Flow-Based Methods in Manifold Reconstruction

Bardia Sadri
Duke University

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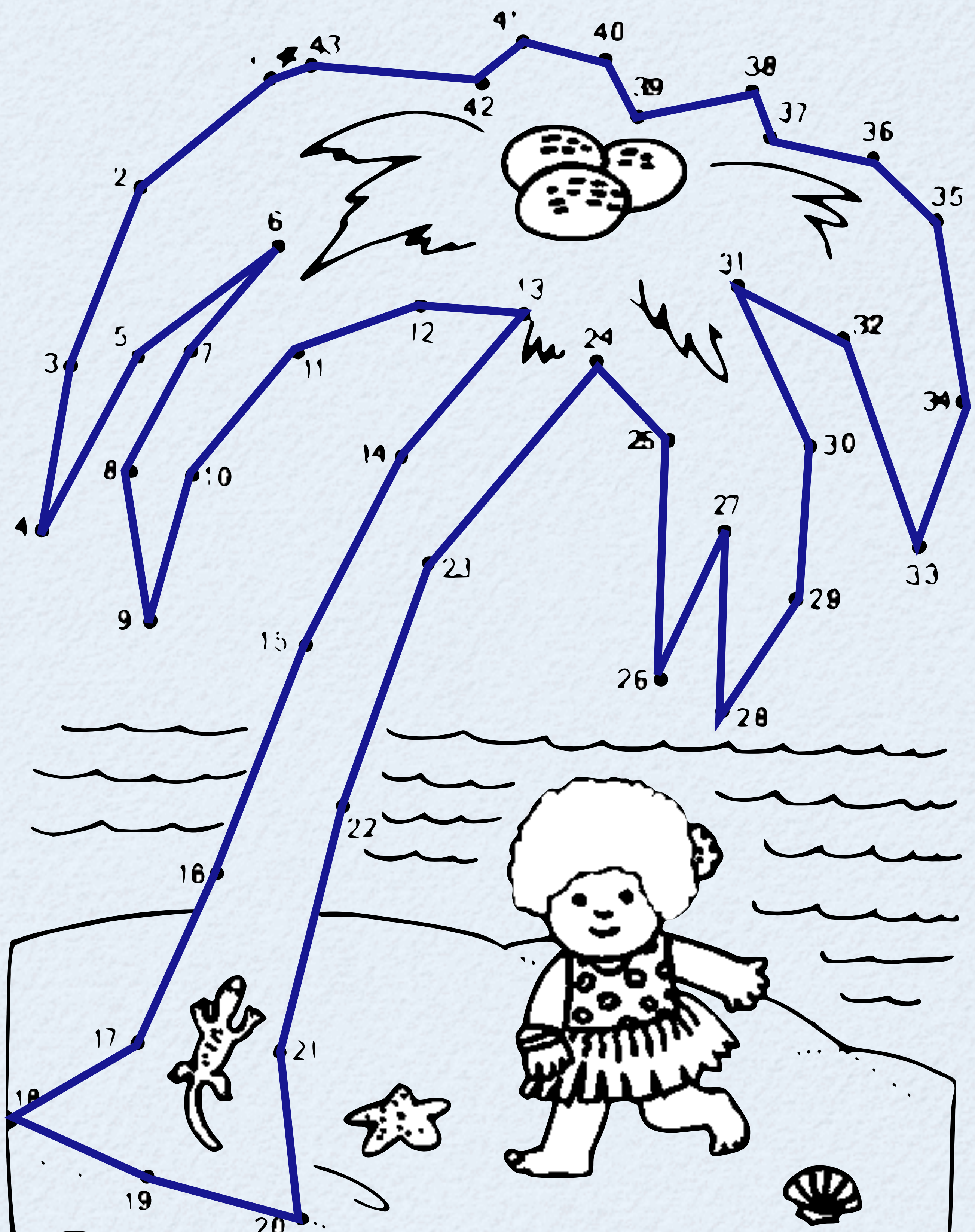
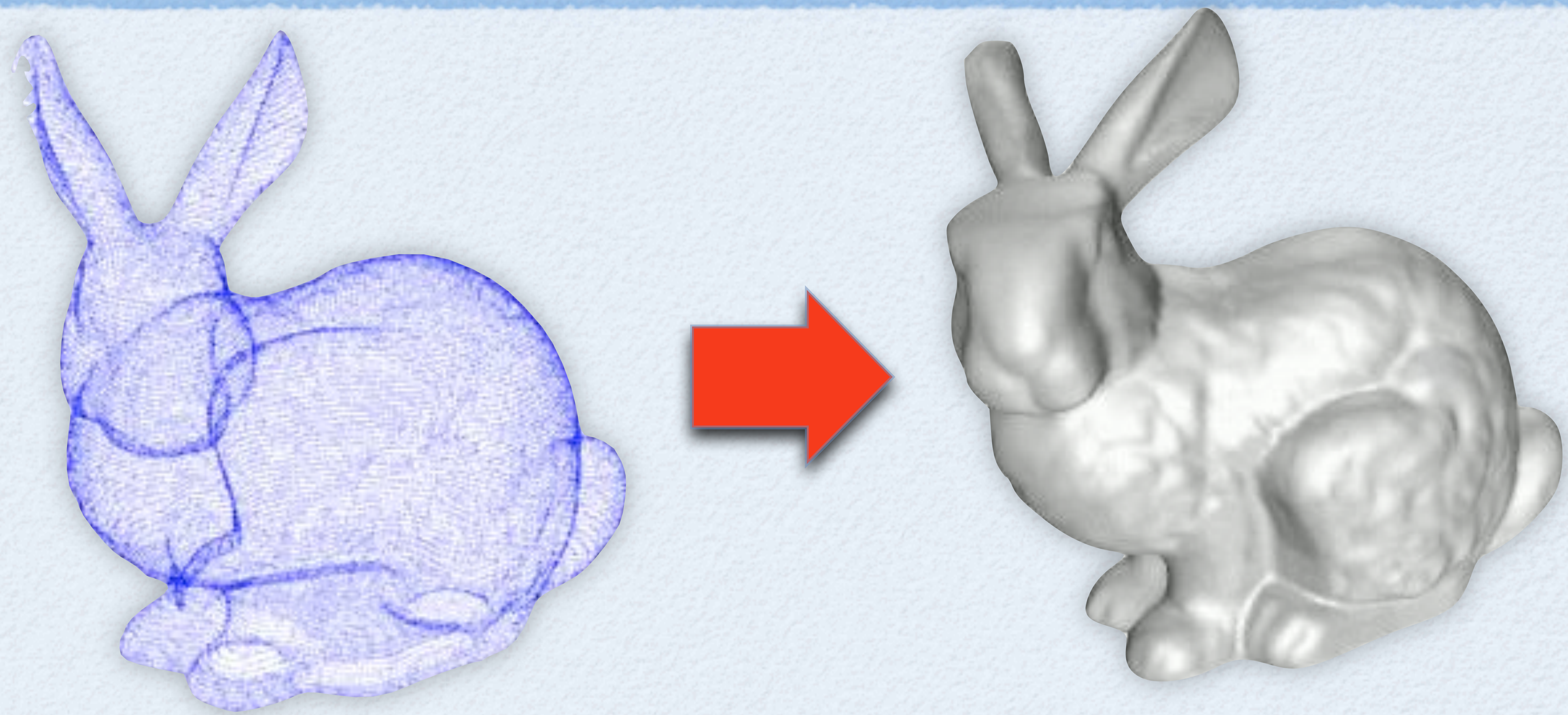


Surface (manifold) reconstruction



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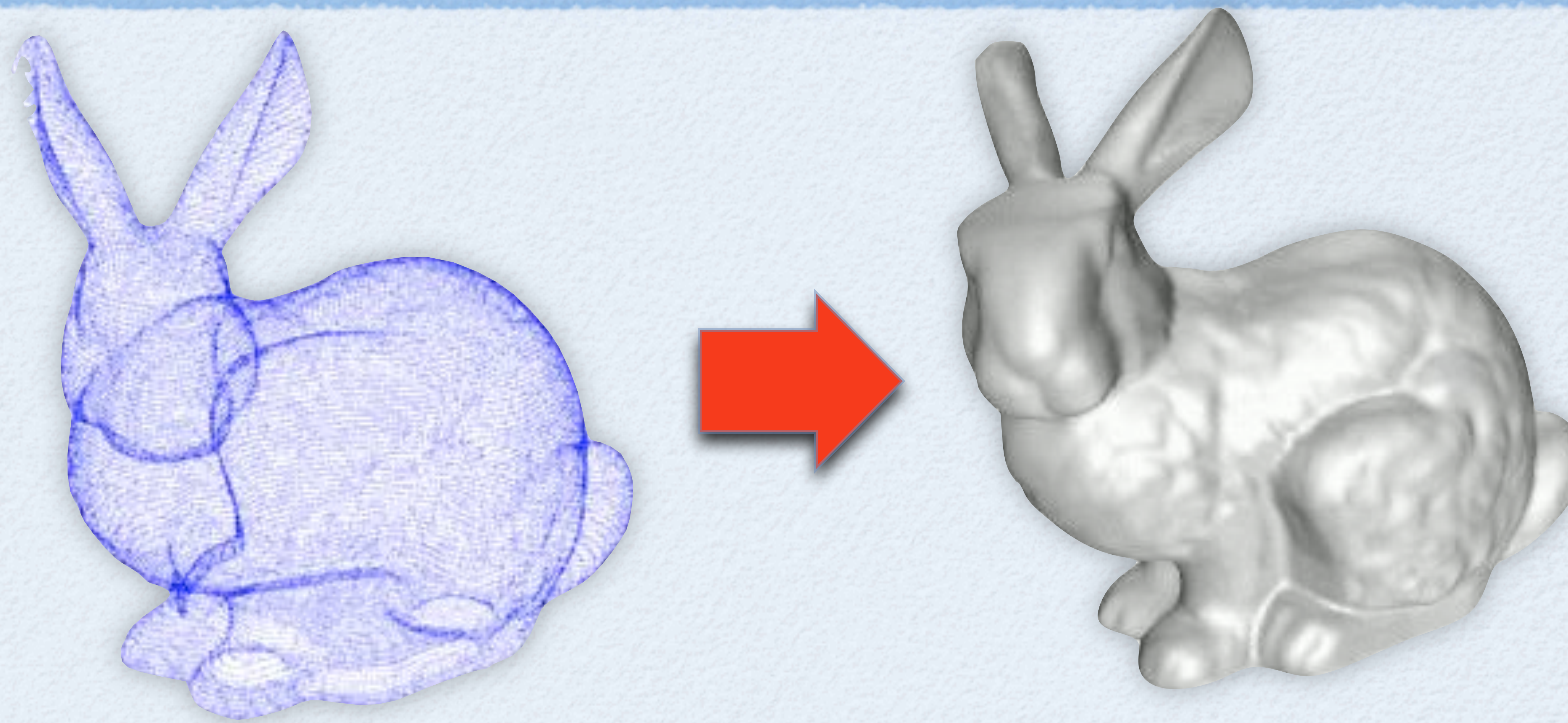

co-dimension 1 submanifold of $\mathbb{R}^n$

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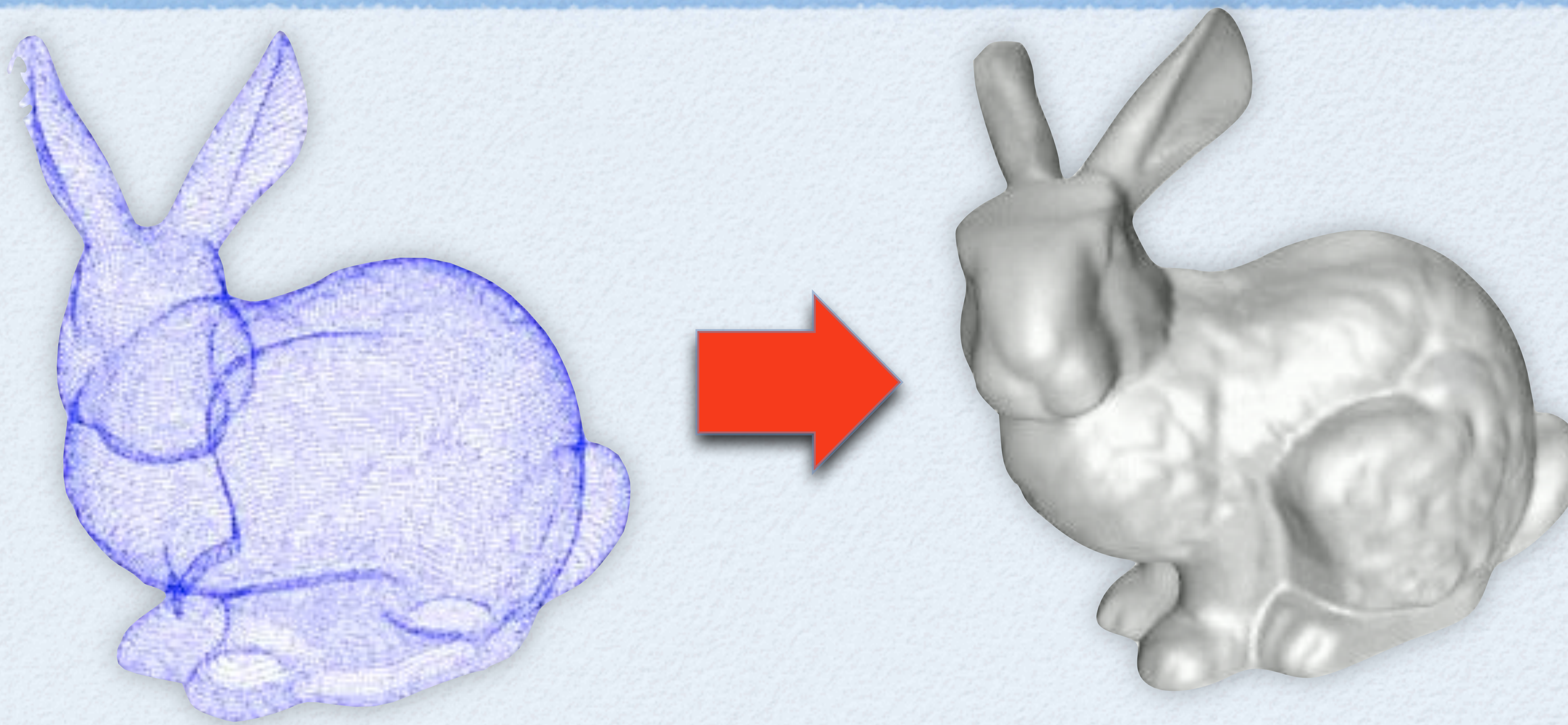

