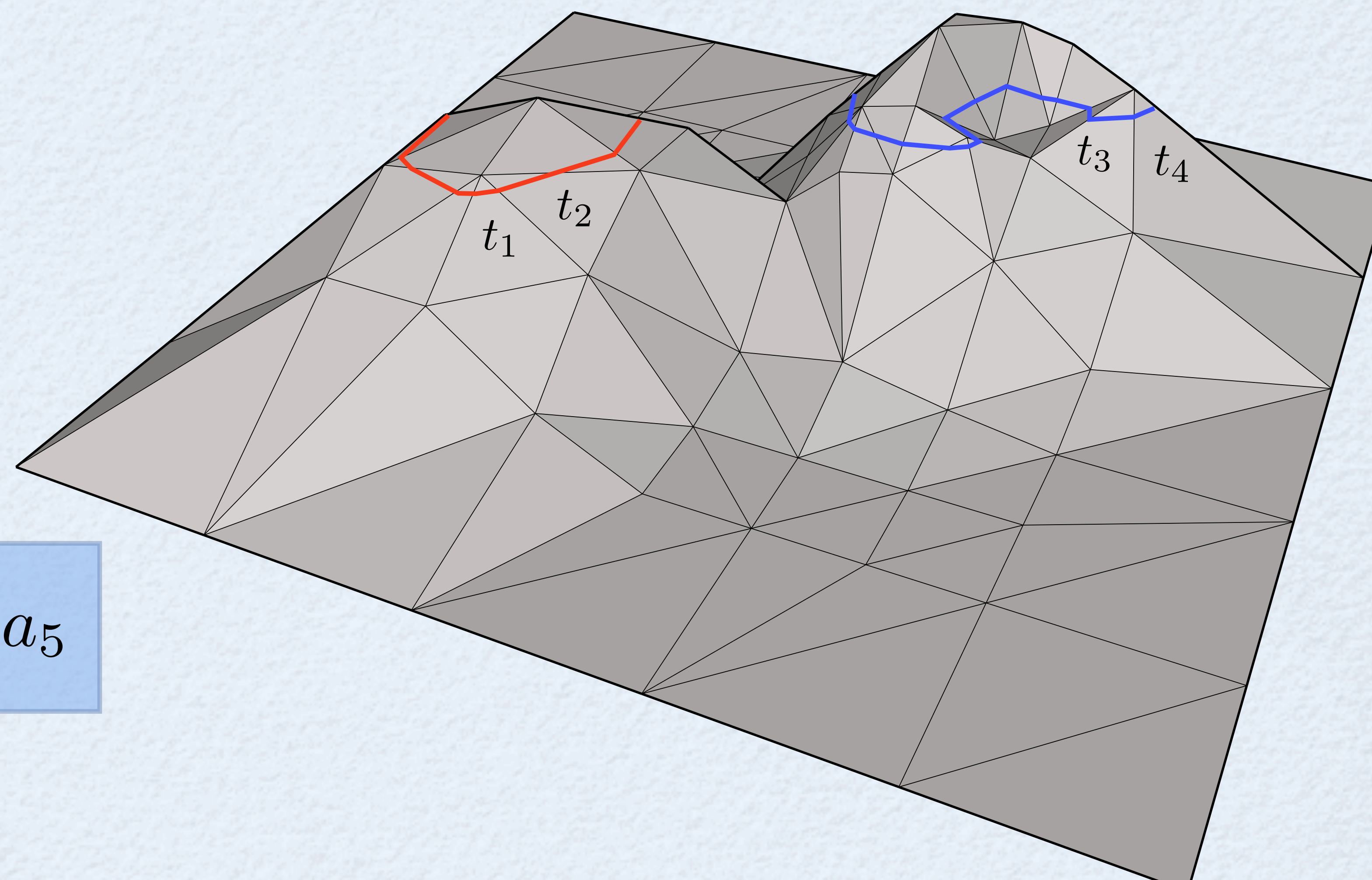

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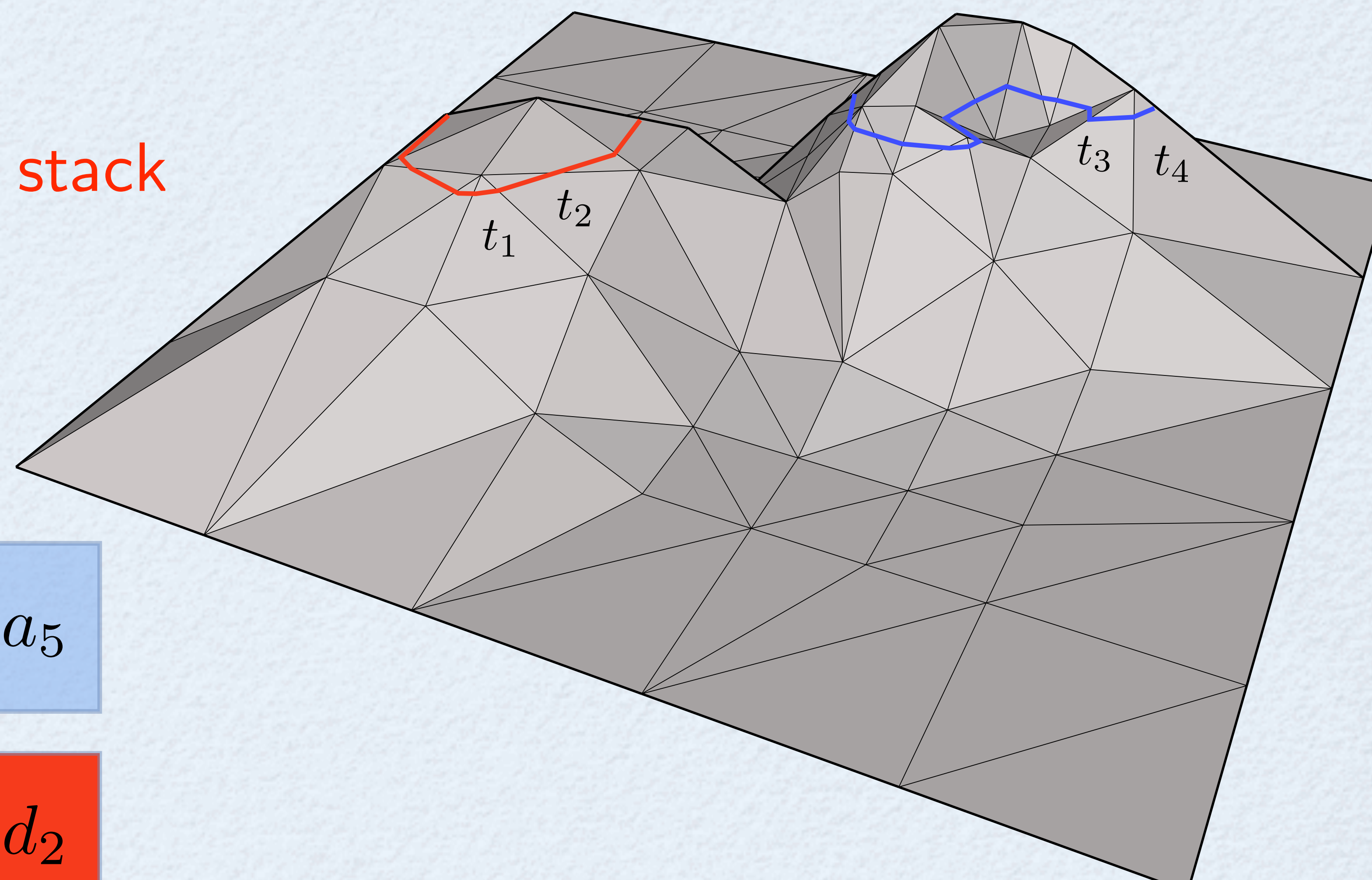
Can separate contours using a stack in $O(T/B)$ I/Os.



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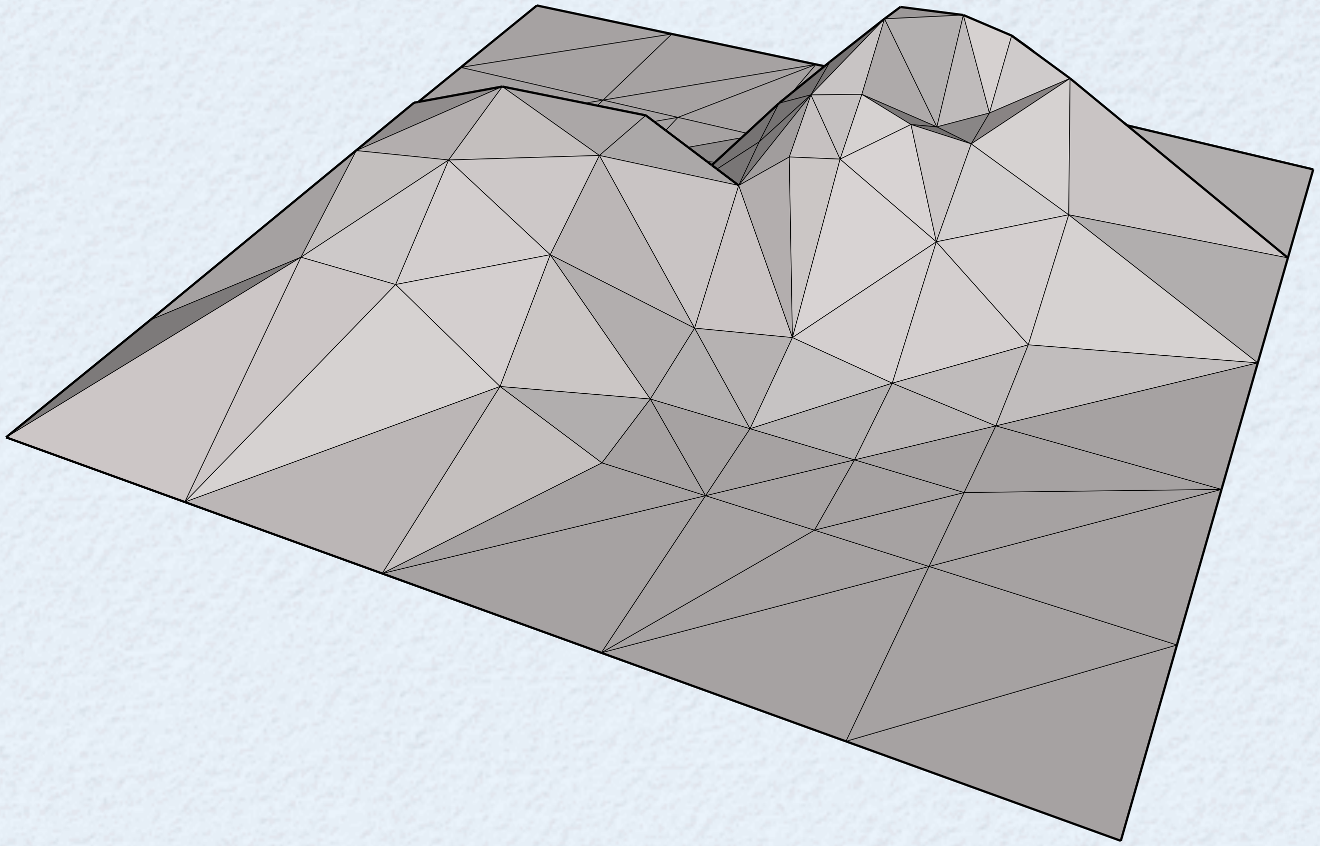
Level Ordering of Elementary Terrains

An elementary terrain has no saddles; thus 1 max and 1 min (boundary).



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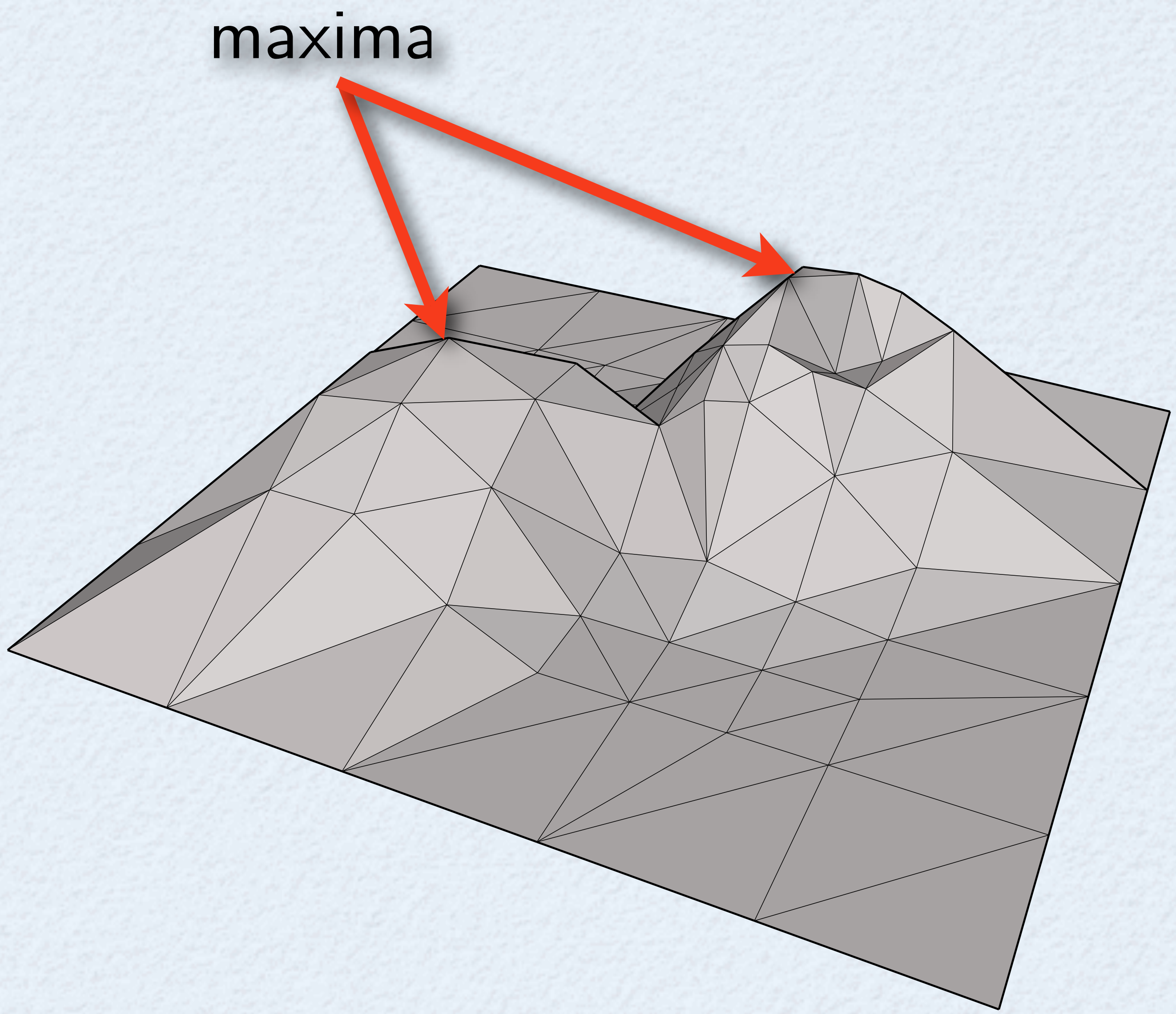

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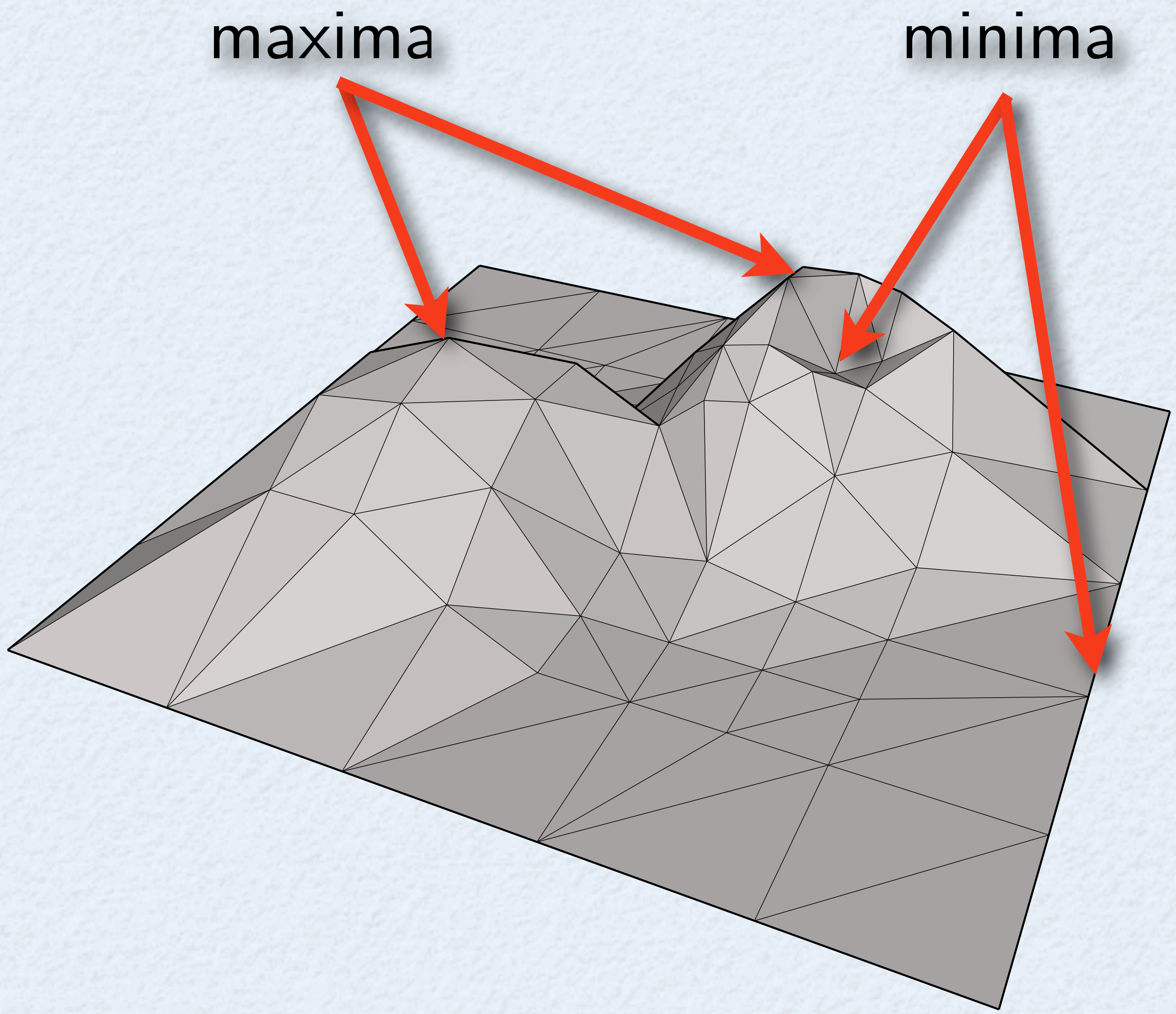

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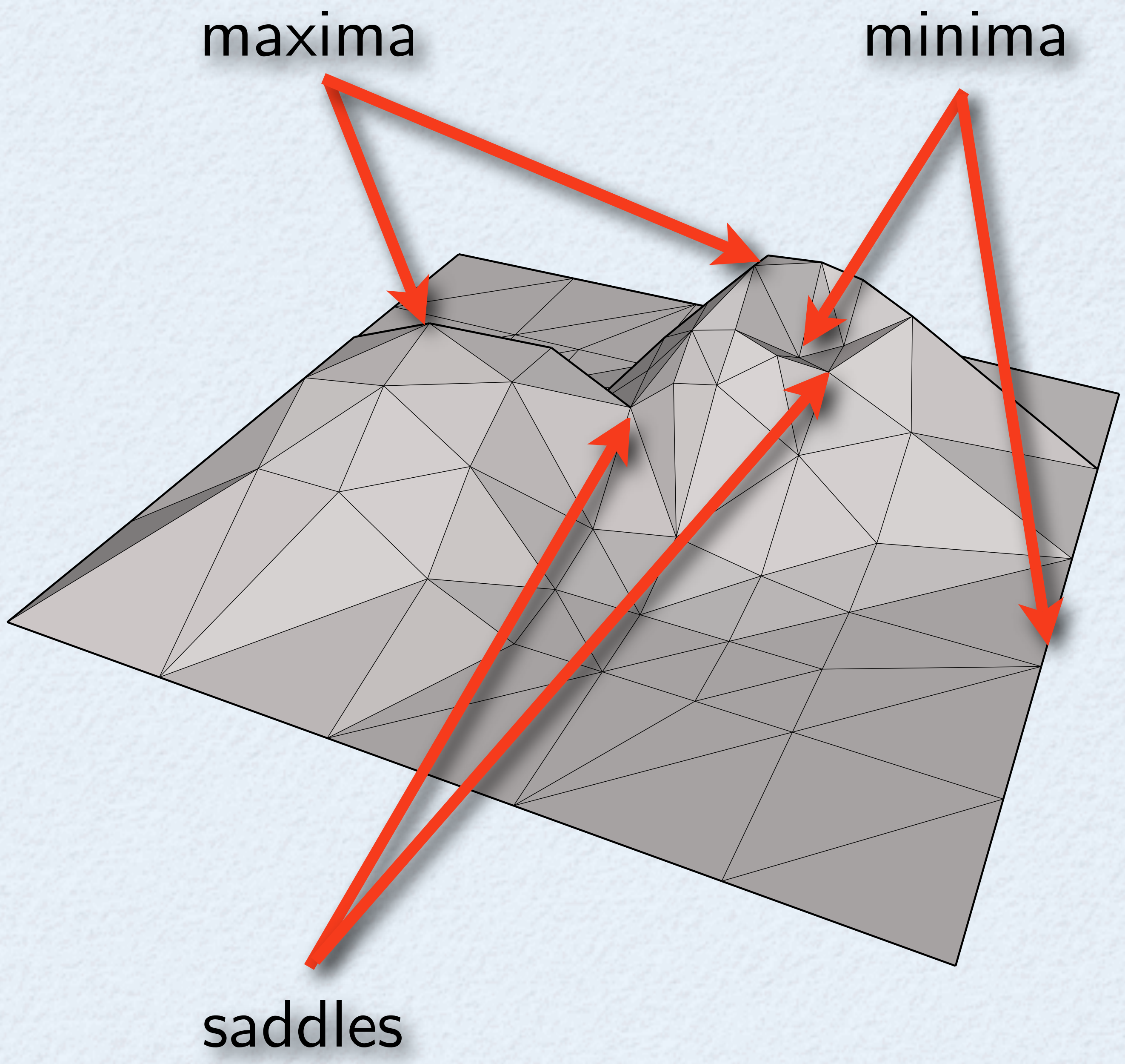

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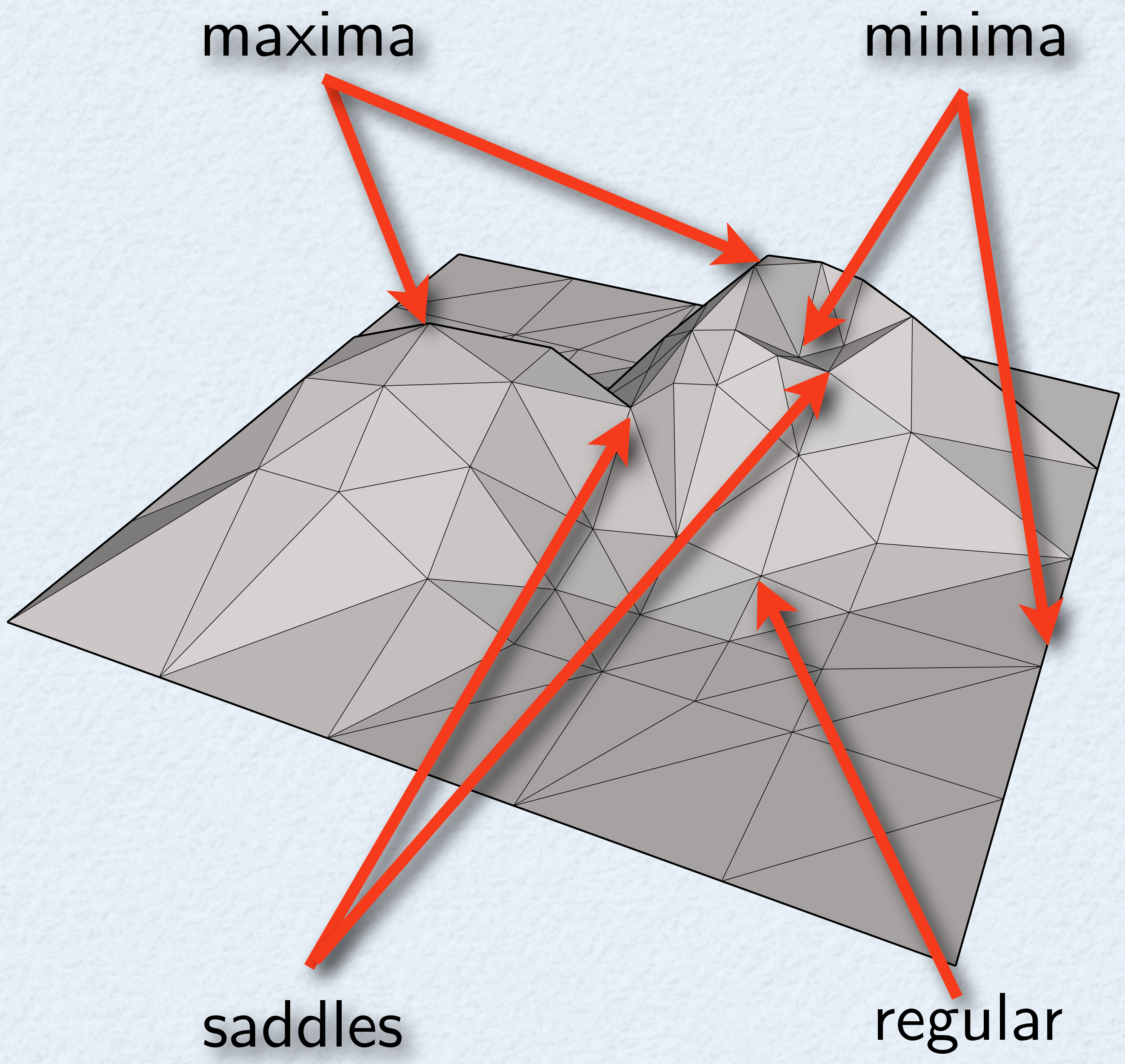

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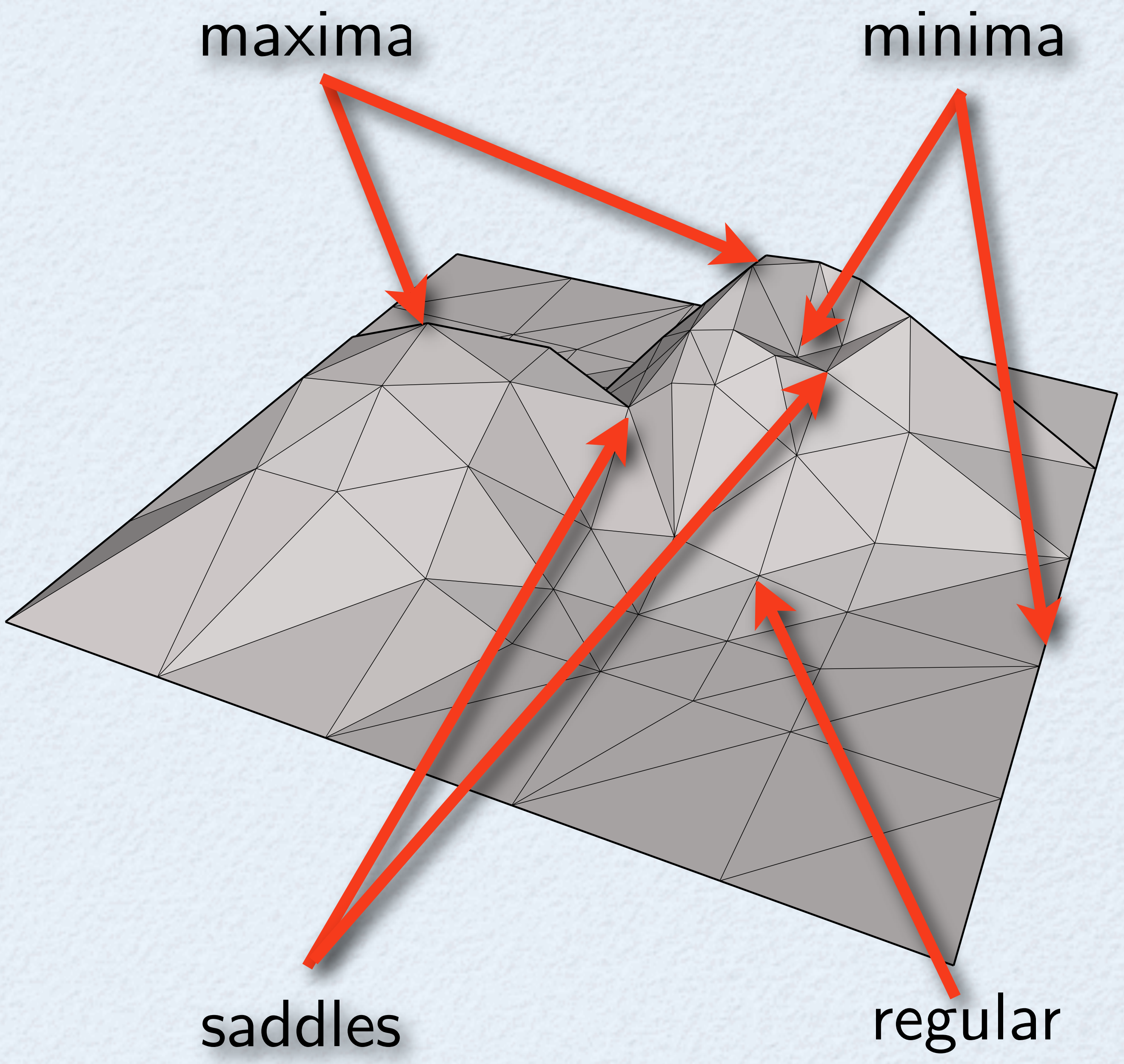

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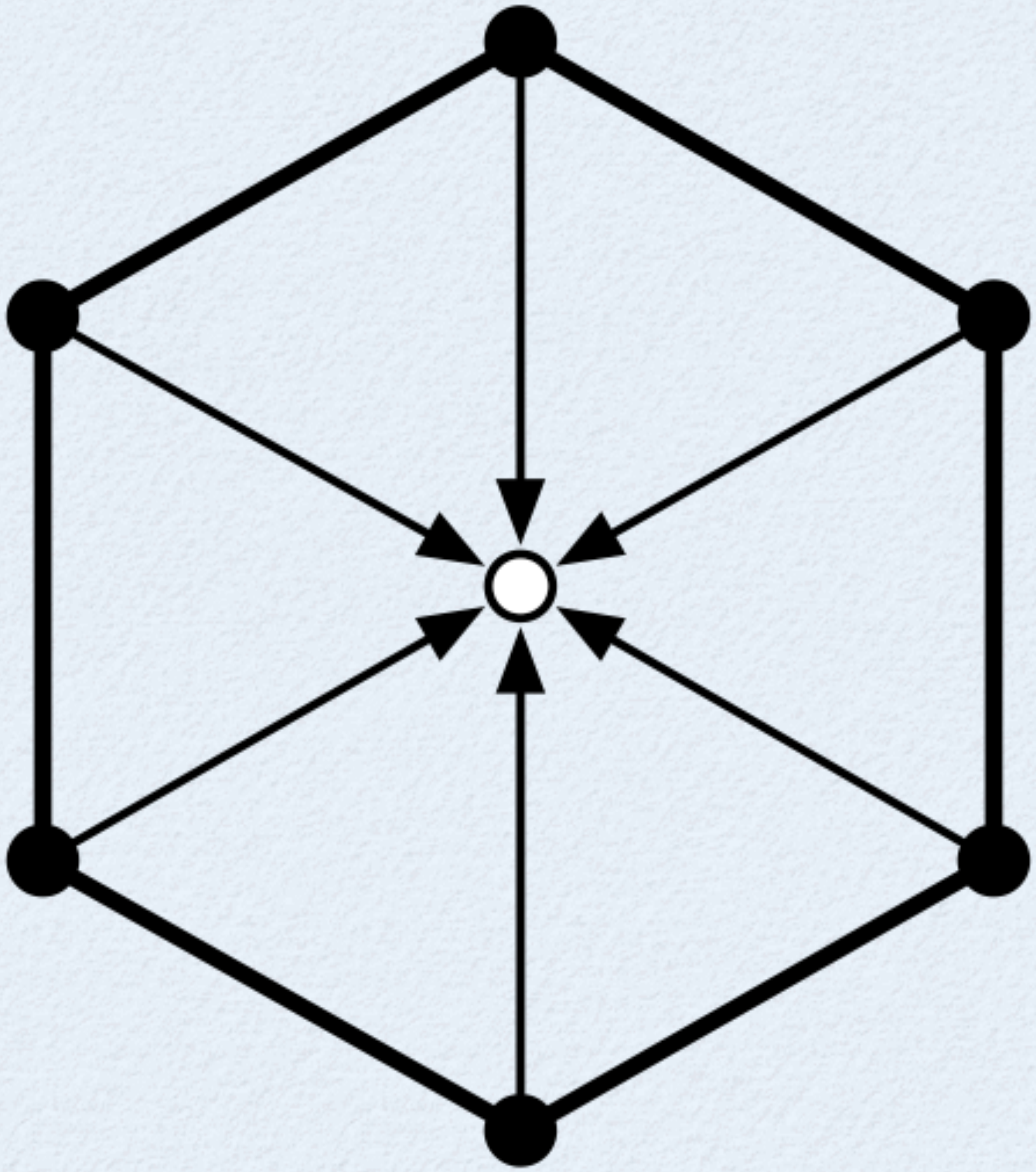

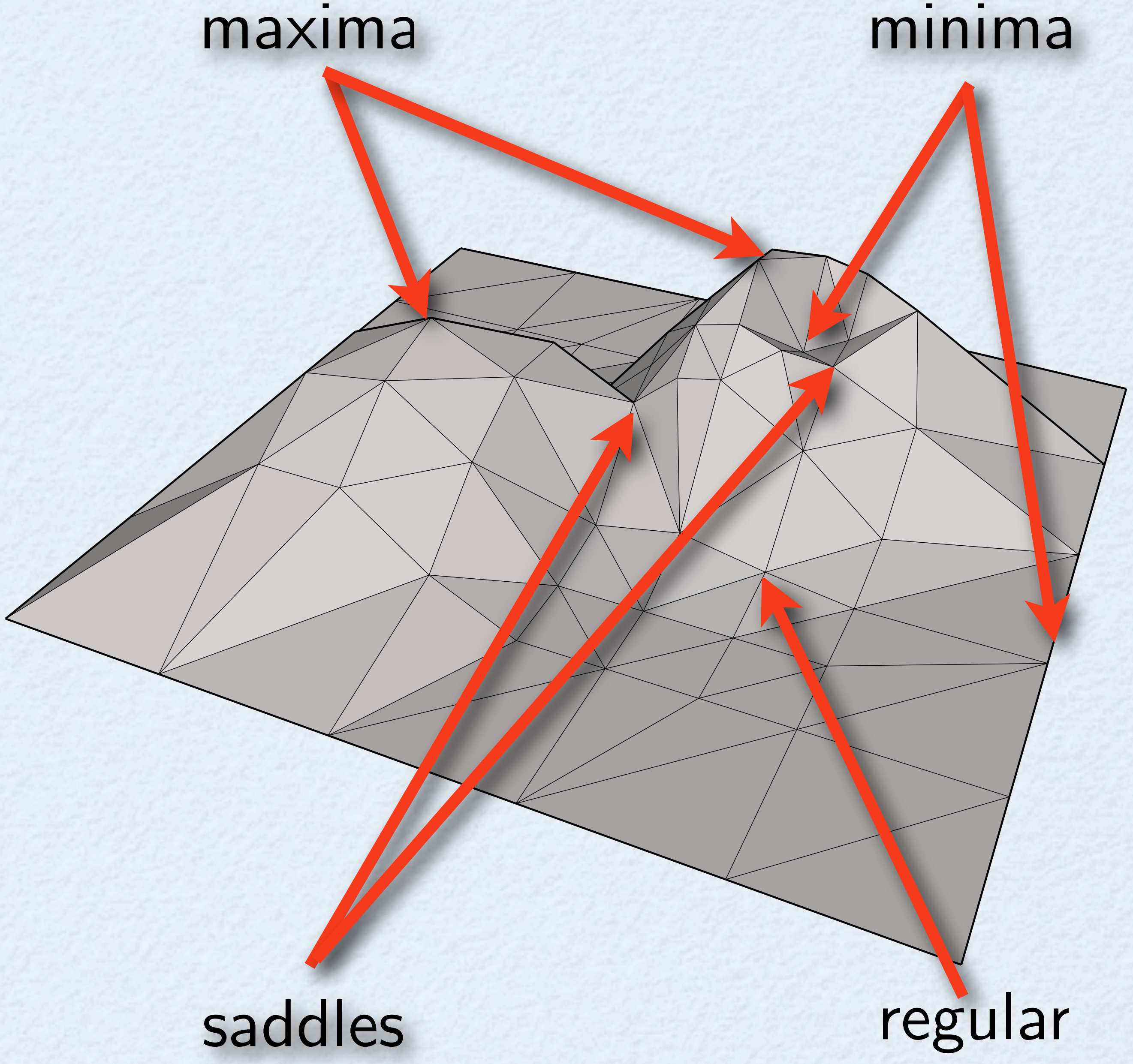

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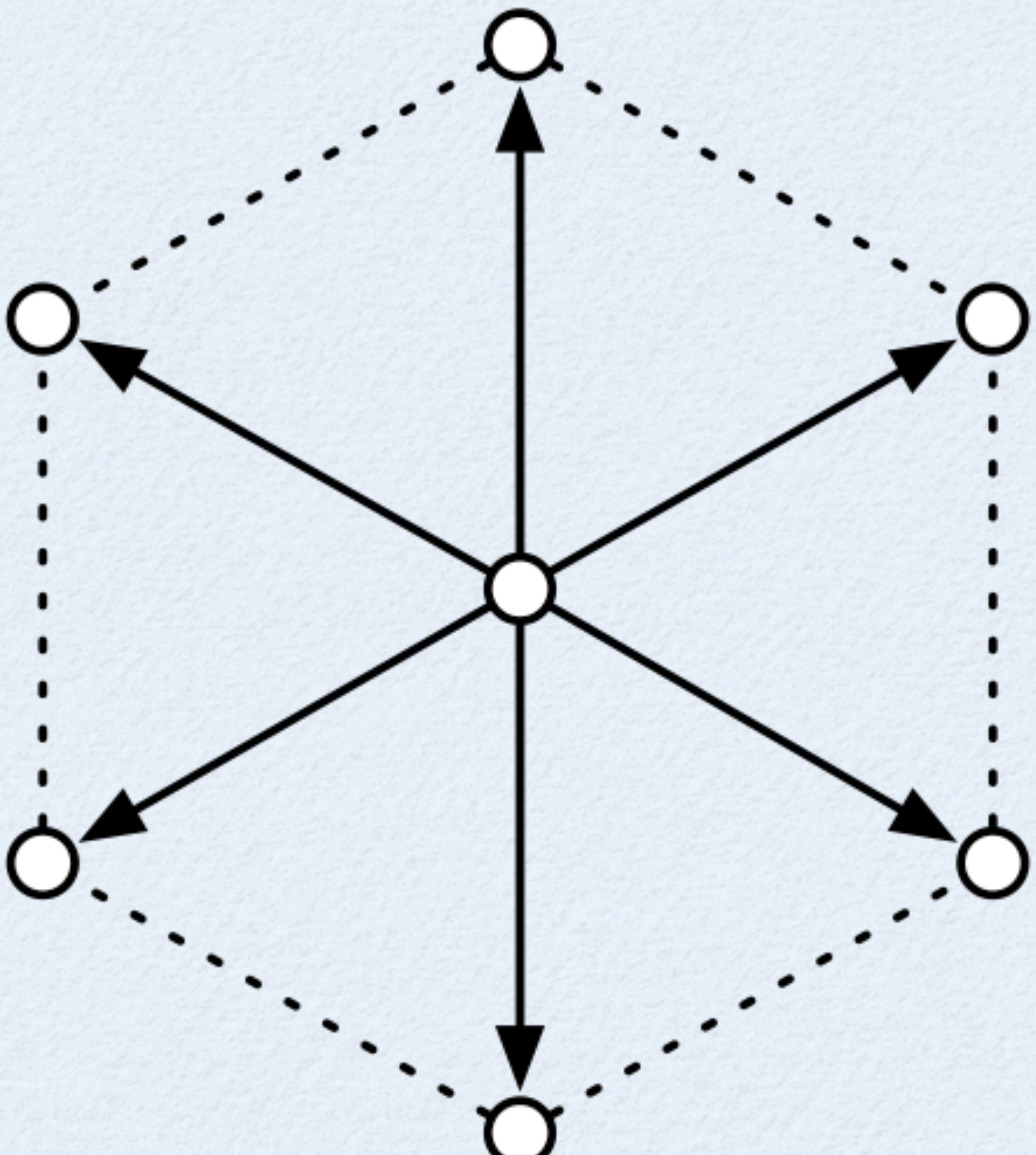

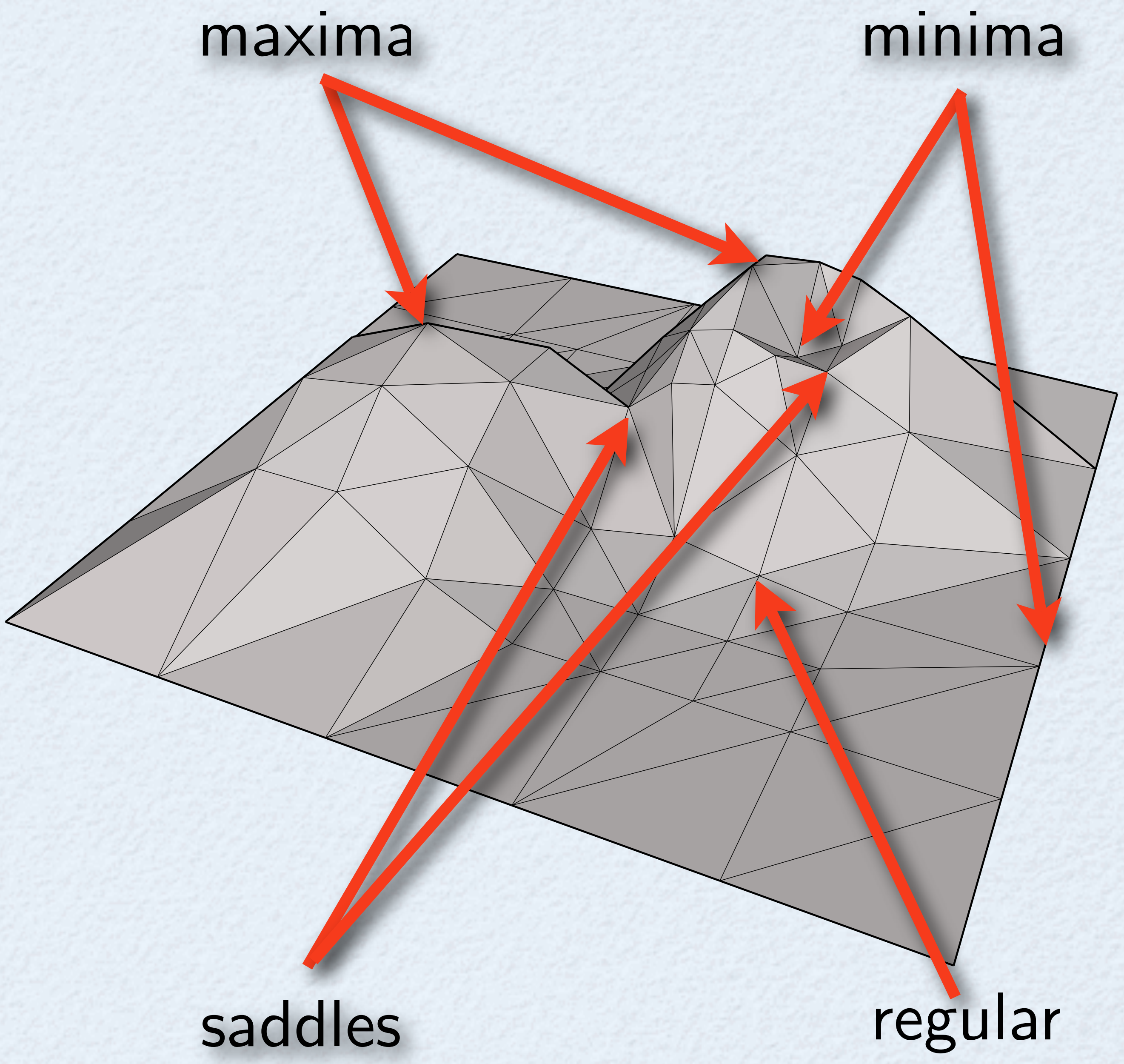

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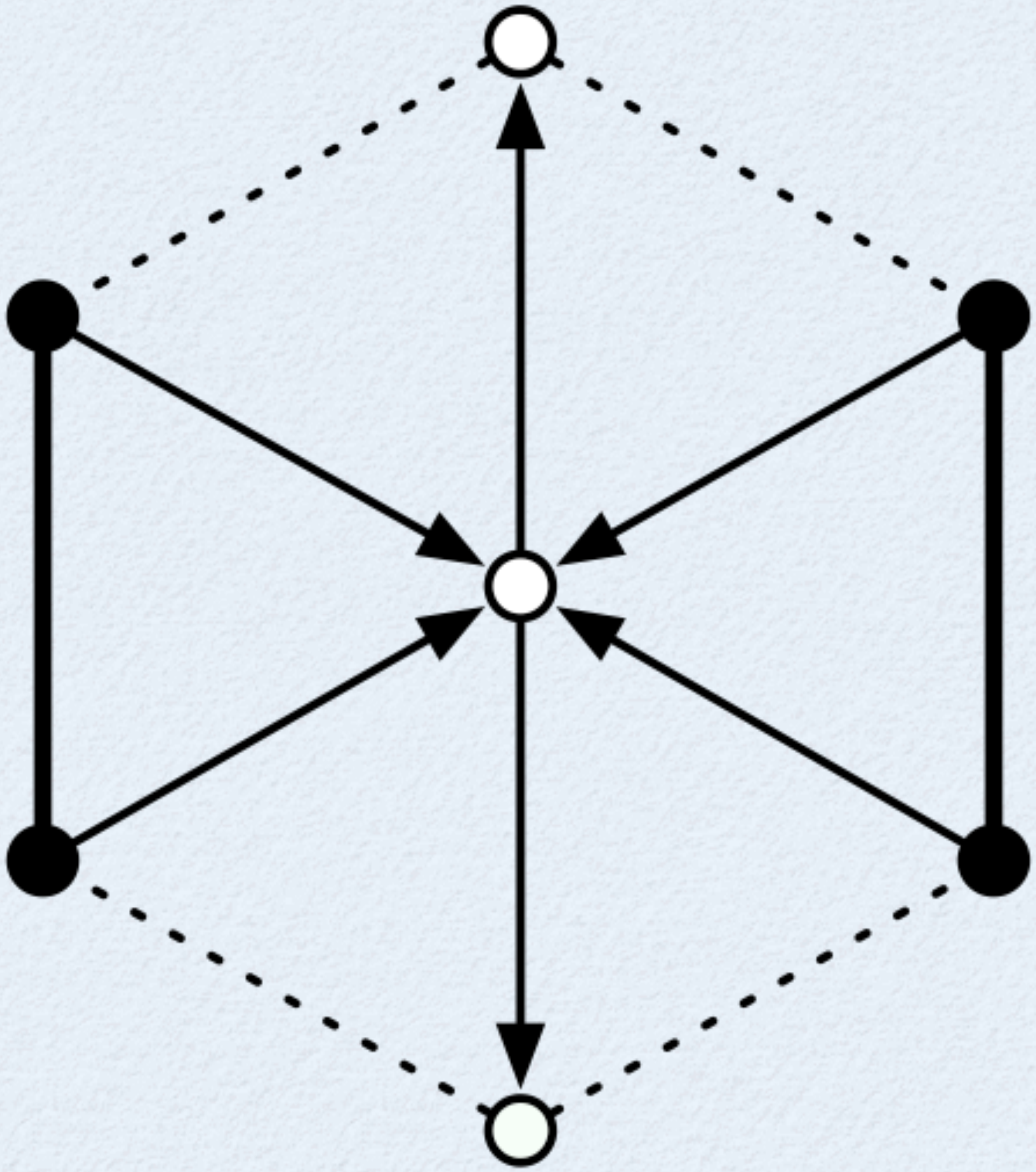

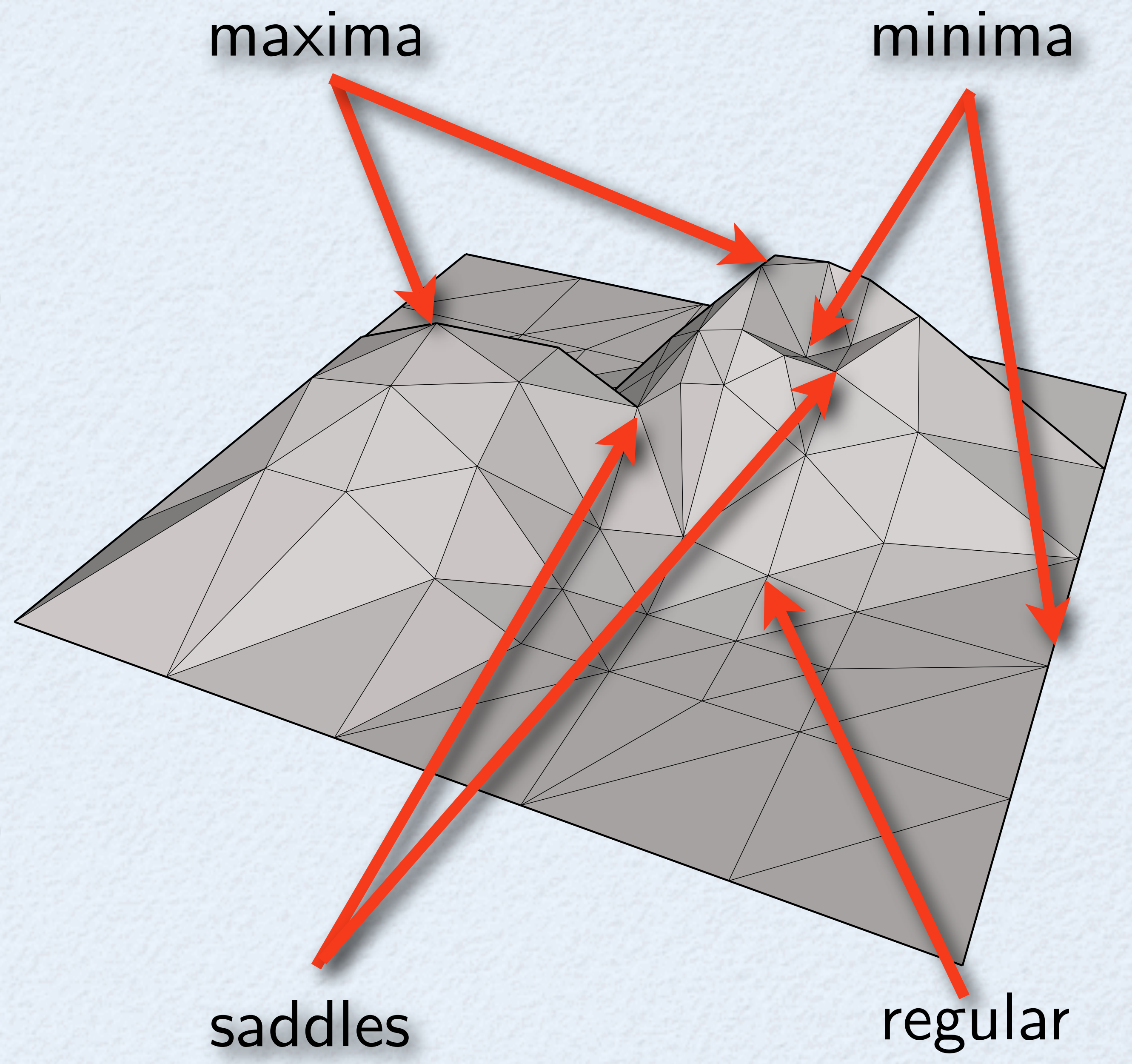

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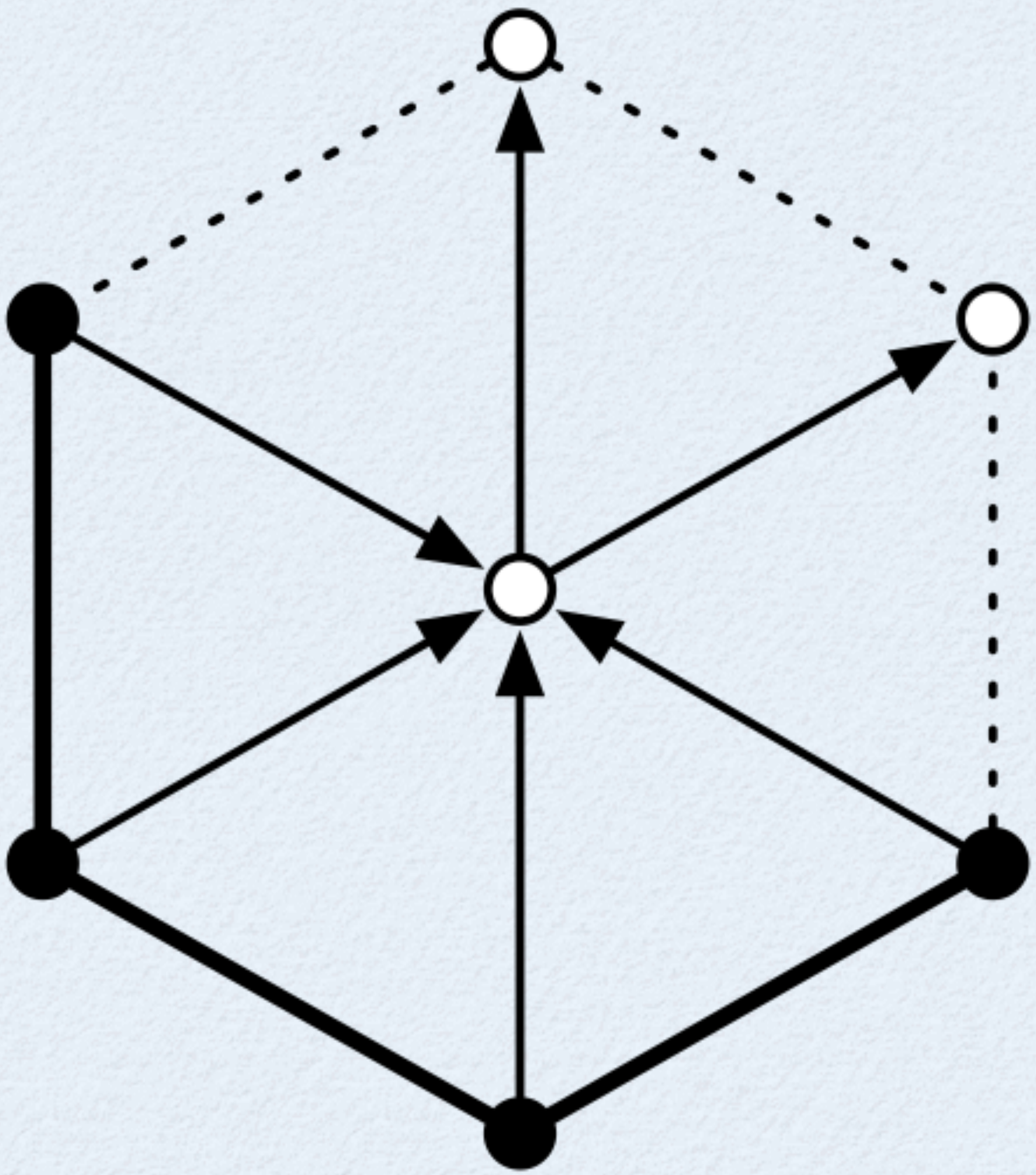

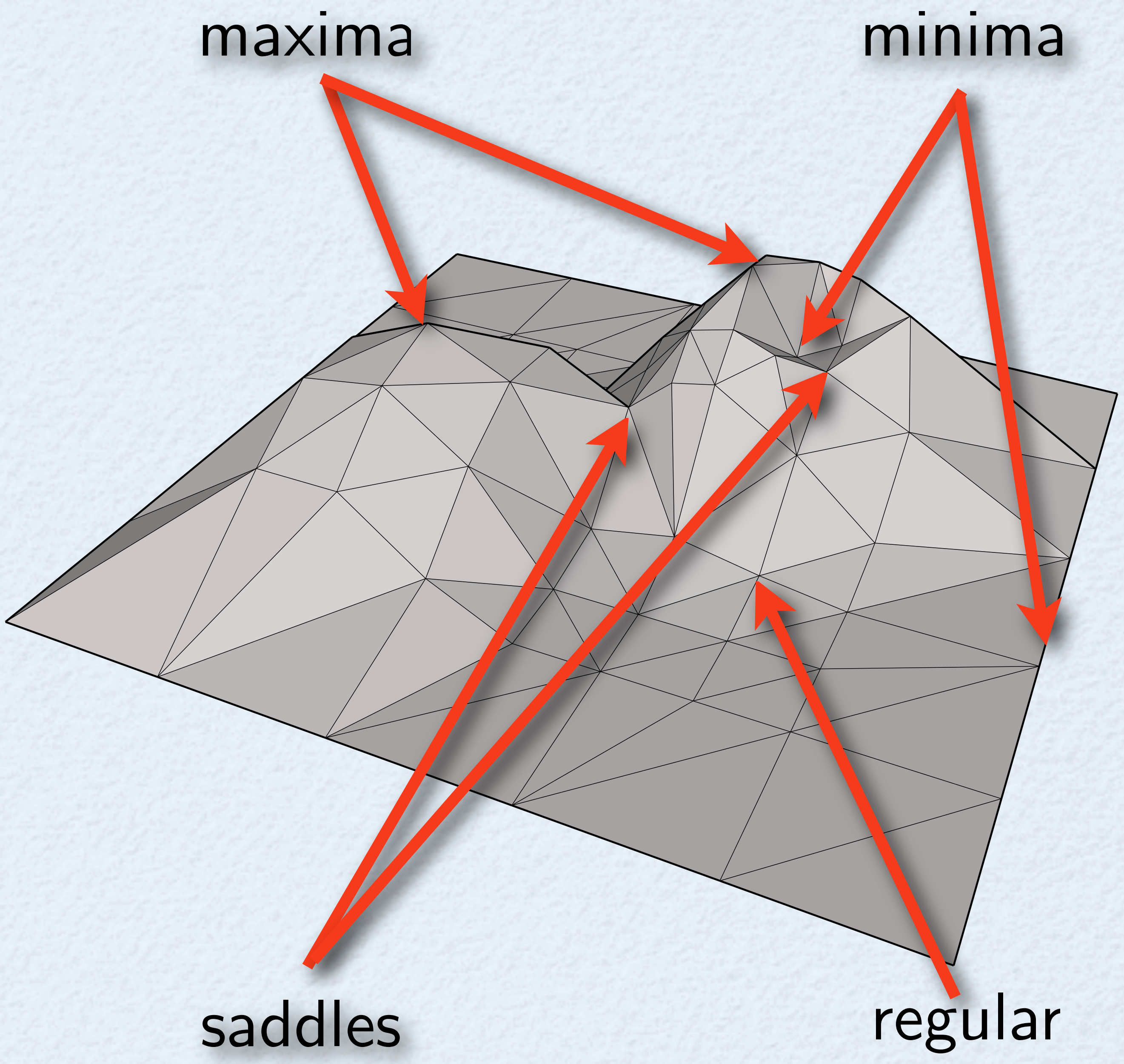

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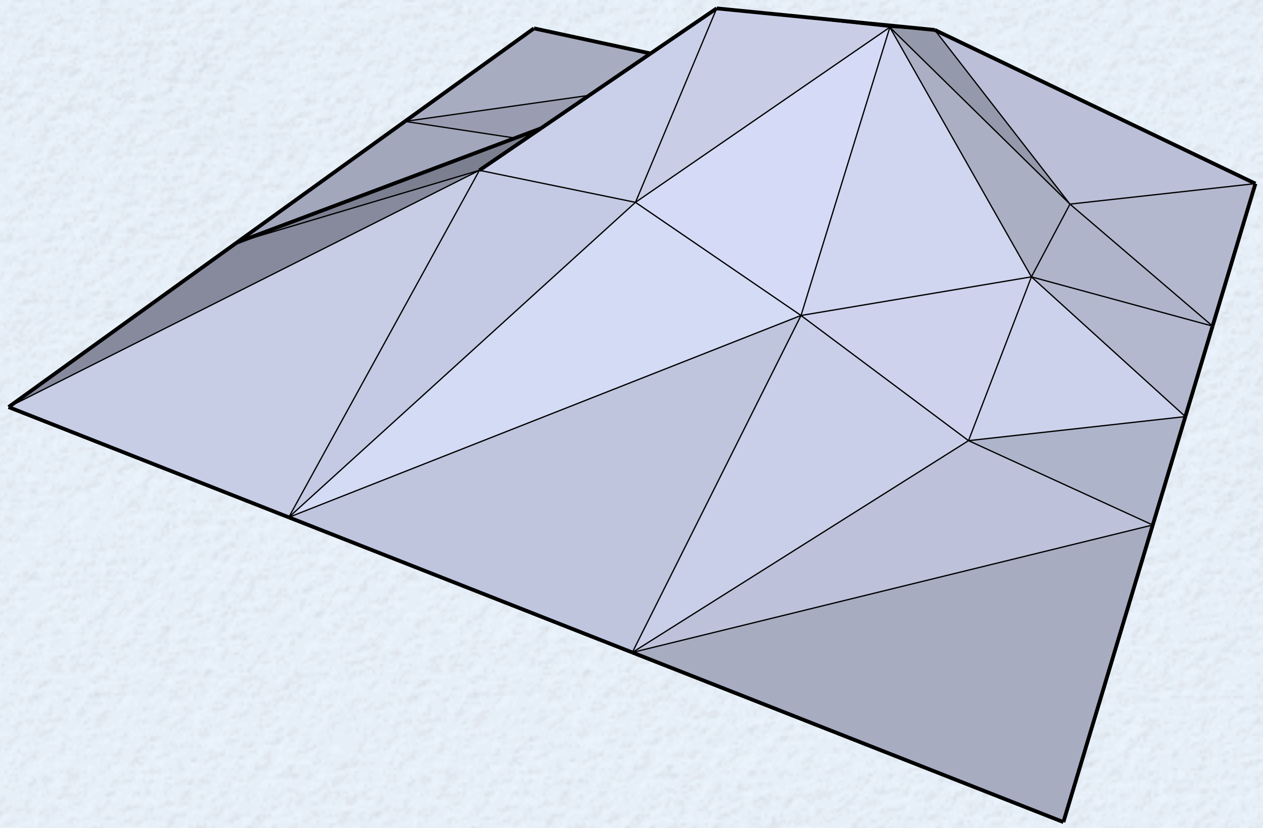

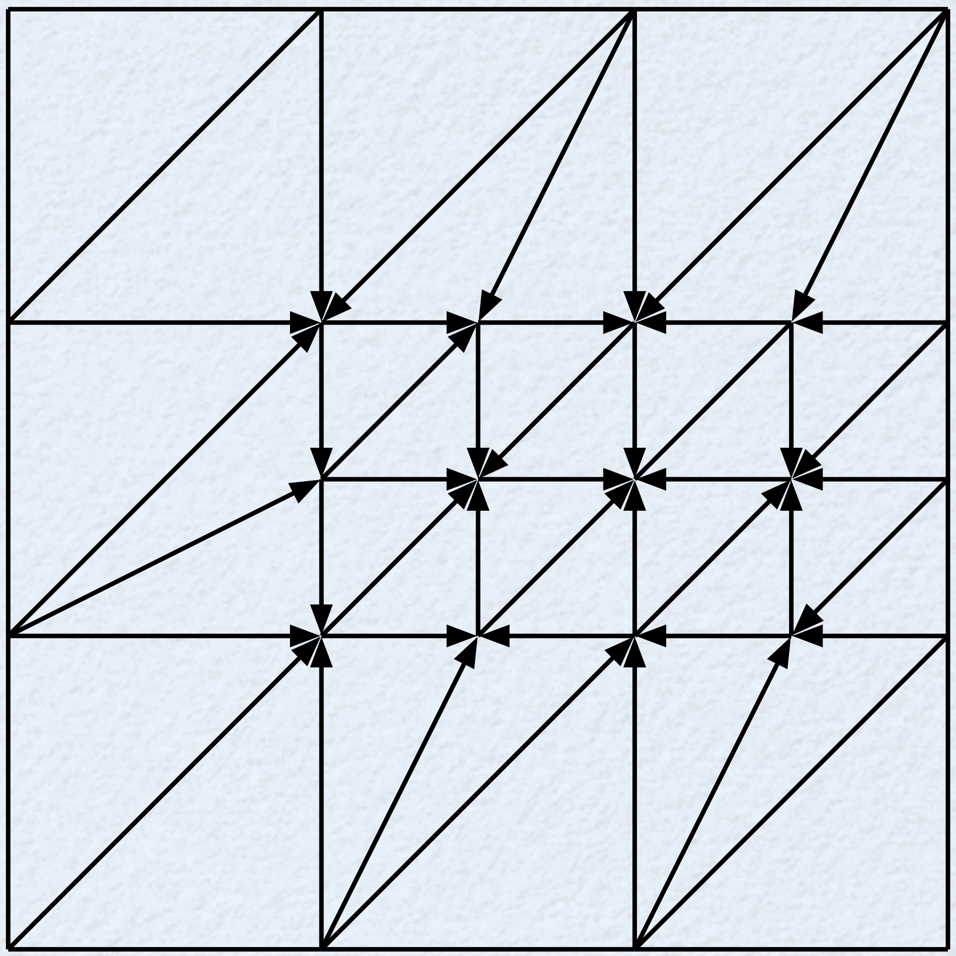

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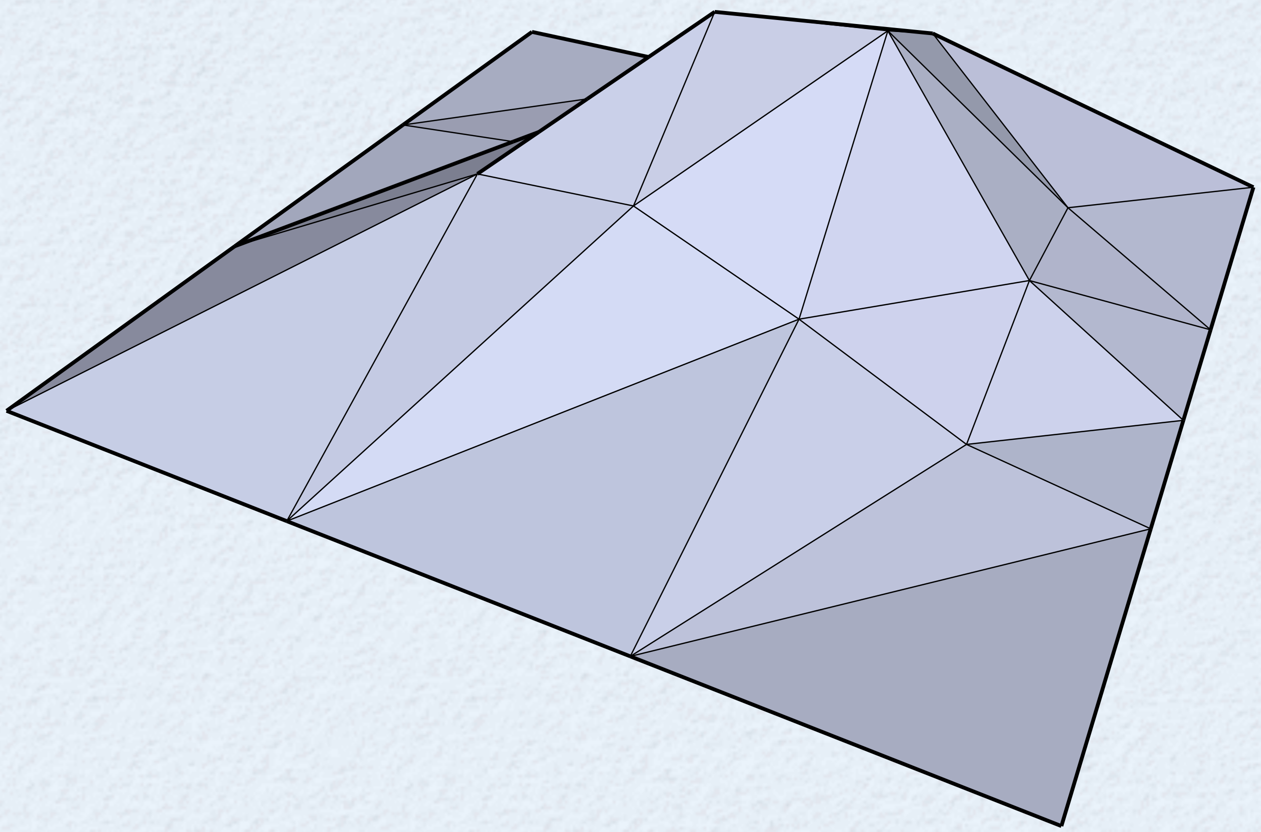

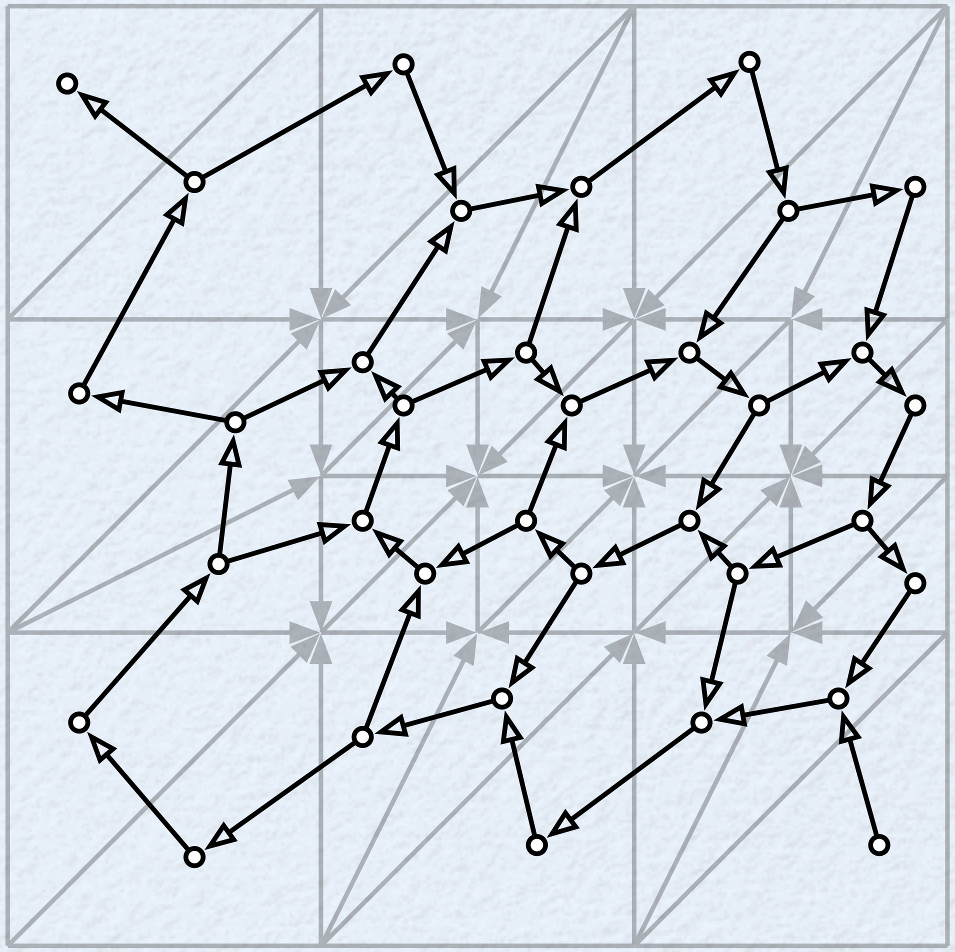

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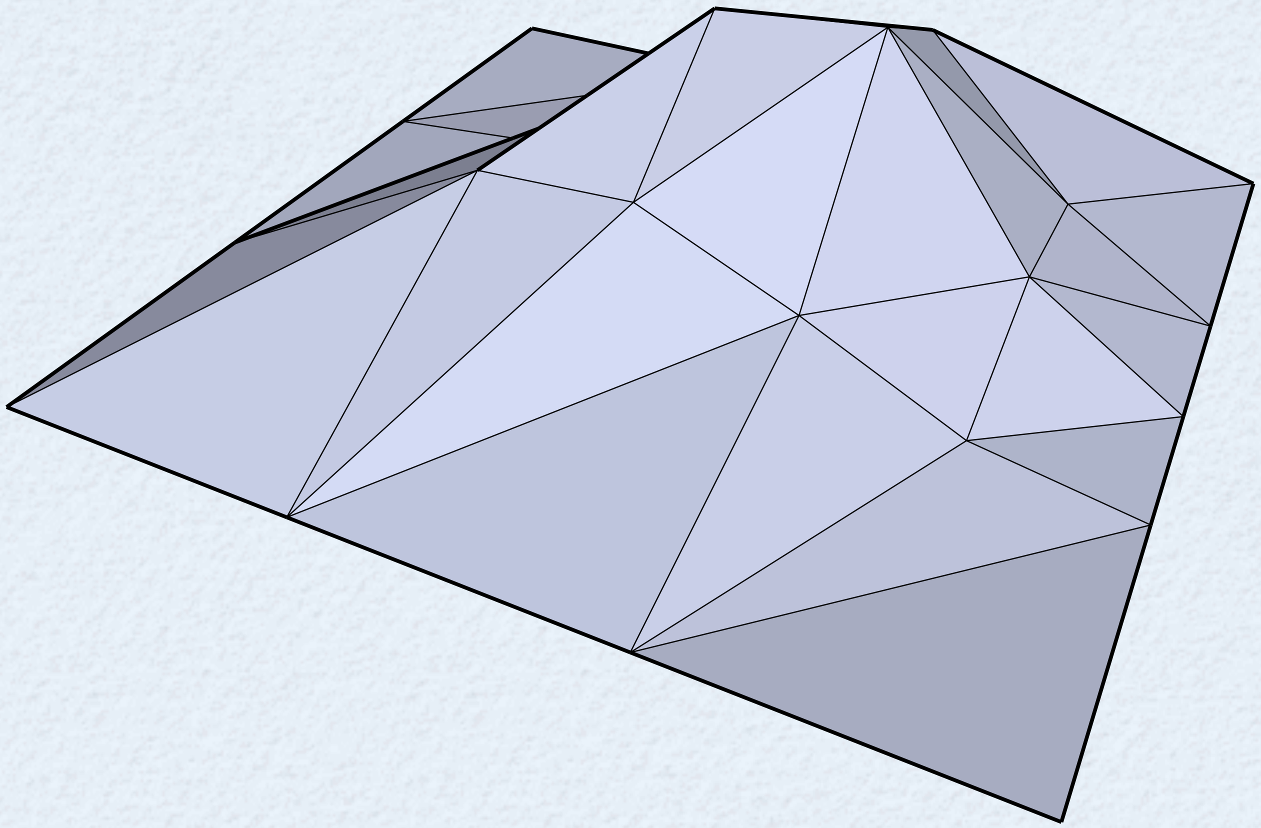

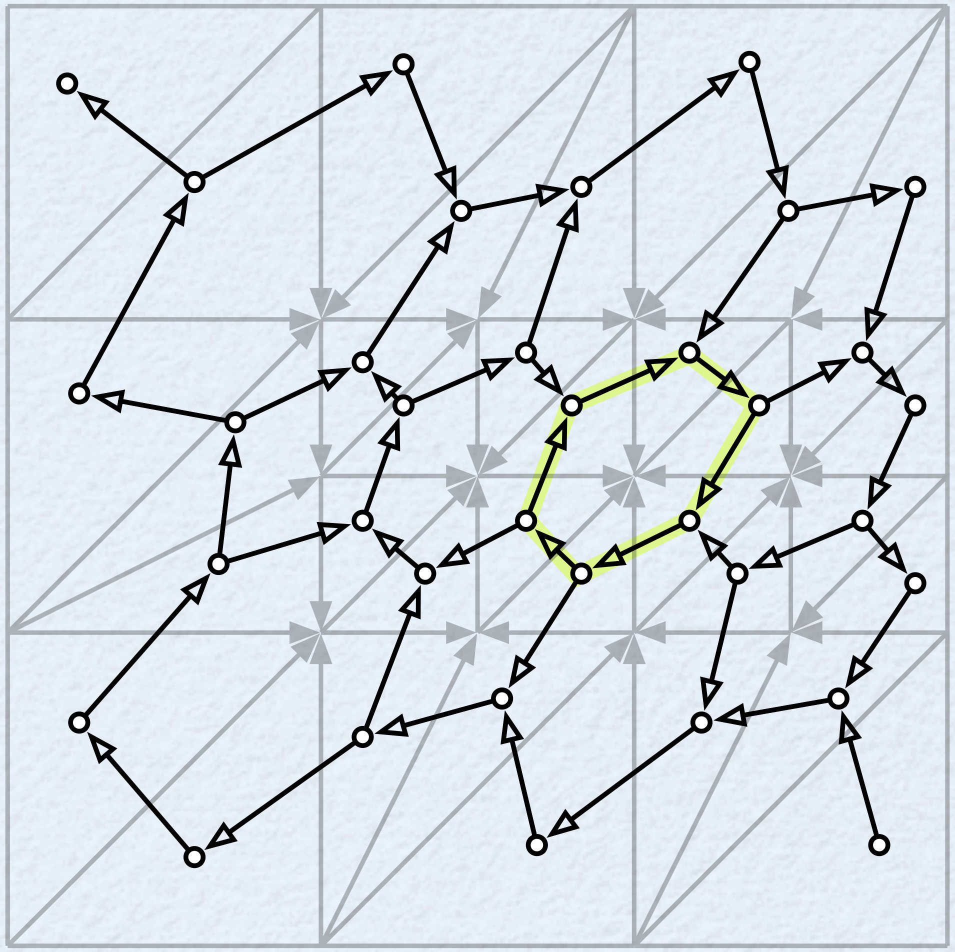

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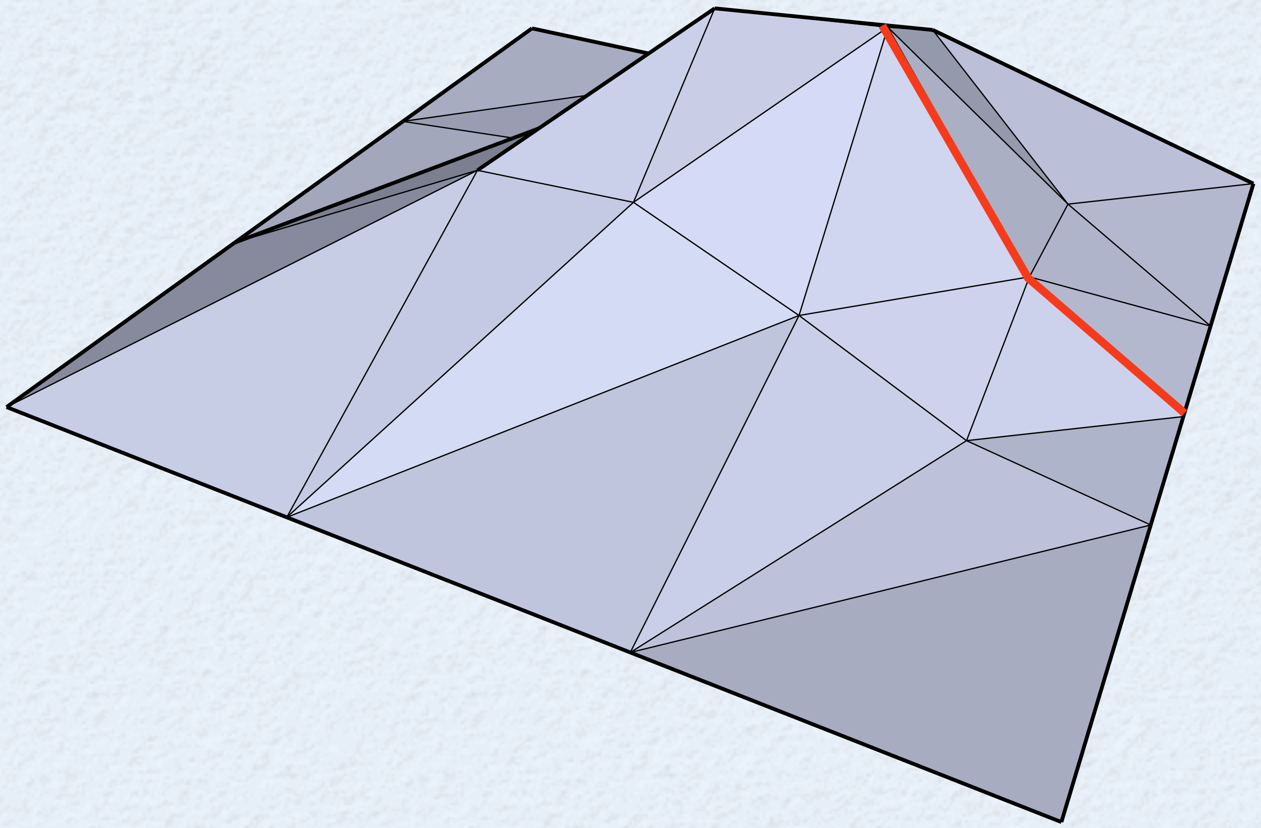
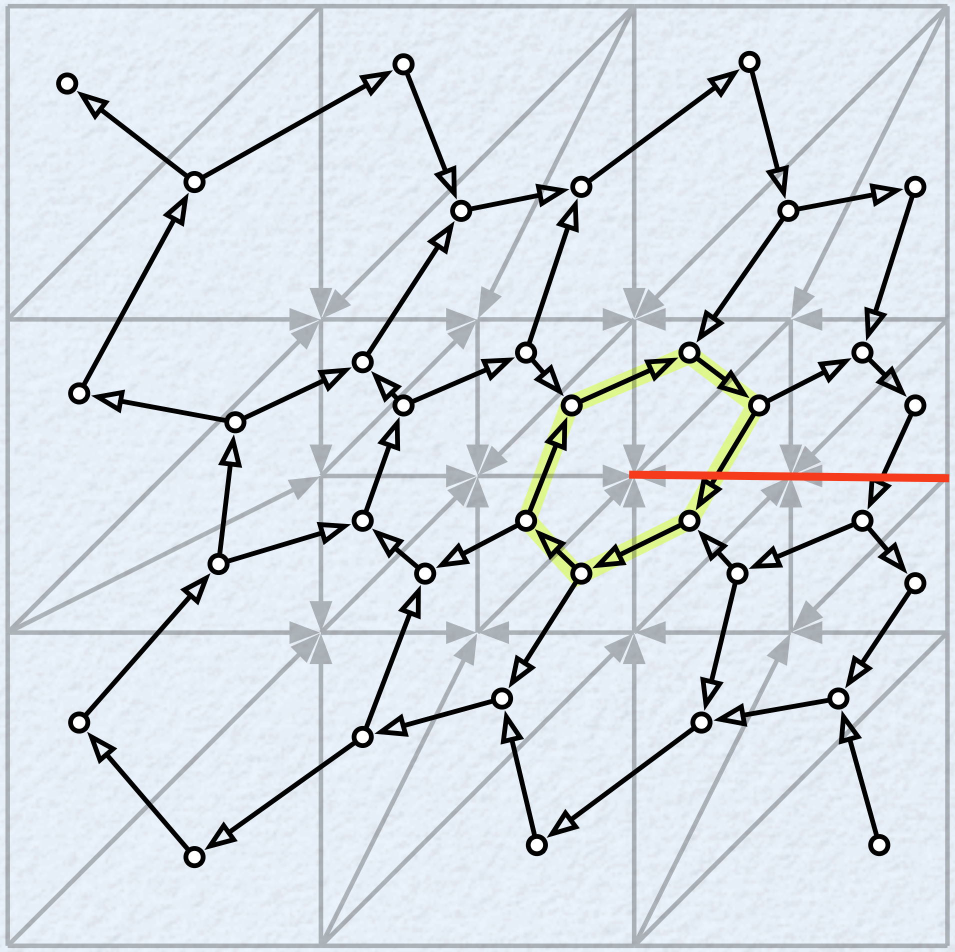
Take a monotone min (bd) to max path P and delete its dual from $\mathbb{M}^*$.