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homeomorphic
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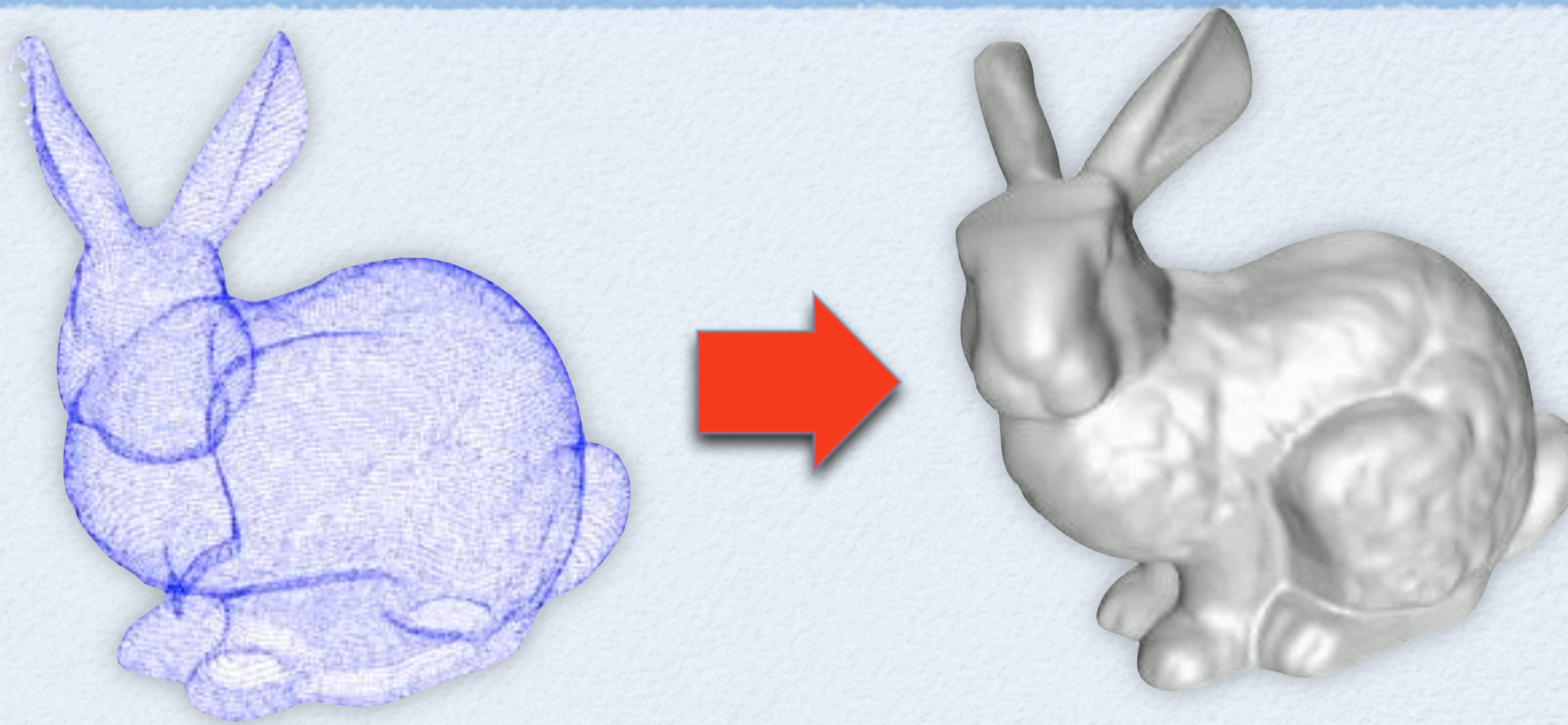

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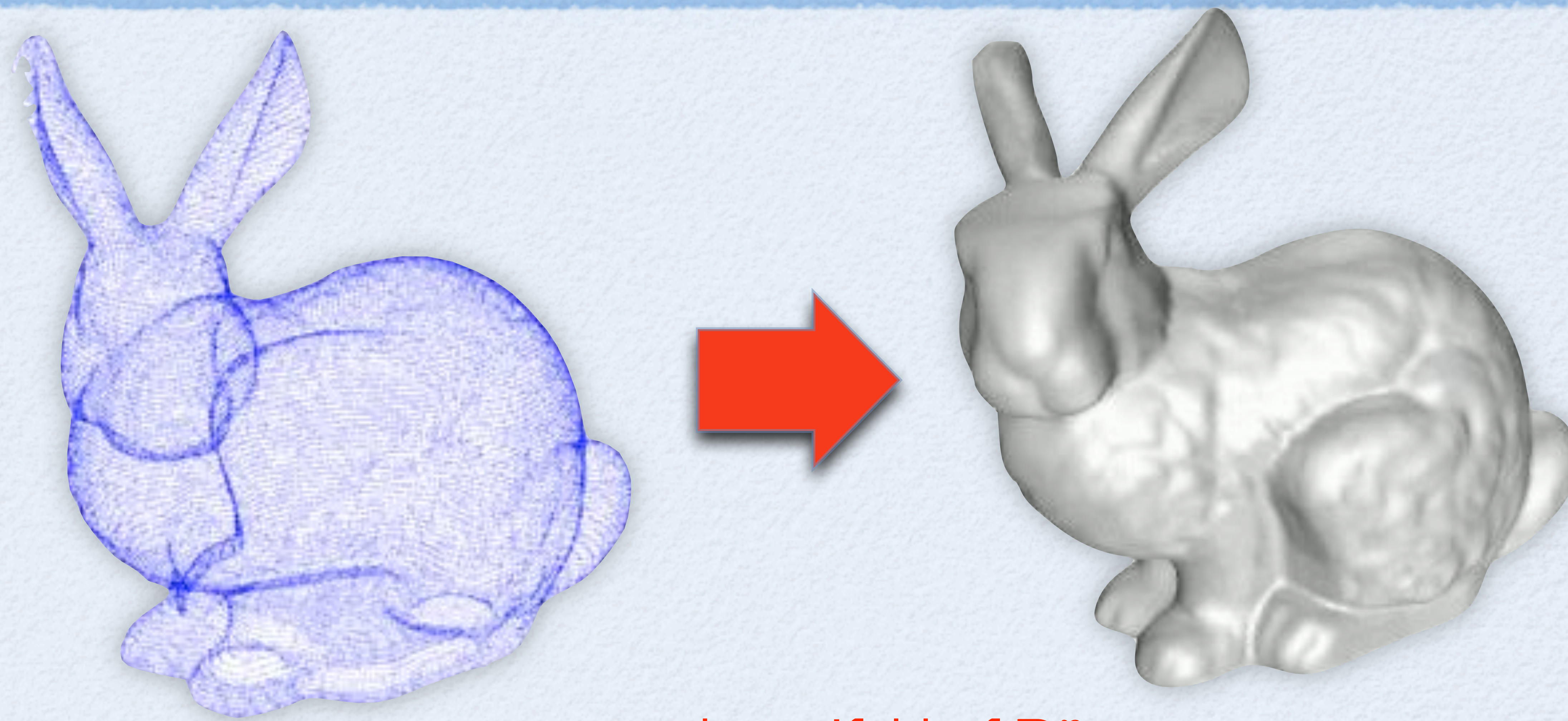

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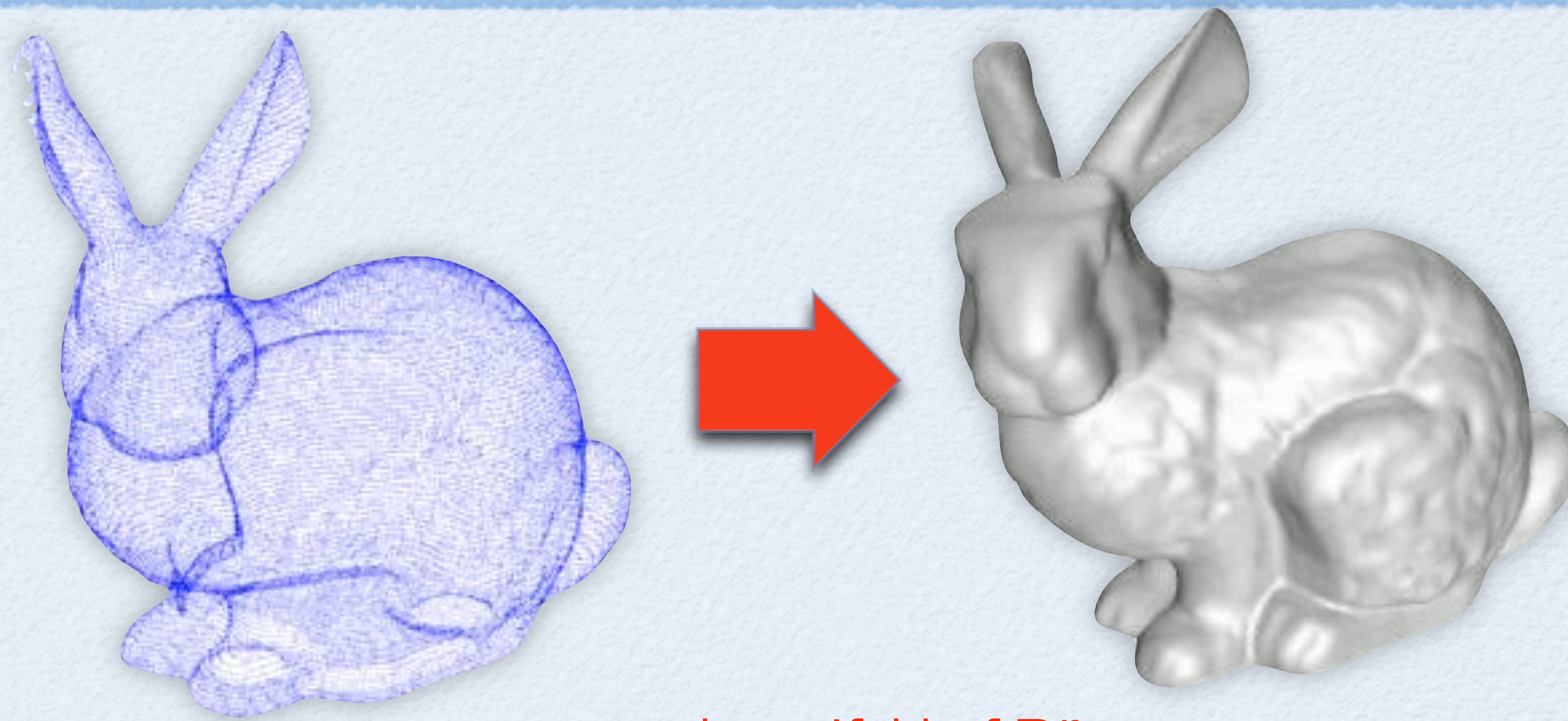

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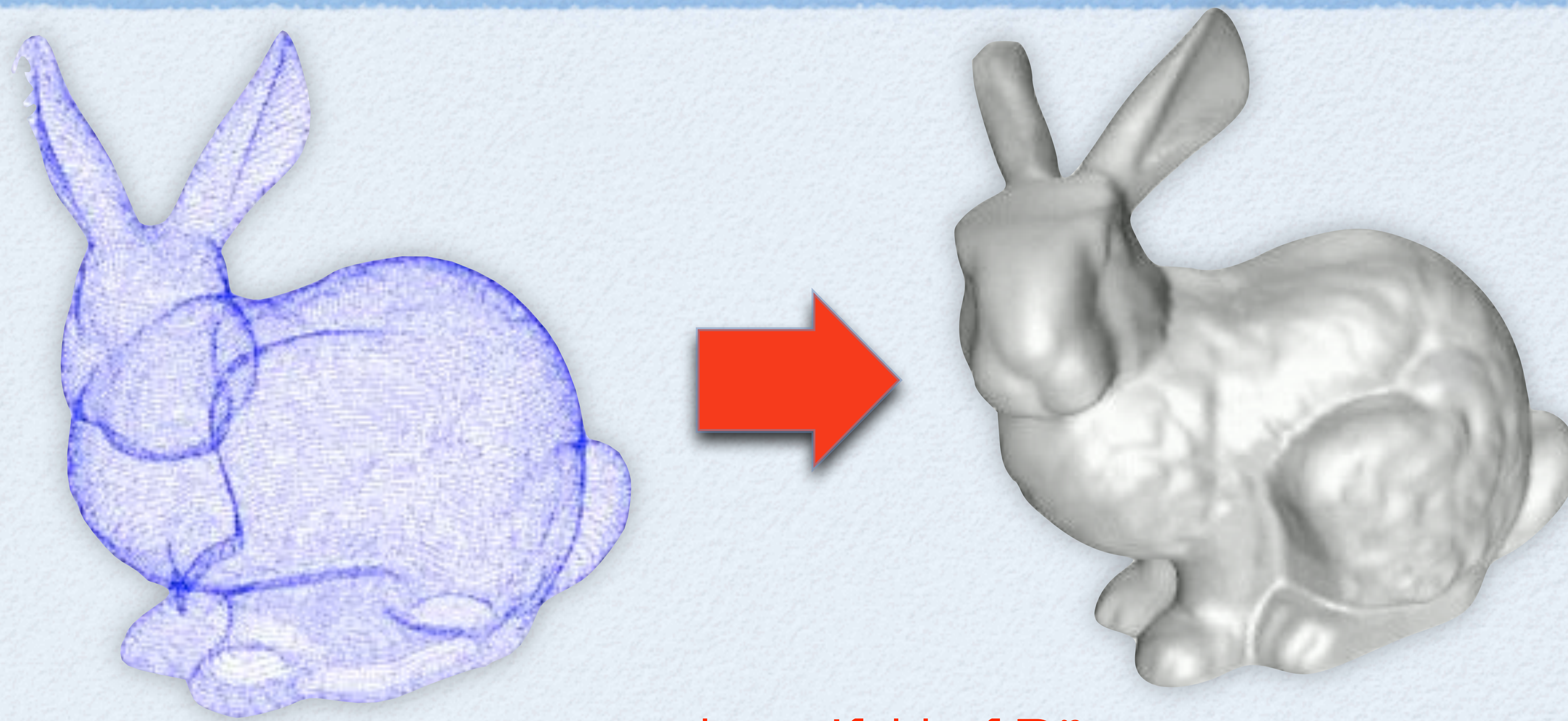

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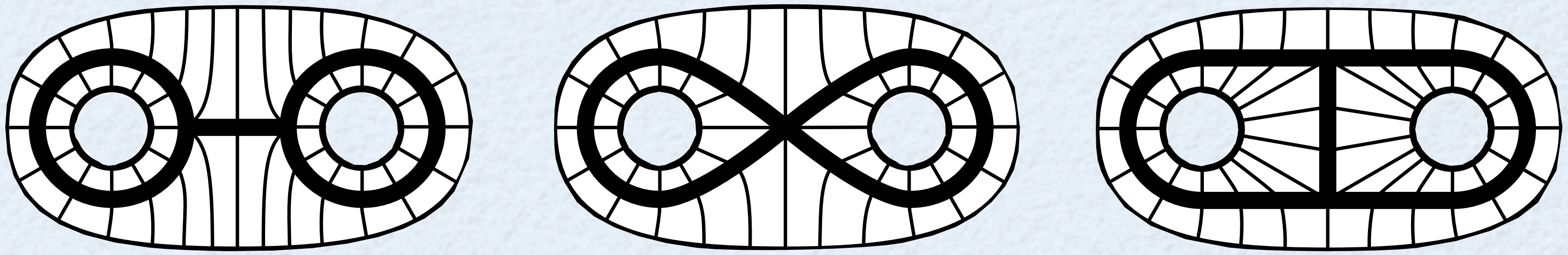
Reconstructing the complement



Unlike homeomorphism, homotopy equivalence does not preserve dimension.