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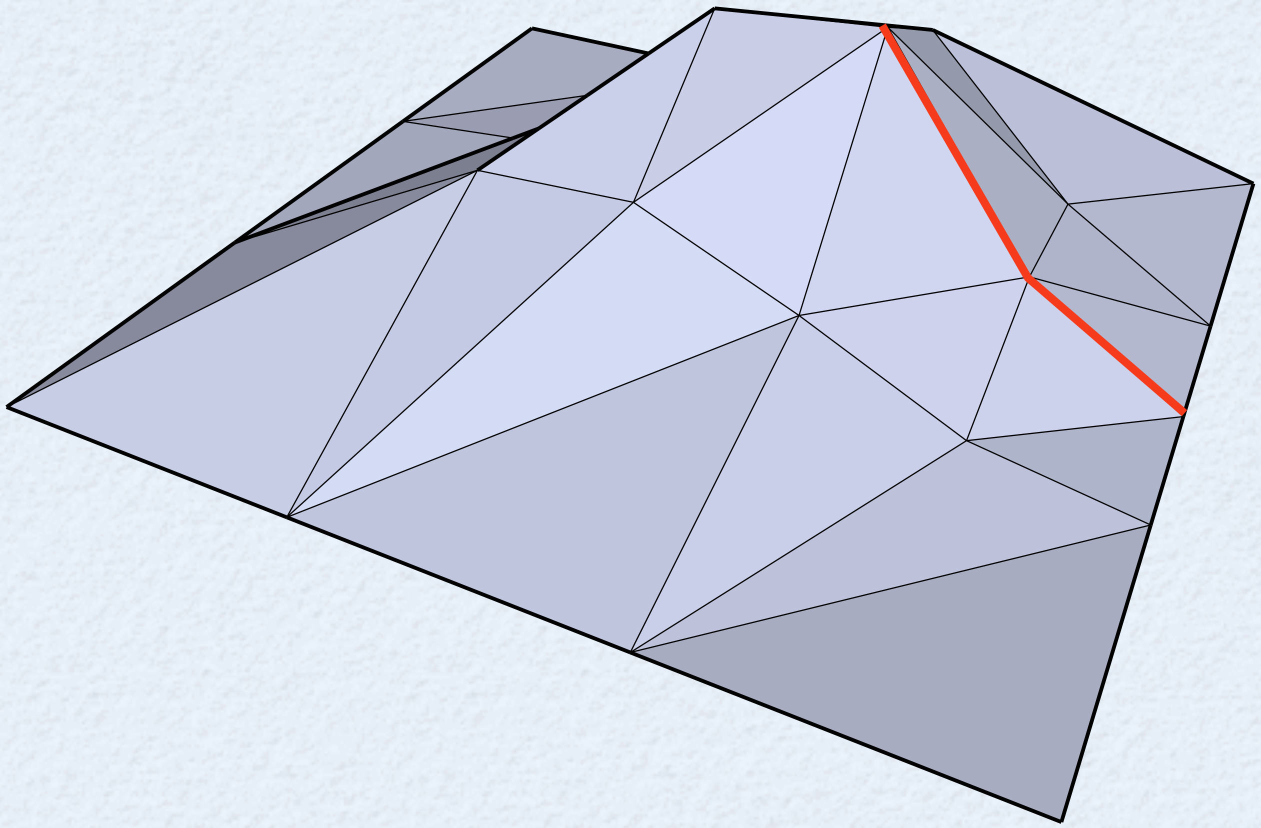

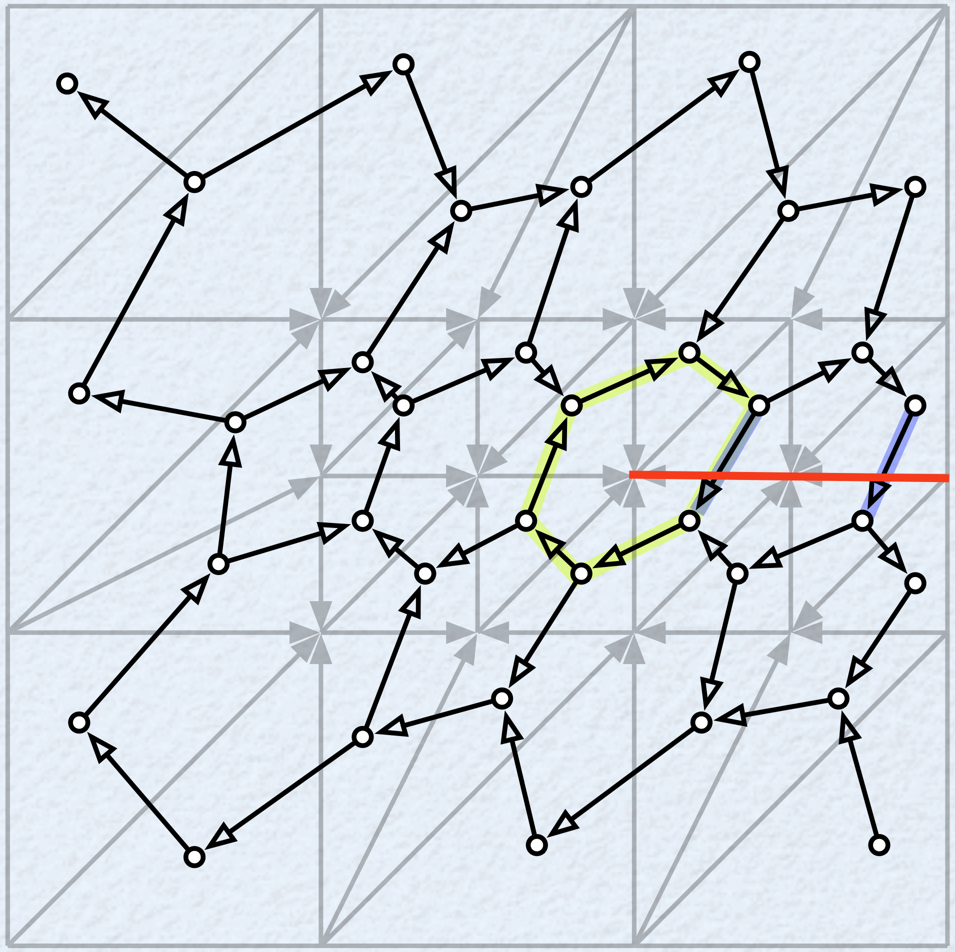

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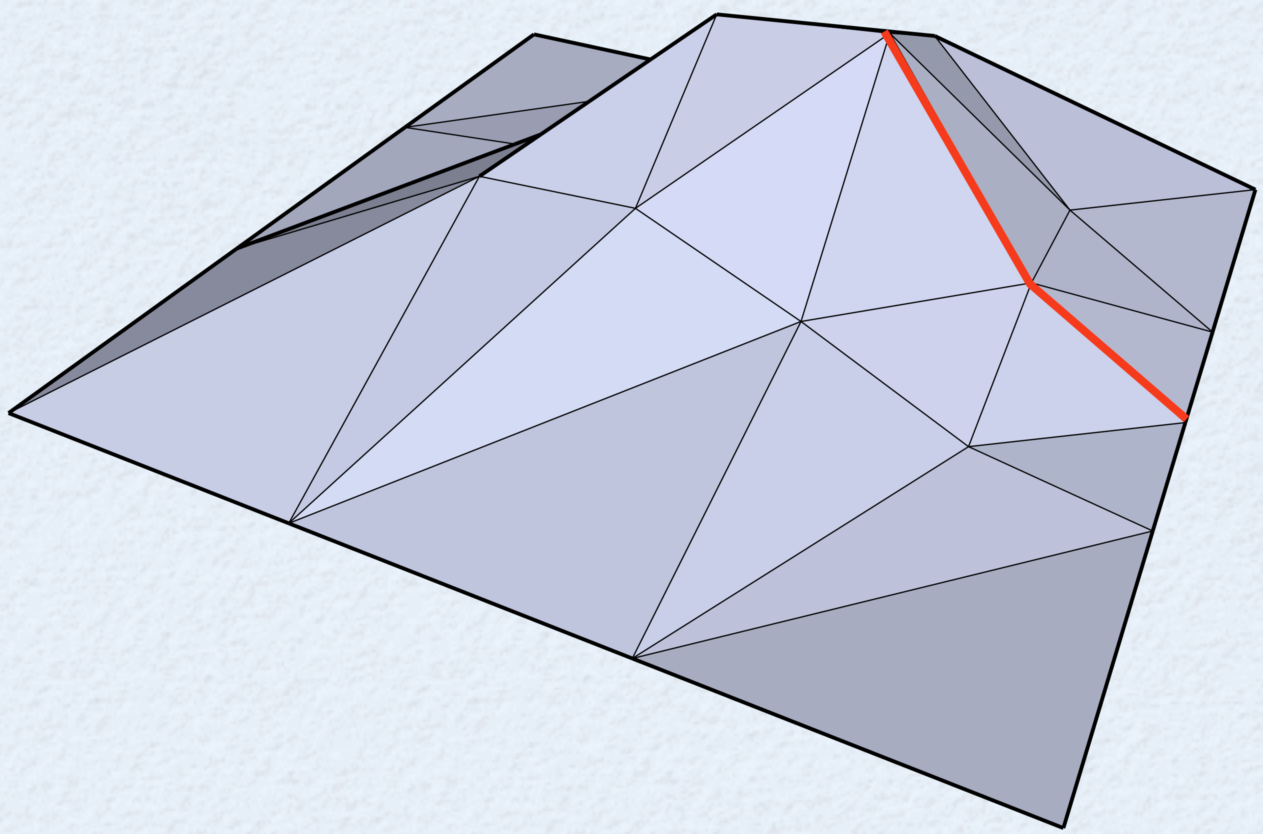

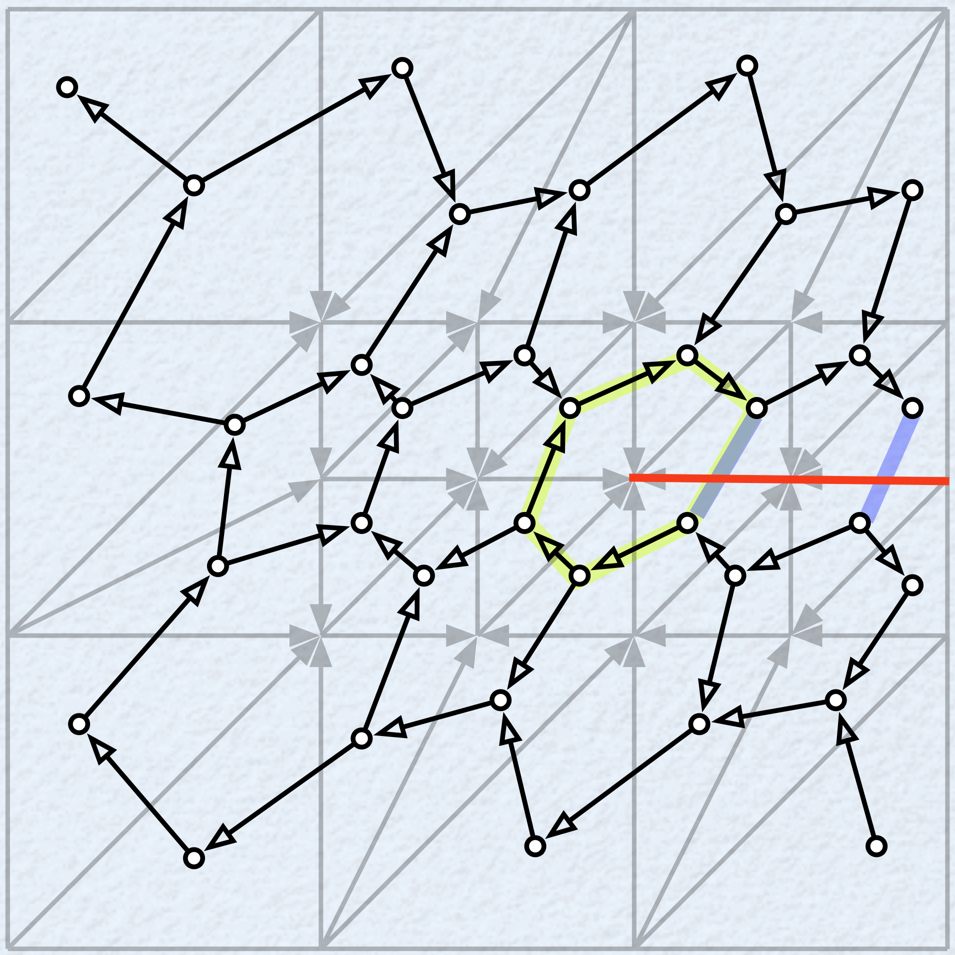

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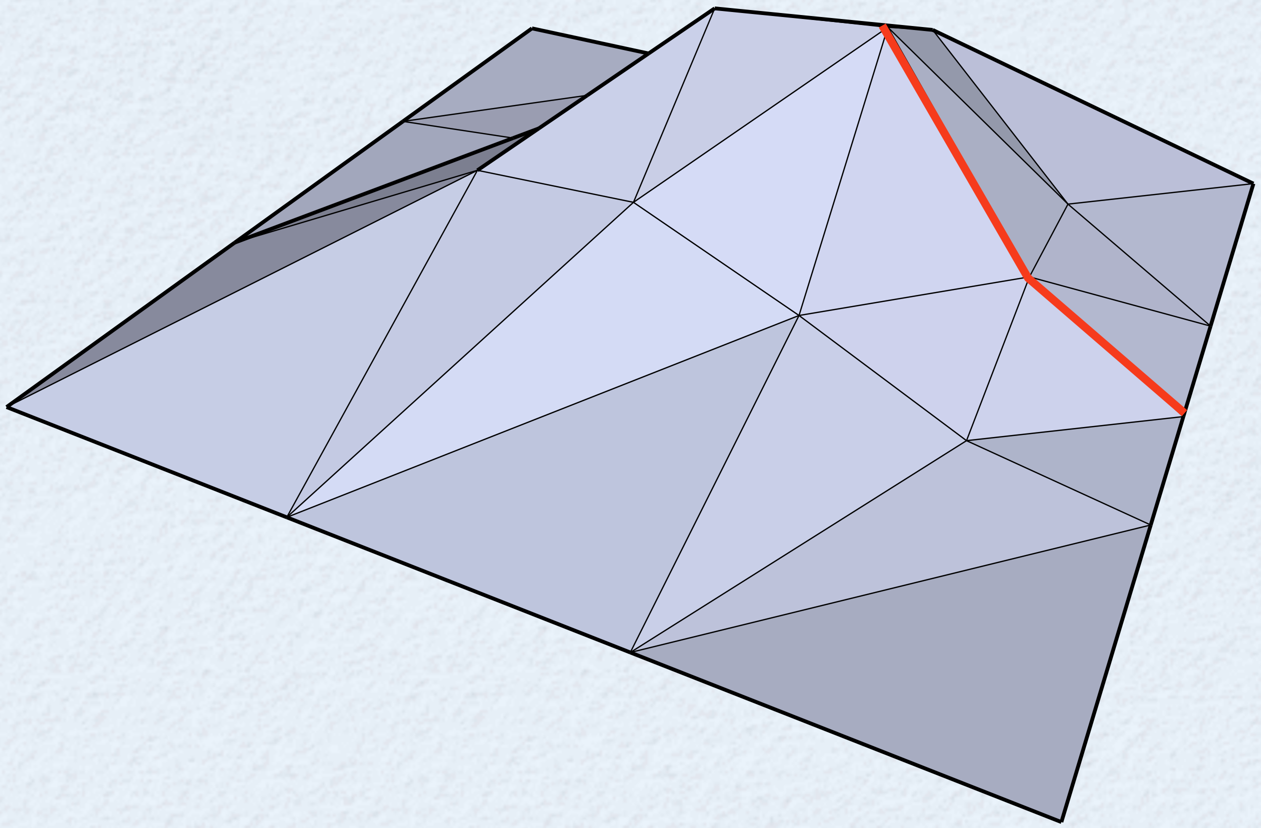

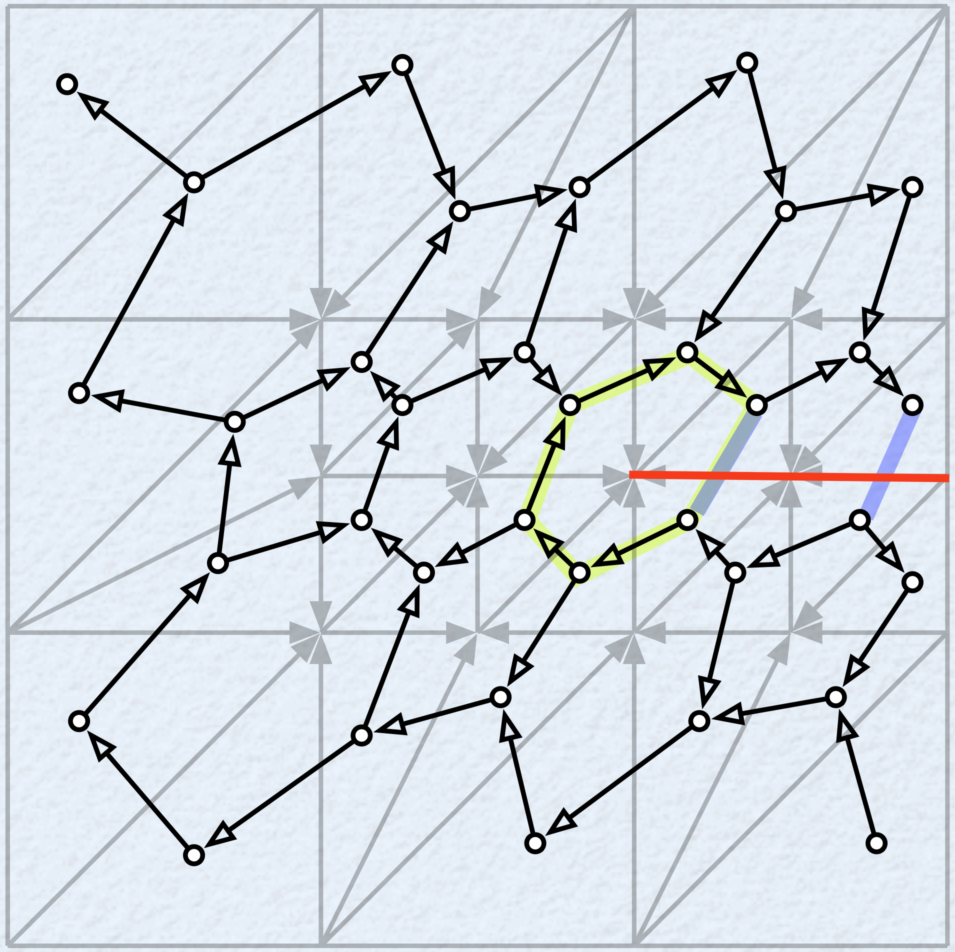

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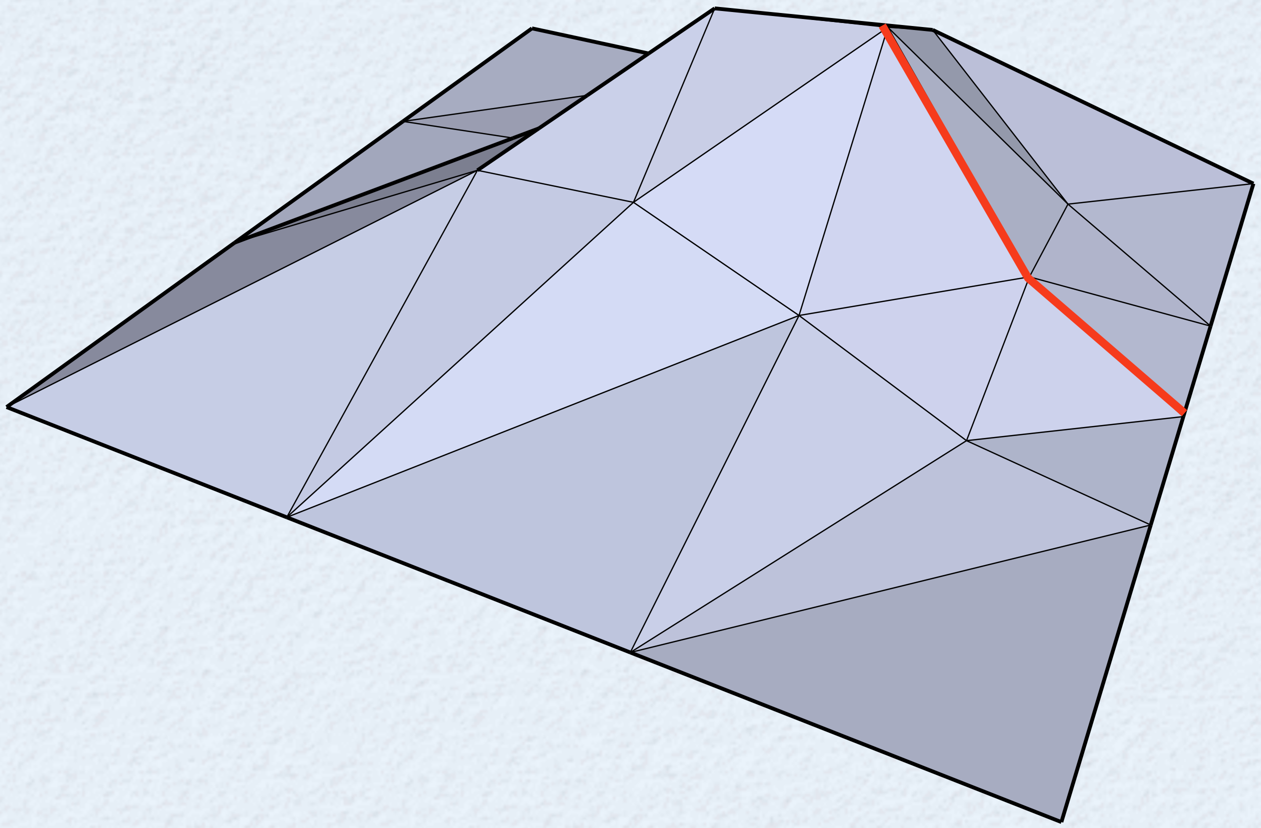

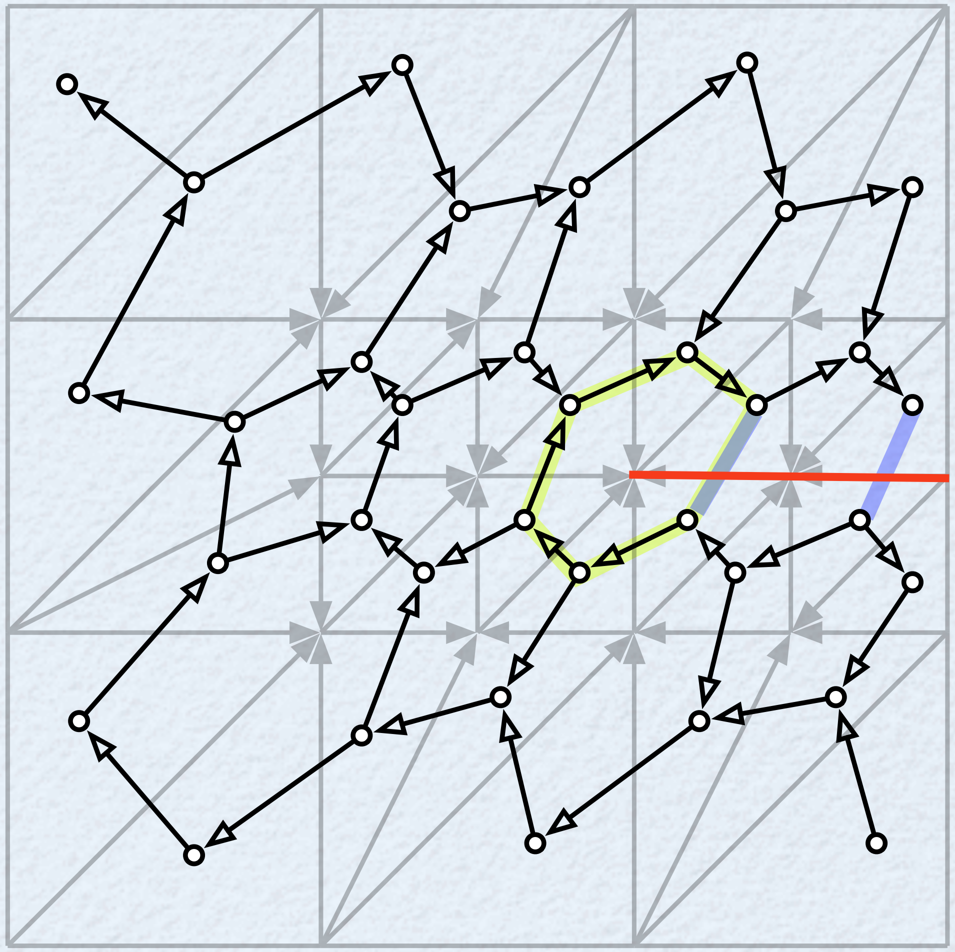

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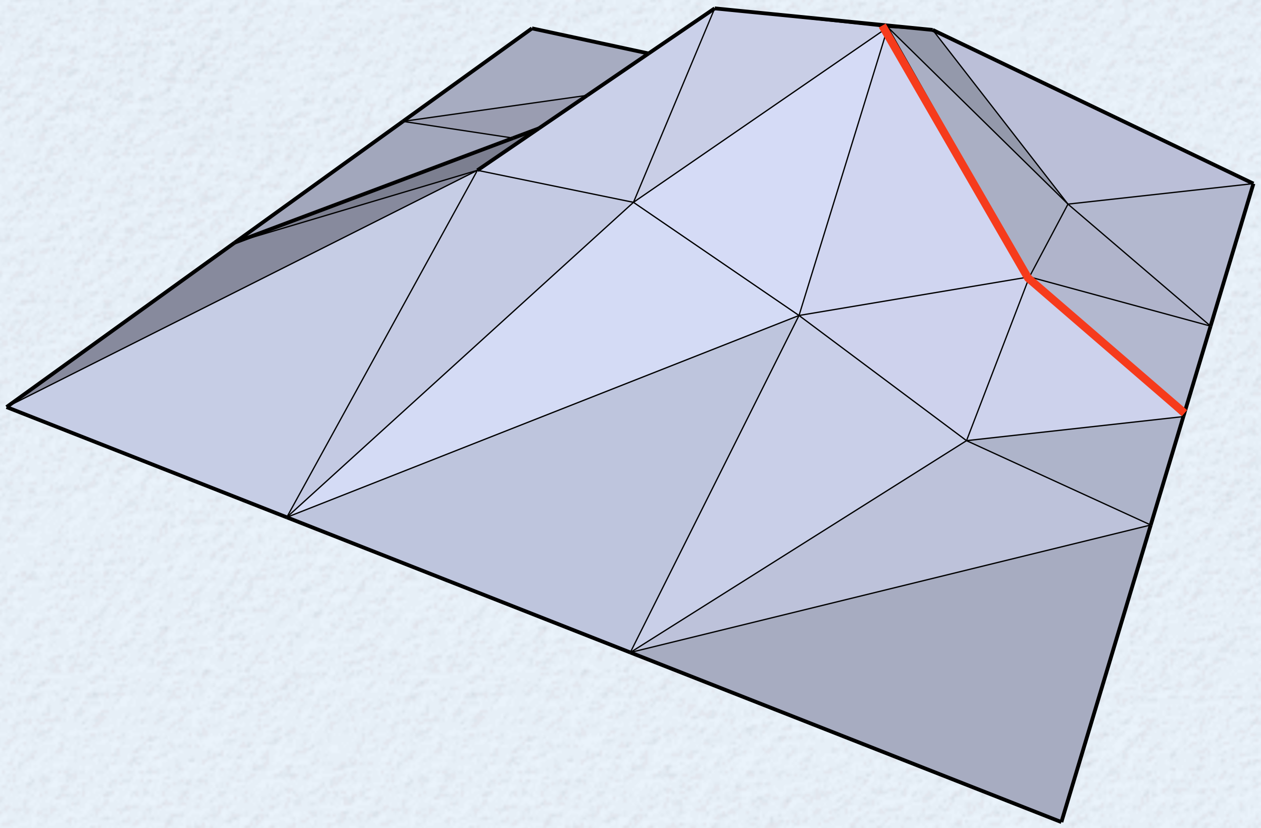

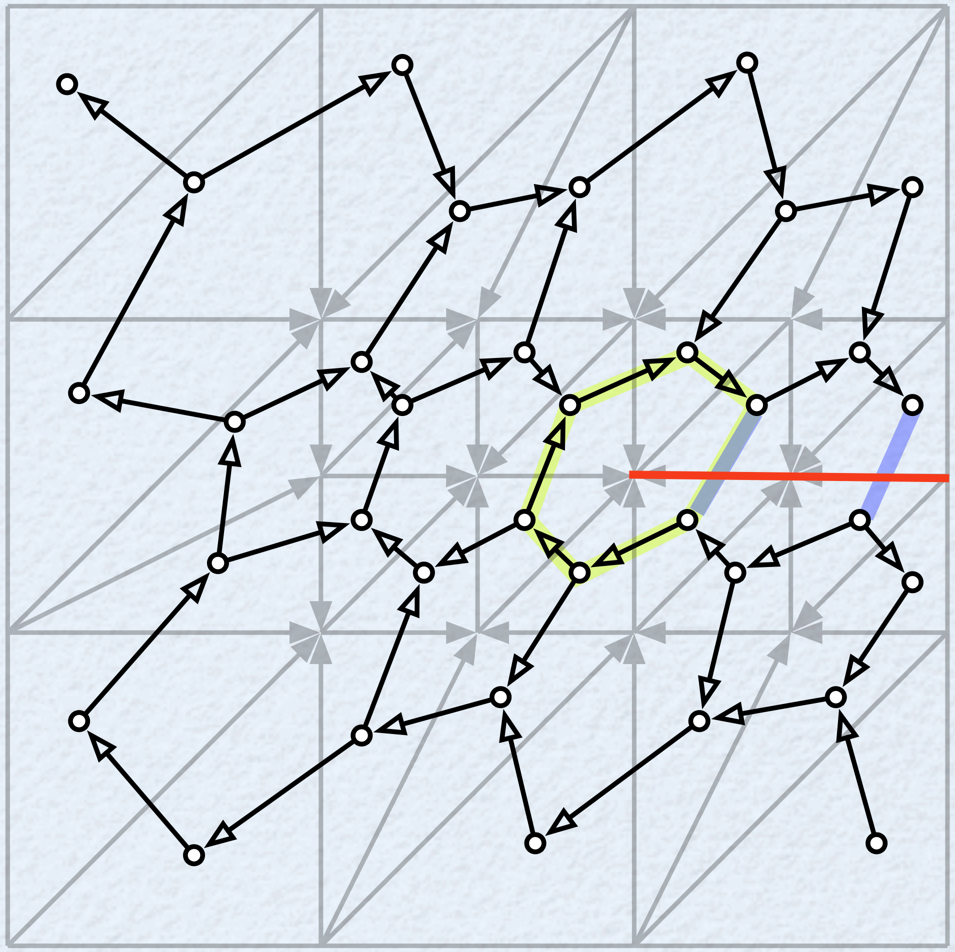

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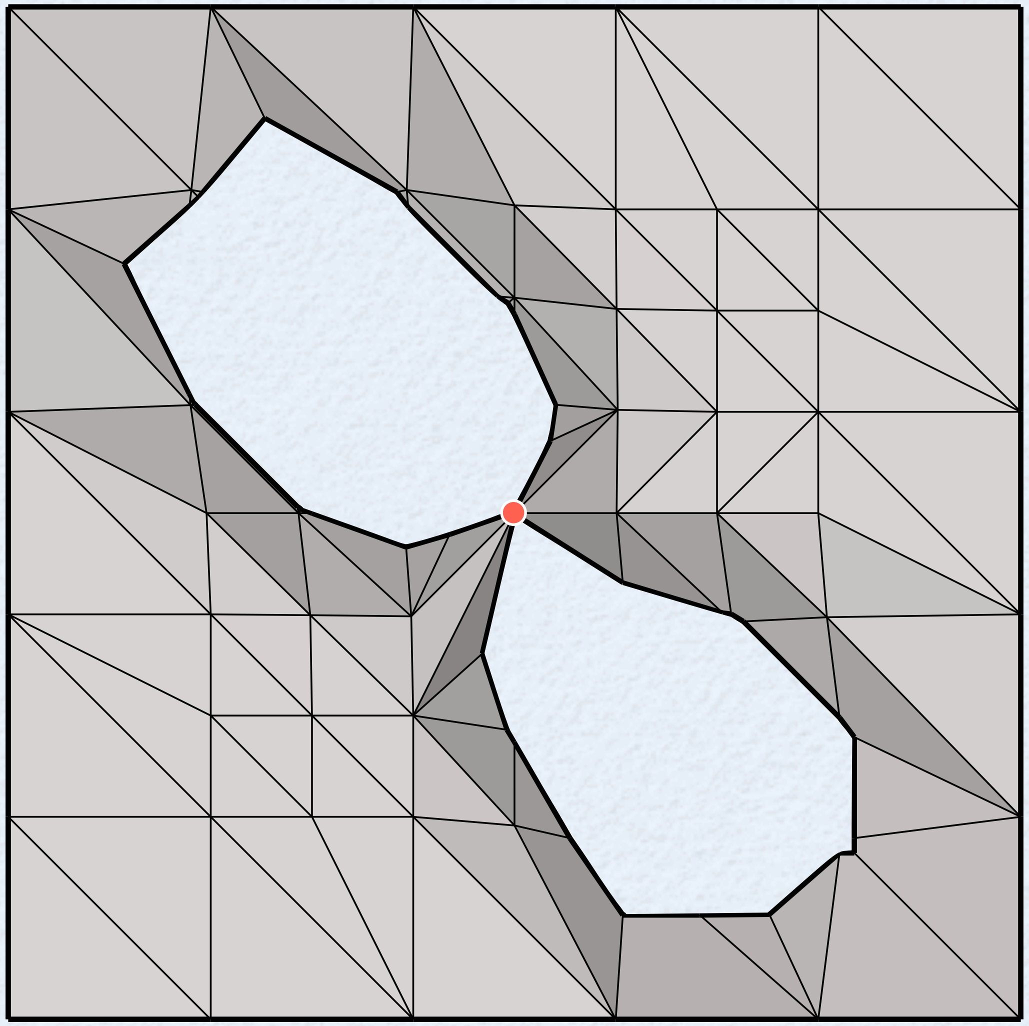
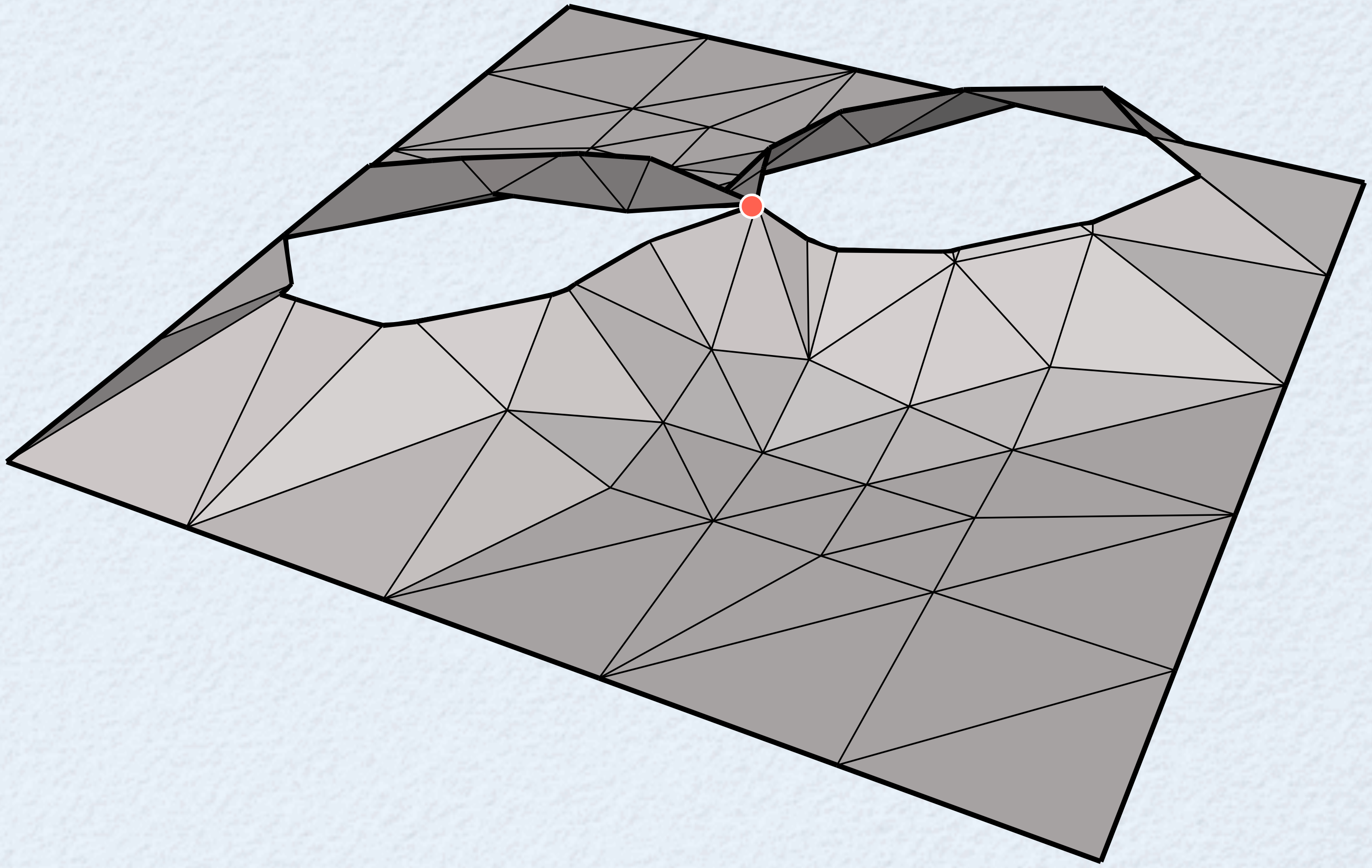
[Arge, Toma, Zeh'03]

What about non-elementary terrains?

A saddle is negative if it joins two disjoint connected components of its sublevel-set and positive otherwise.





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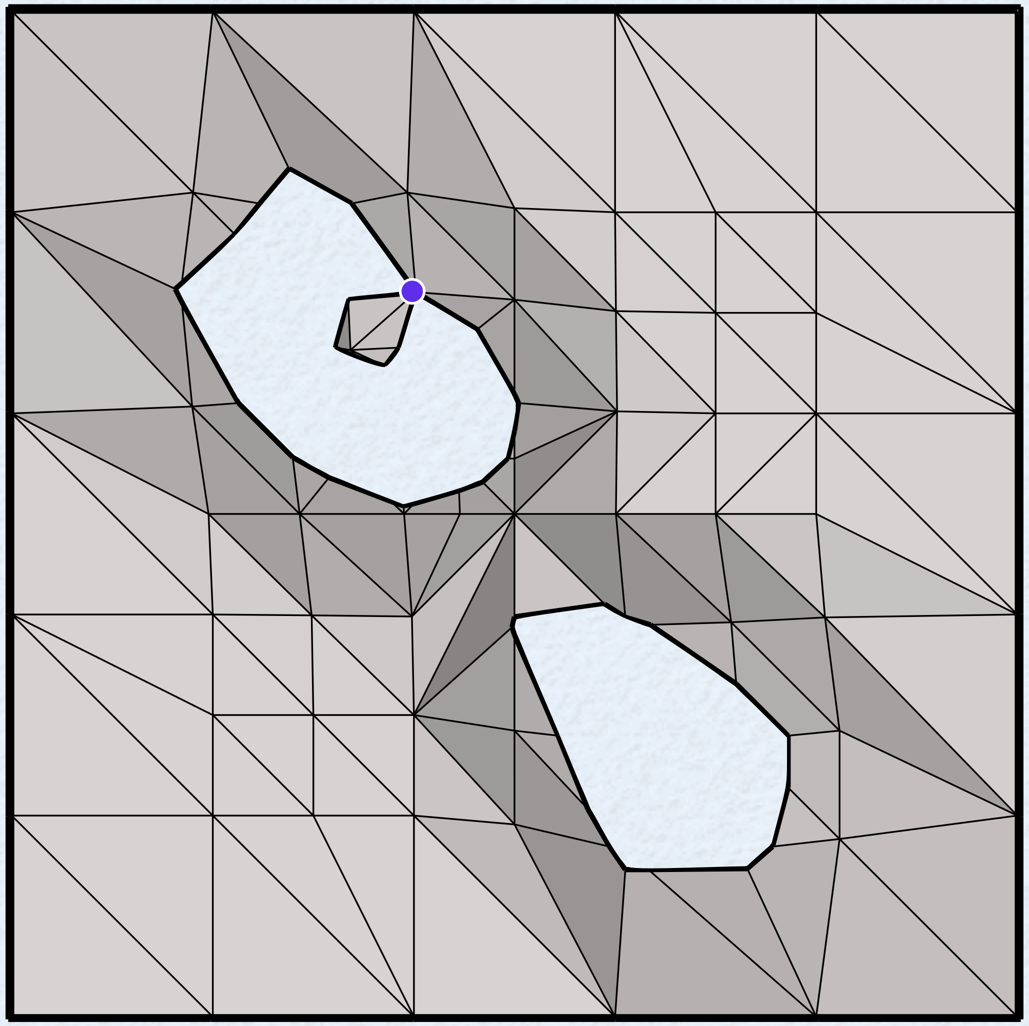

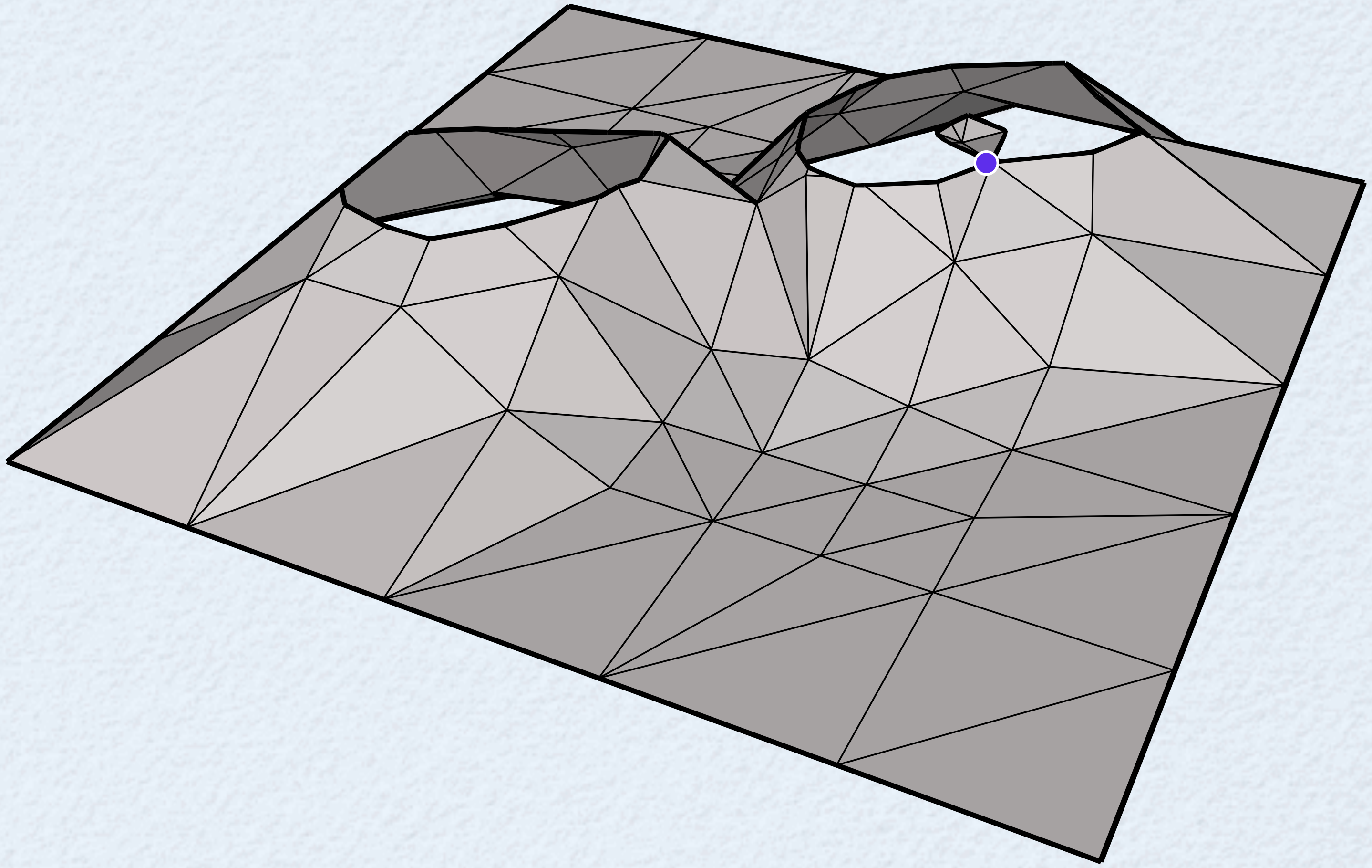

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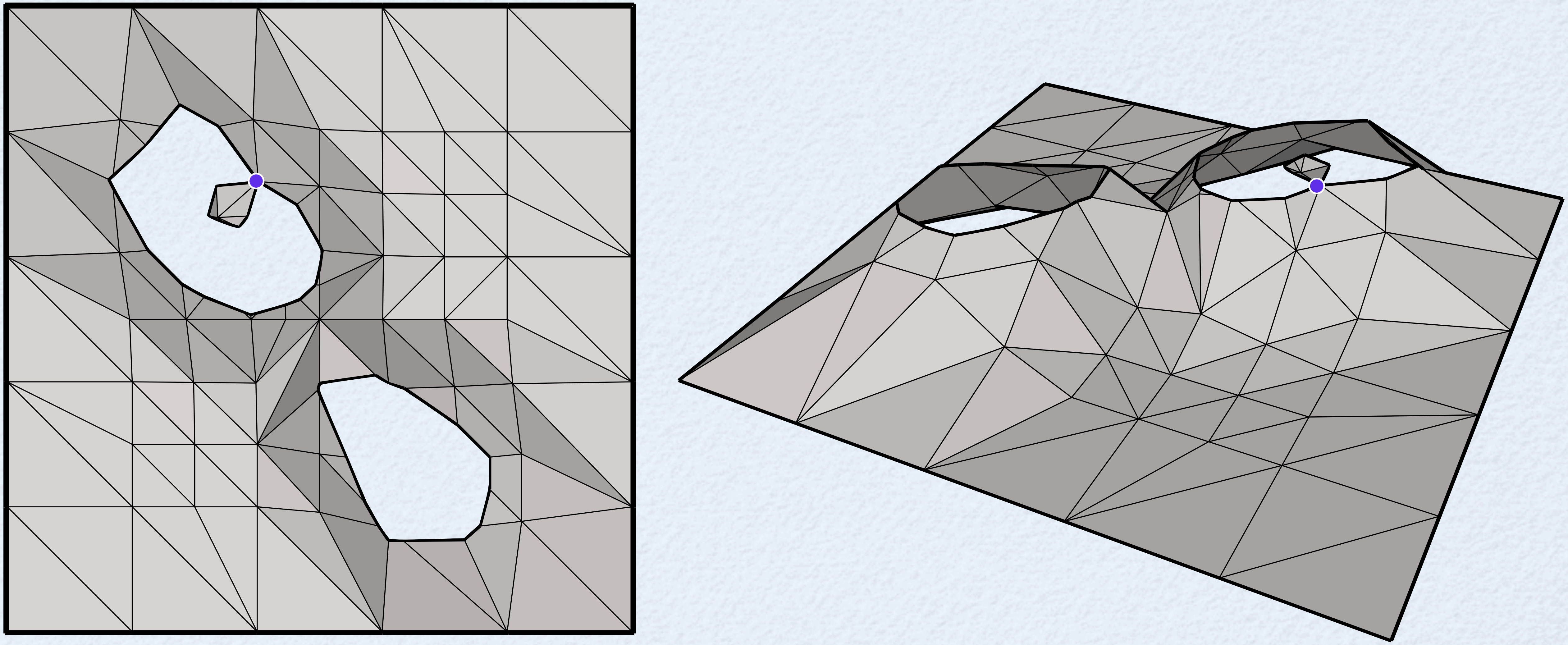

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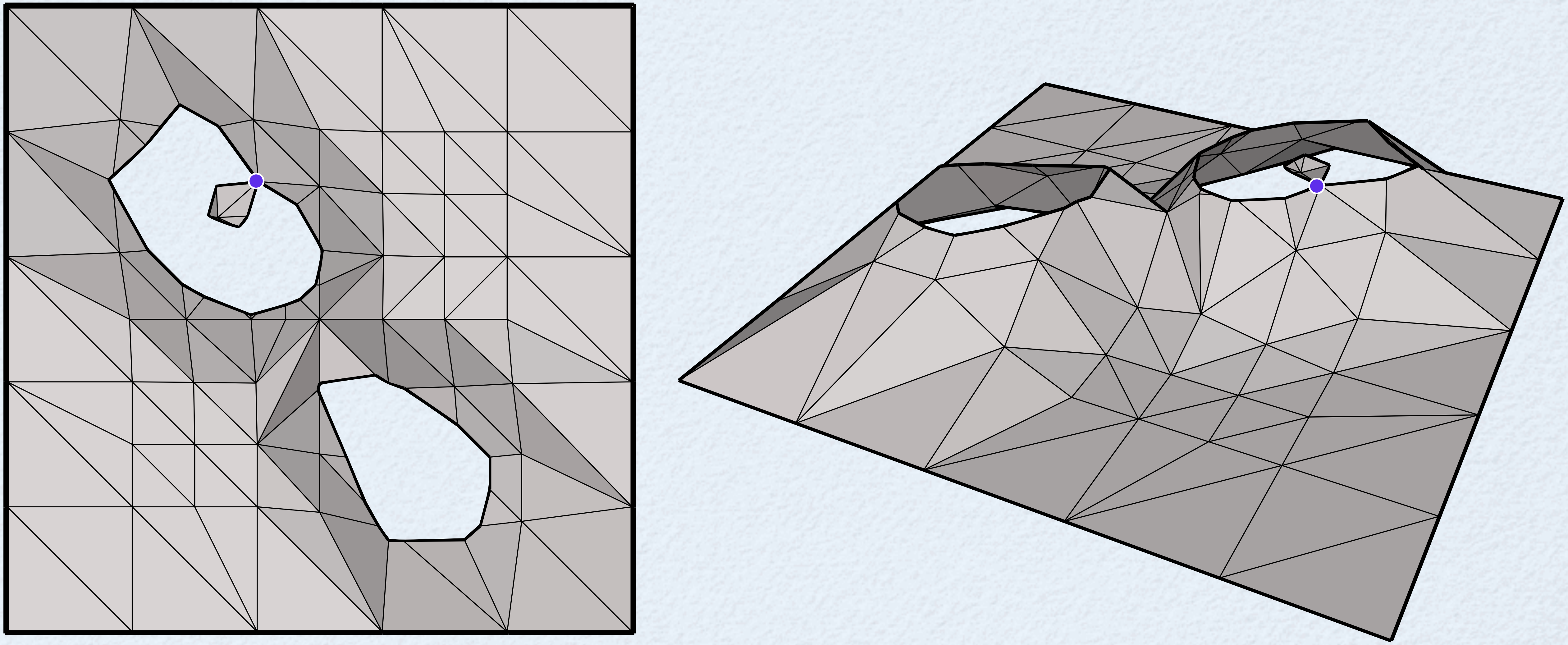

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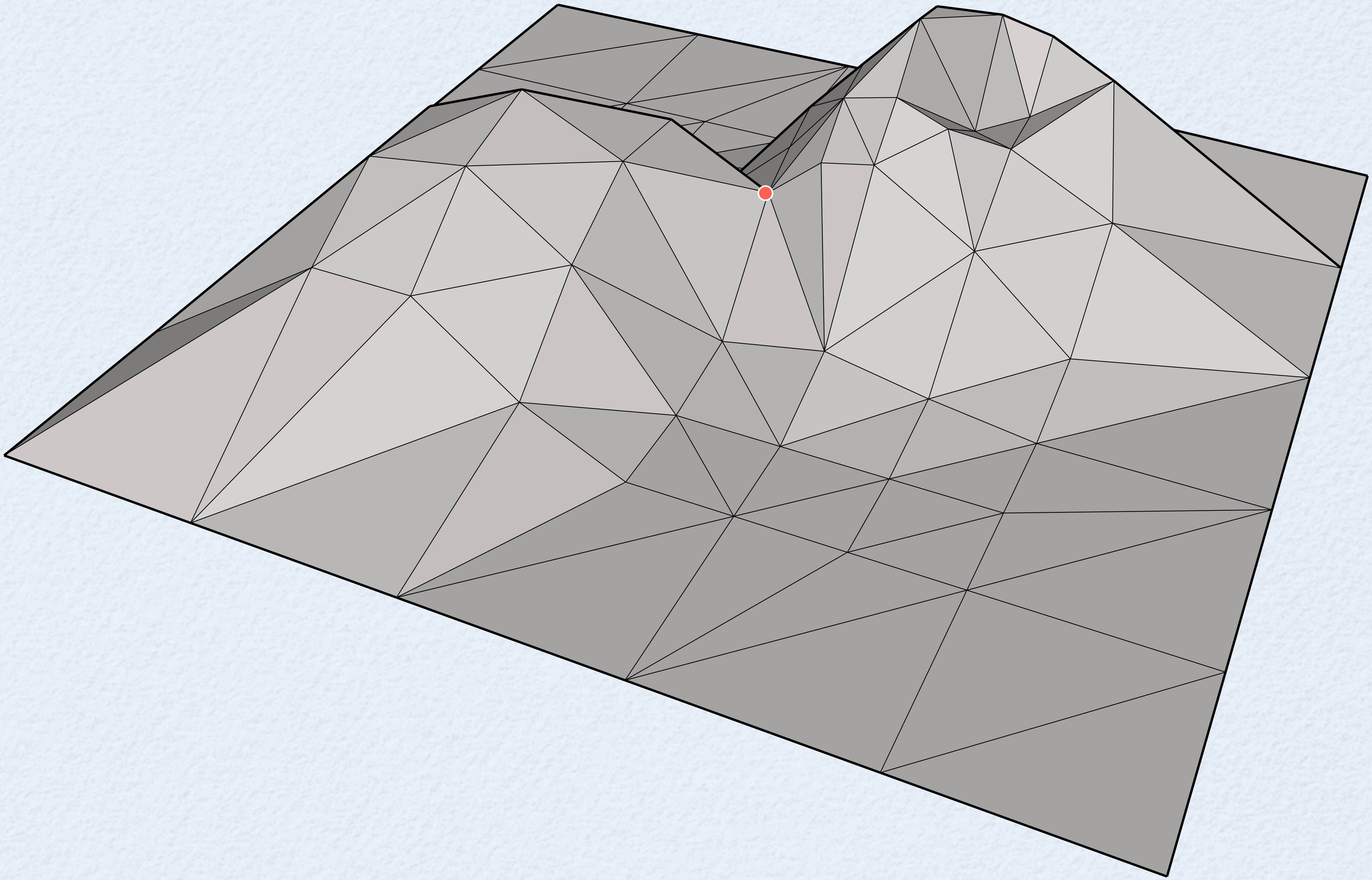
[Agarwal, Arge, Yi '06] Positive and negative saddle points can be found in $O(\mathrm{Sort}(N))$ I/Os.

Positive and Negative Cut-Trees

Positive Cut-Tree: follow an ascending path in every connected component of the upper link of every positive saddle, joining paths when they collide.



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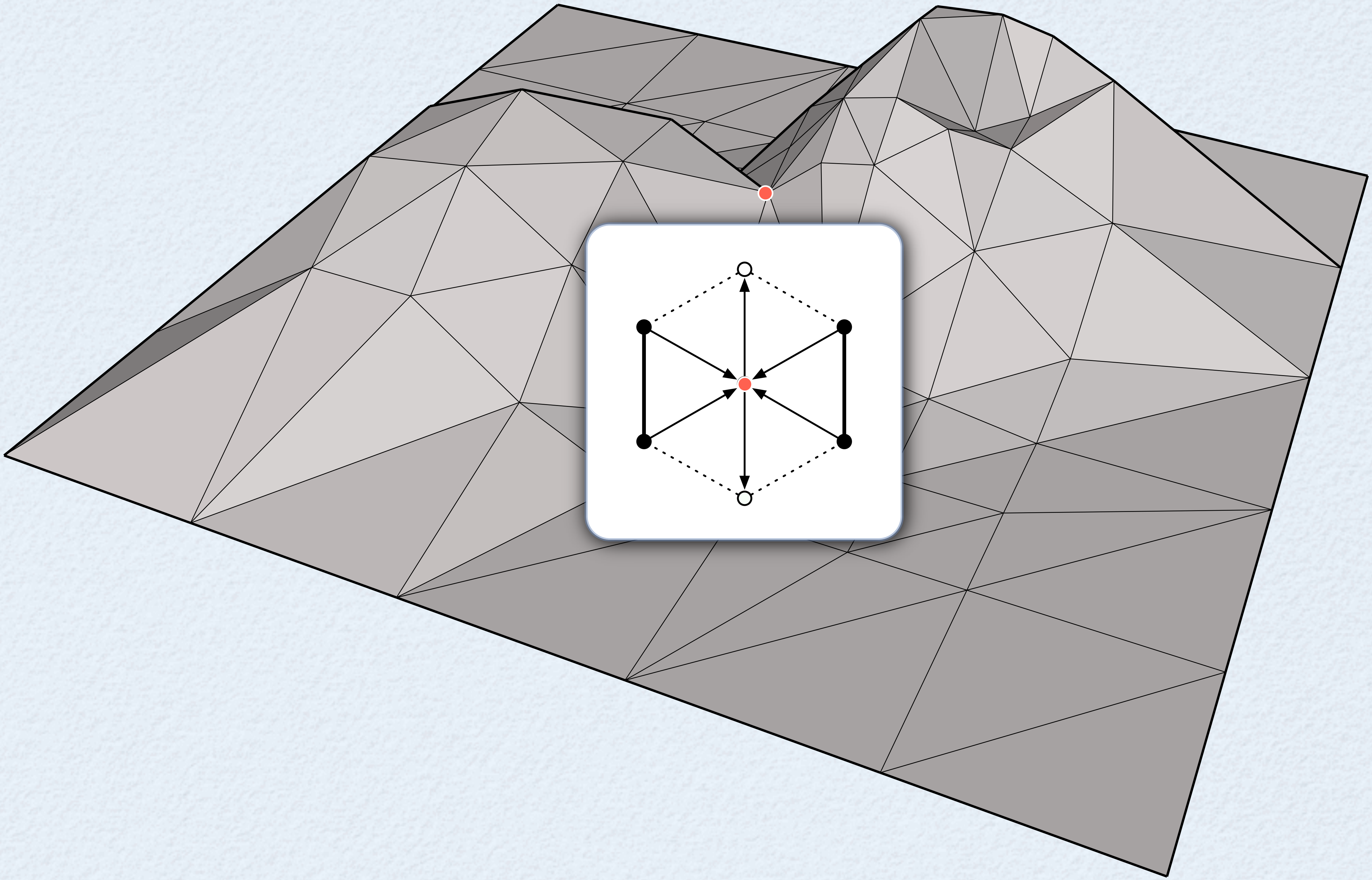

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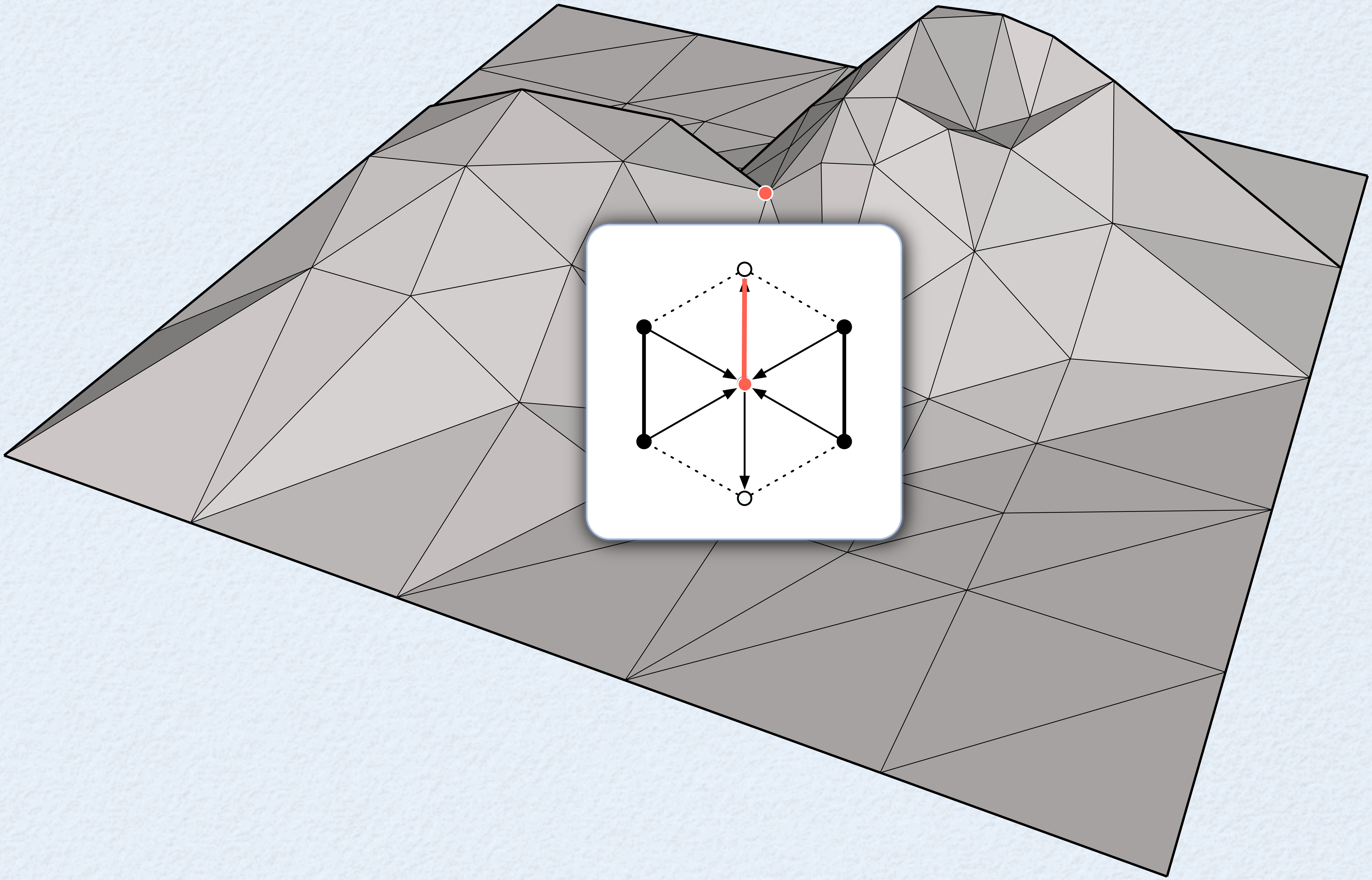

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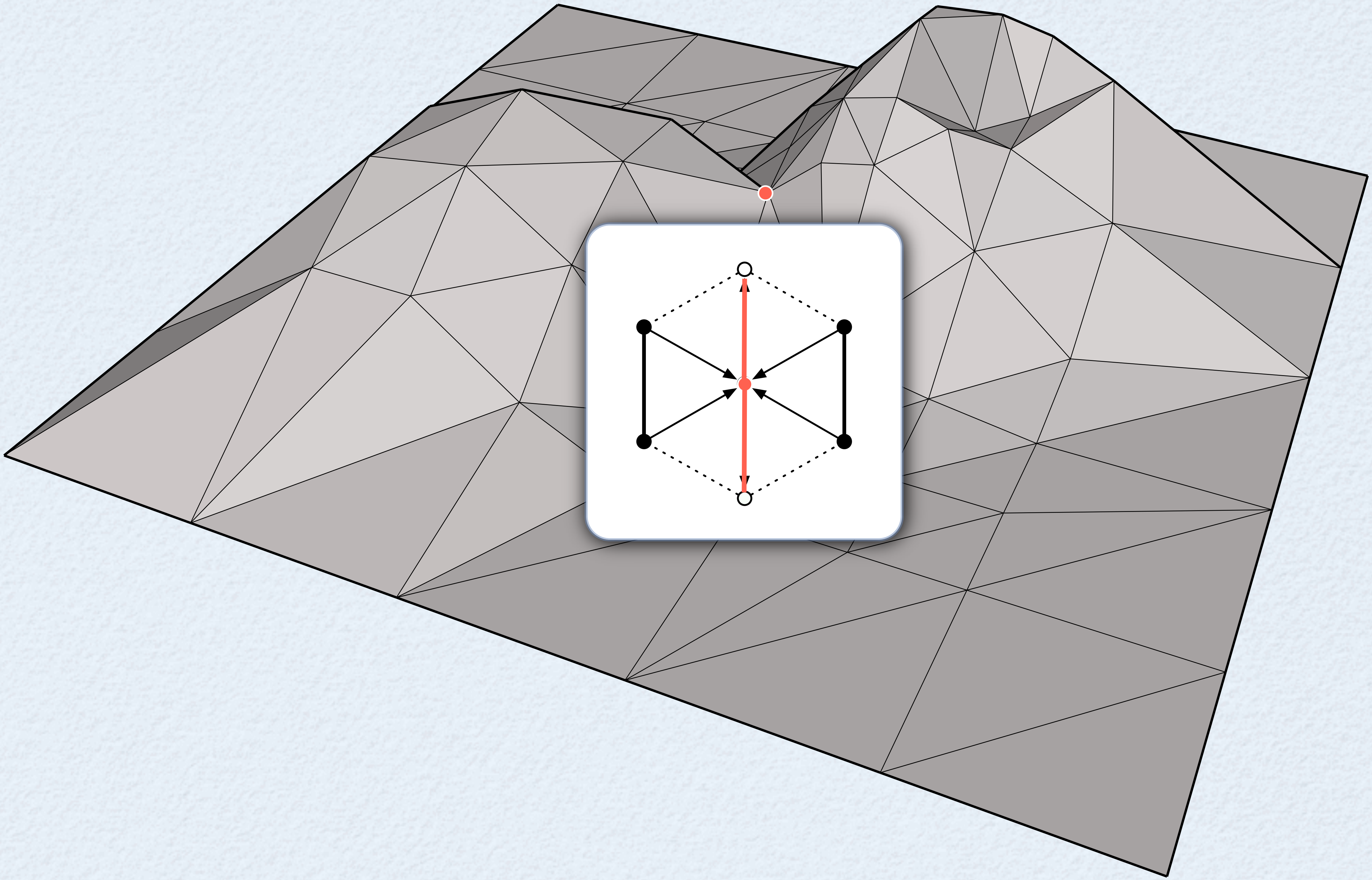

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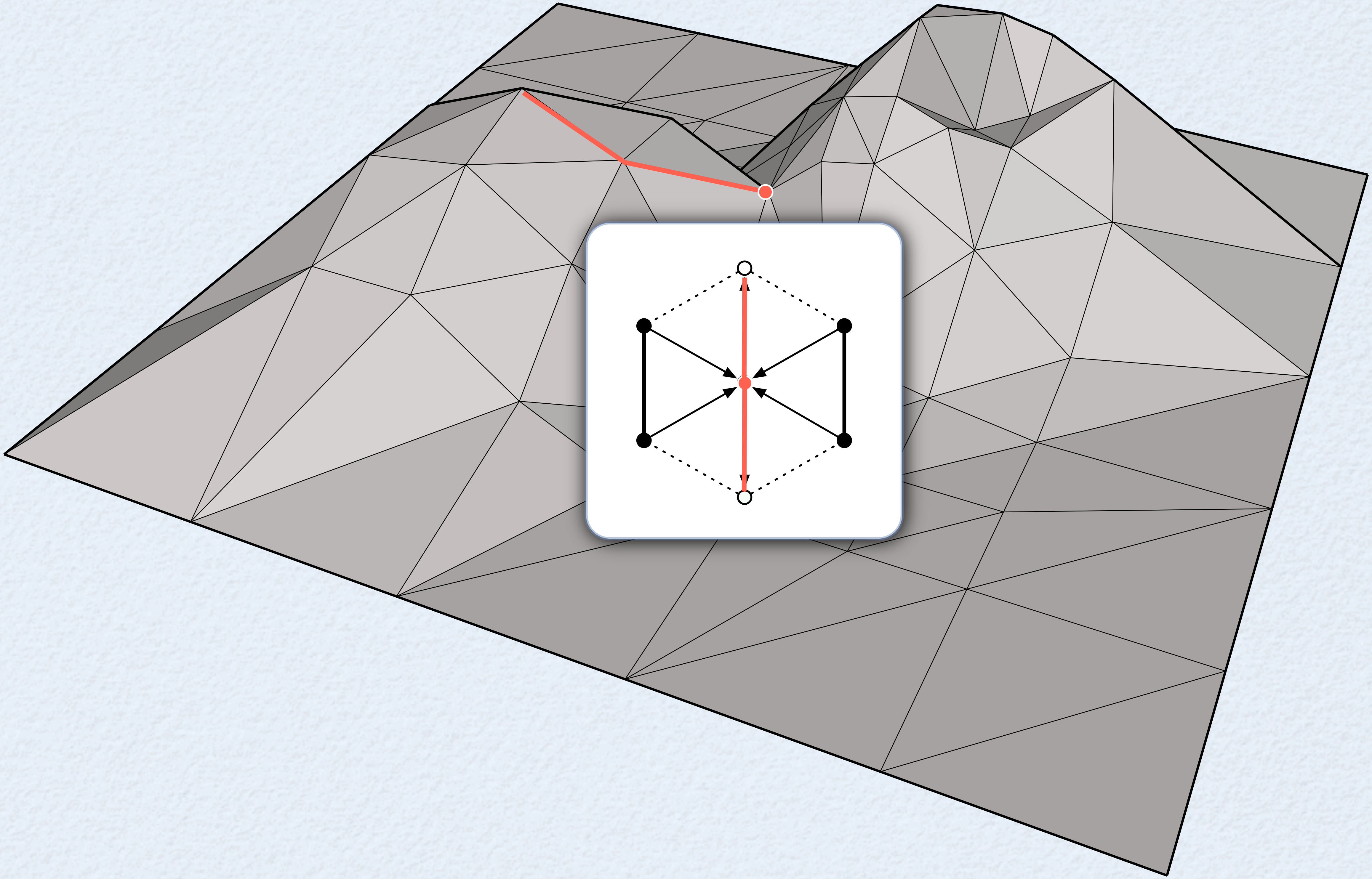

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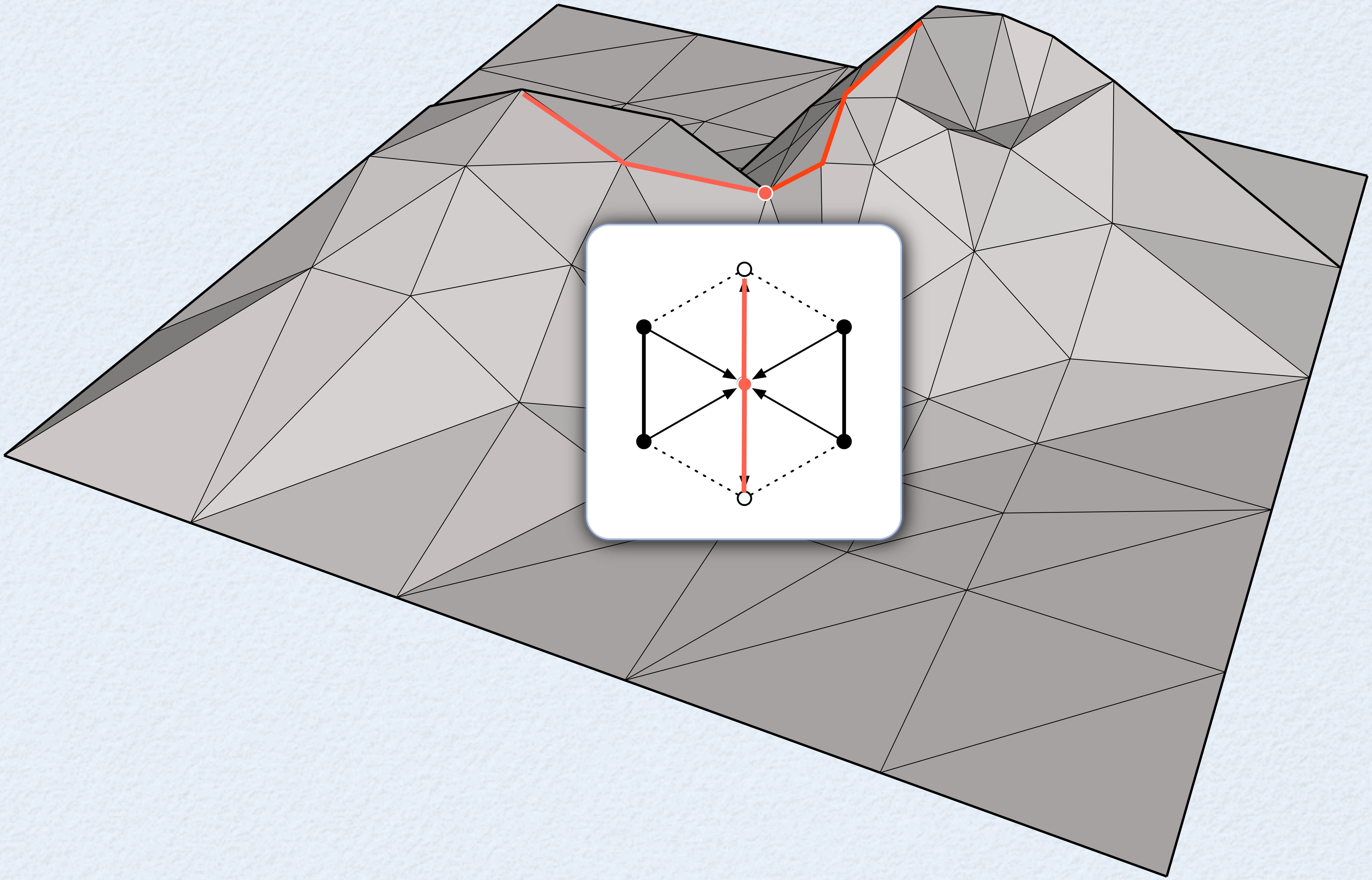

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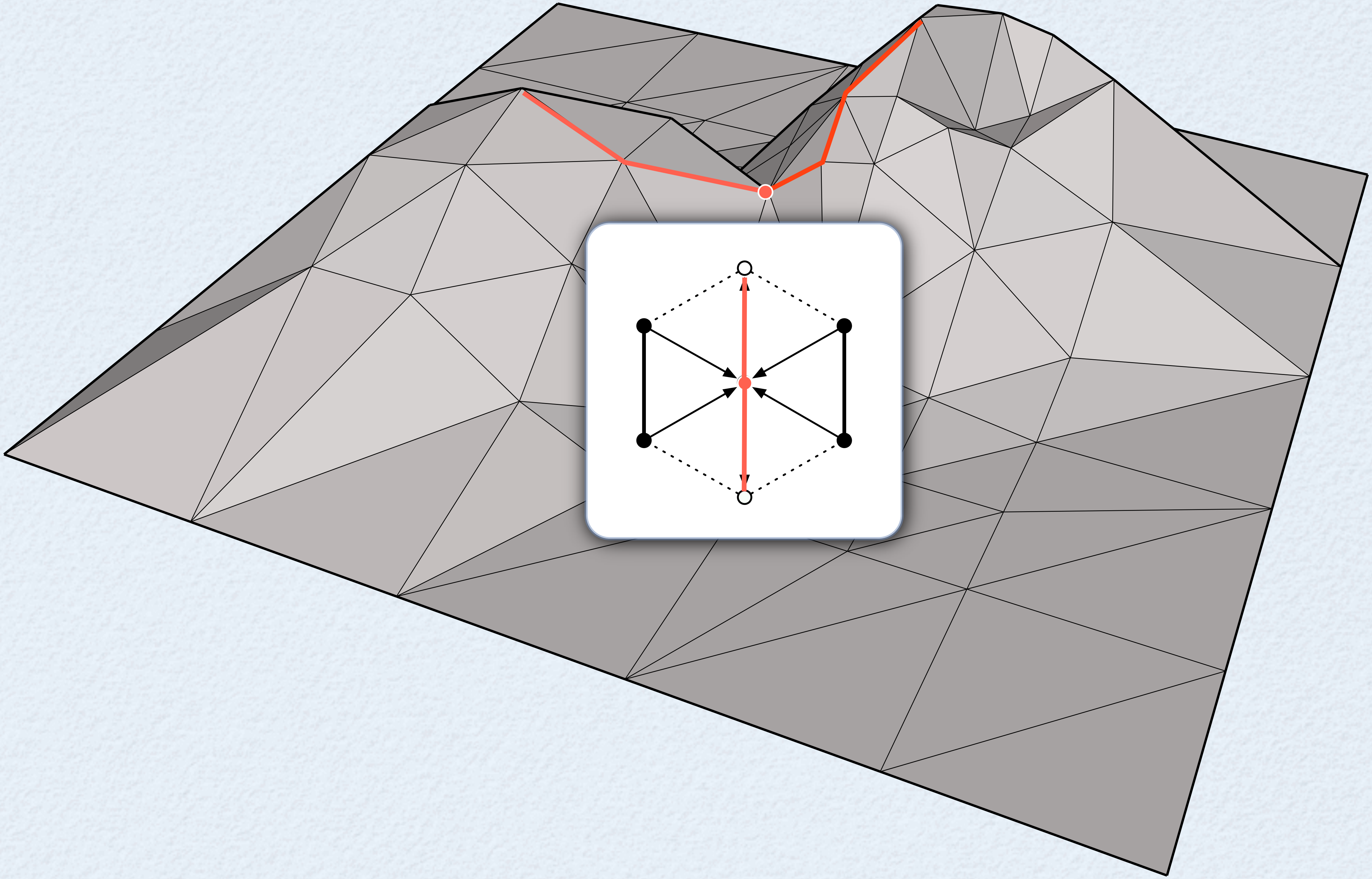

Lemma. The result is a tree (not just forest) that reaches every maximum.