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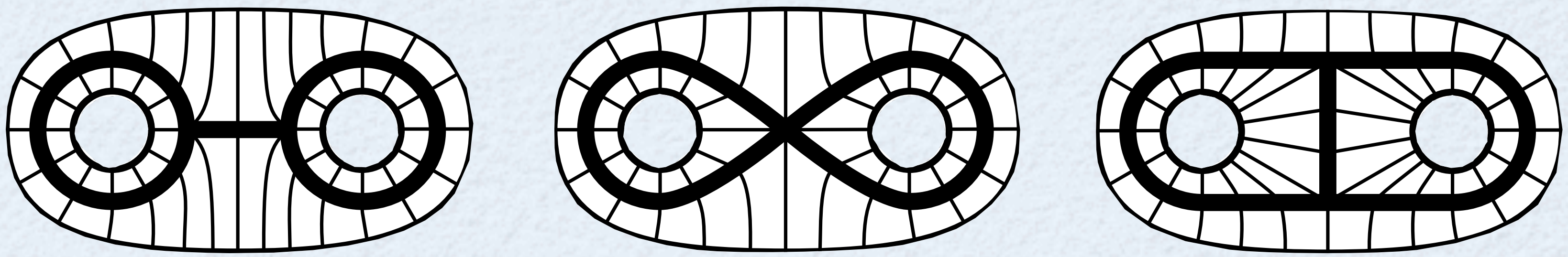

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All these knots have the same homotopy type, but not their complements.

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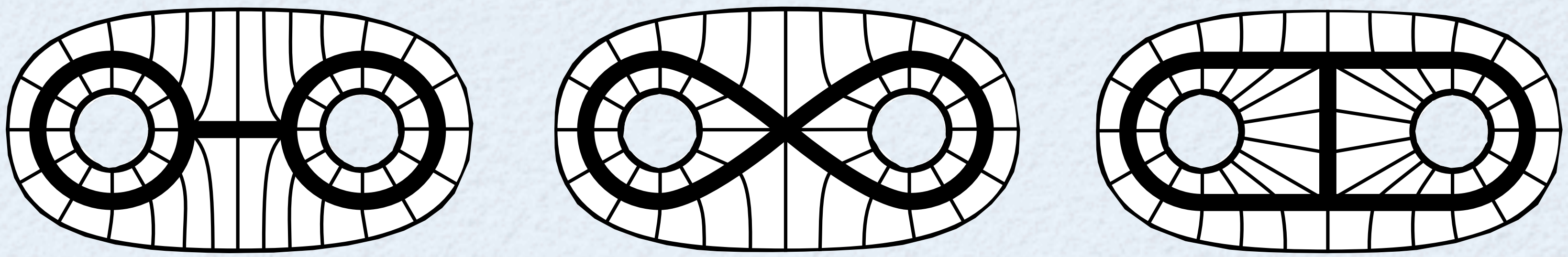

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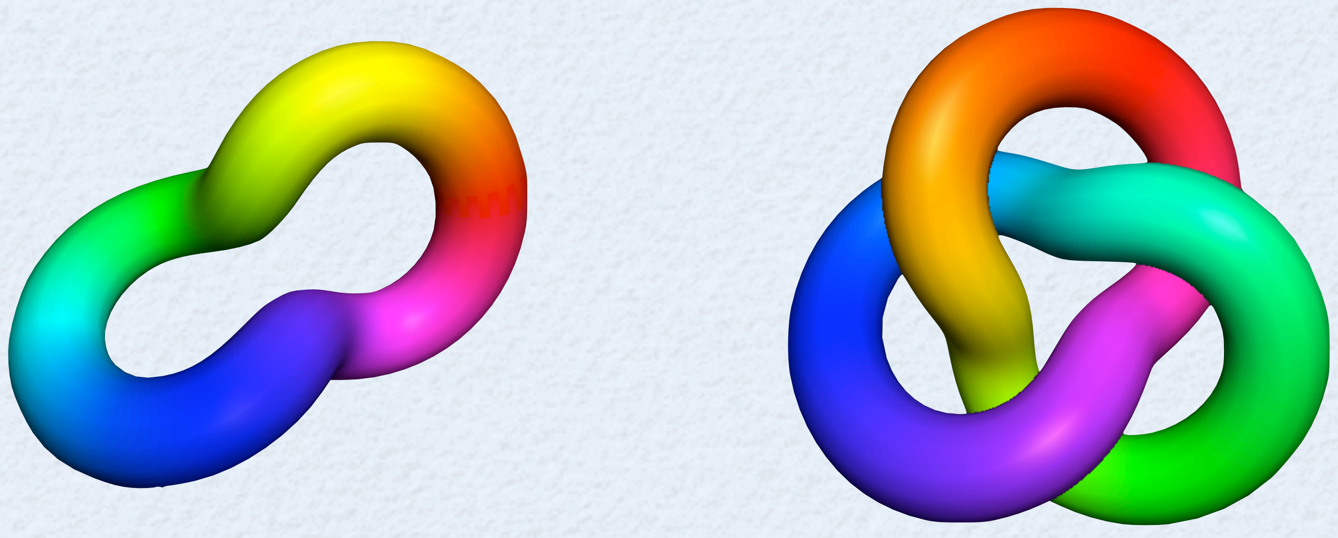
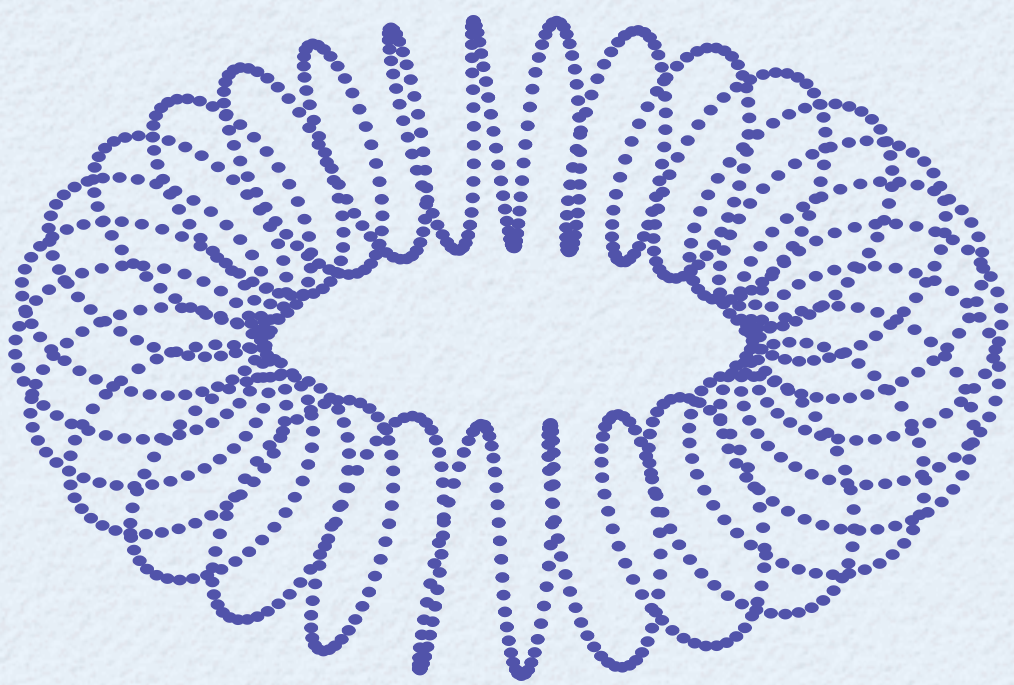


What if we don't know the submanifold's dimension?

Naturally occurring data may be generated by structured systems with much fewer degrees of freedom than the ambient dimension.



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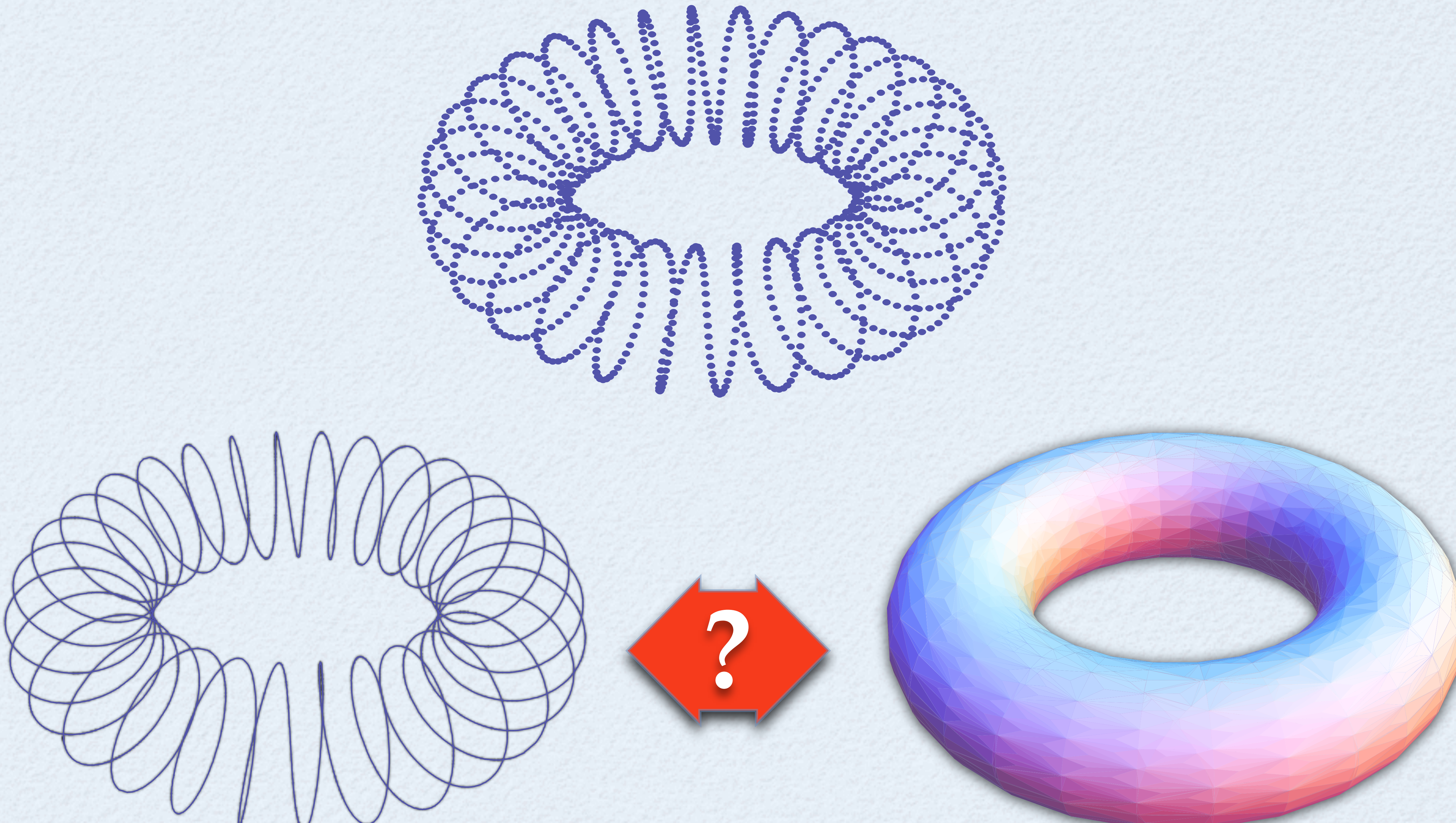

Very rich area!

- Reconstruction as an iso-surfaces:
  0-set of signed dist functs [Hoppe et al'92, Curless et al'96]
  Nearest Neighbor Interpolation [Boissonnat-Cazals'02]
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- Delaunay Methods:
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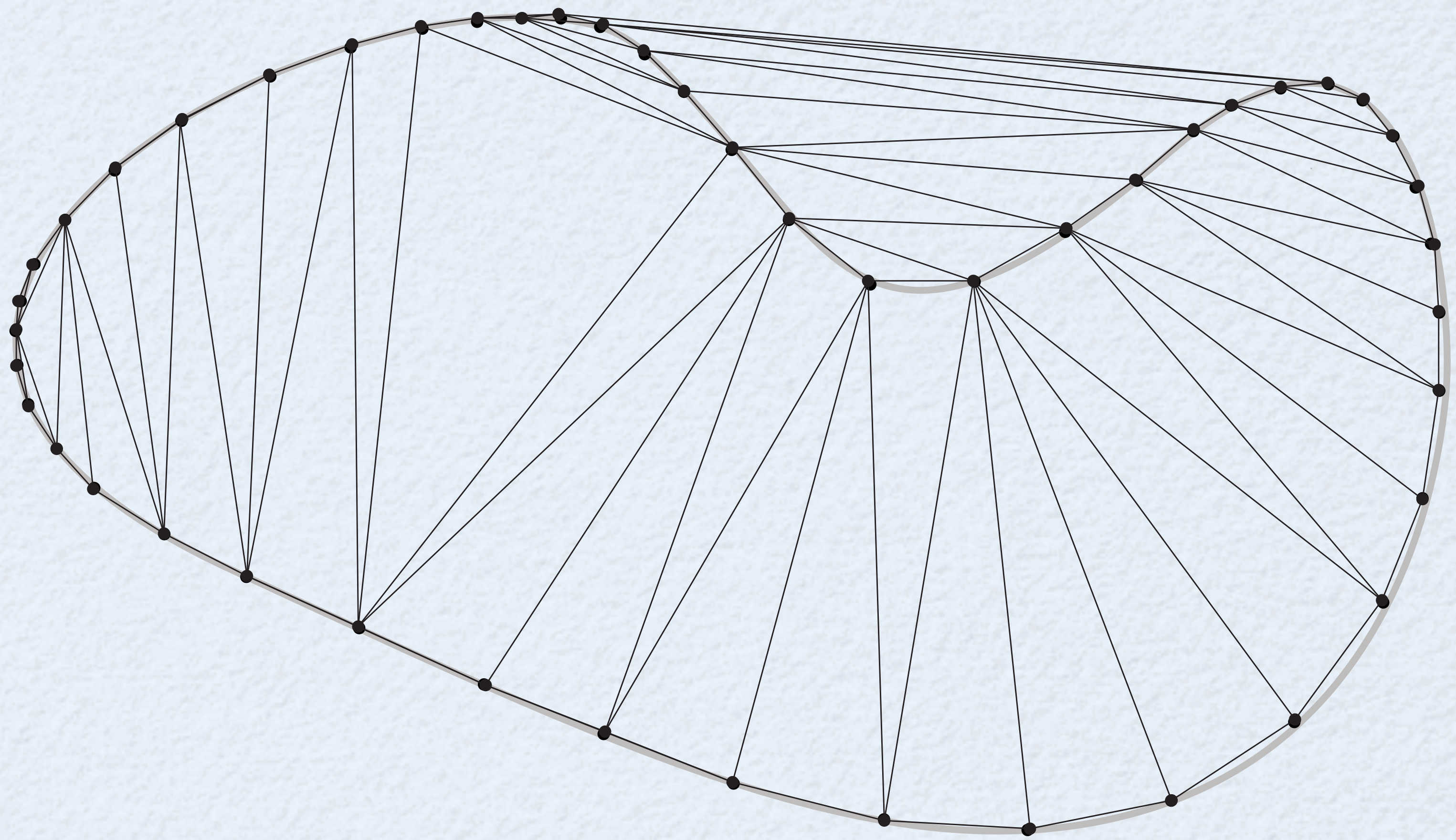