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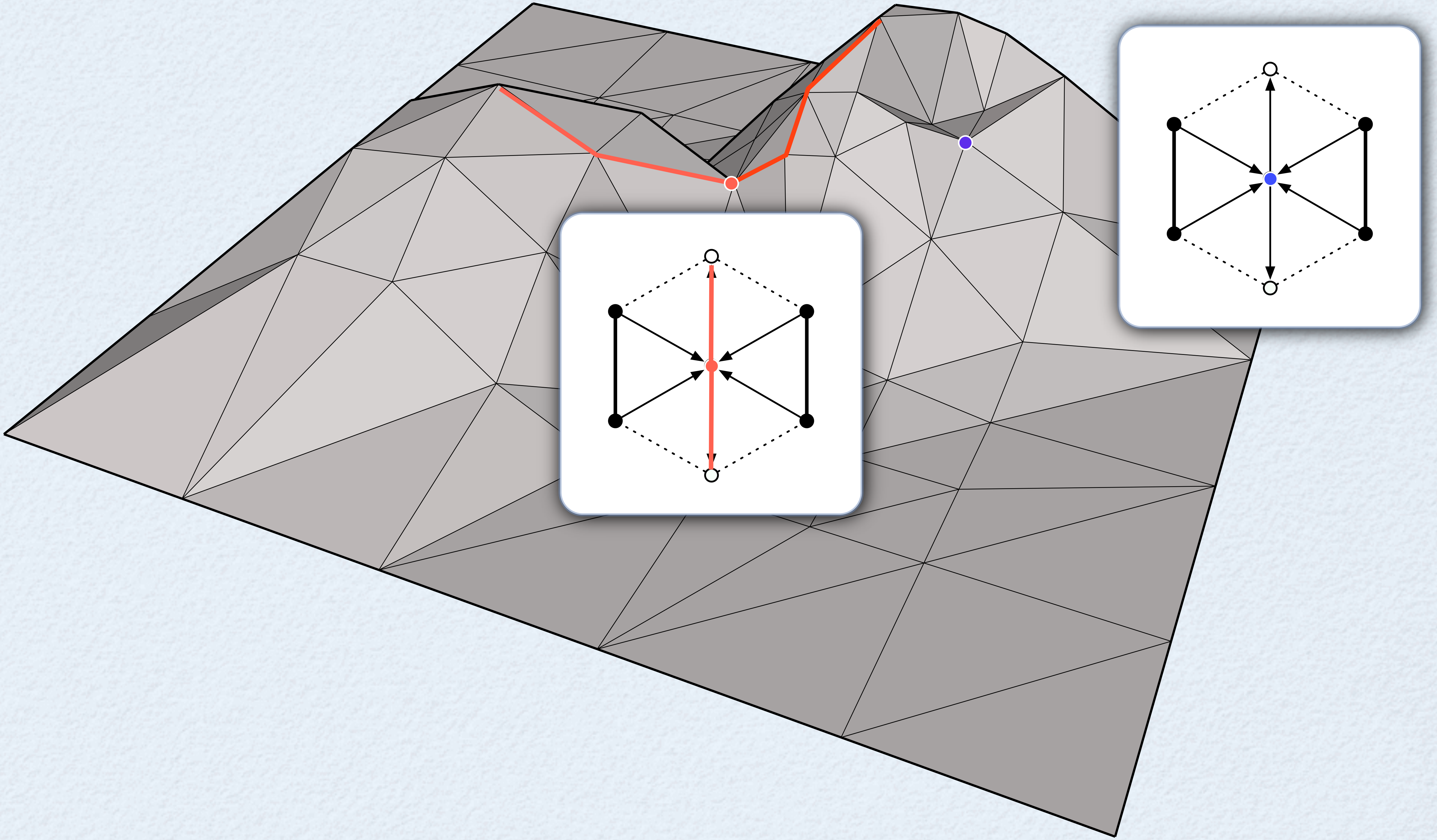

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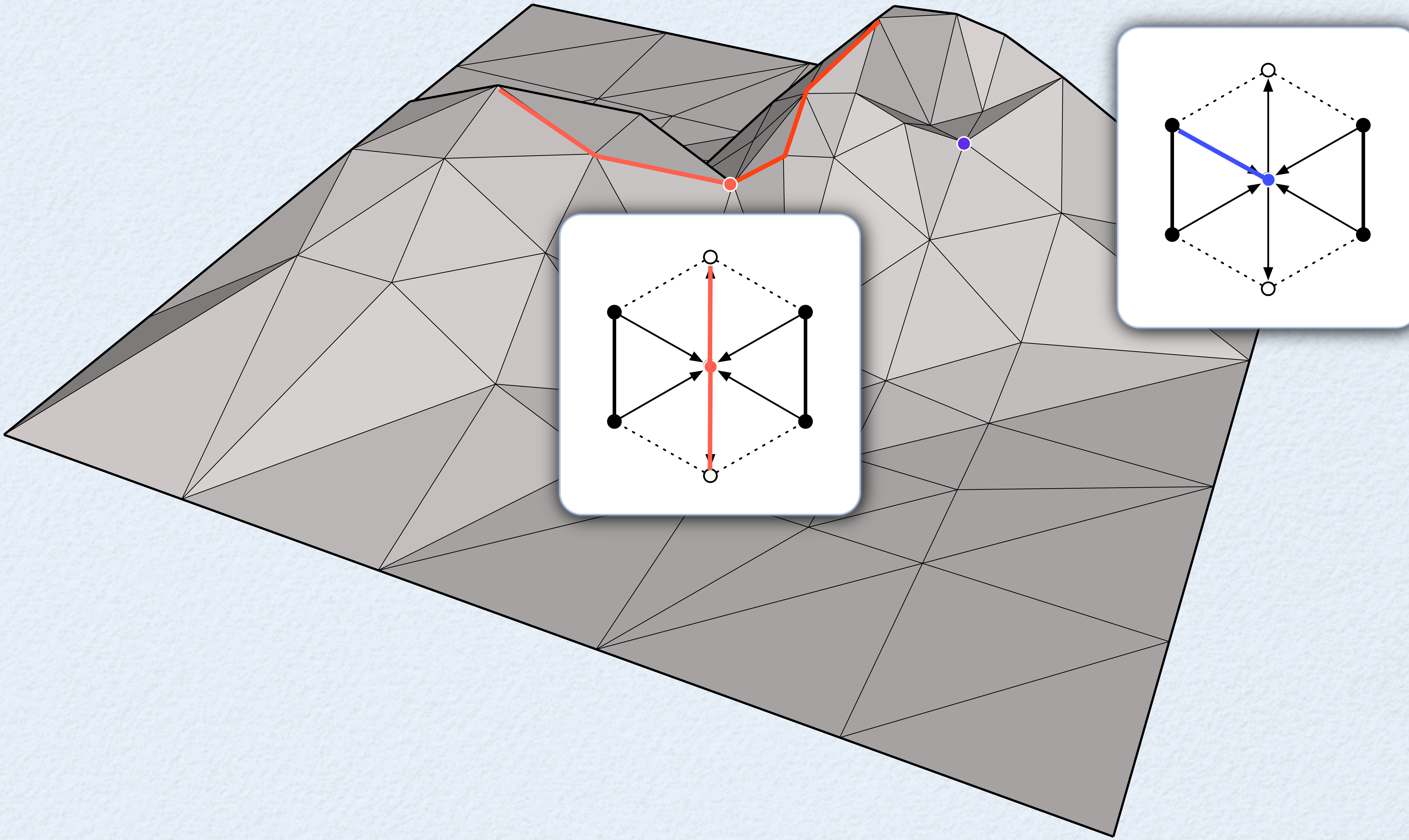

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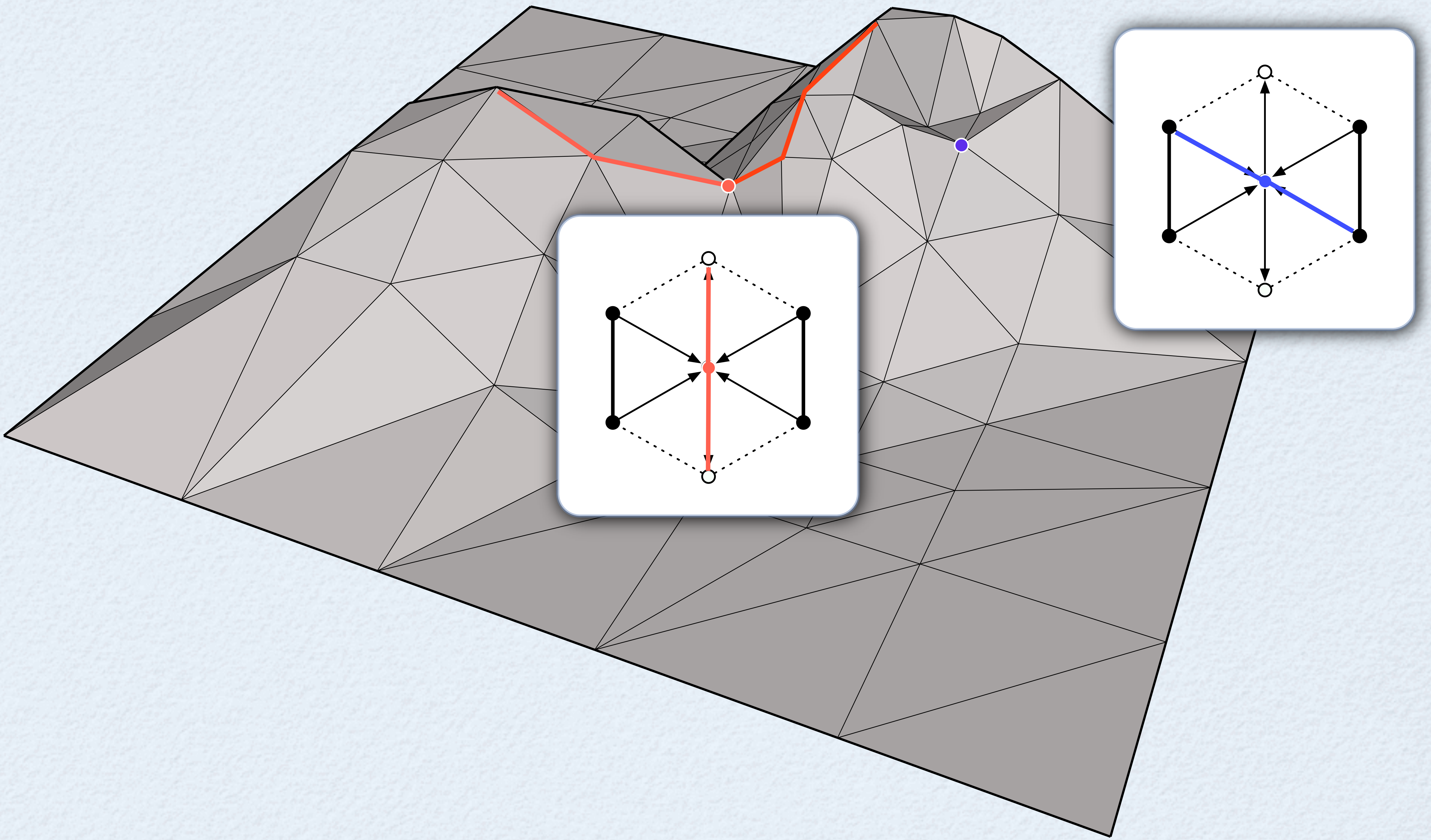

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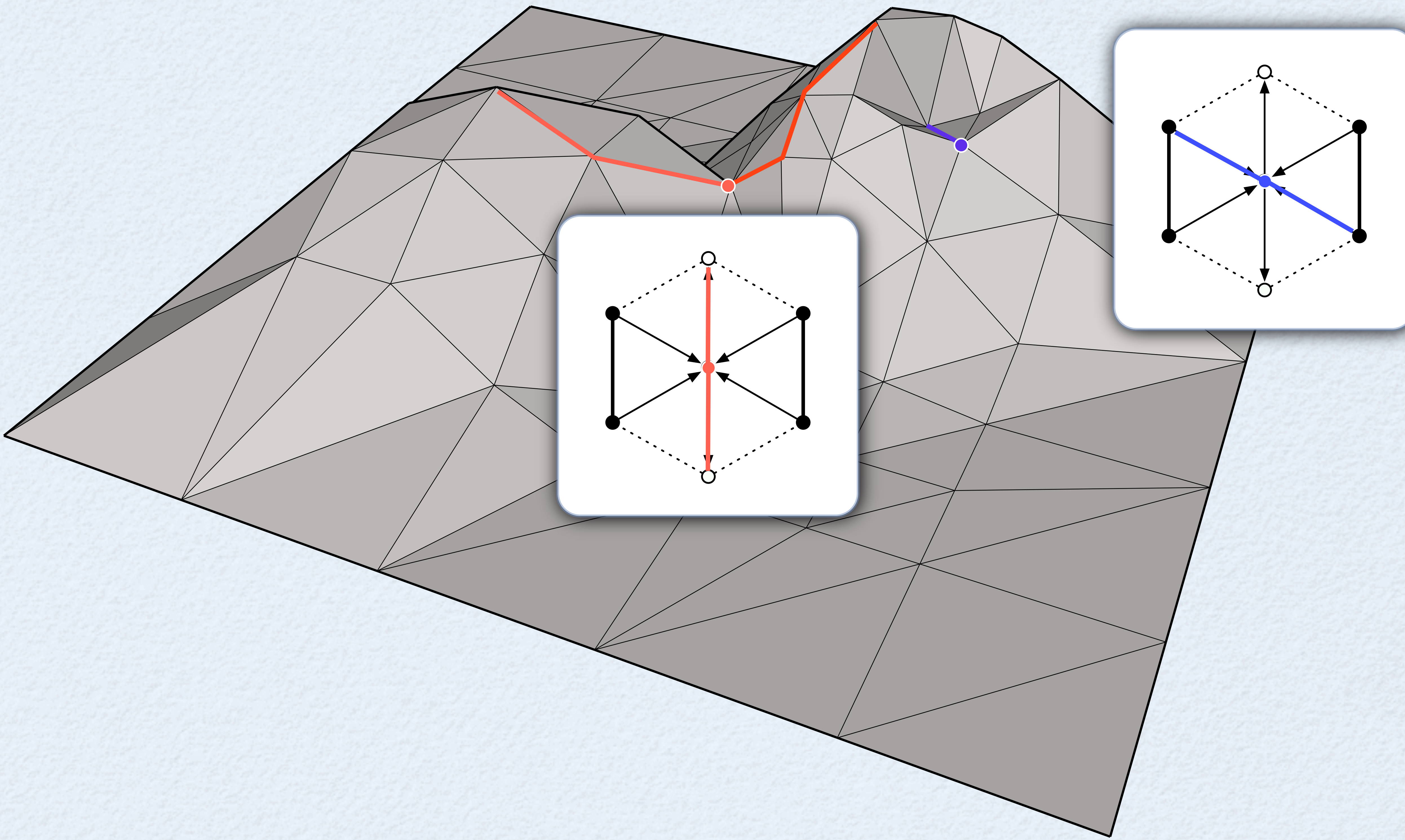

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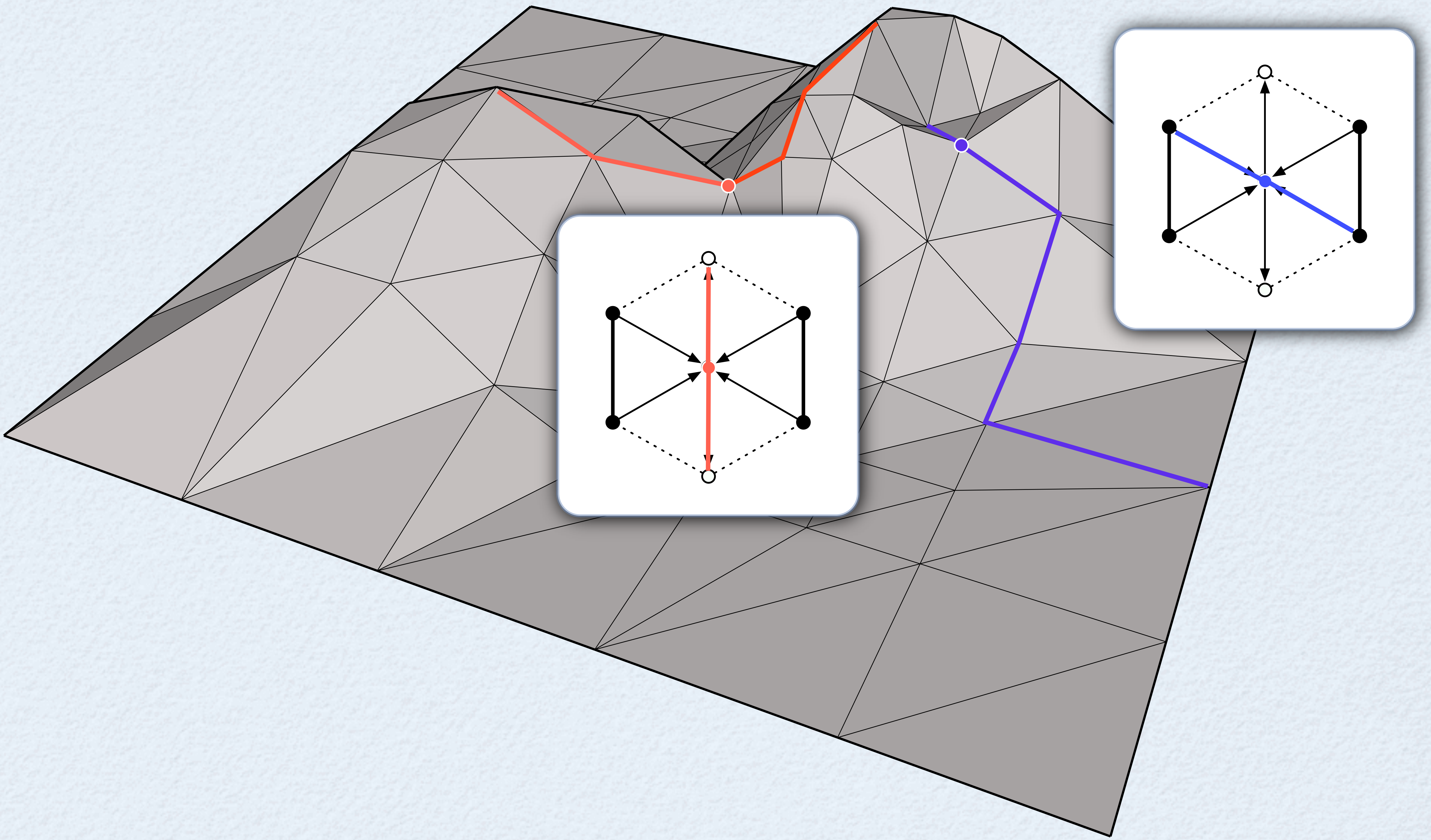

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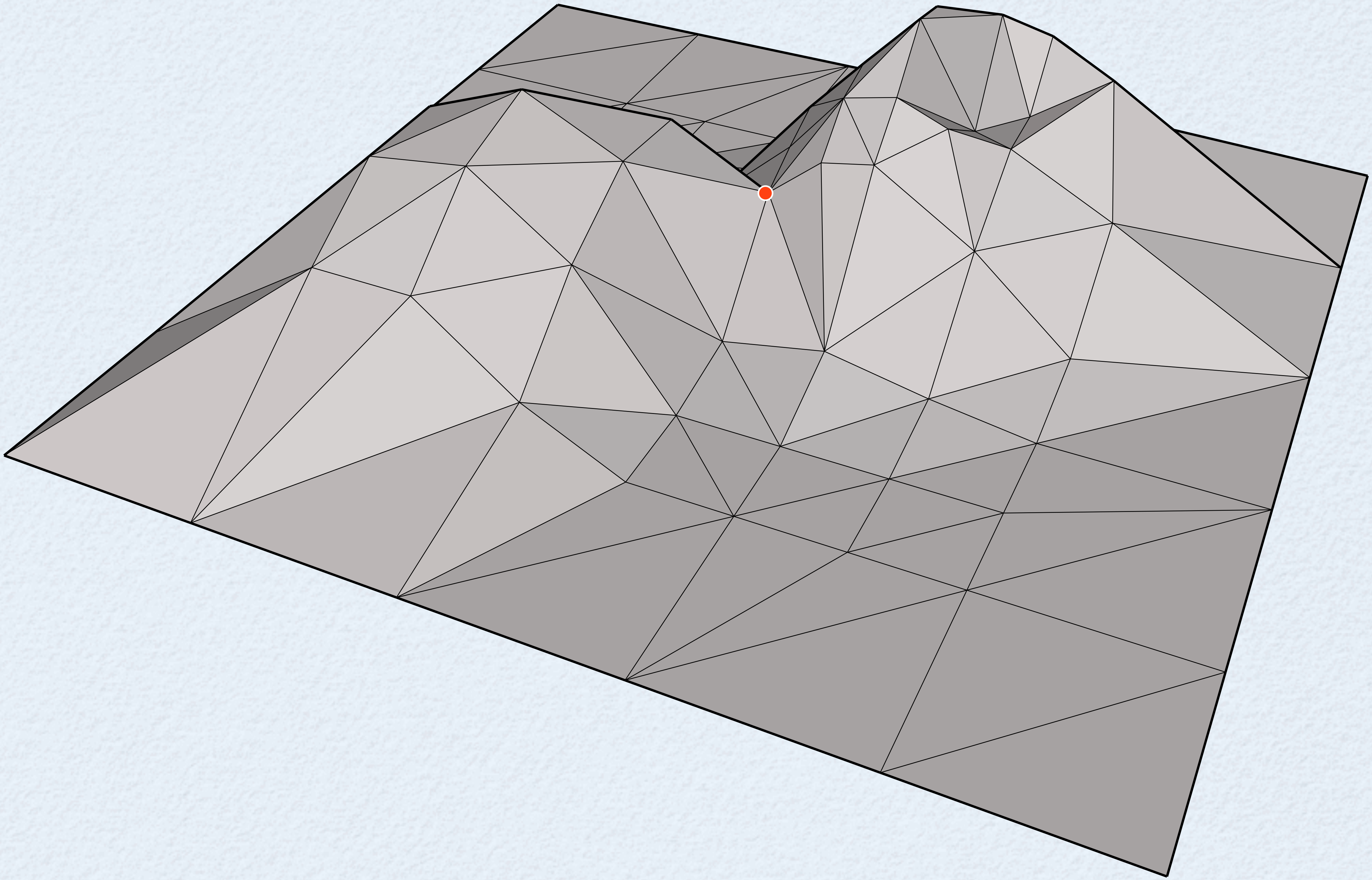
Turning a terrain into an elementary one by surgery



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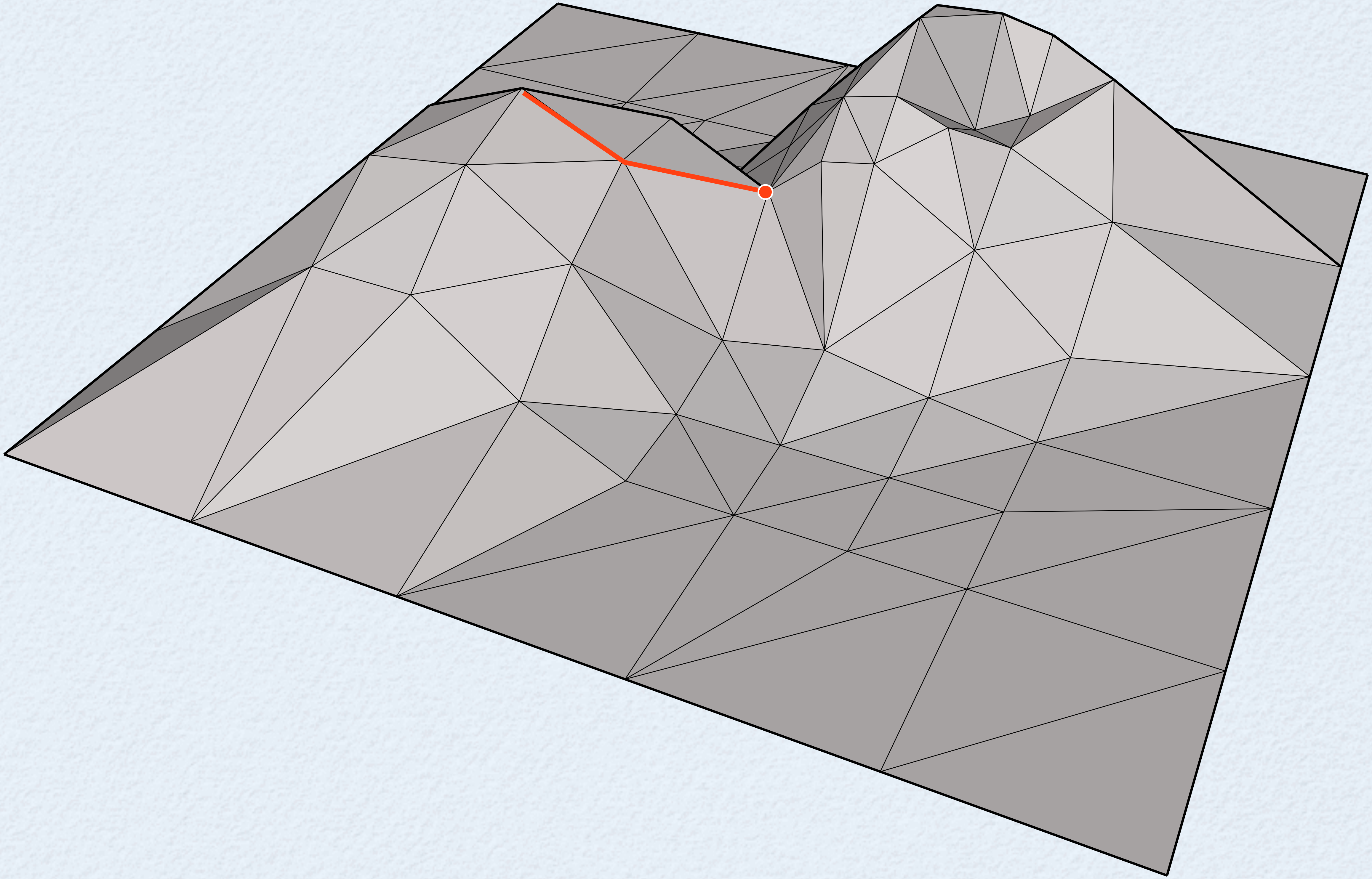

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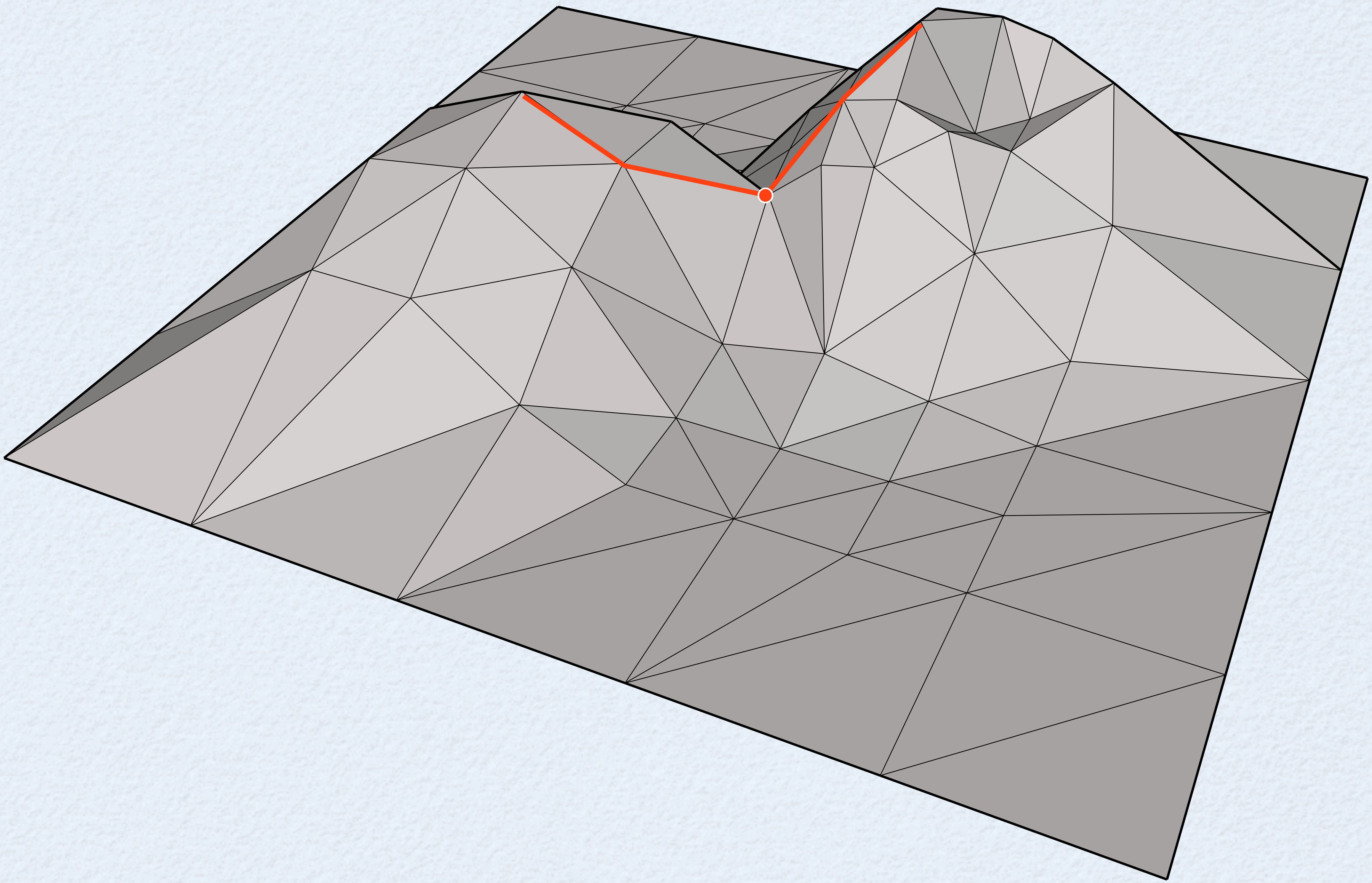

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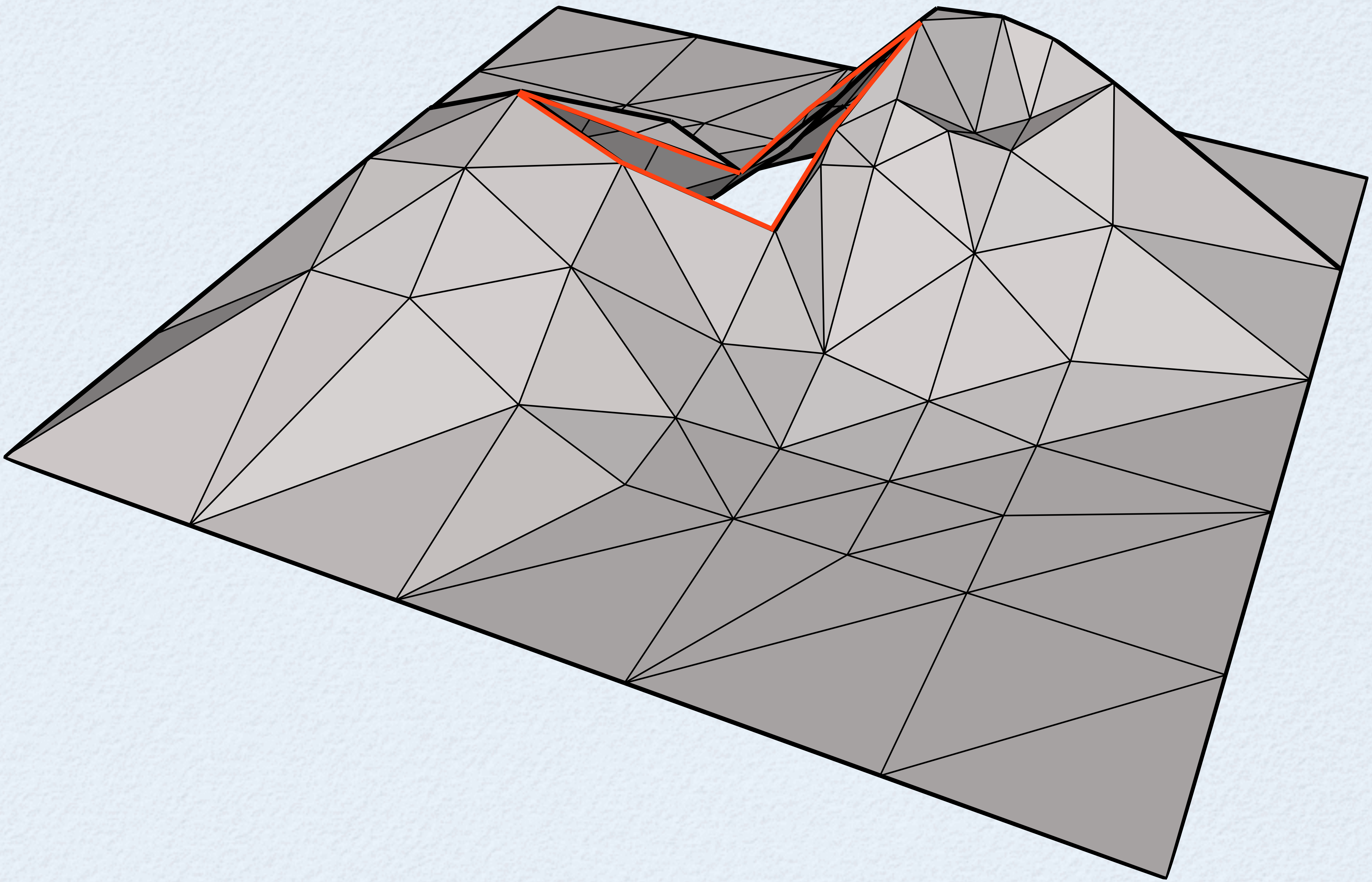