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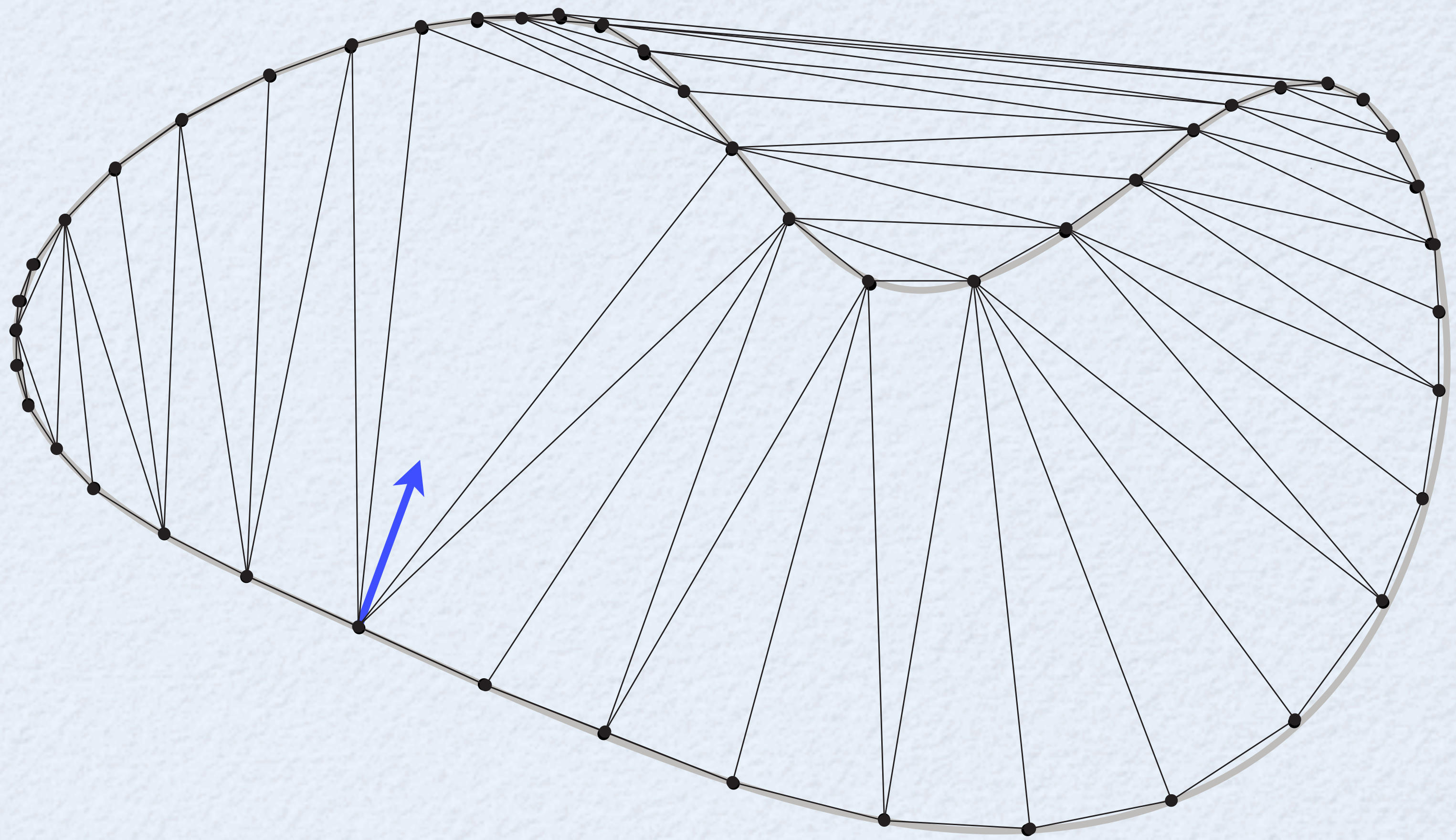

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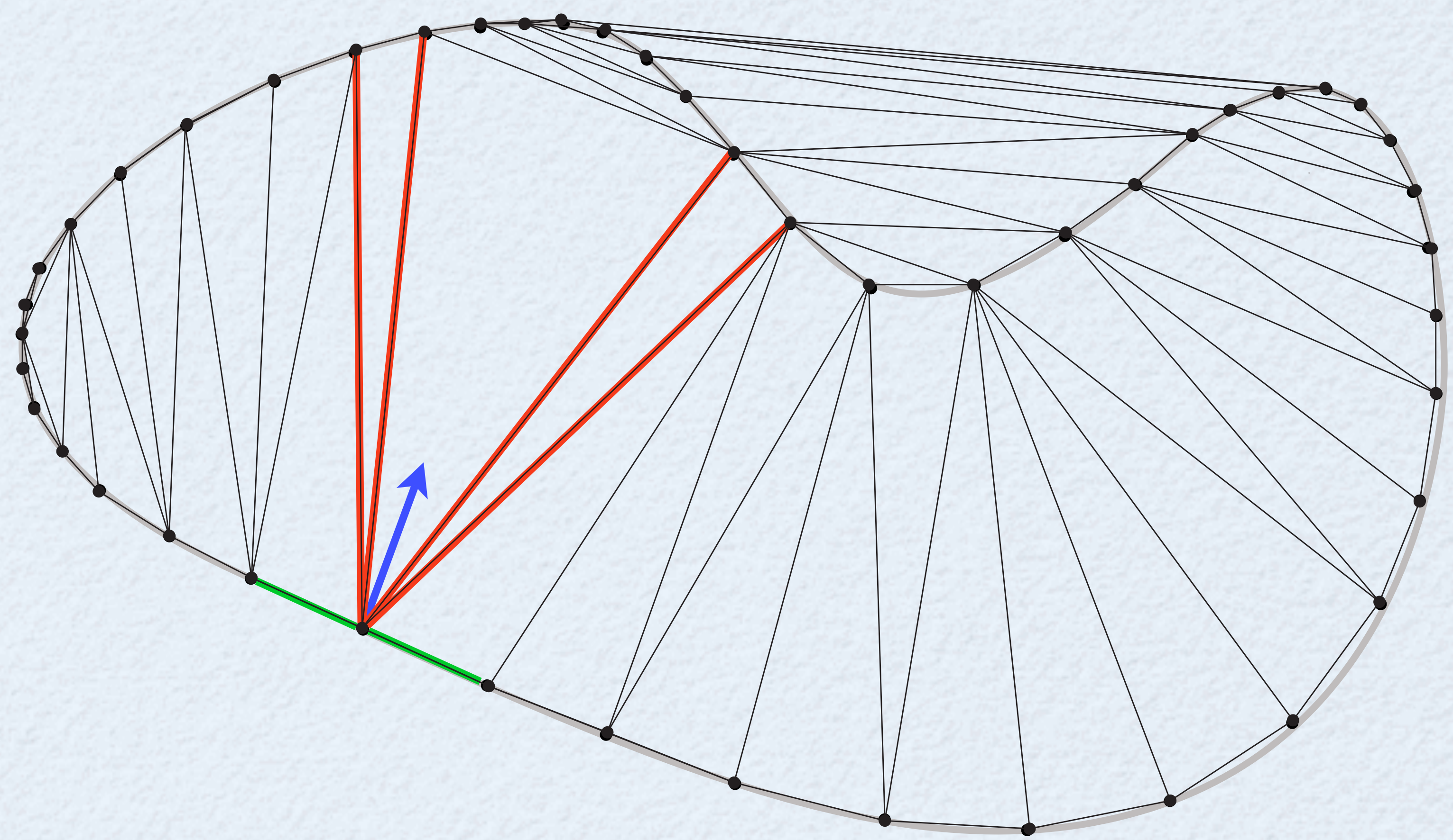

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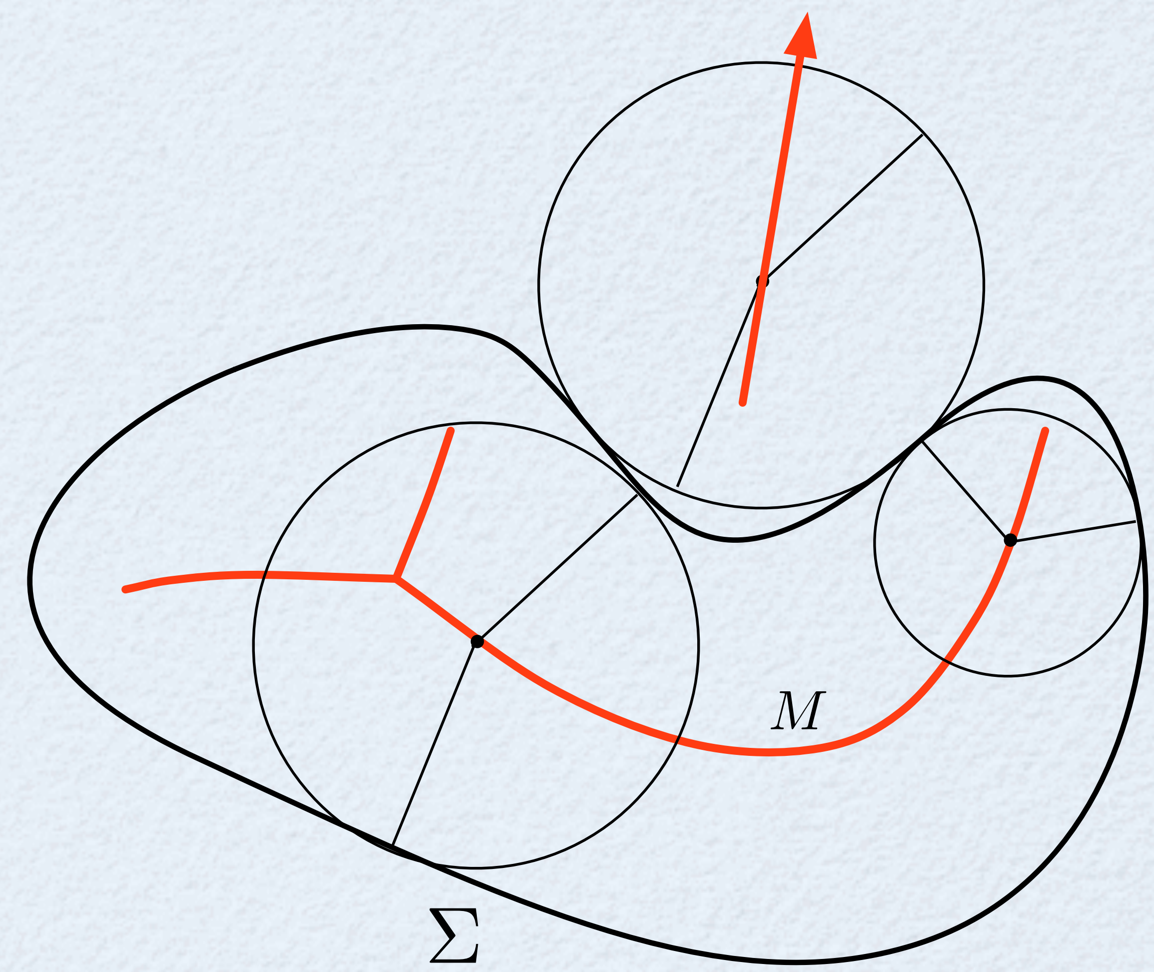
Medial axis & sampling assumption

The medial axis (MA) of Σ is the set of points $M \subset \mathbb{R}^n$ that have ≥ 2 closest points in Σ.



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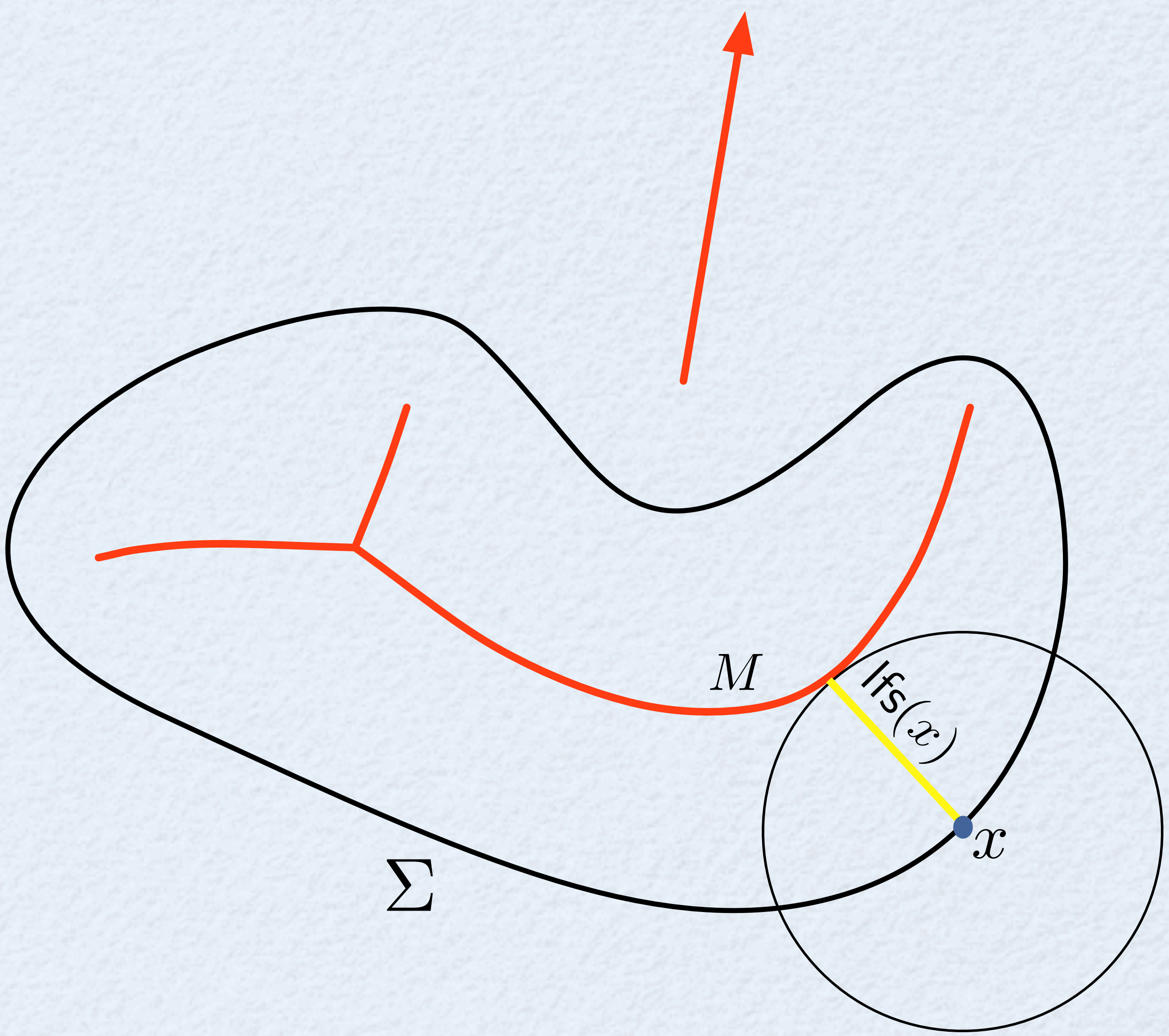