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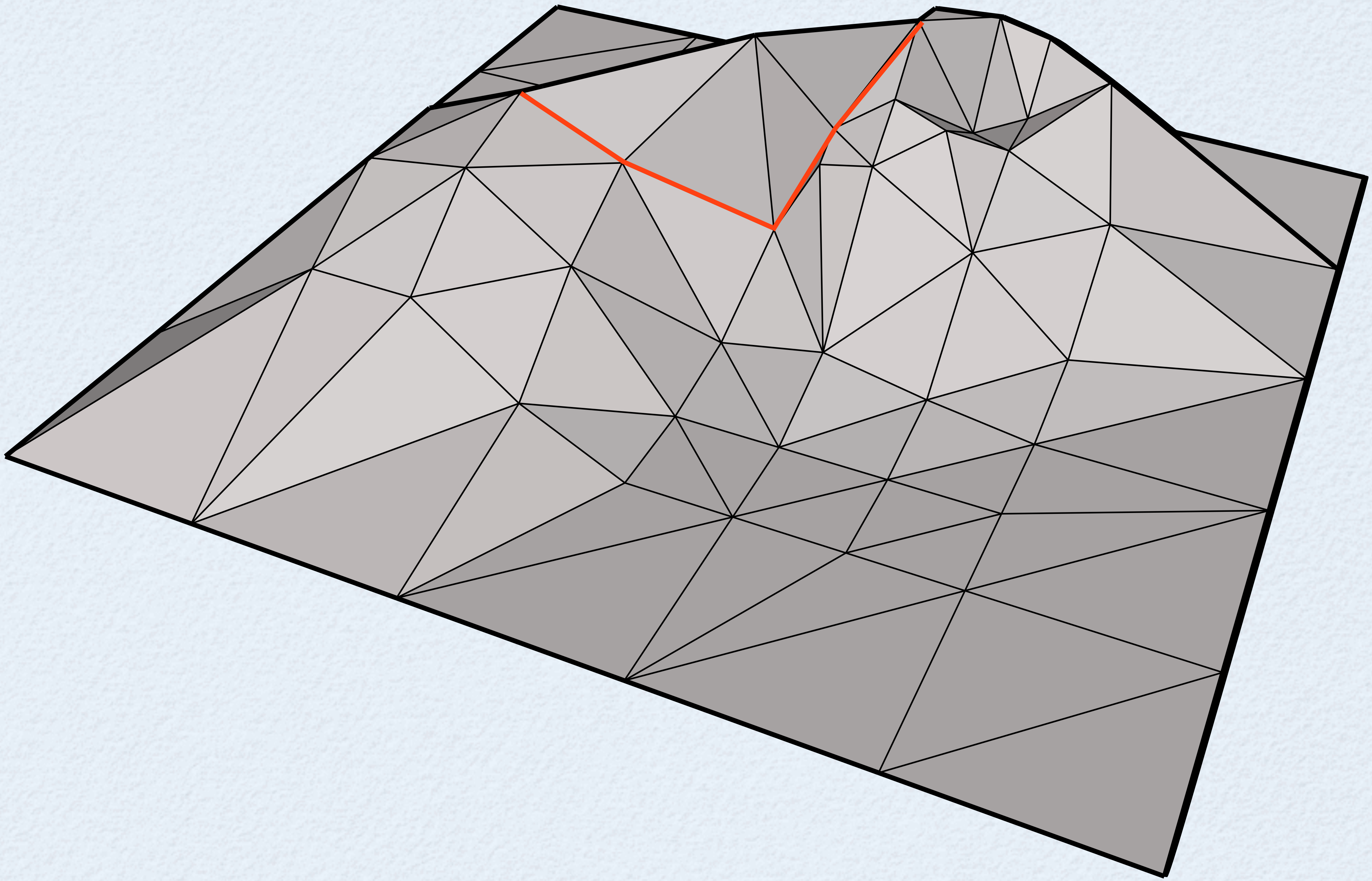

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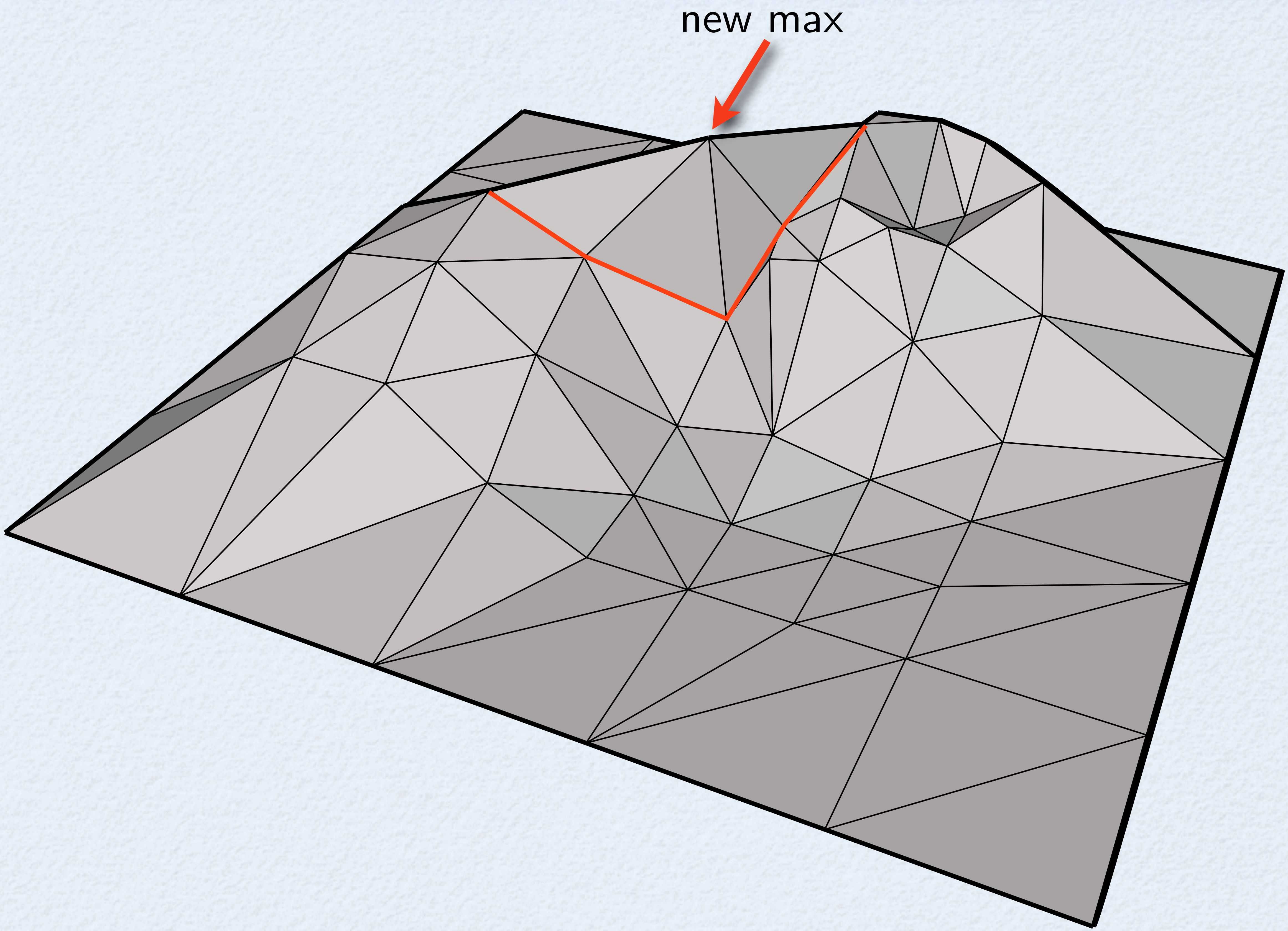

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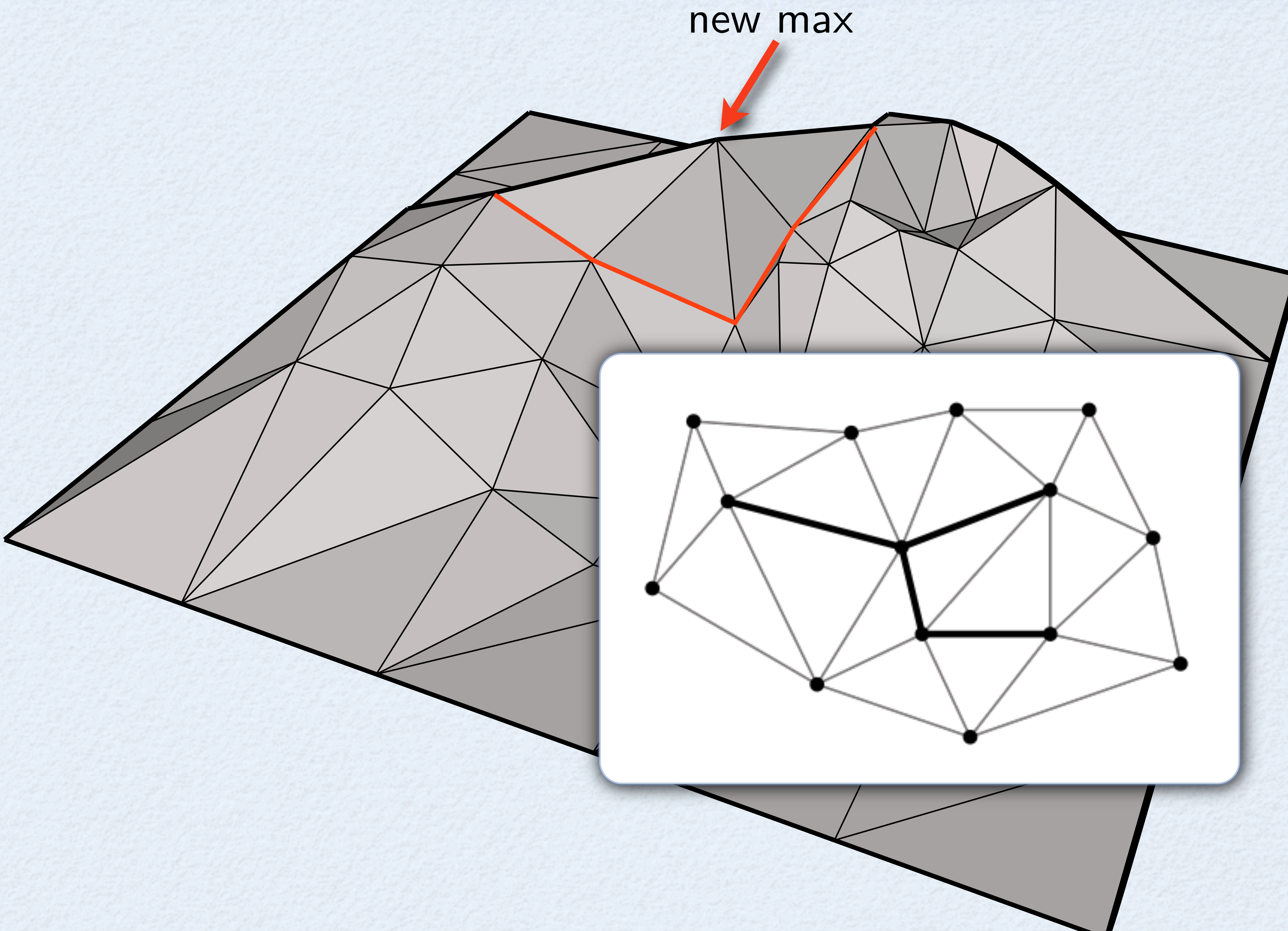

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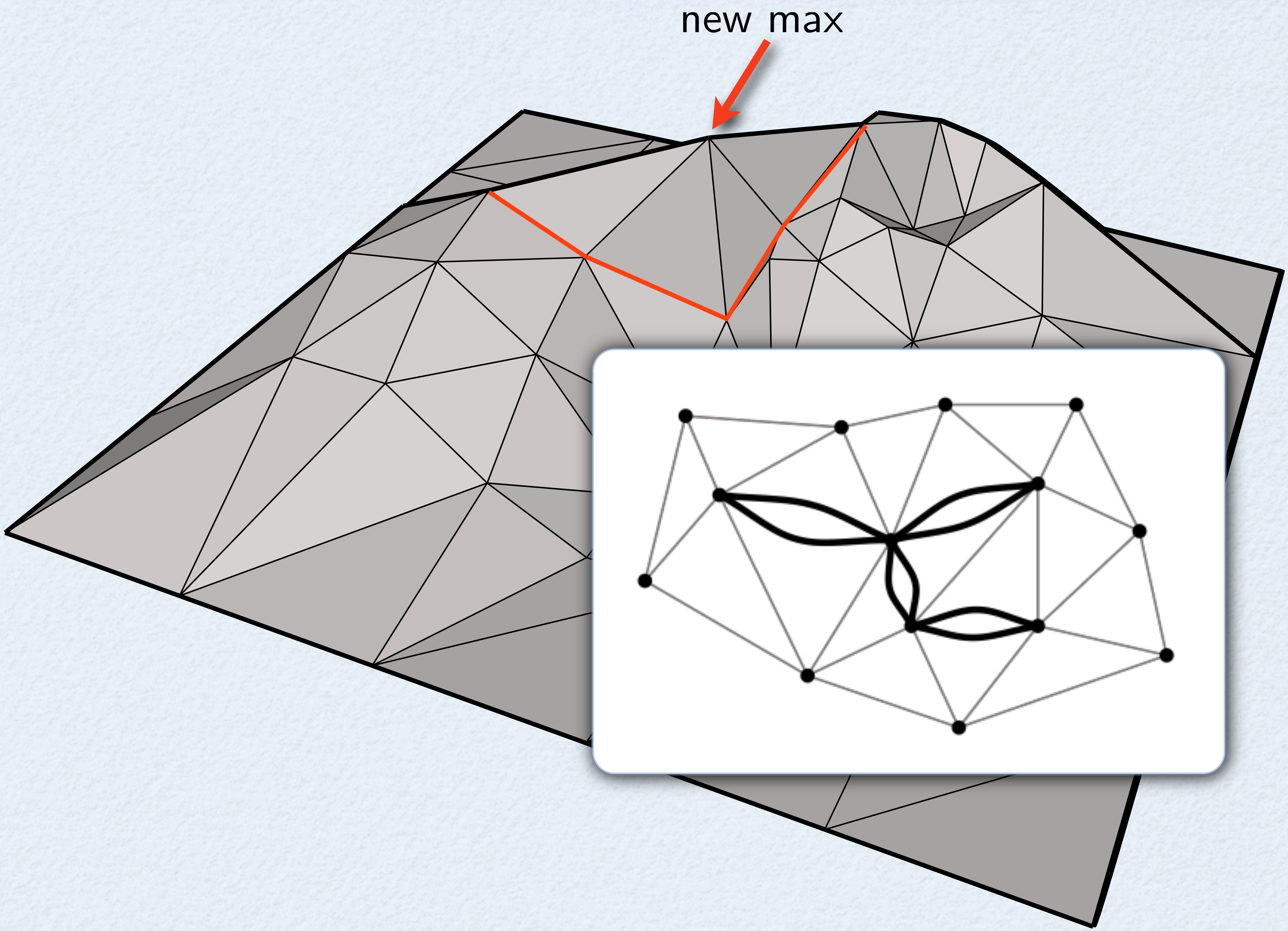

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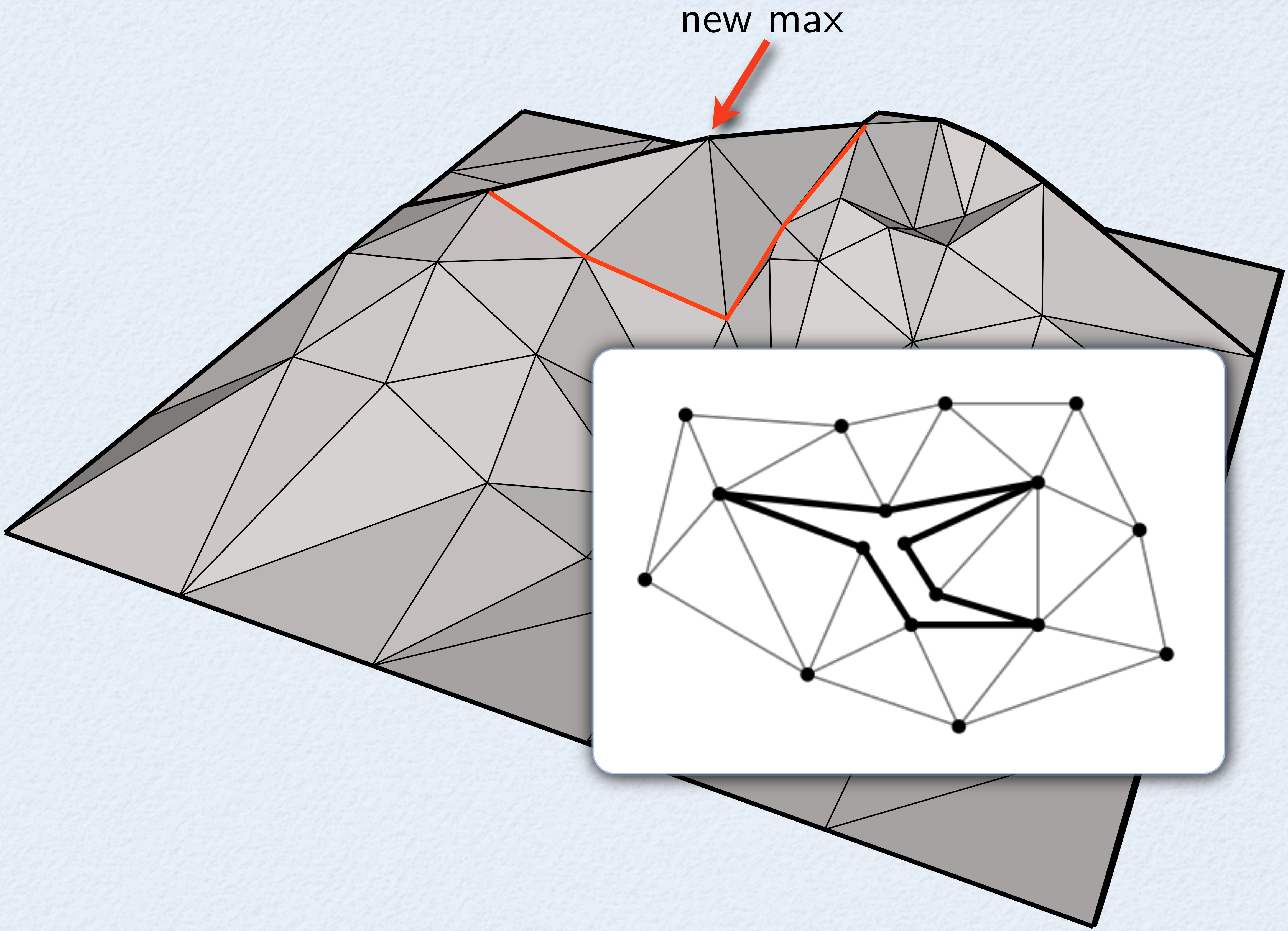

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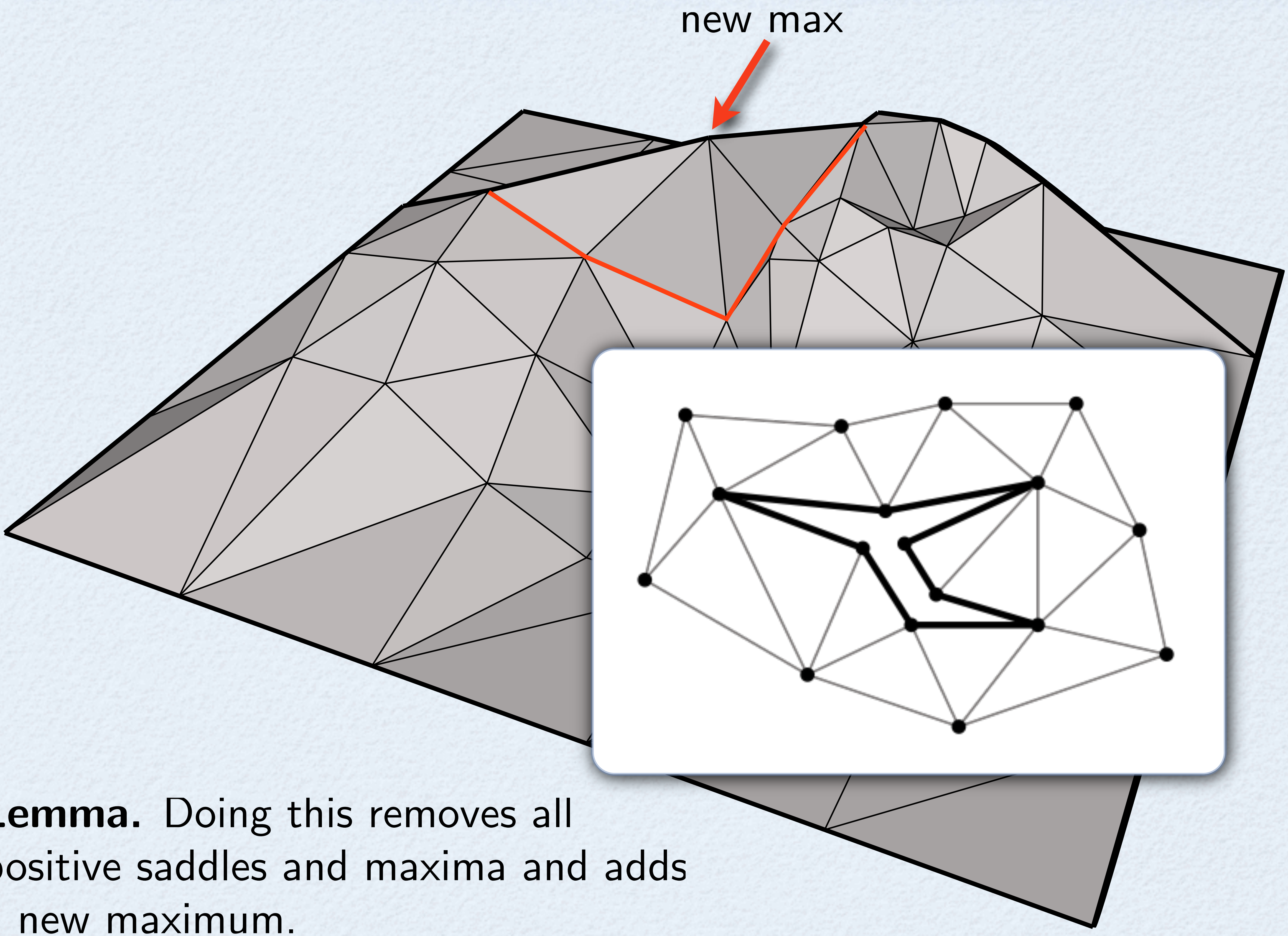
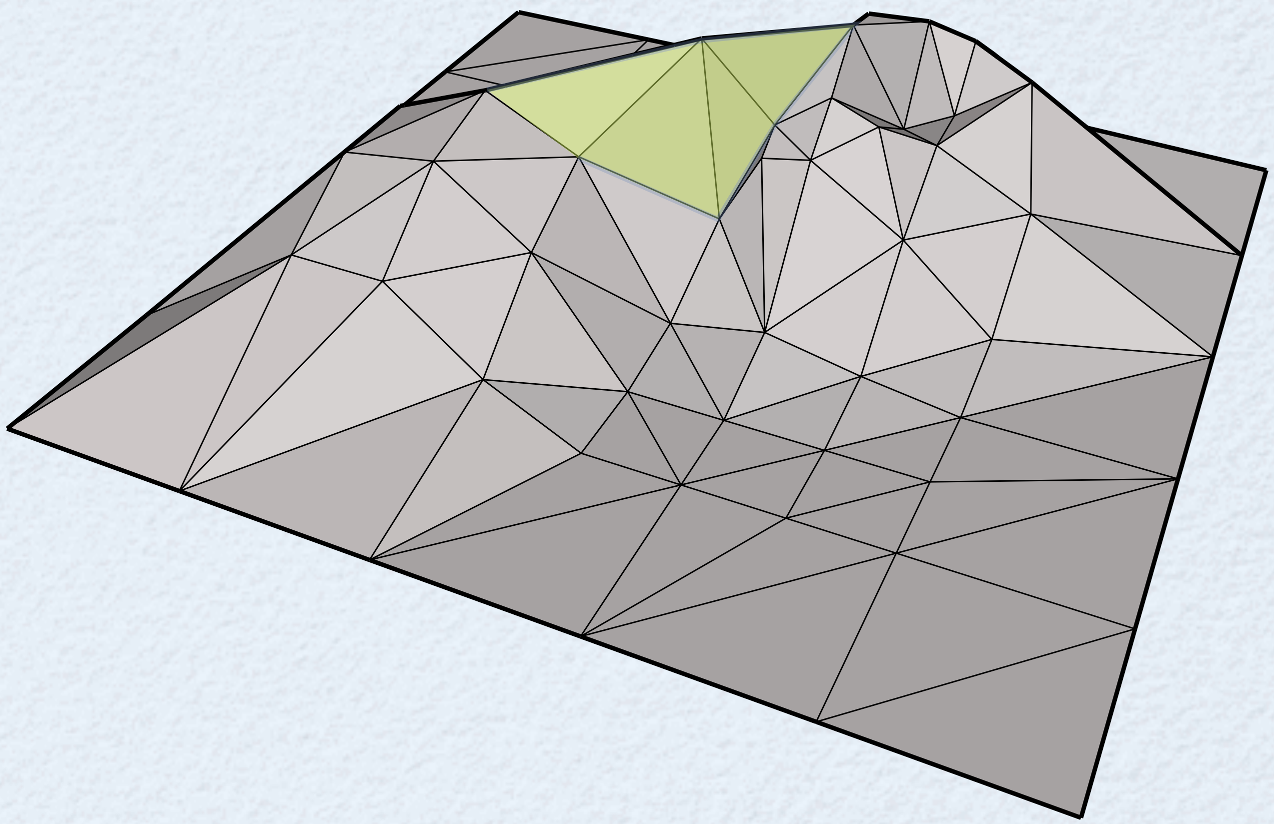


Lemma. Doing this removes all positive saddles and maxima and adds a new maximum.

What surgery does to contours?

The elementary terrain $\mathbb{M}'$ has all the triangles of $\mathbb{M}$ plus some of "new" triangles.



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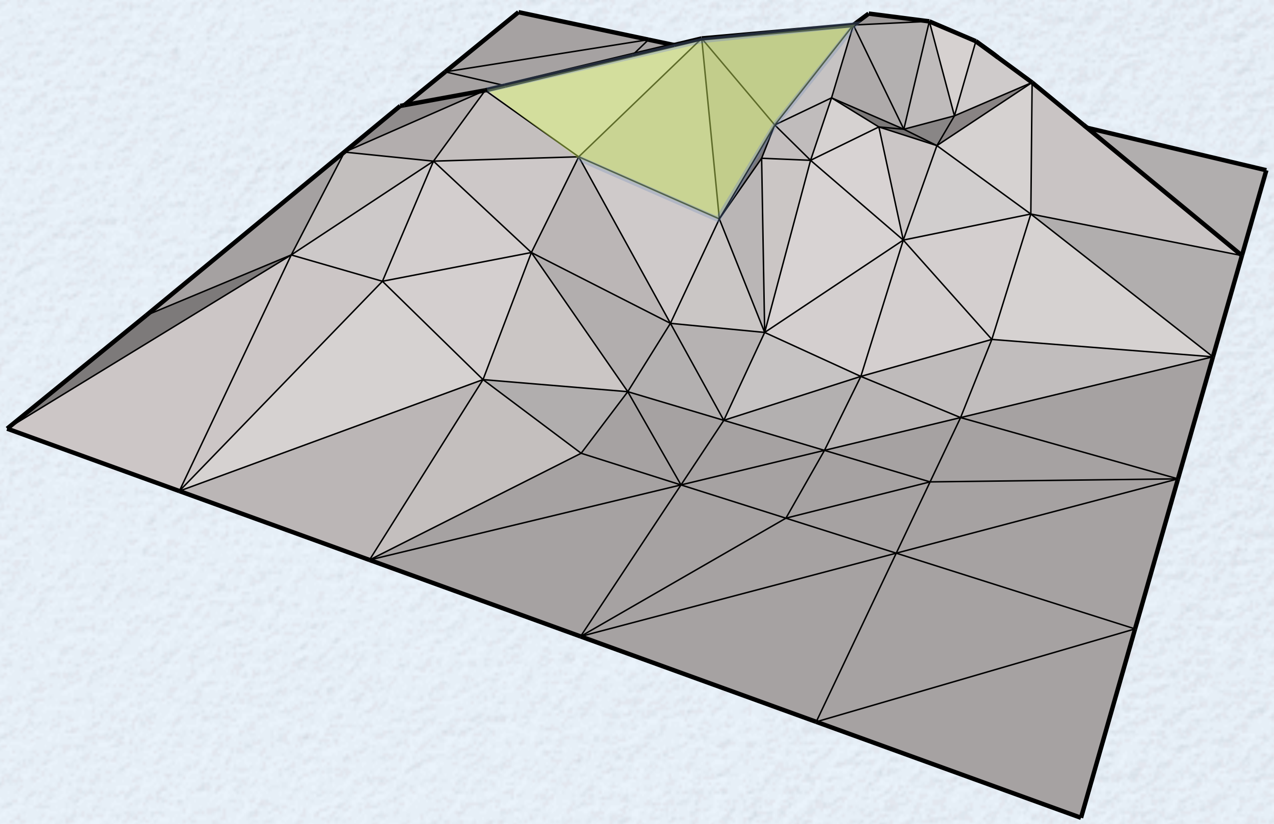

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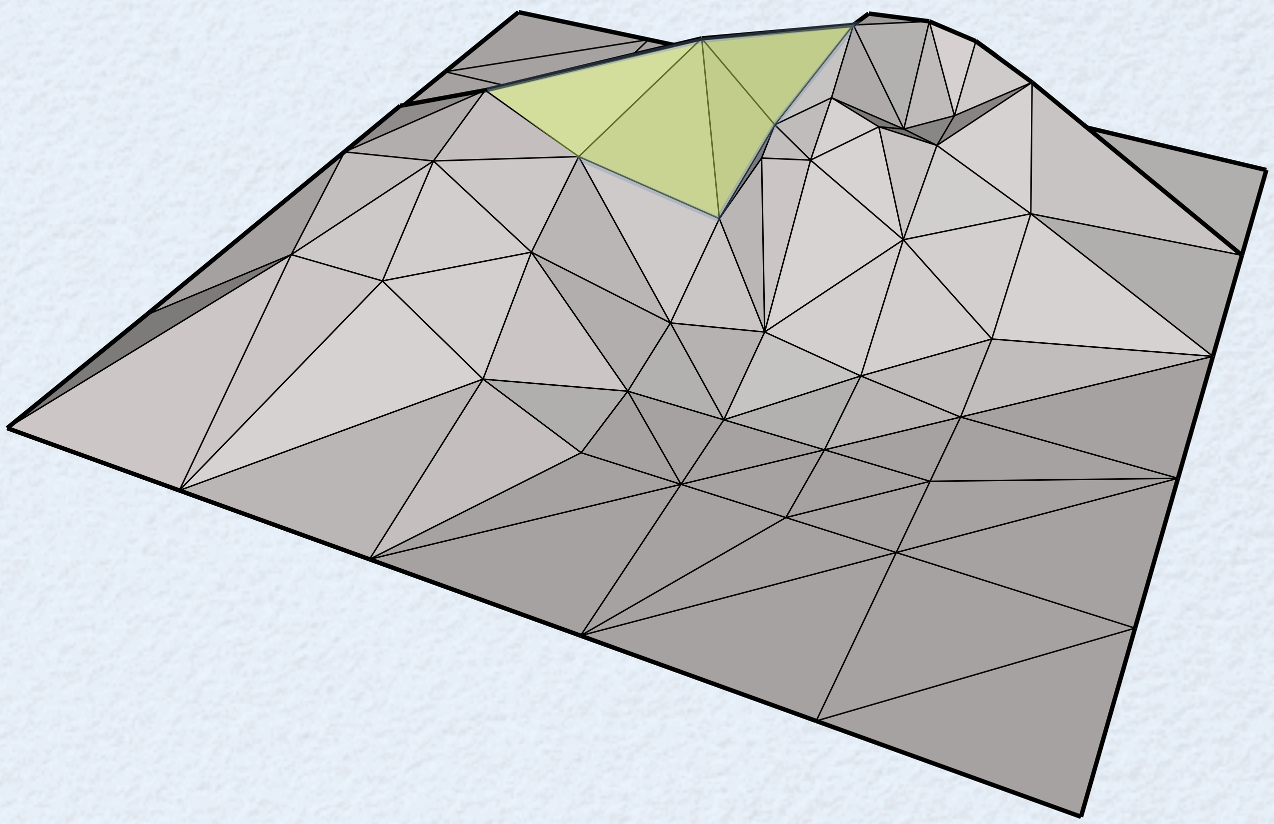

Theorem. In a contour of $\mathbb{M}'$, corresponding contours of $\mathbb{M}$ are broken (by segments from new triangles) in a nested (parenthesized) manner.

$$a_1 a_2 * * b_1 * * * c_1 c_2 c_3 * * * * b_2 b_3 * d_1 d_2 * * b_4 * * a_3 a_4 a_5$$

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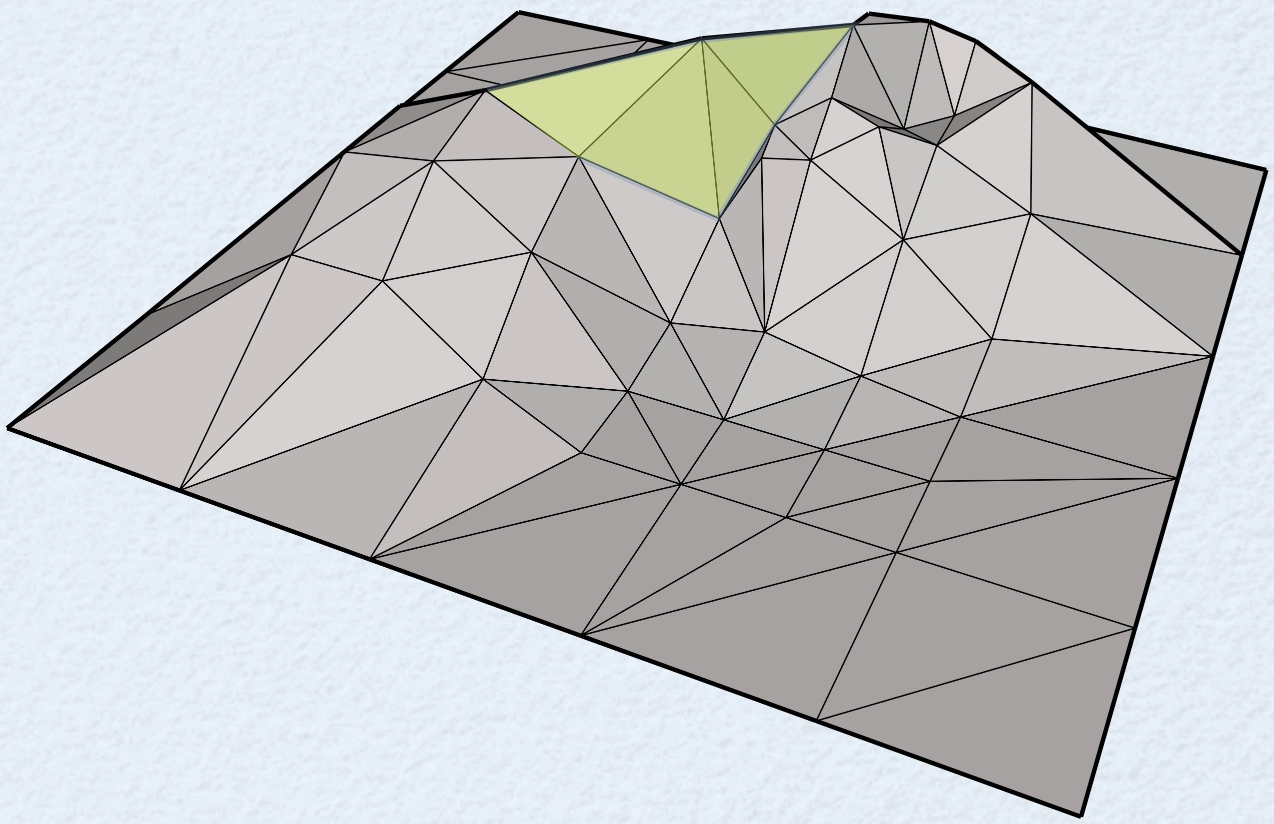

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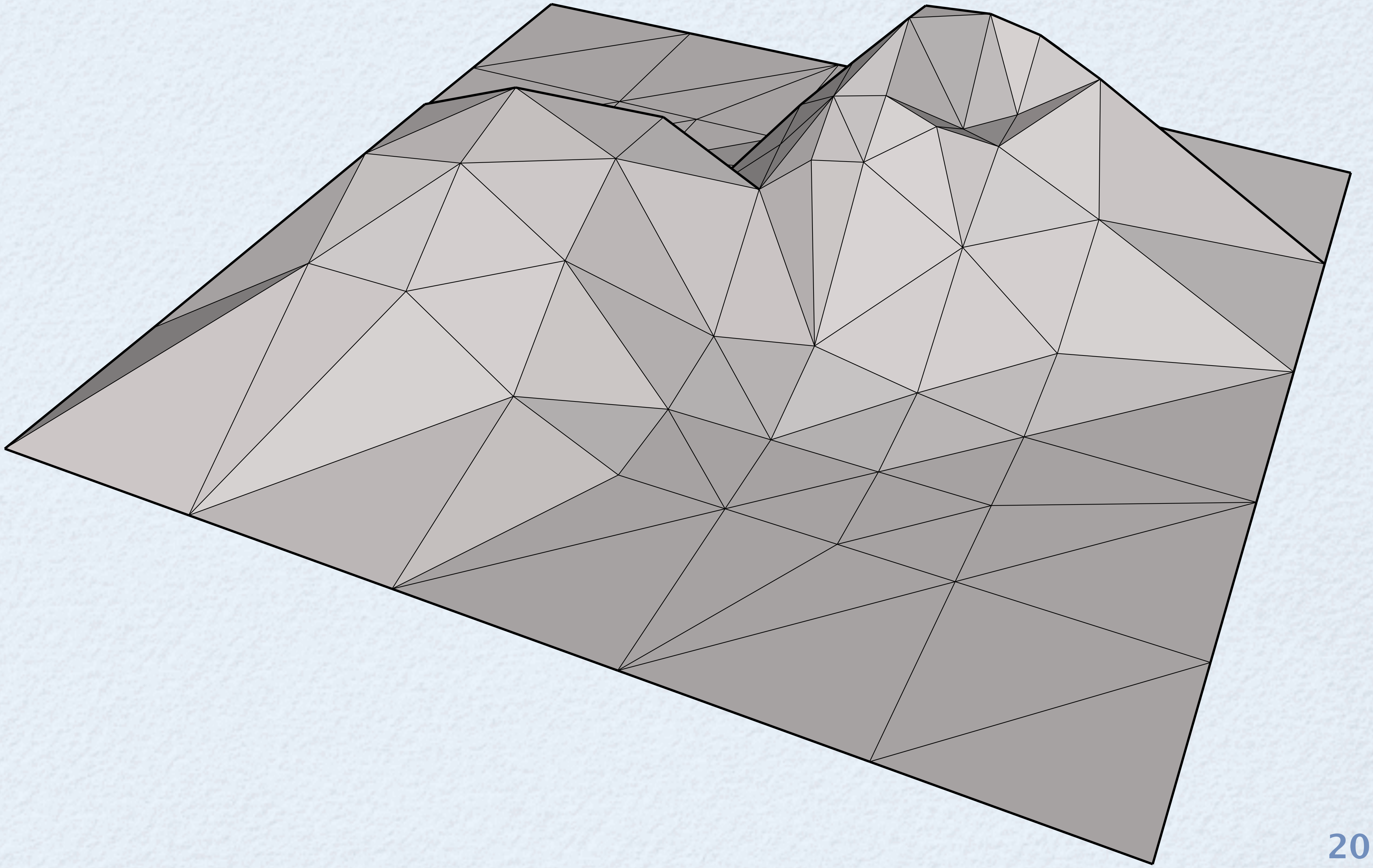

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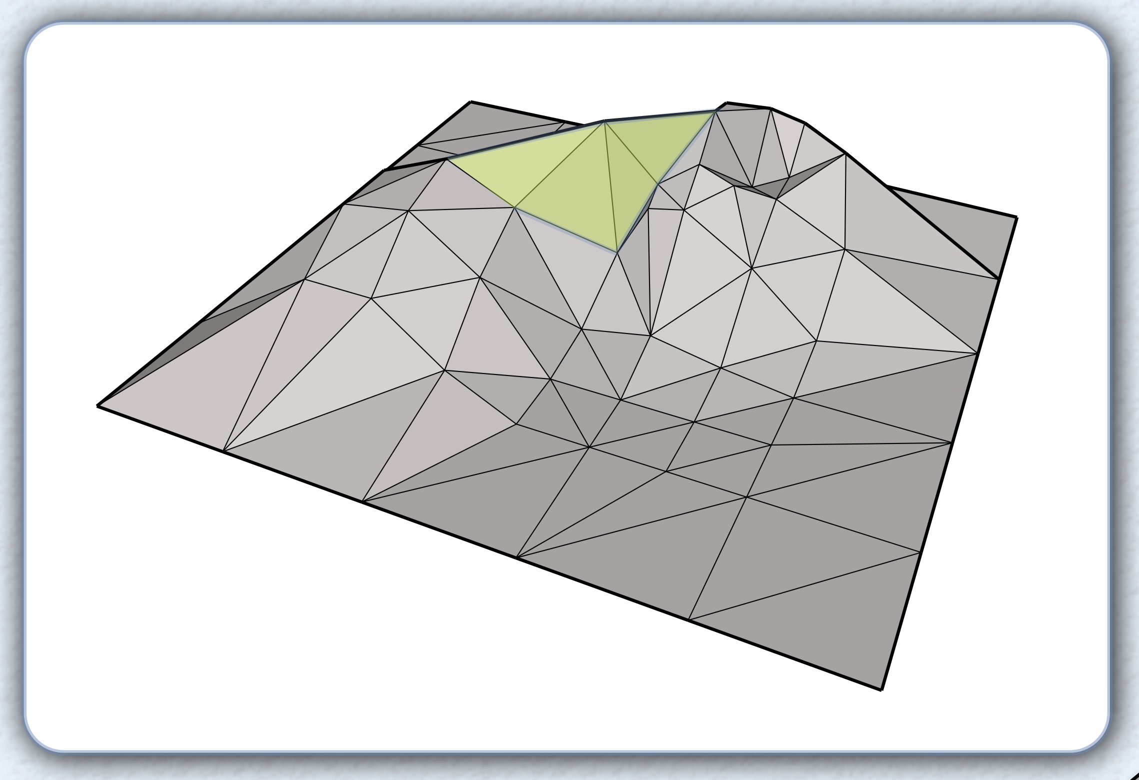

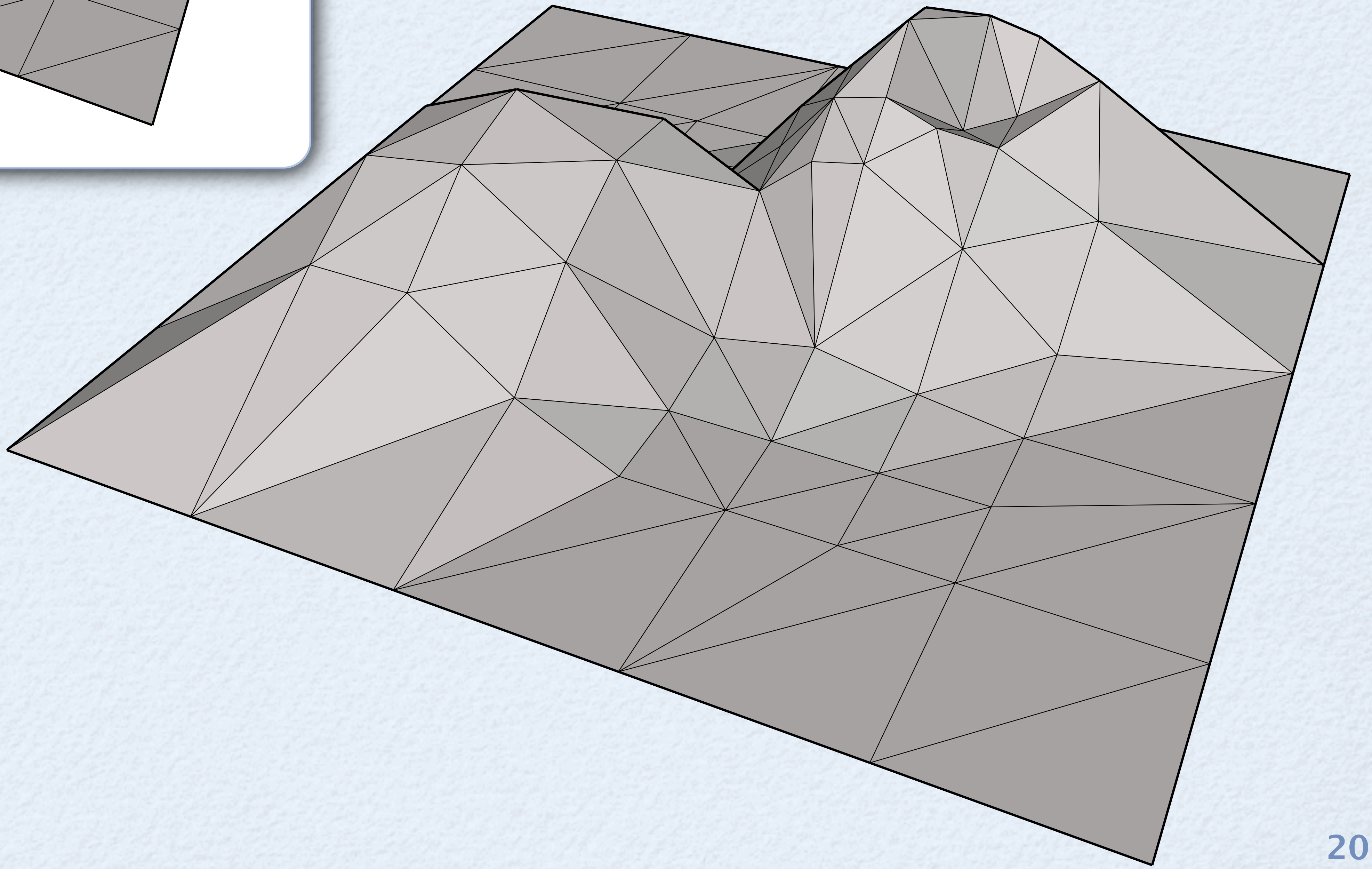

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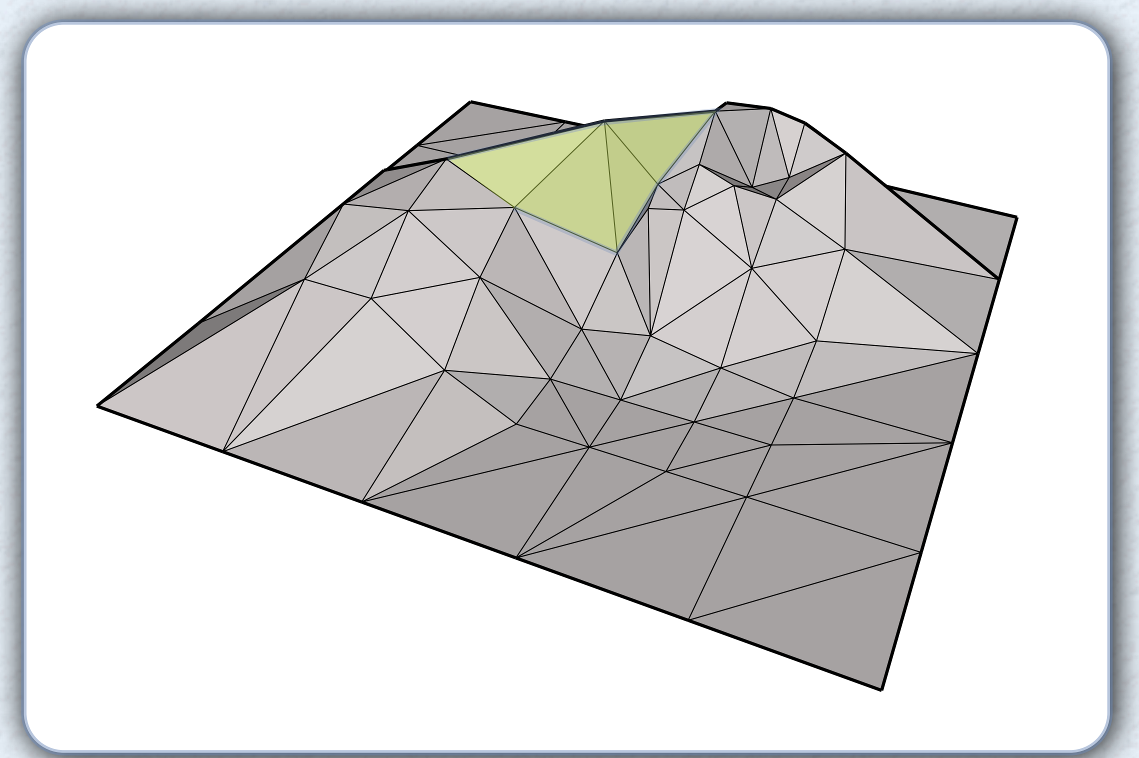

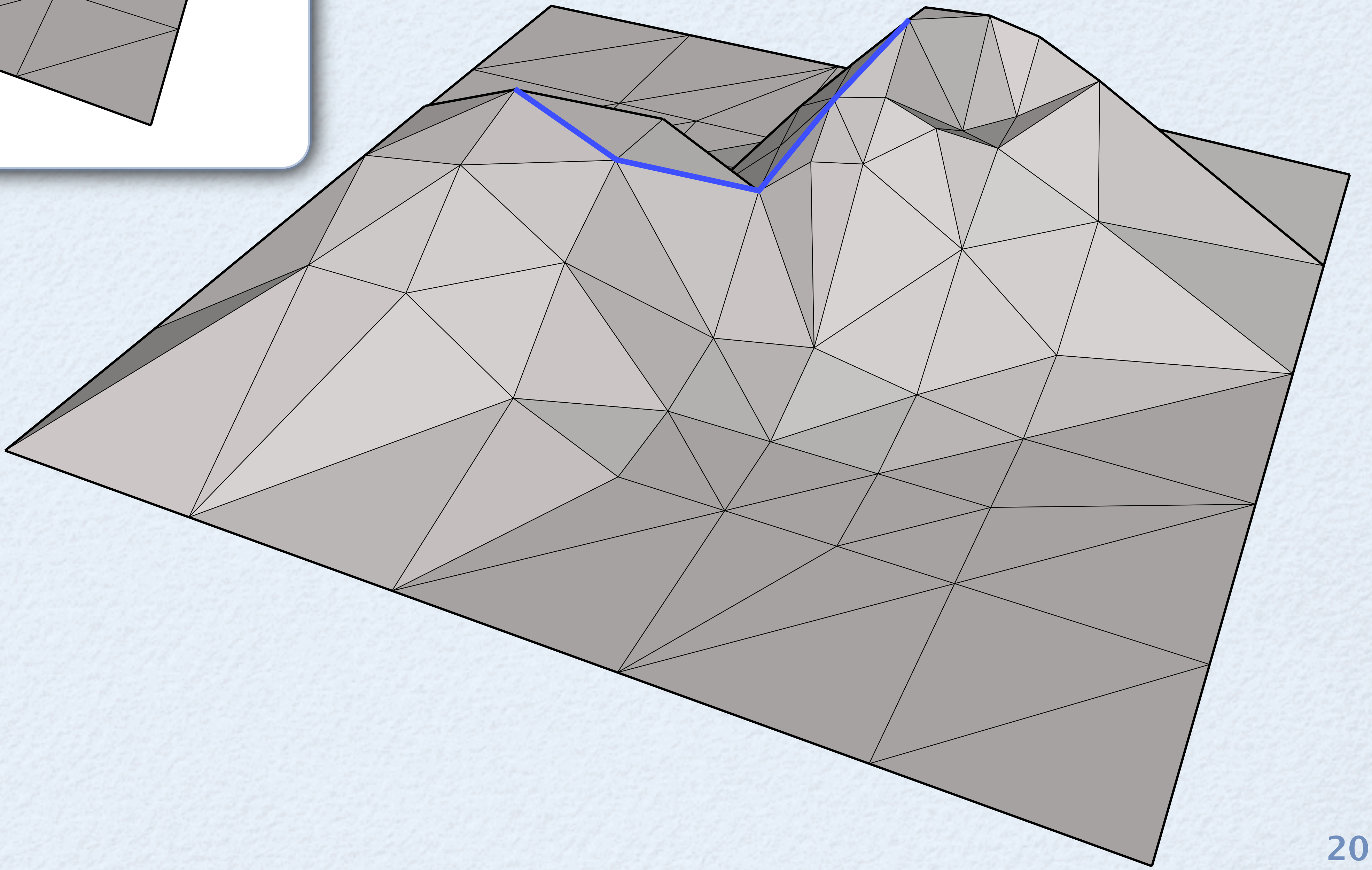

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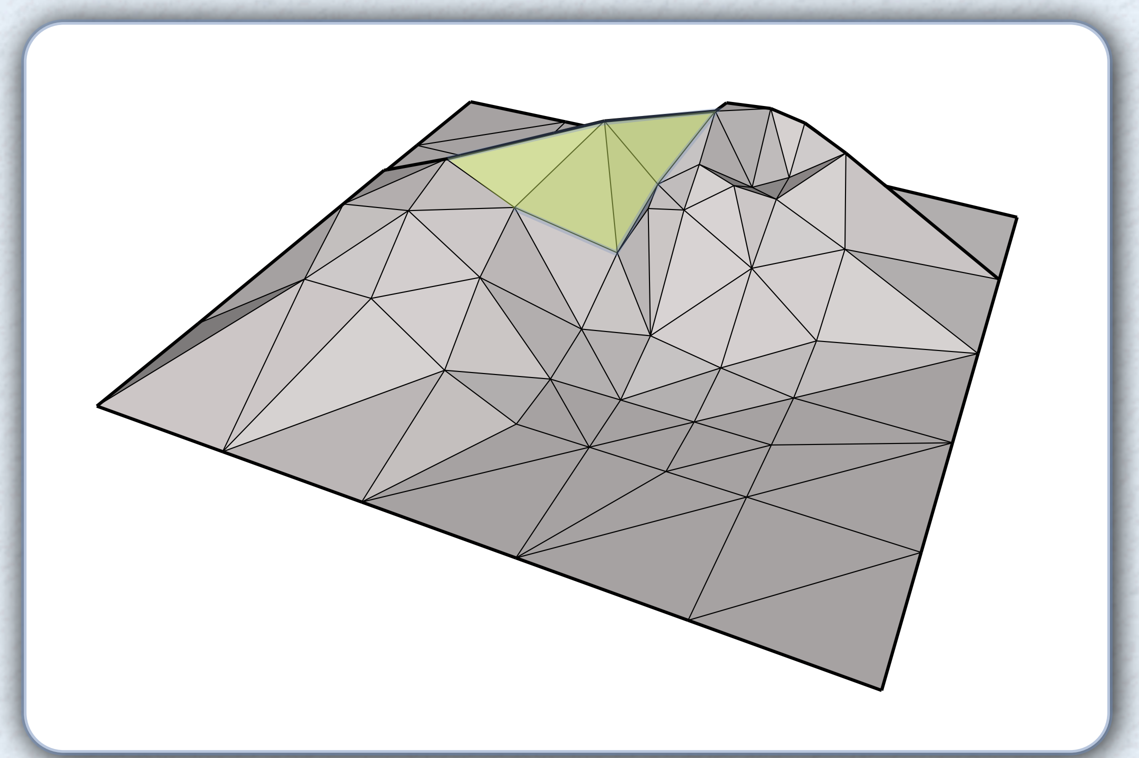

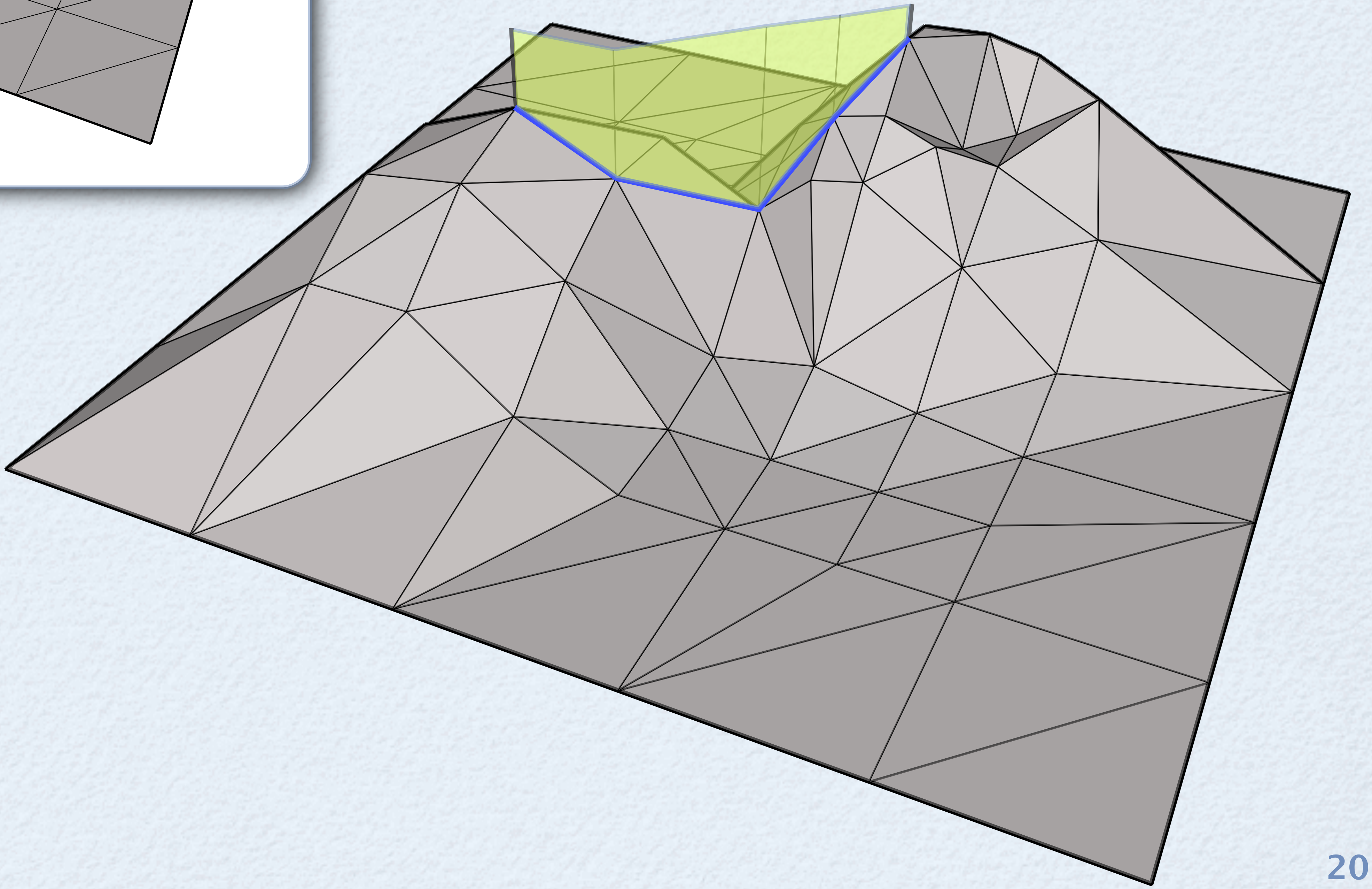

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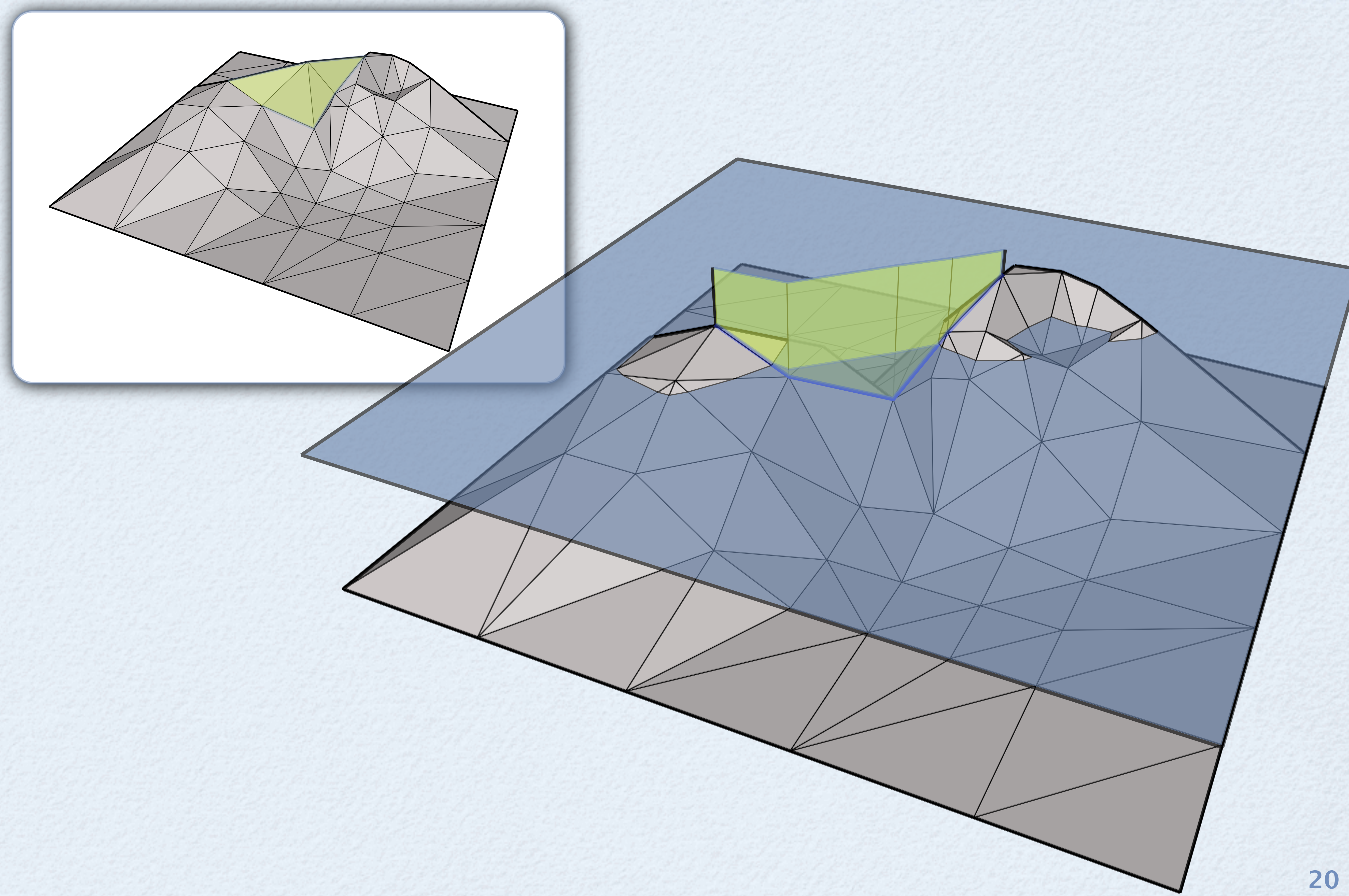

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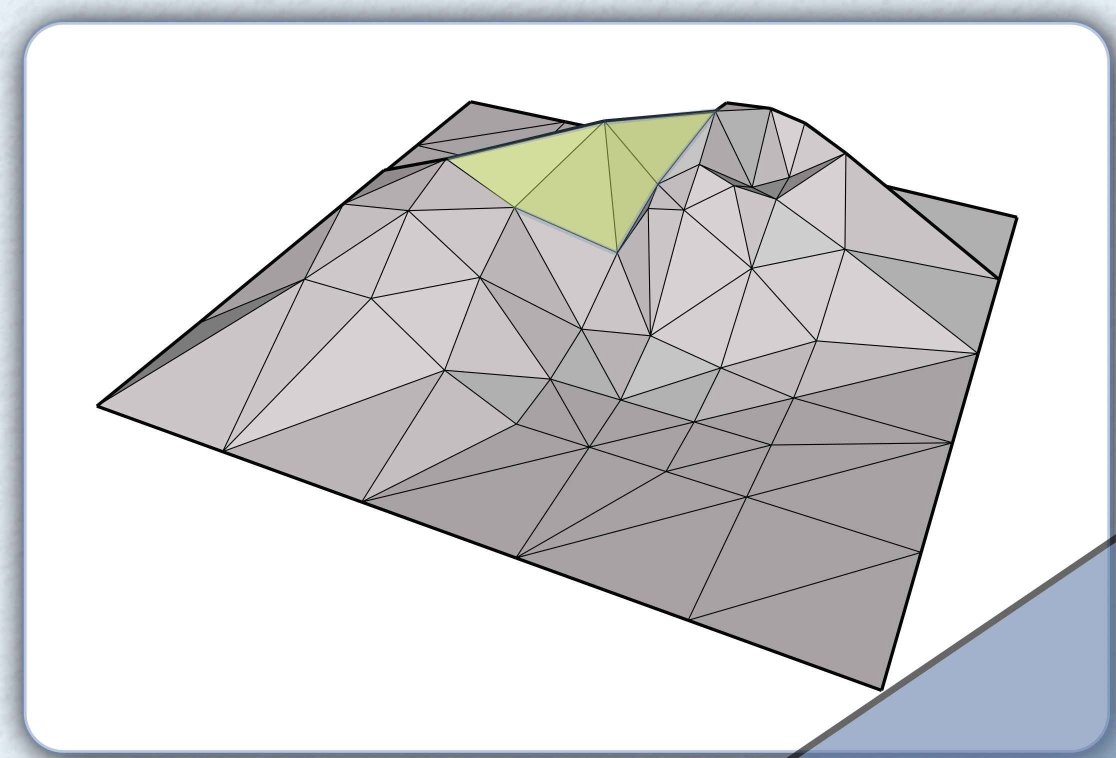
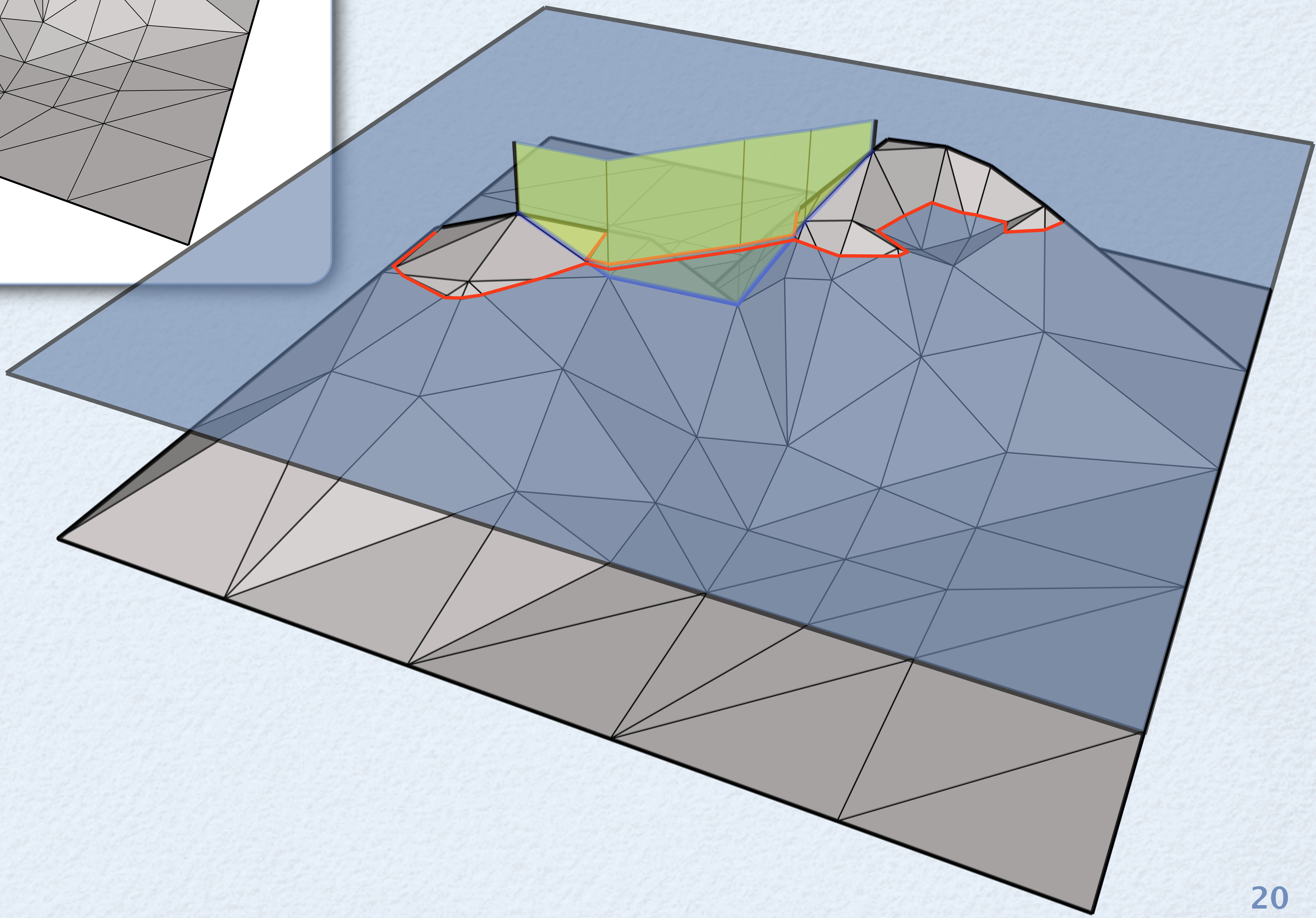
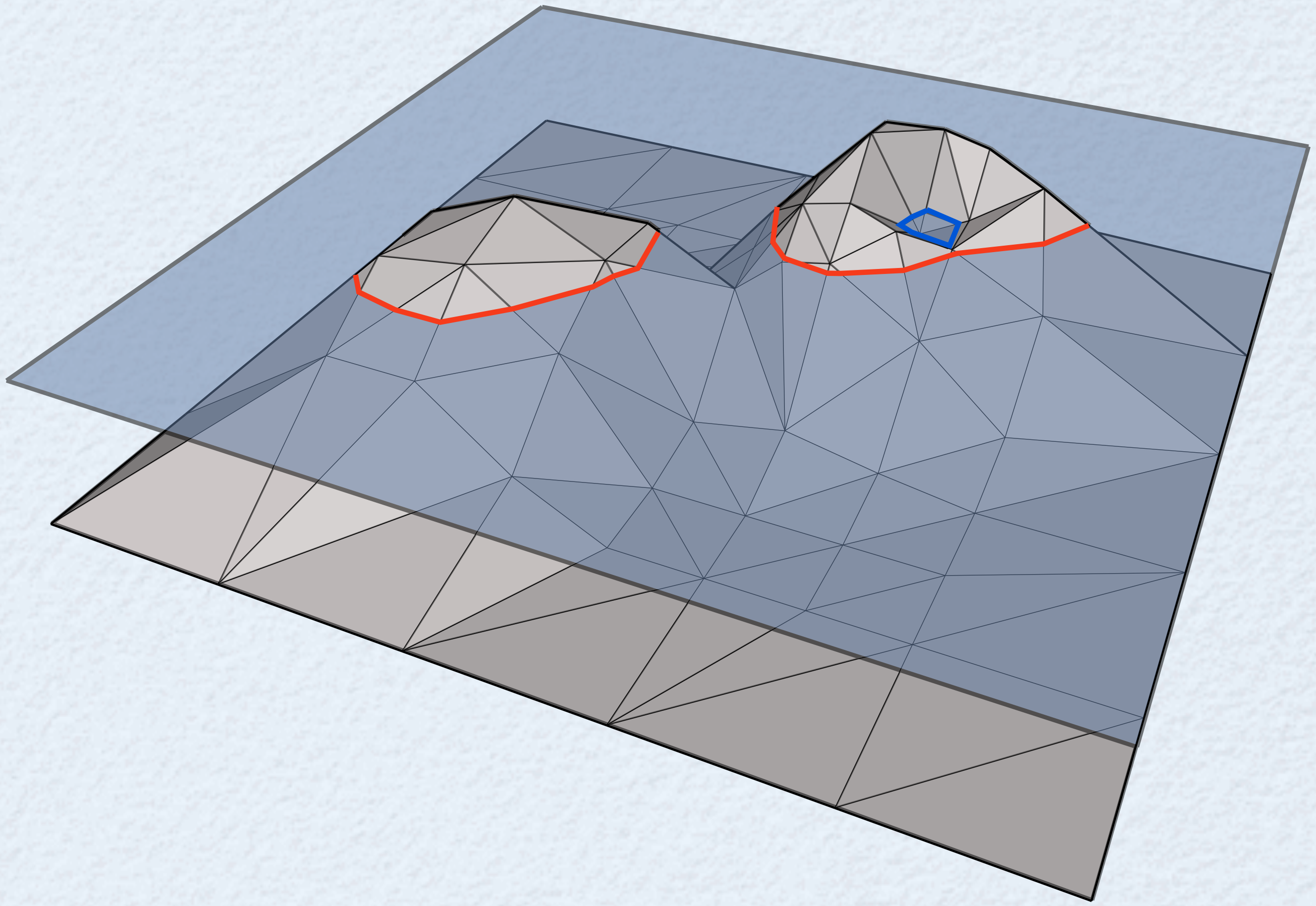




Red and Blue Contours

A contour is red (blue) if "locally" the sublevel set is in its outside (inside).



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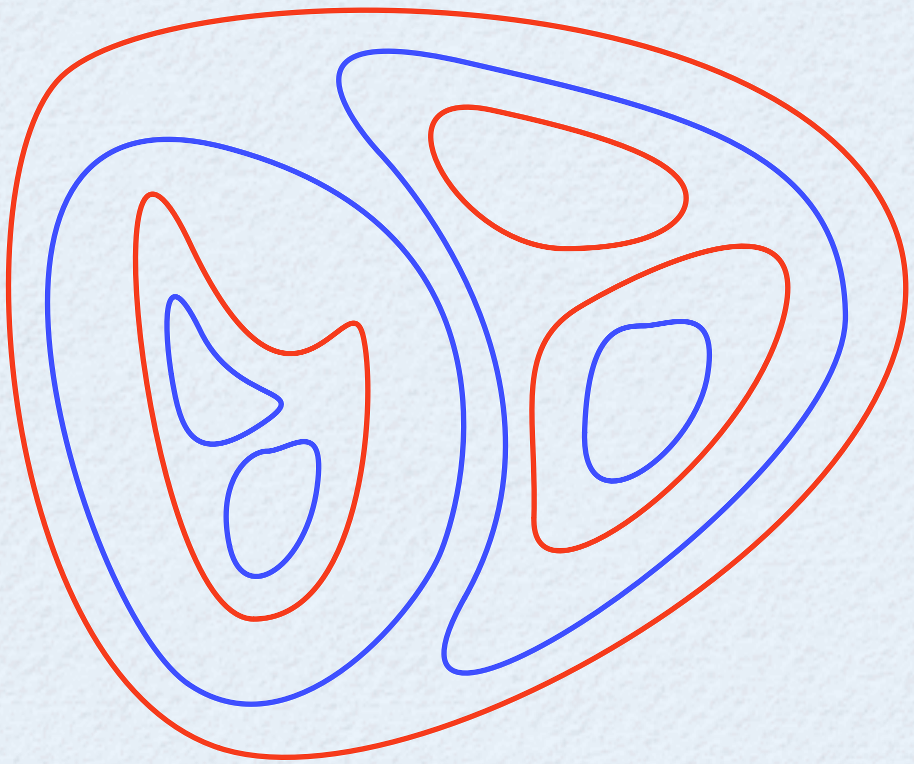

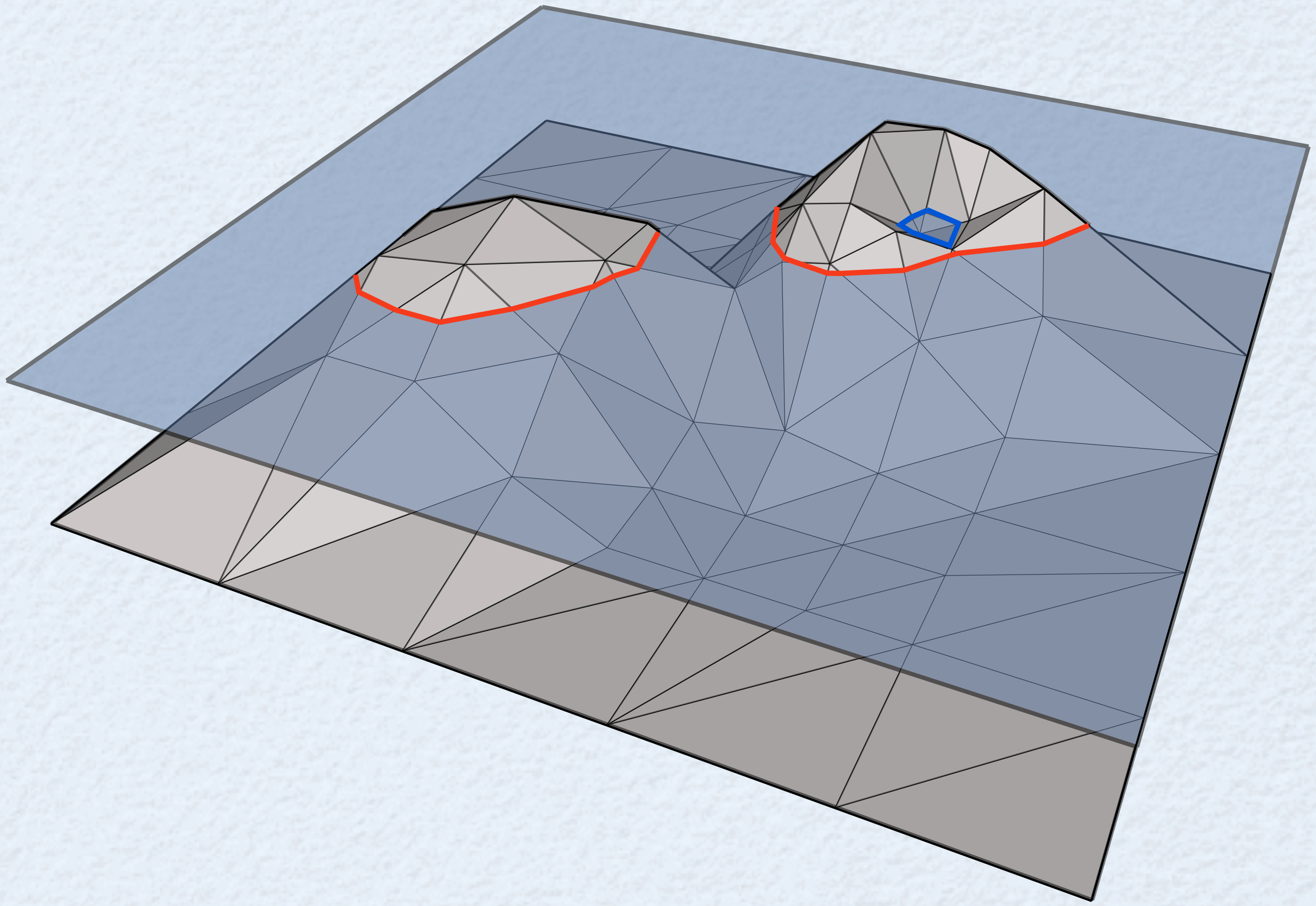

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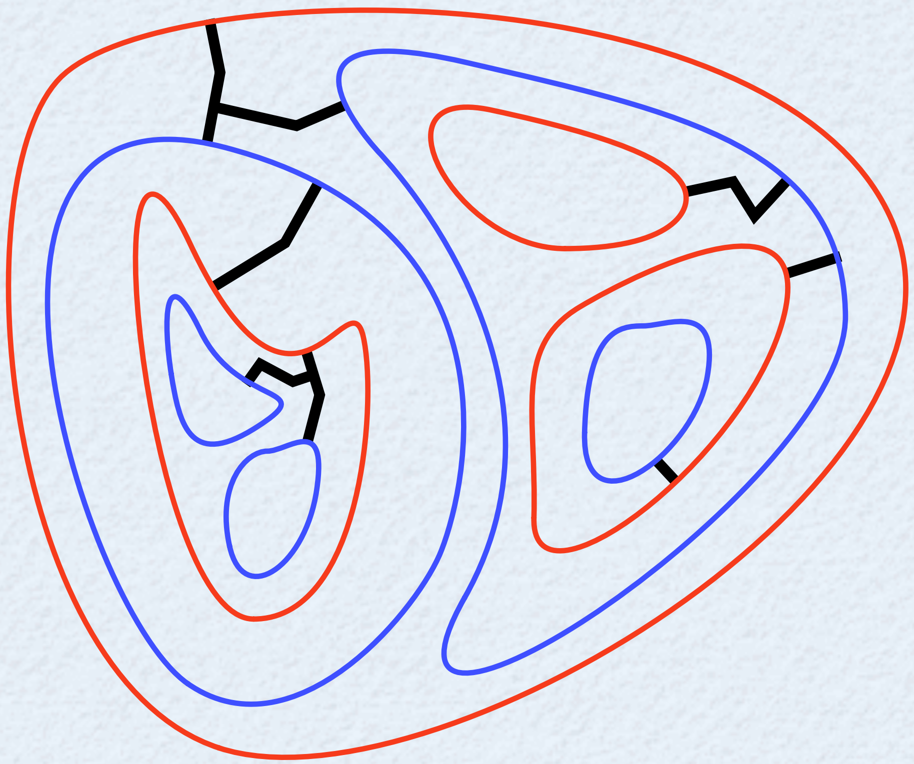