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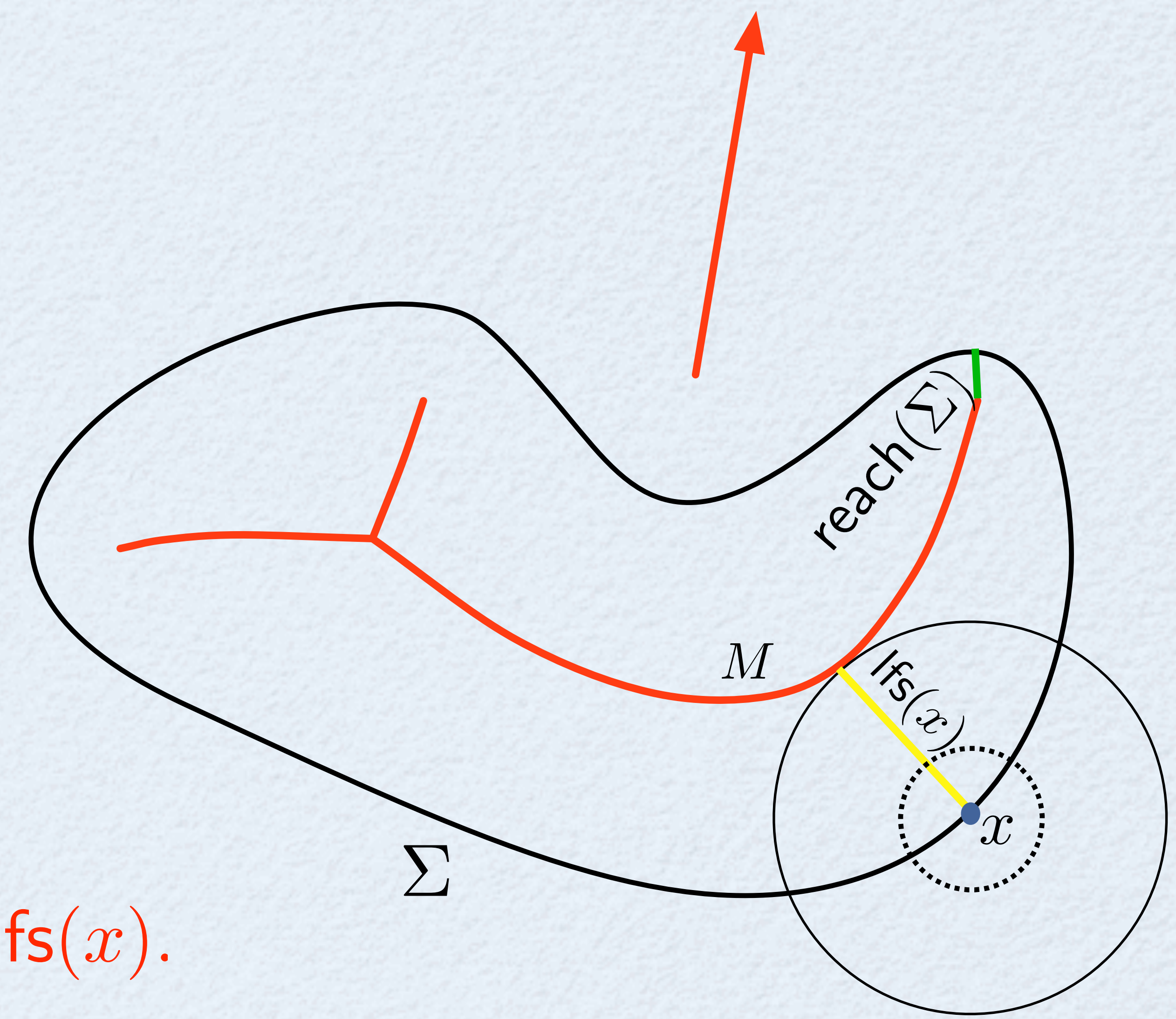

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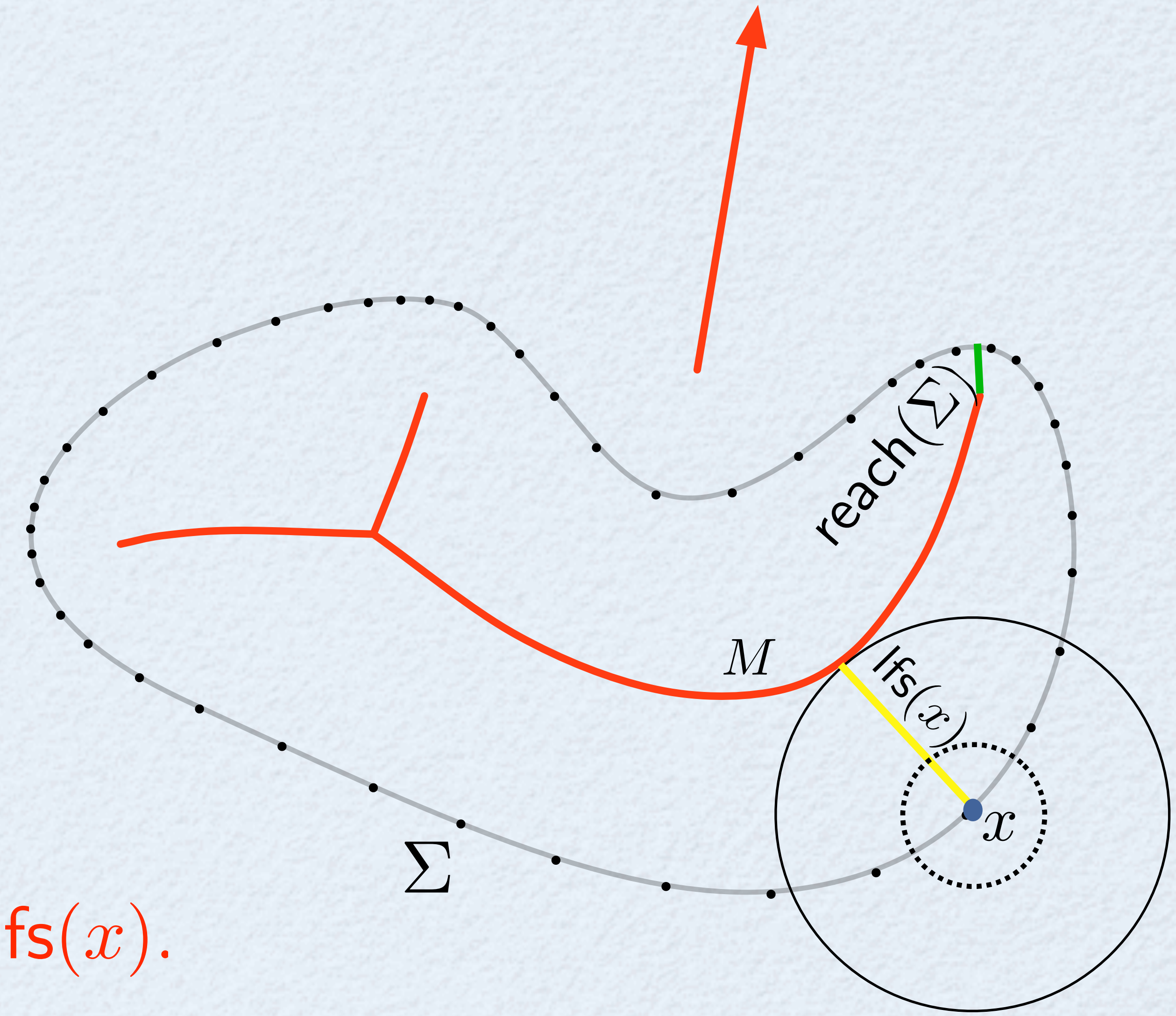

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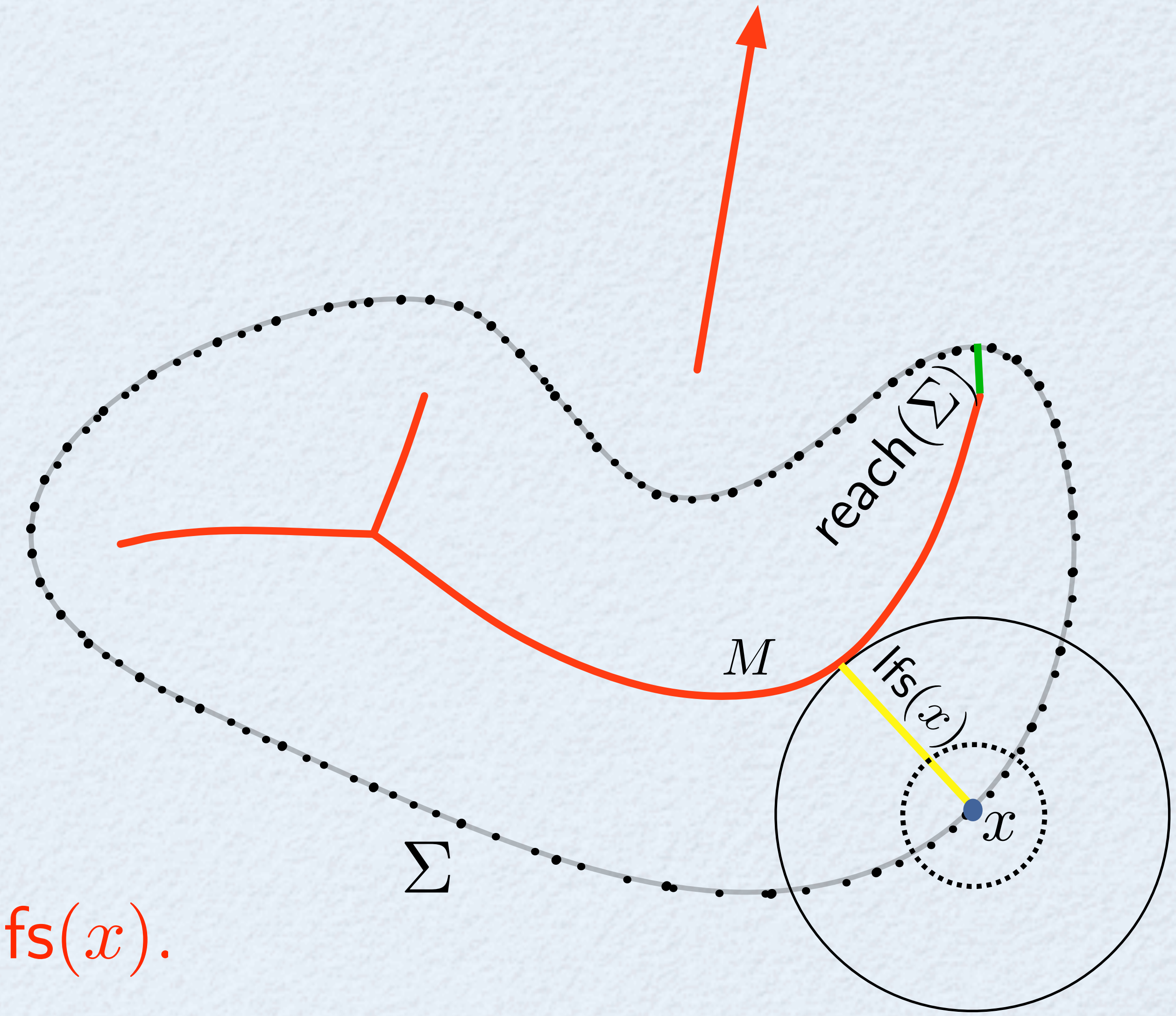

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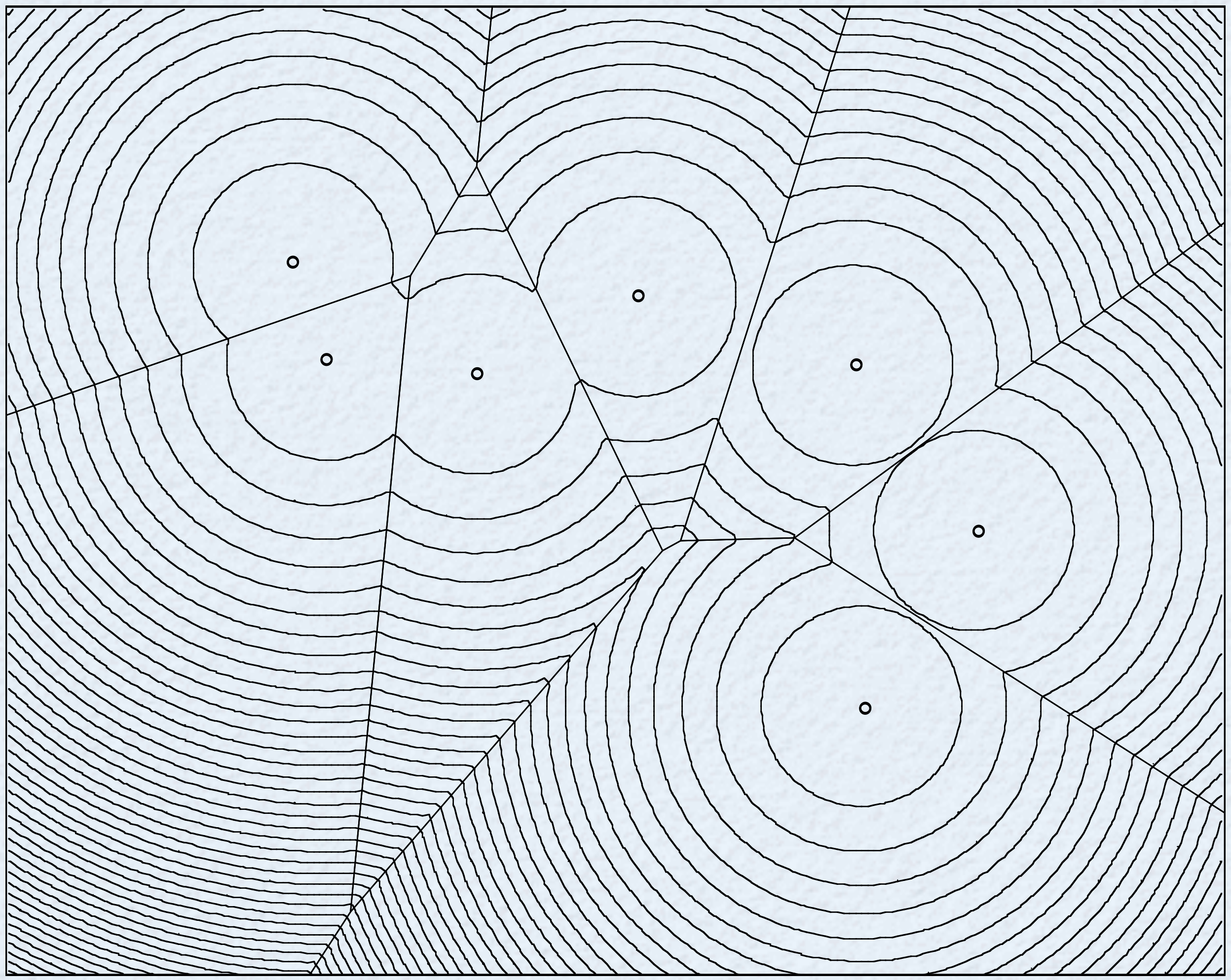

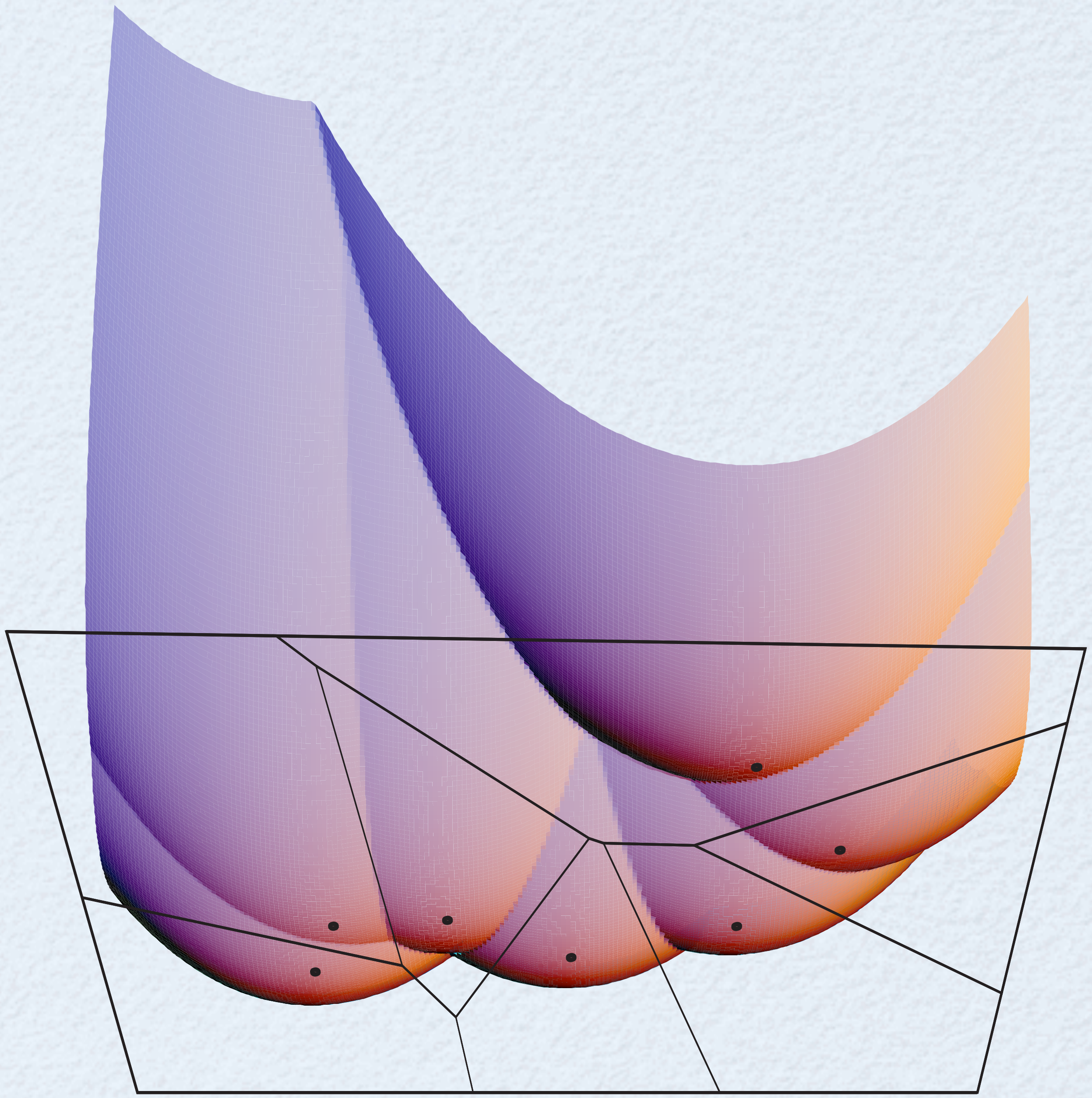

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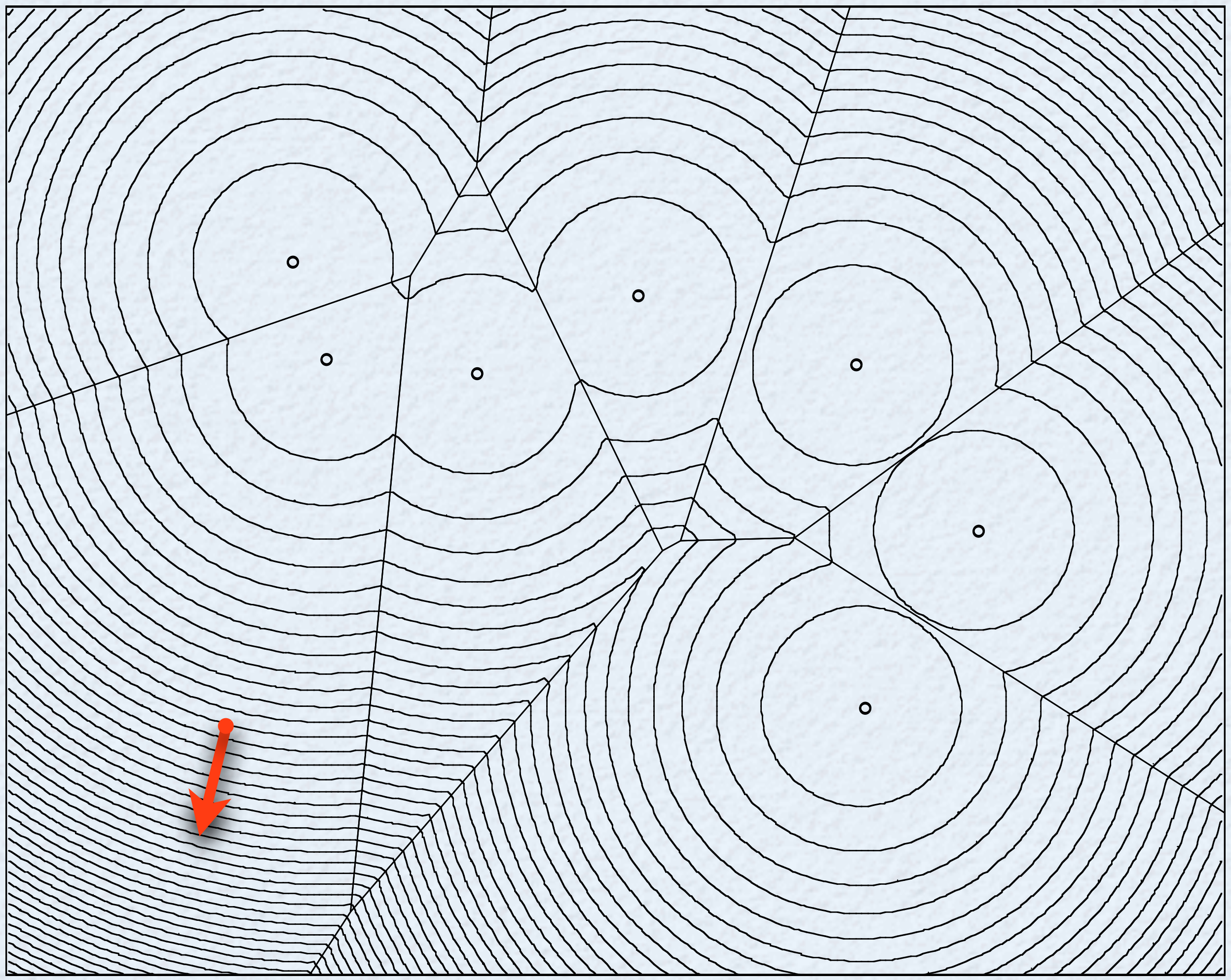

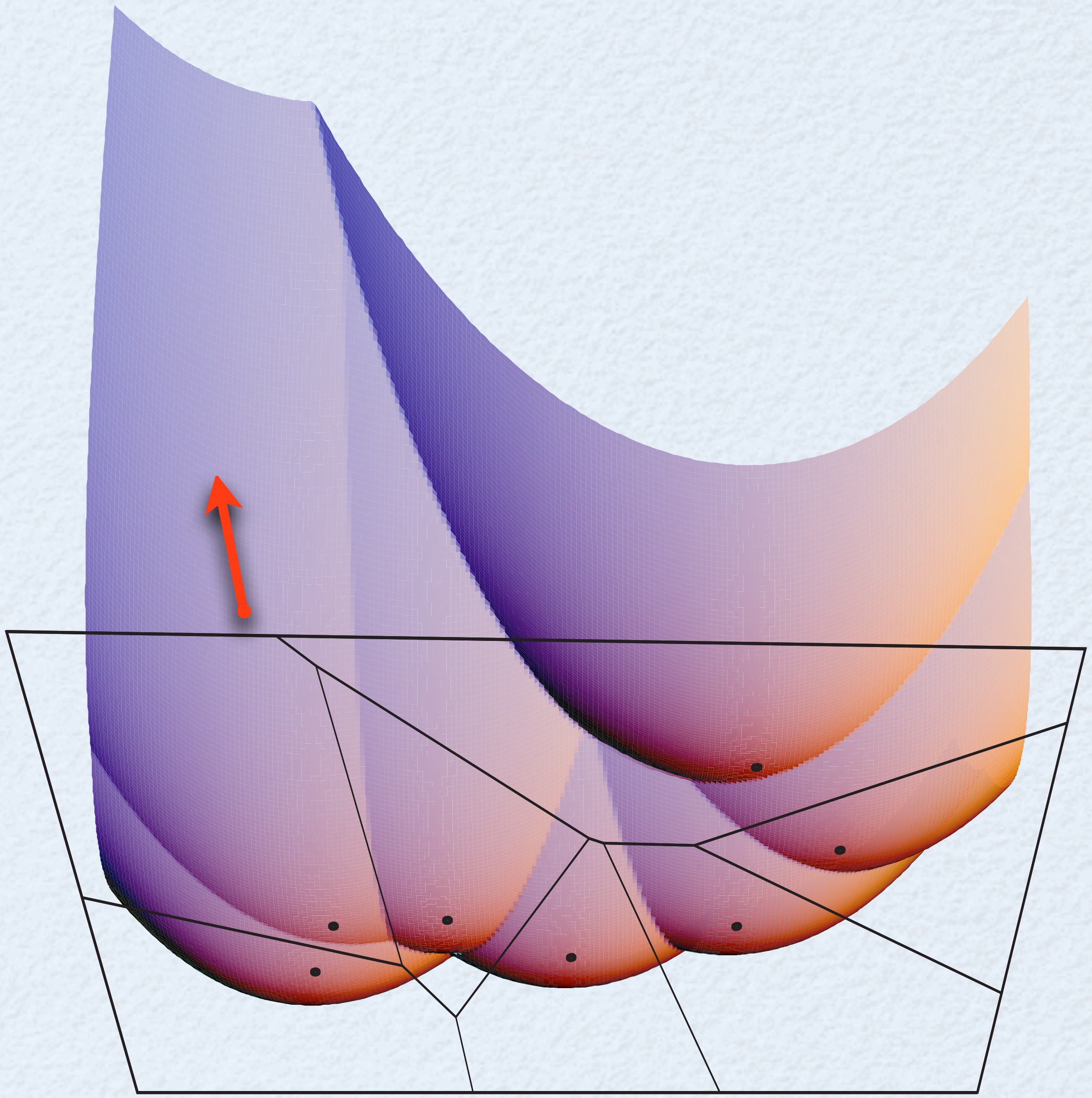

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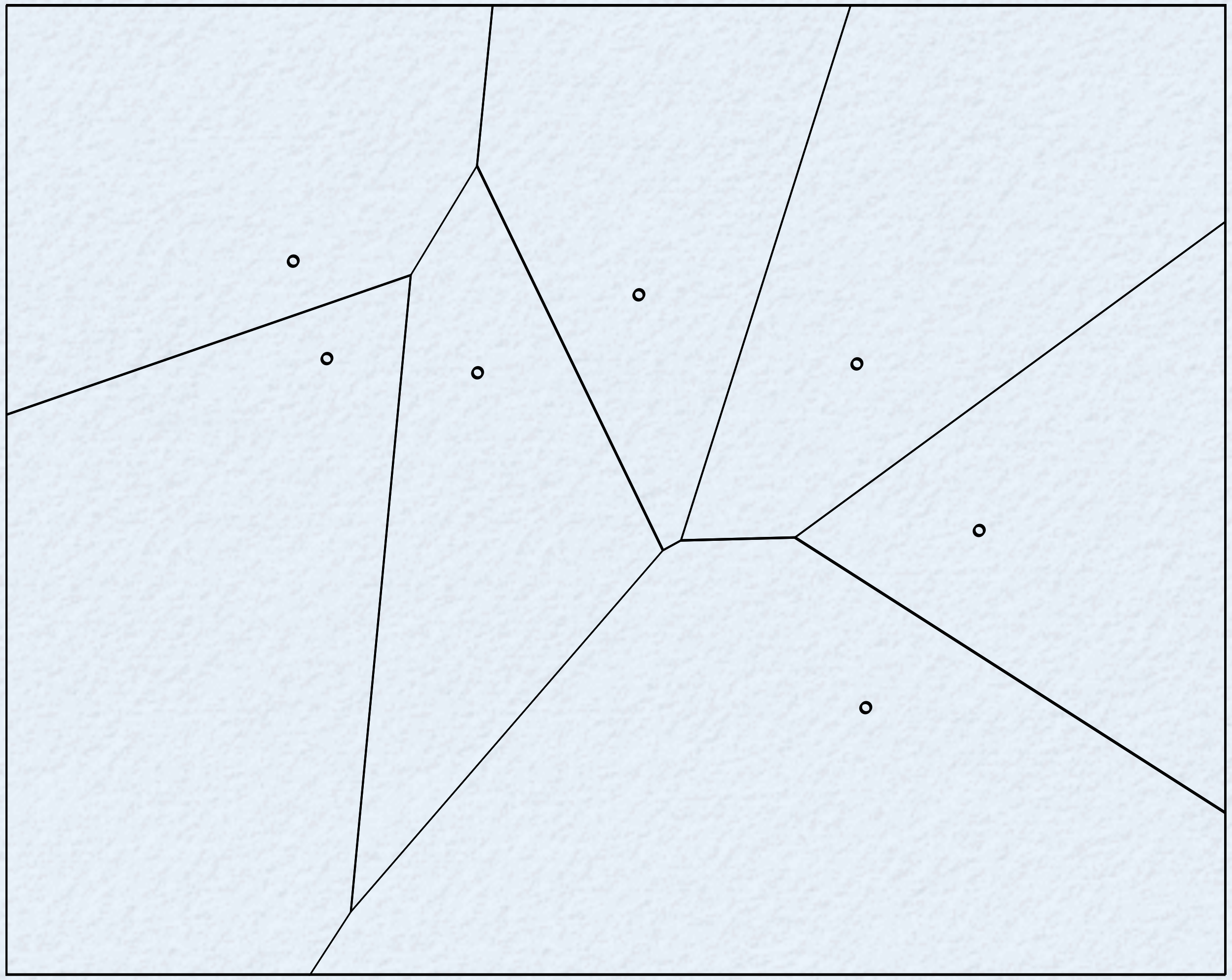