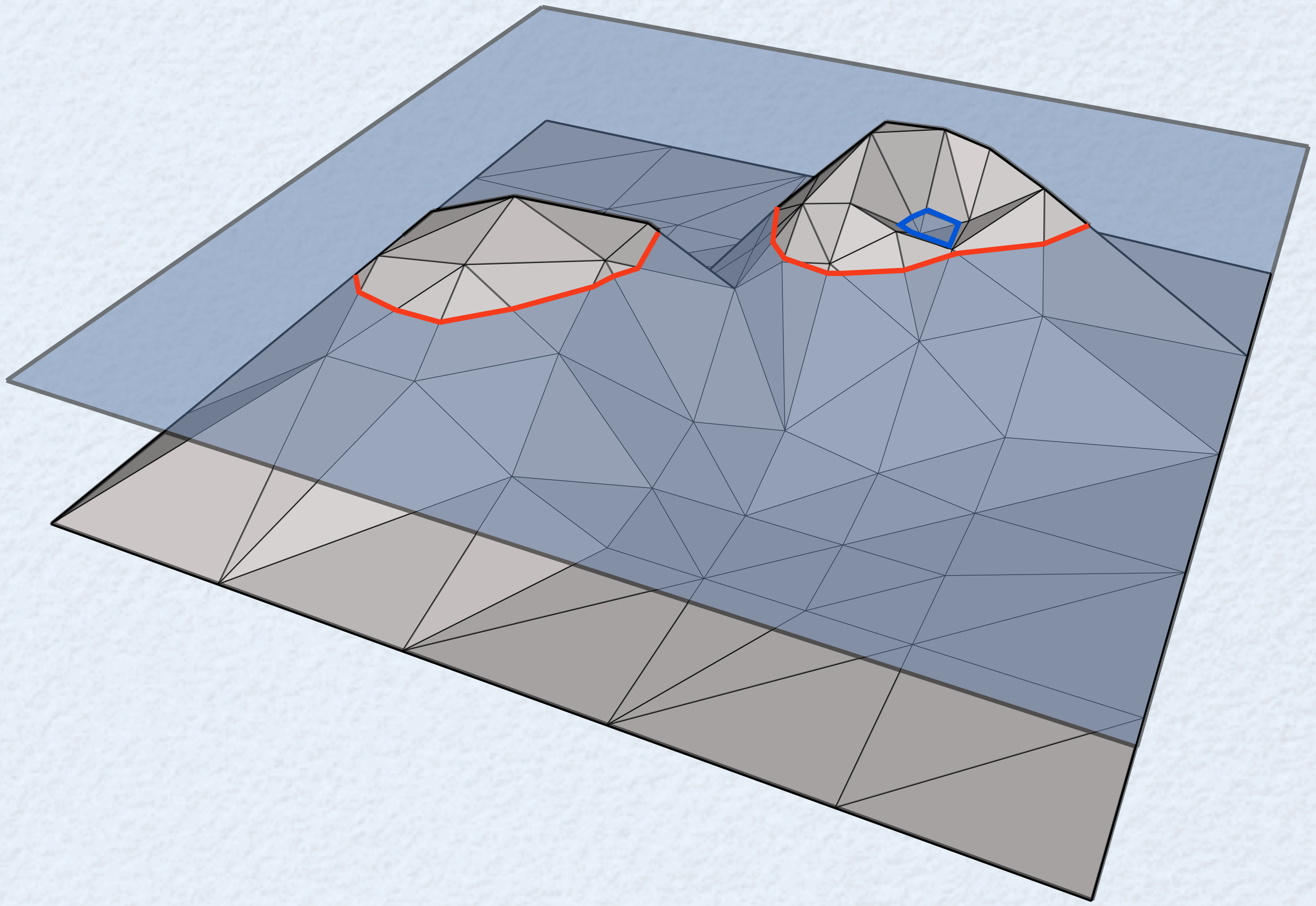

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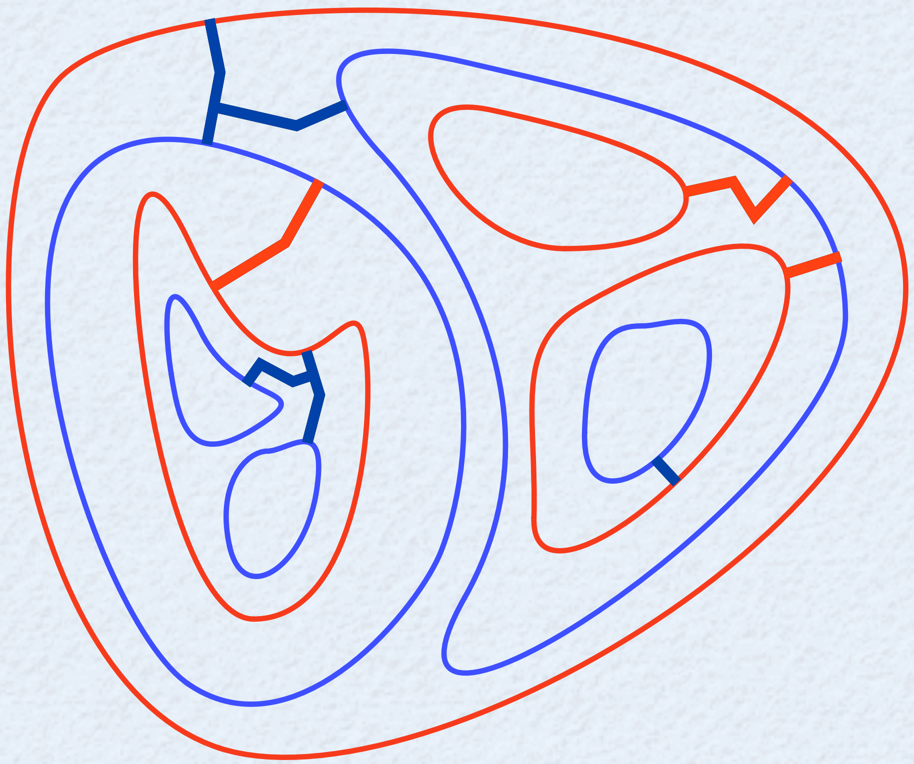

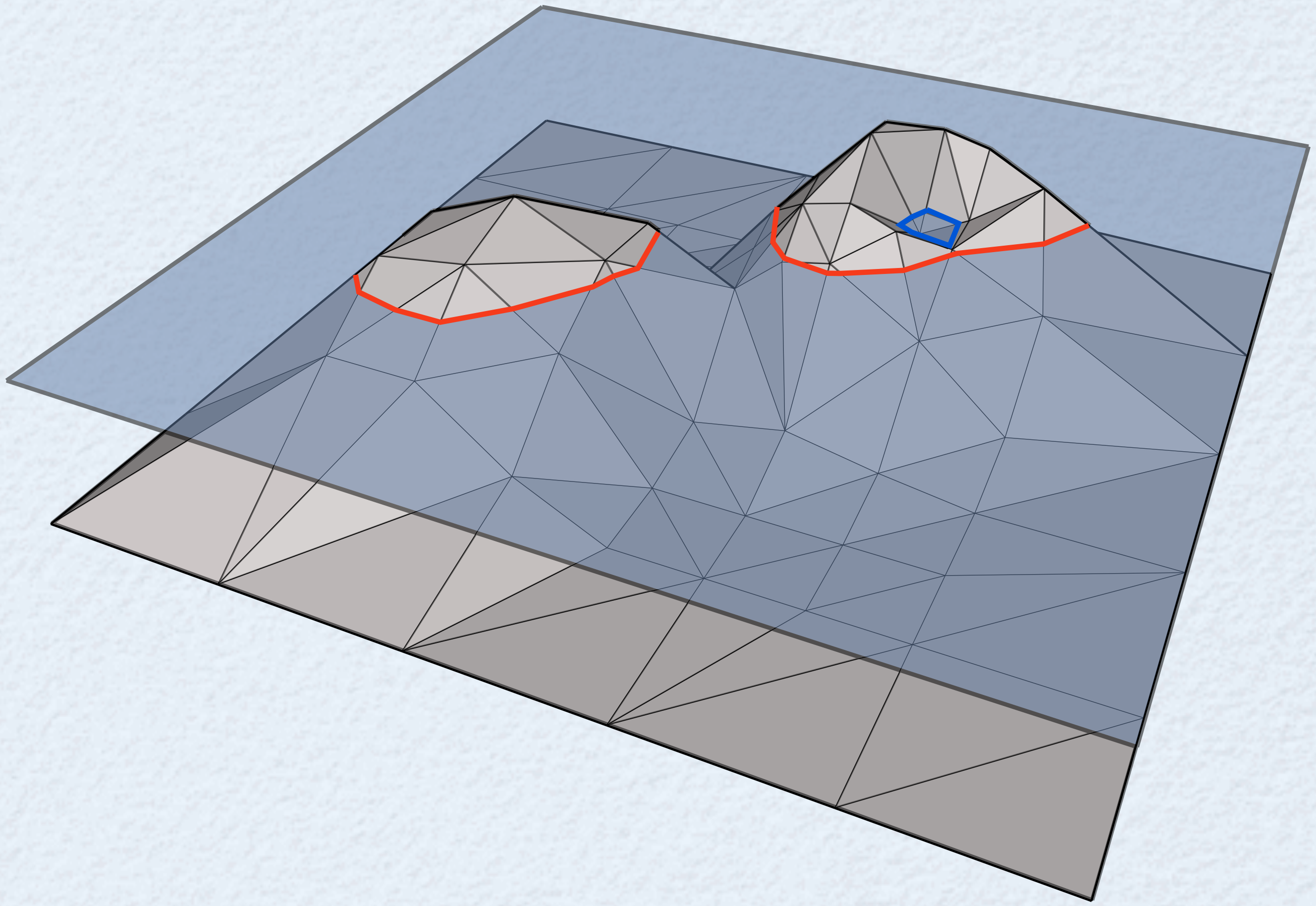

Theorem. Only the red tree connects a blue contour and its red children.

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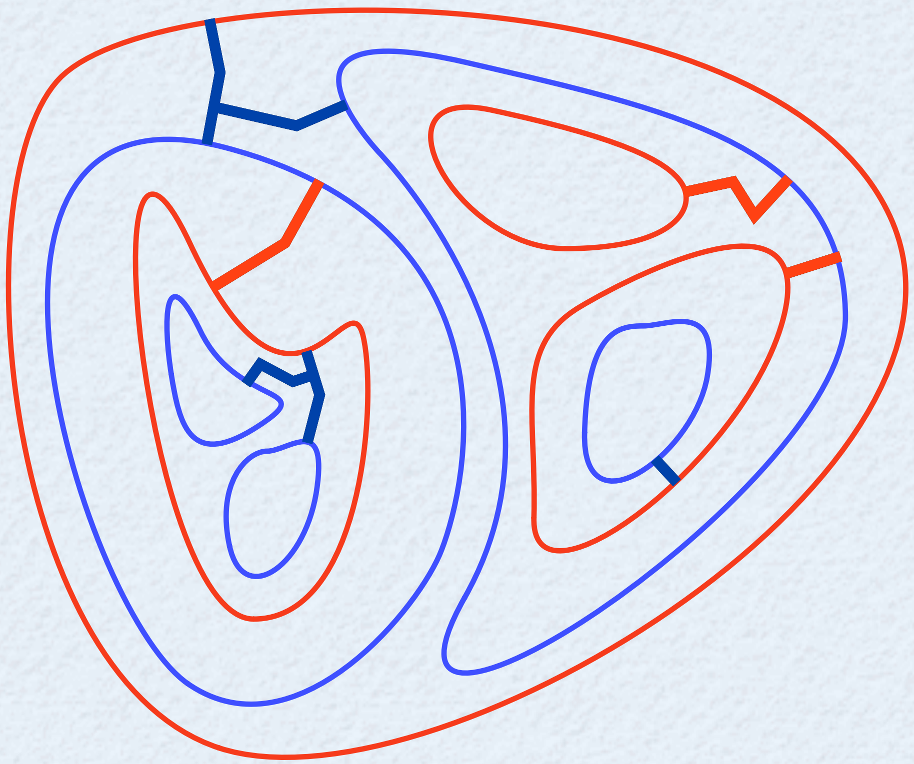

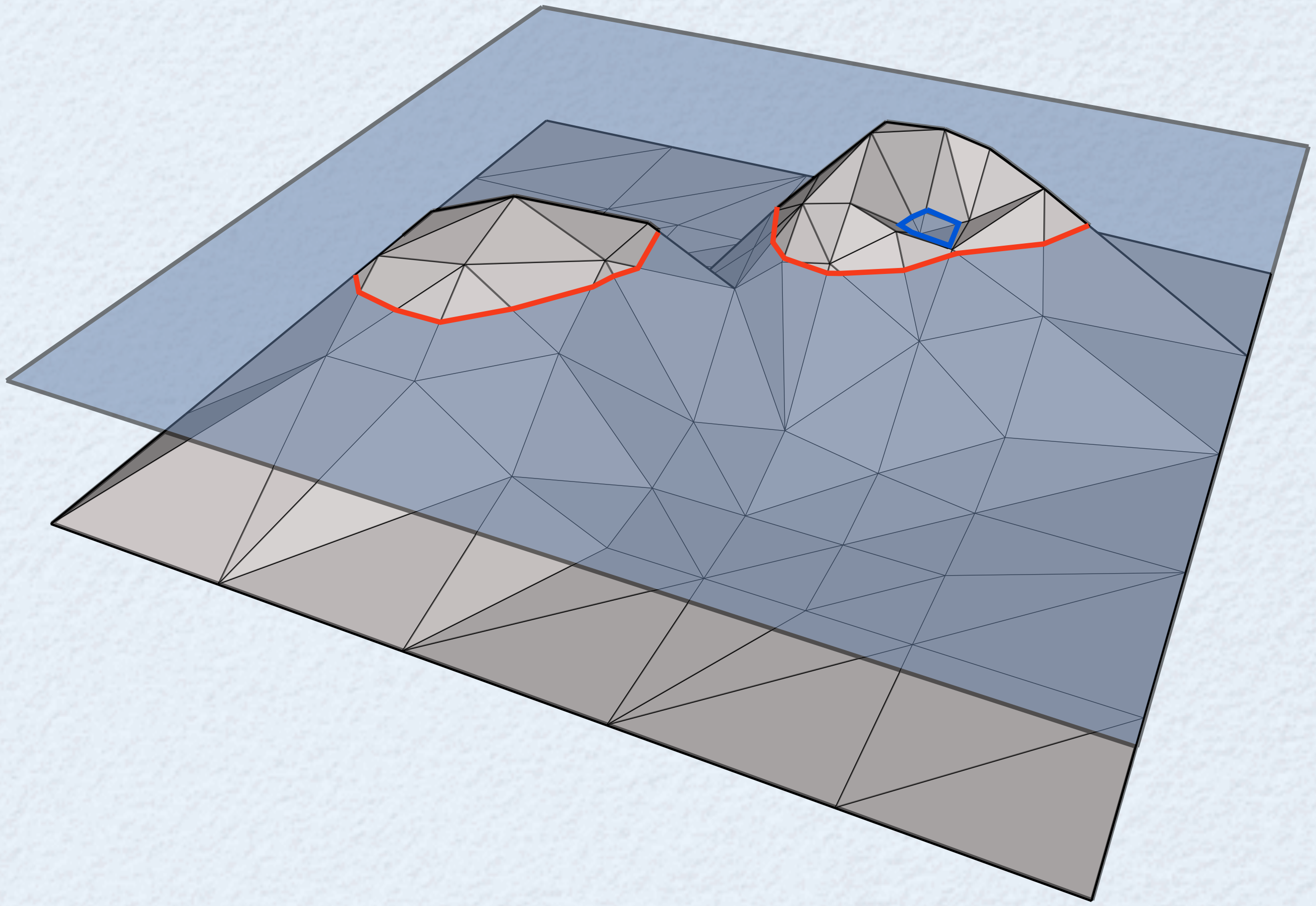

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Theorem. If we contract each contour to a point, the result is a tree.

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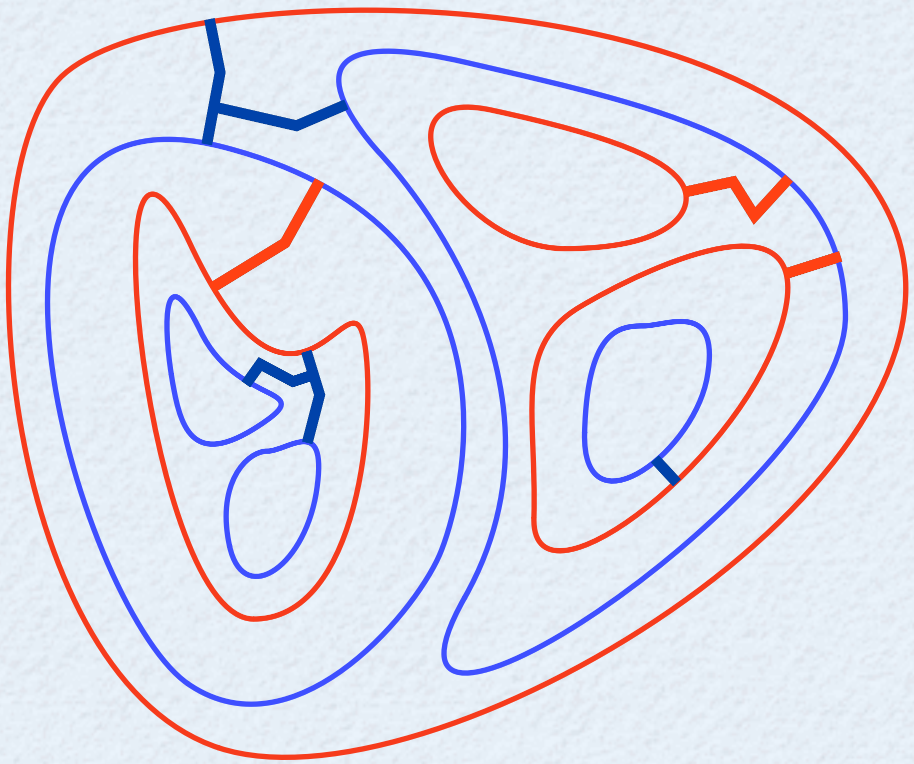
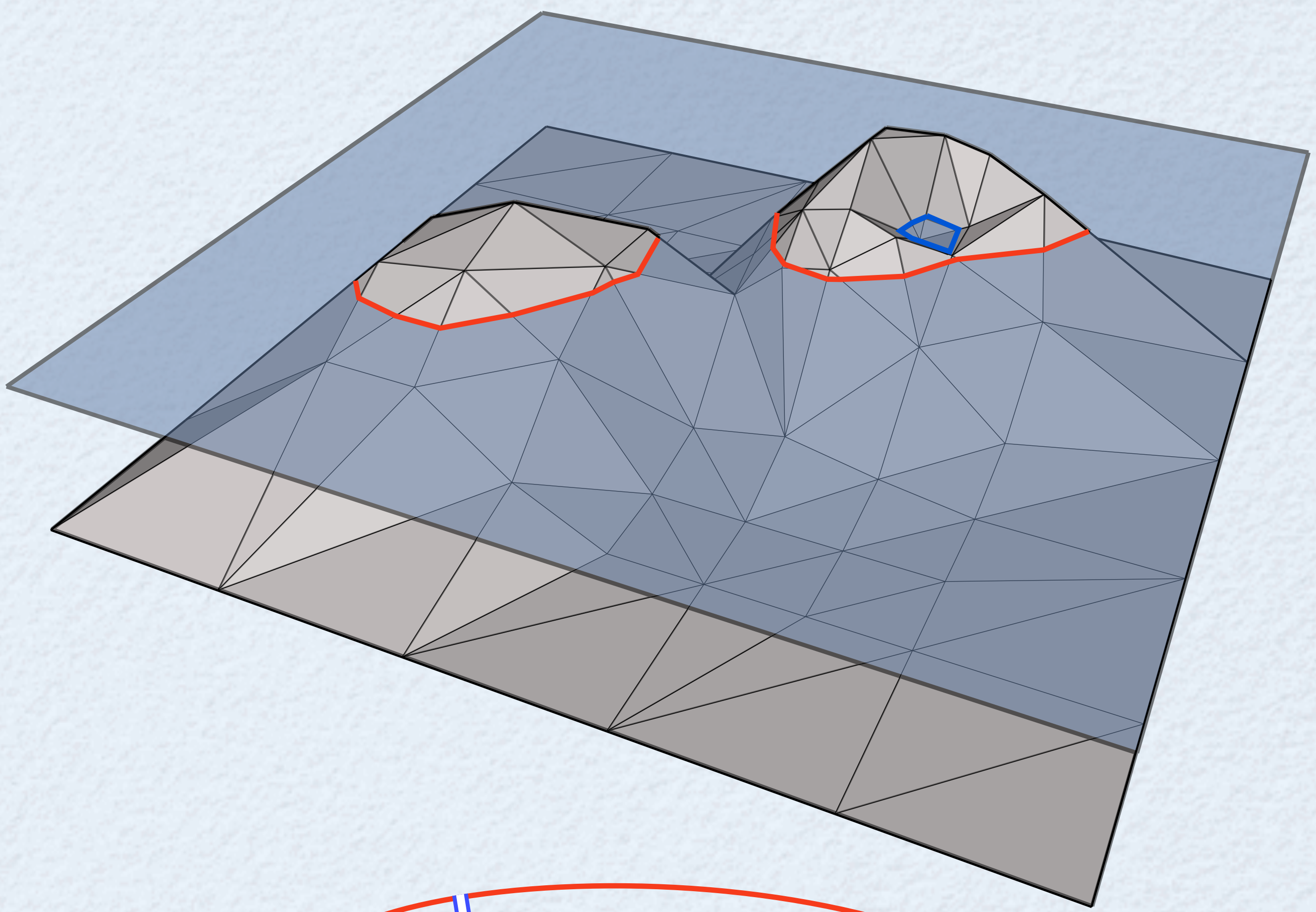
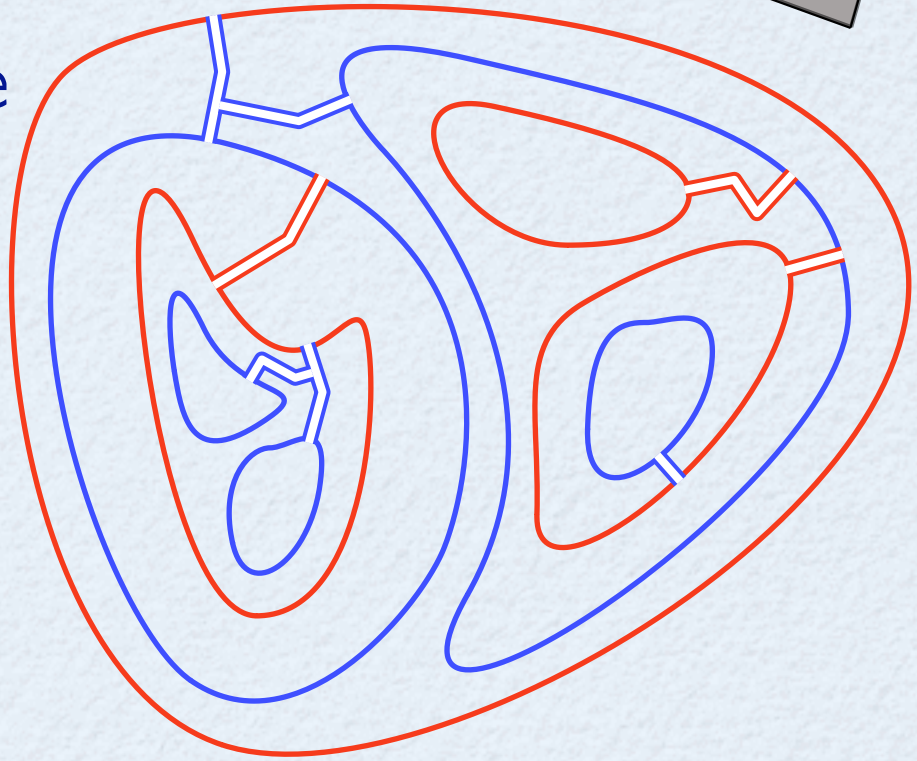




Theorem. Only the red tree connects a blue contour and its red children.

Theorem. If we contract each contour to a point, the result is a tree.



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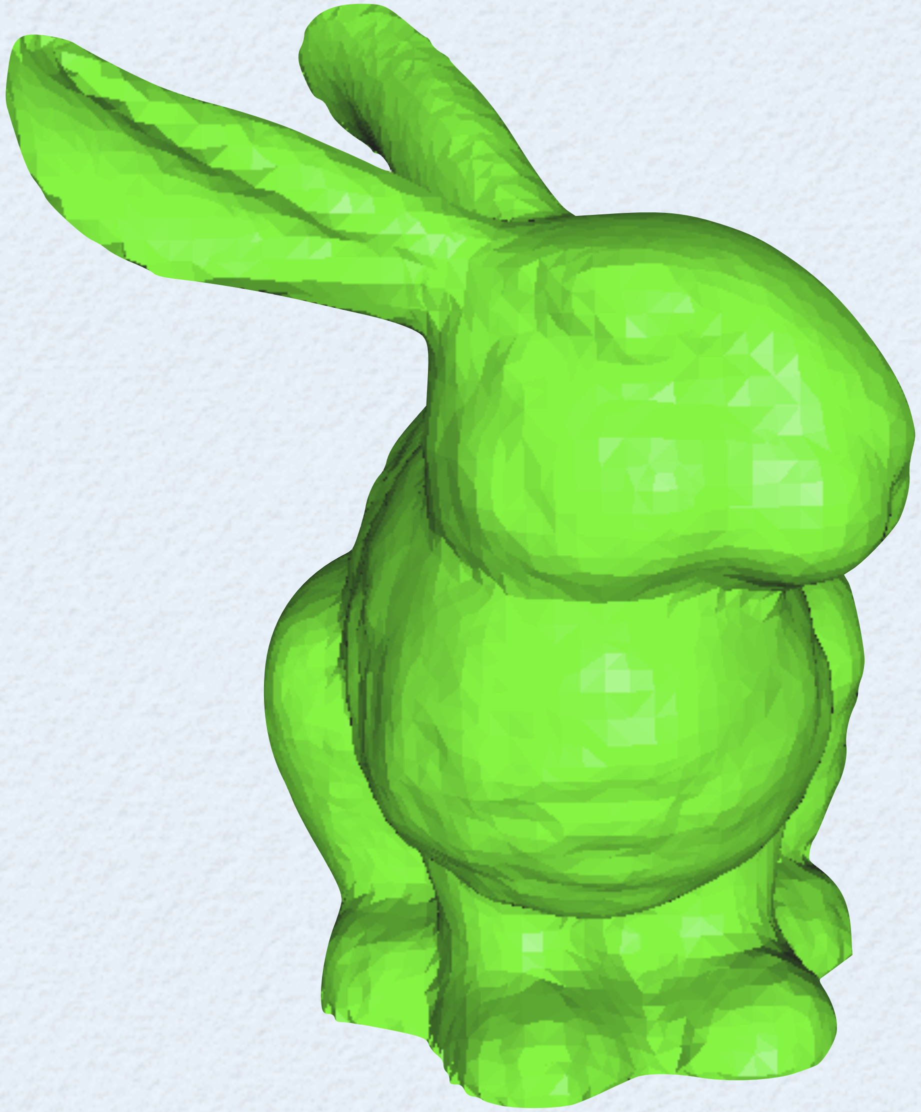

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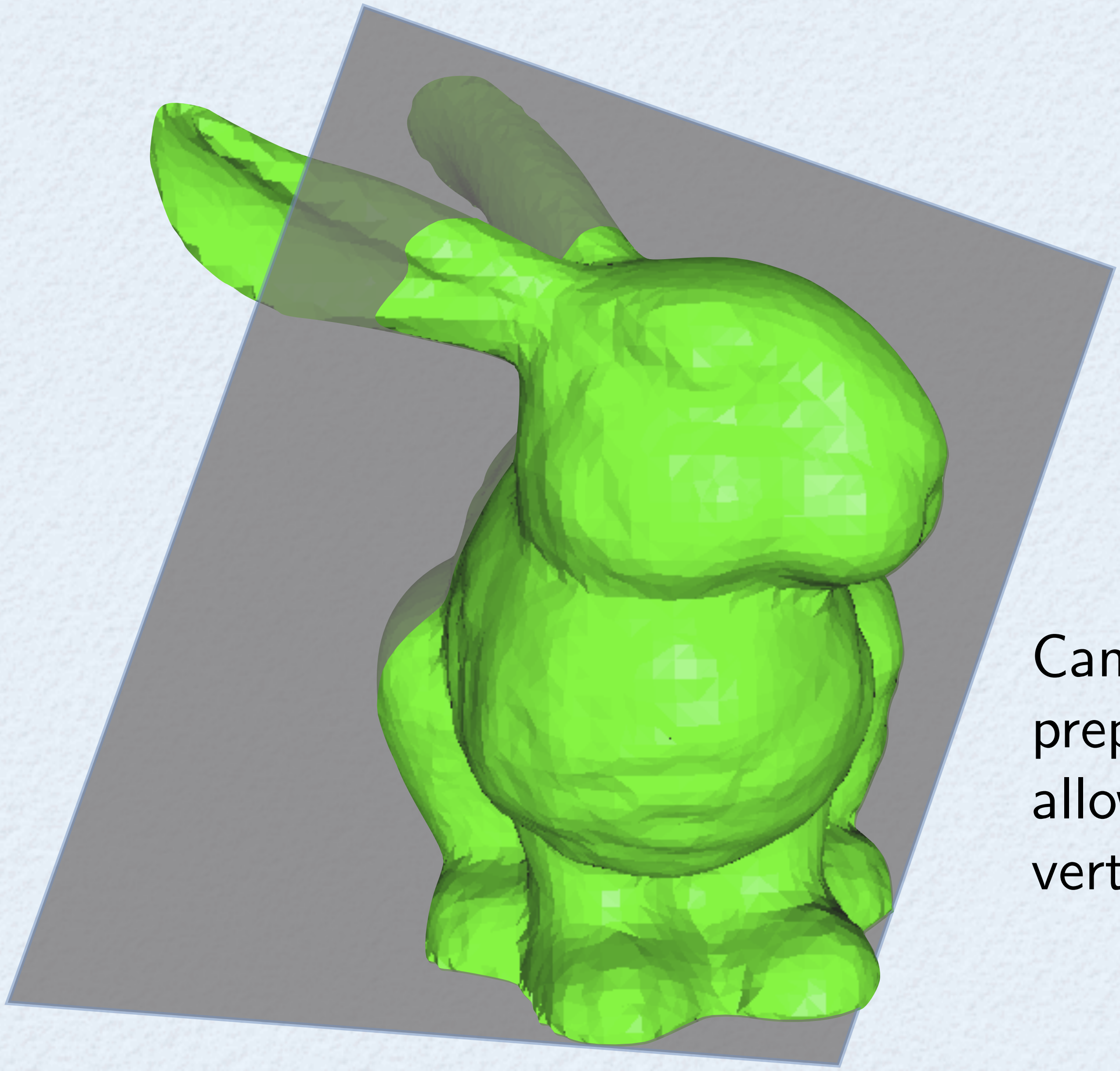

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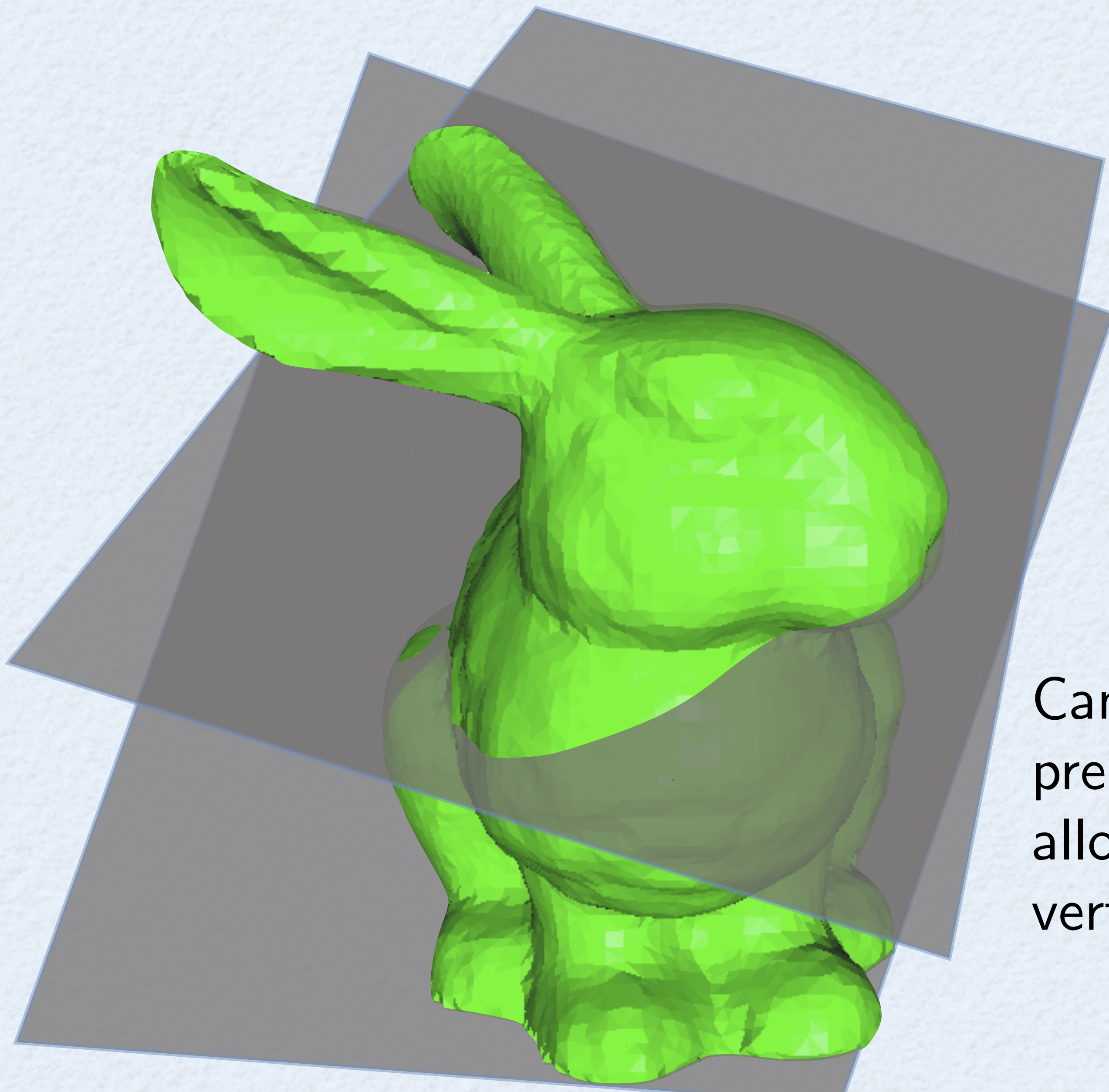

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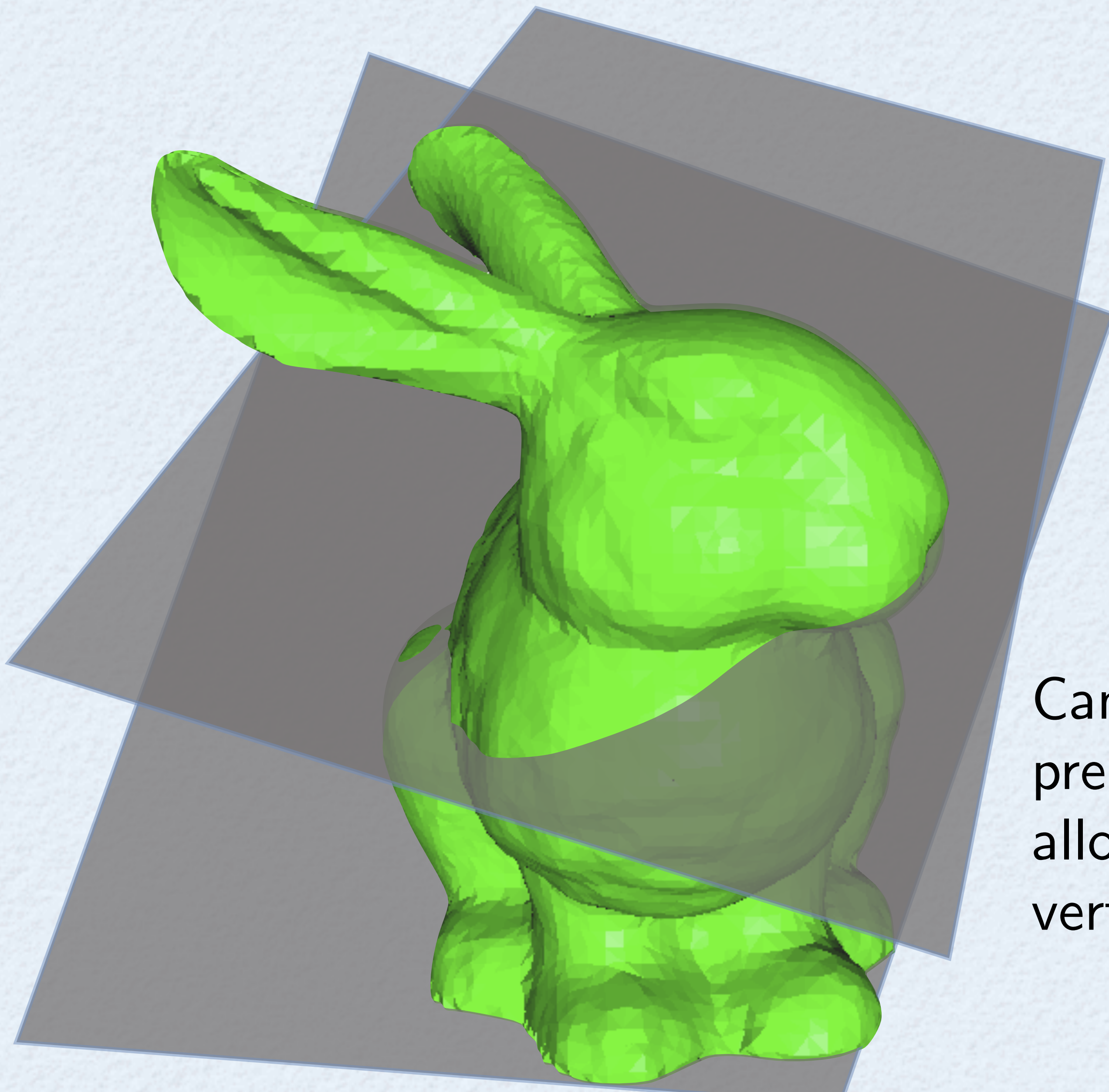

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Thank You!