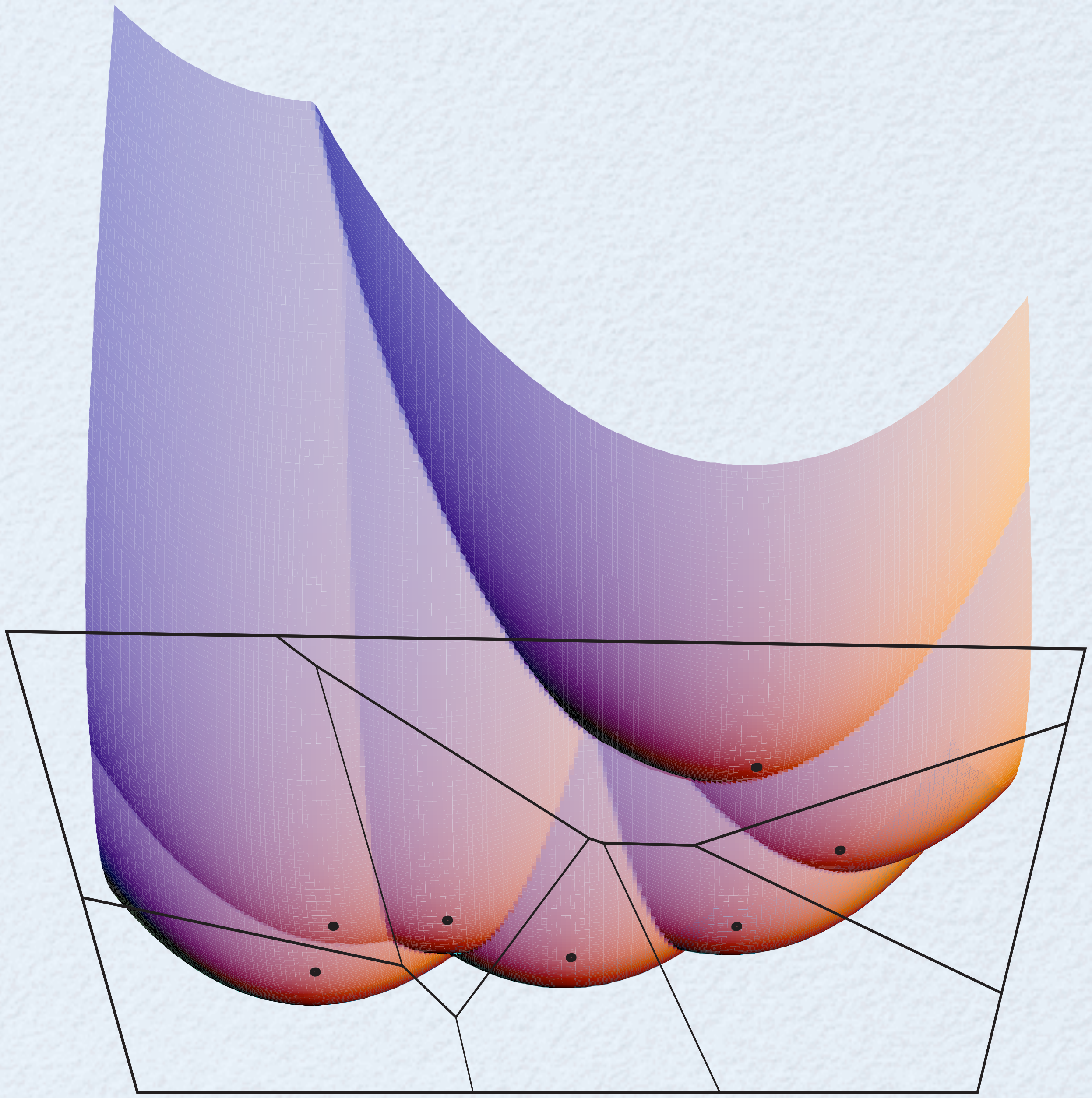

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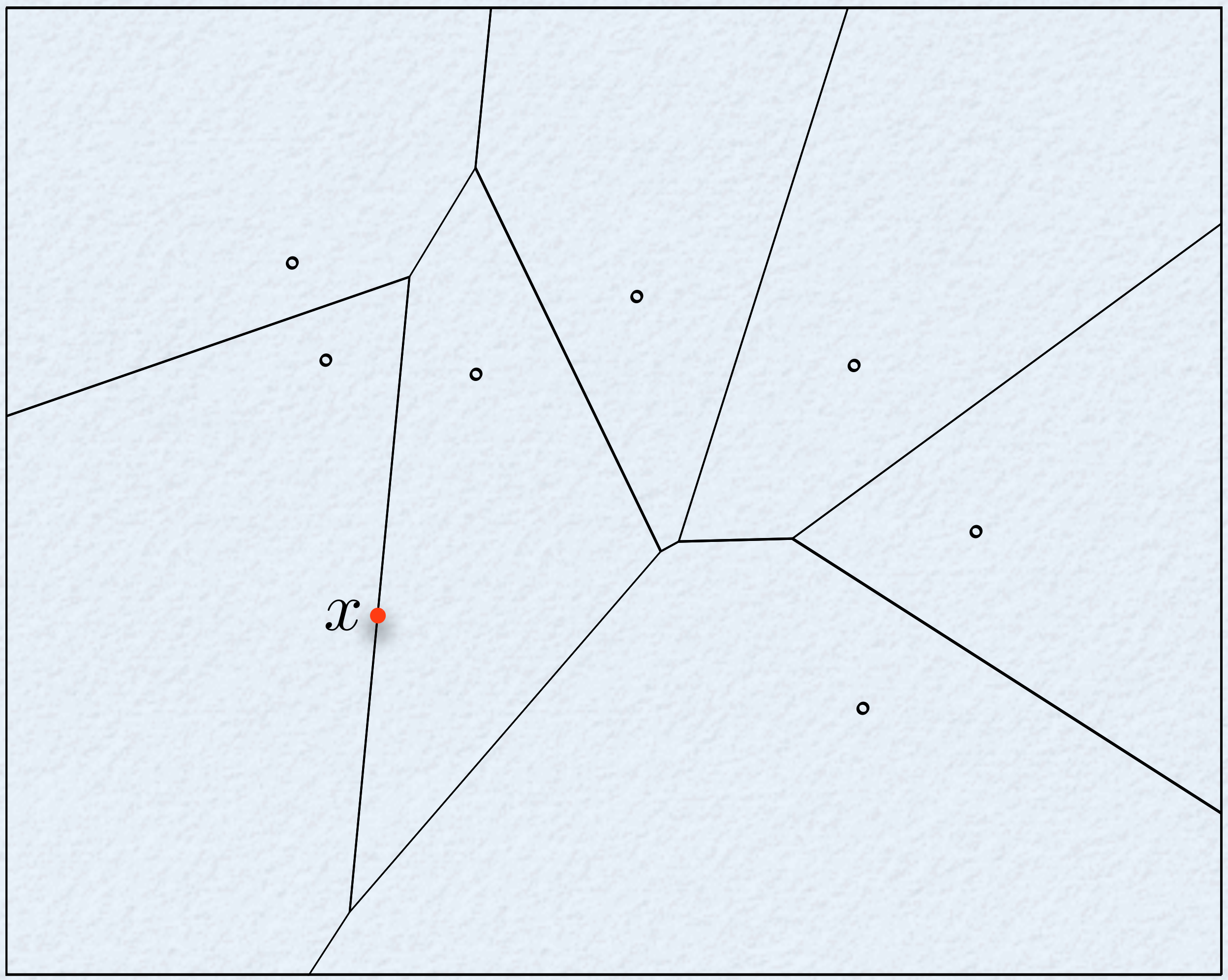

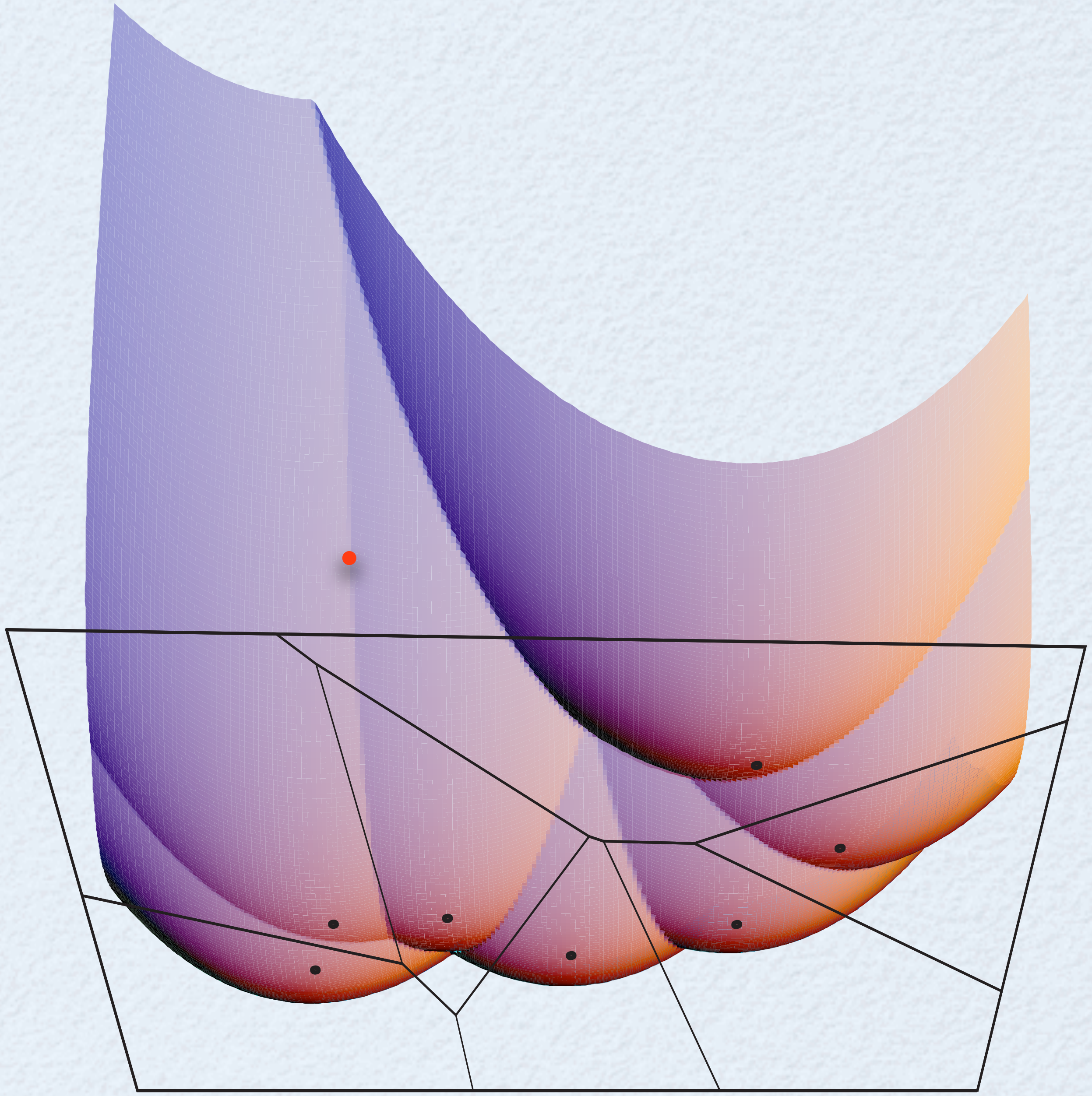

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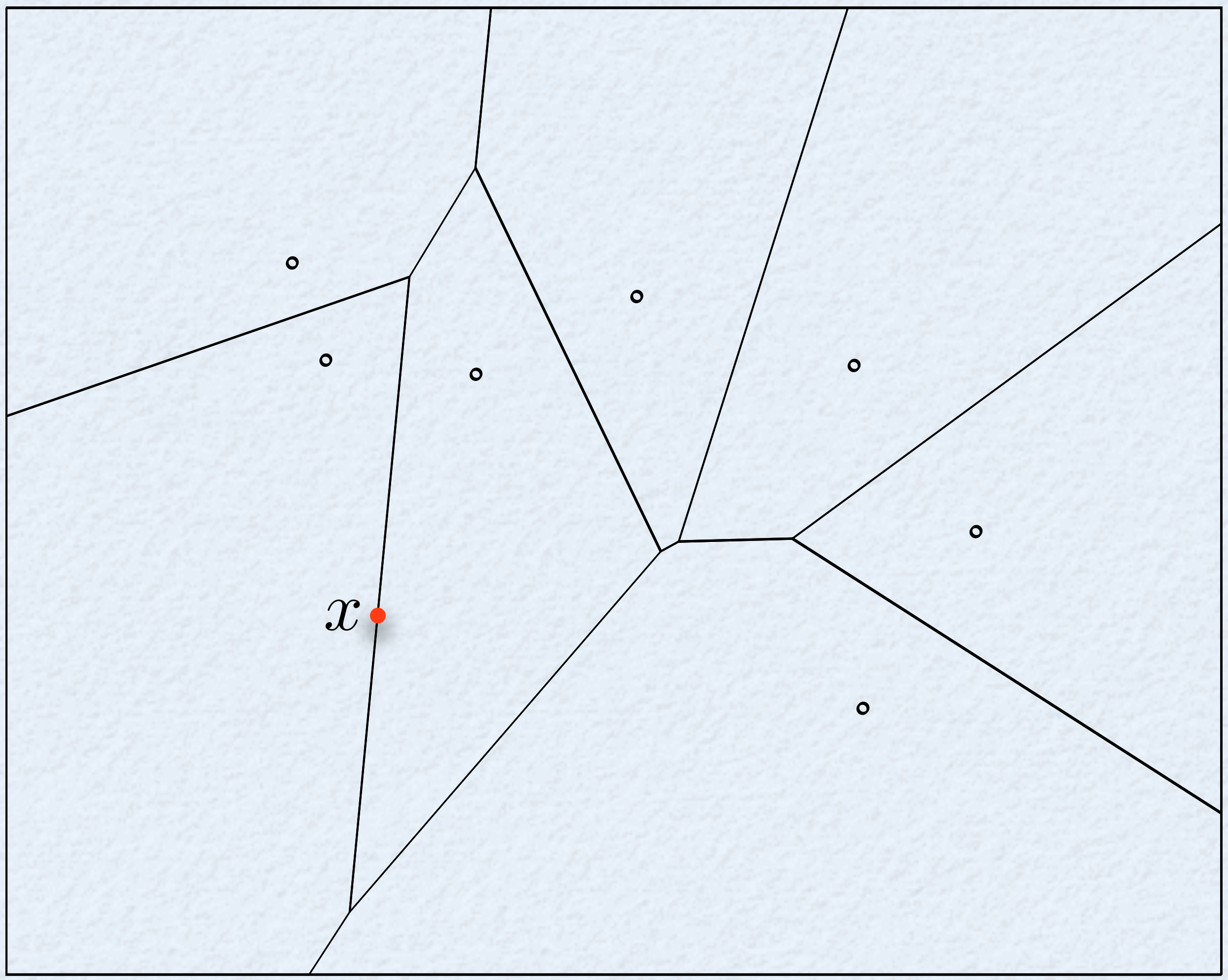

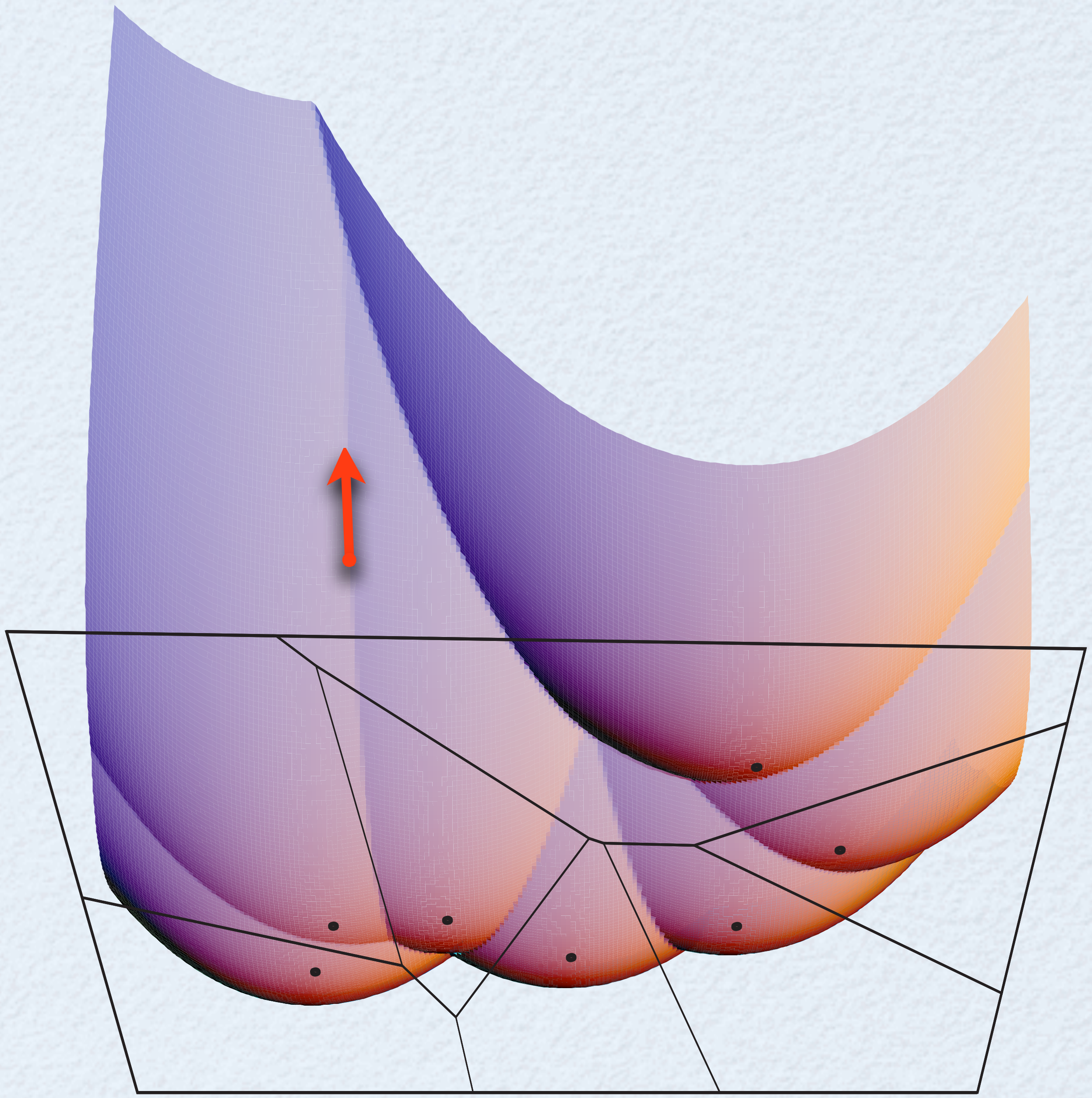

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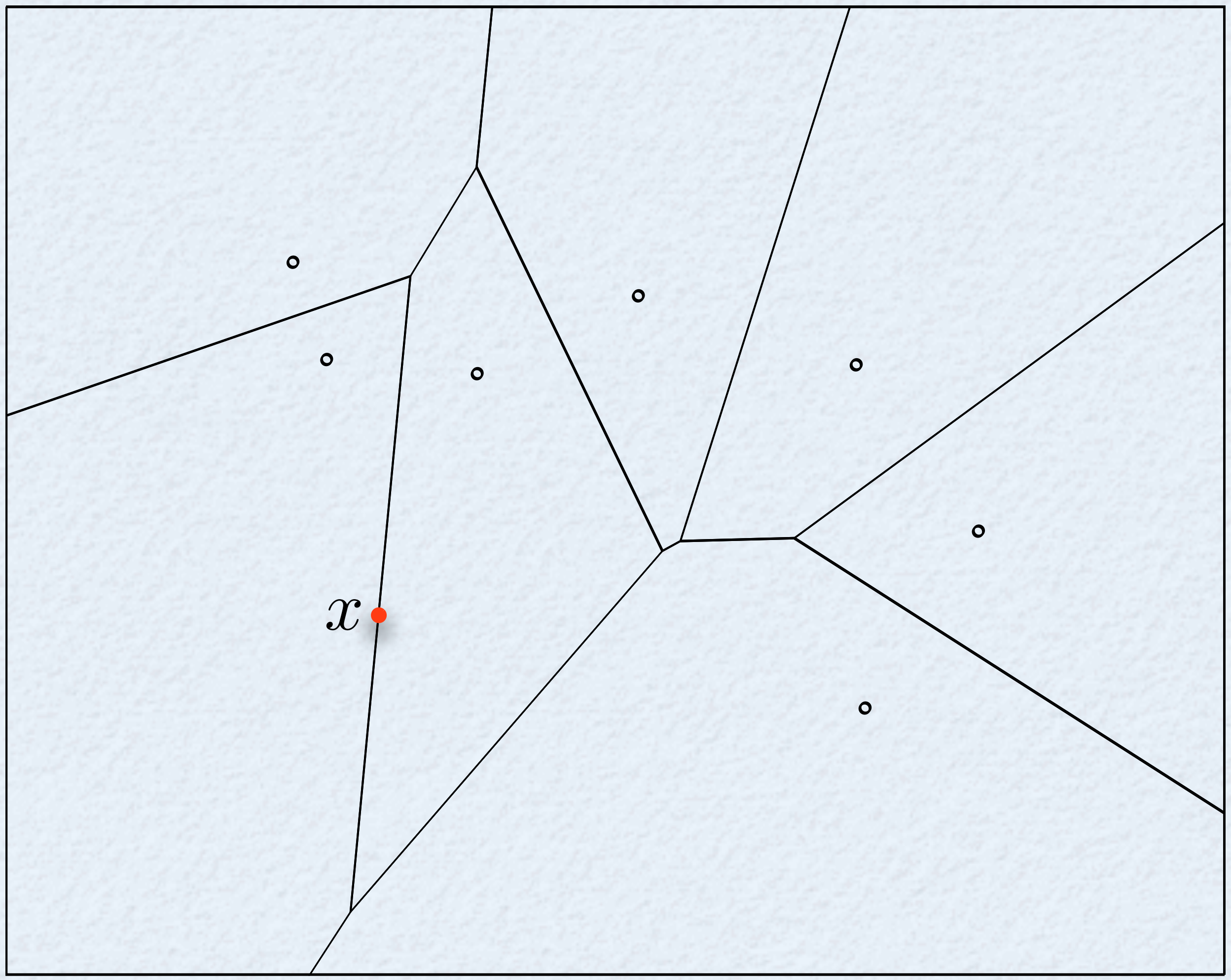

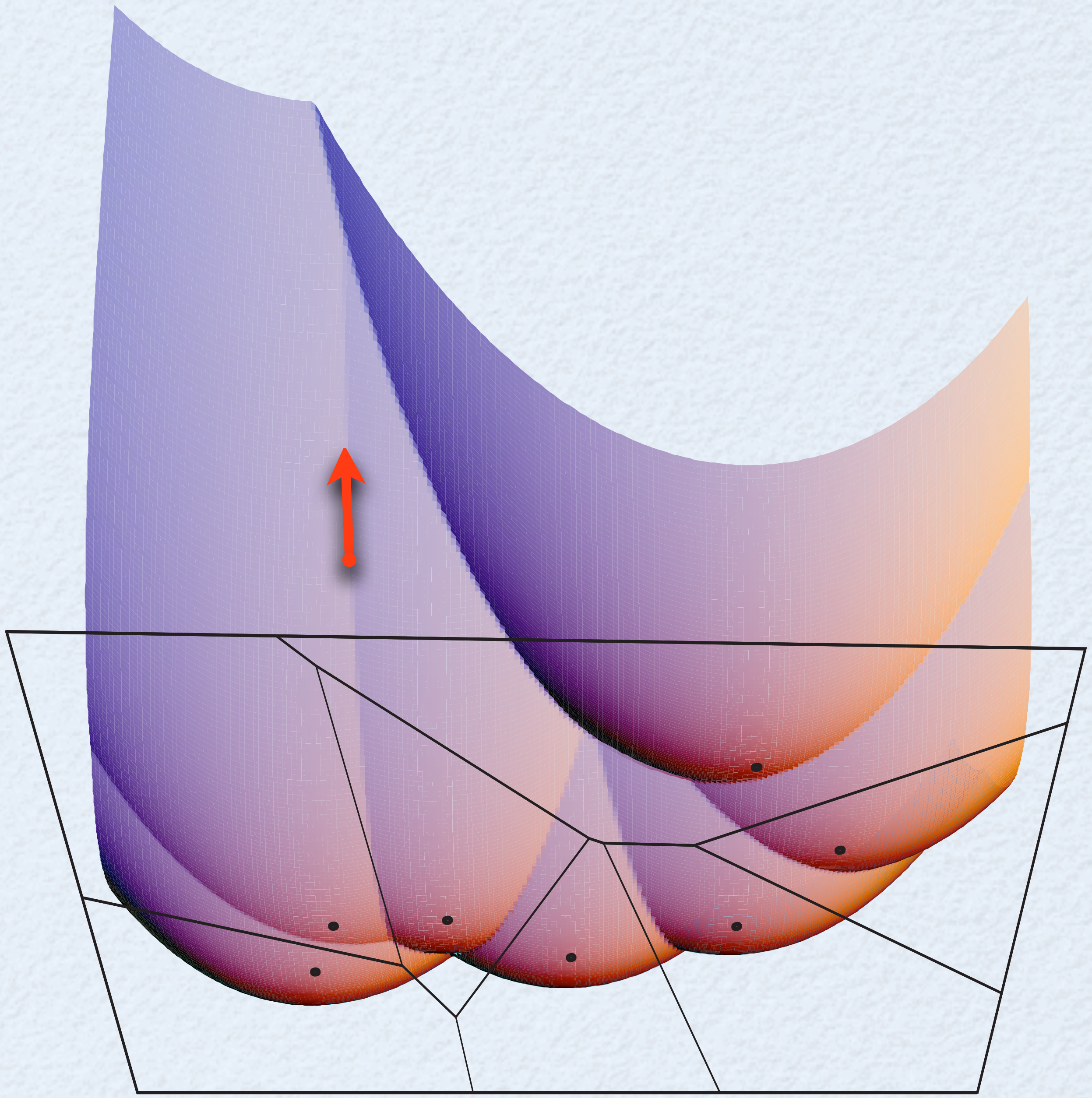

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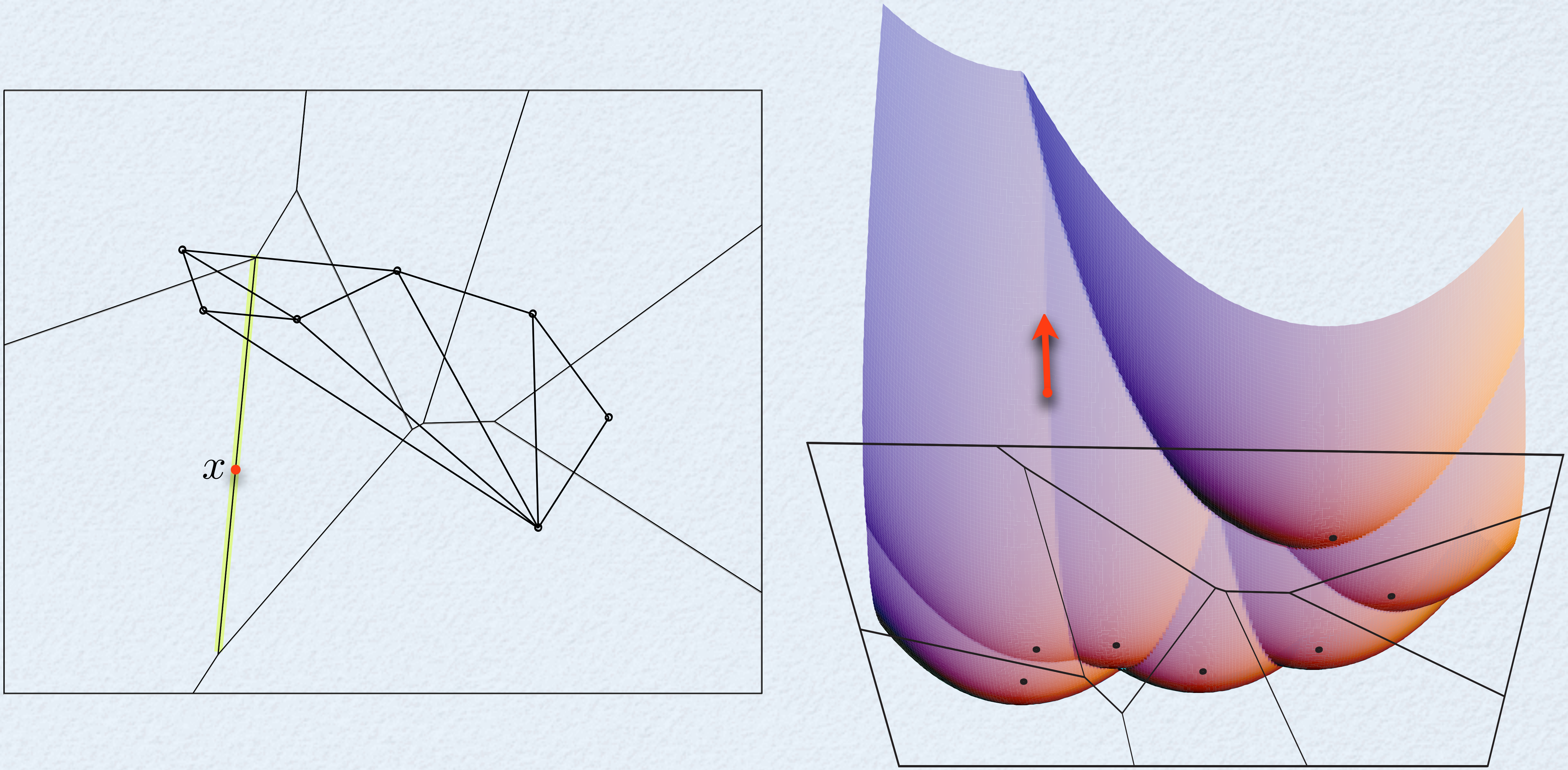

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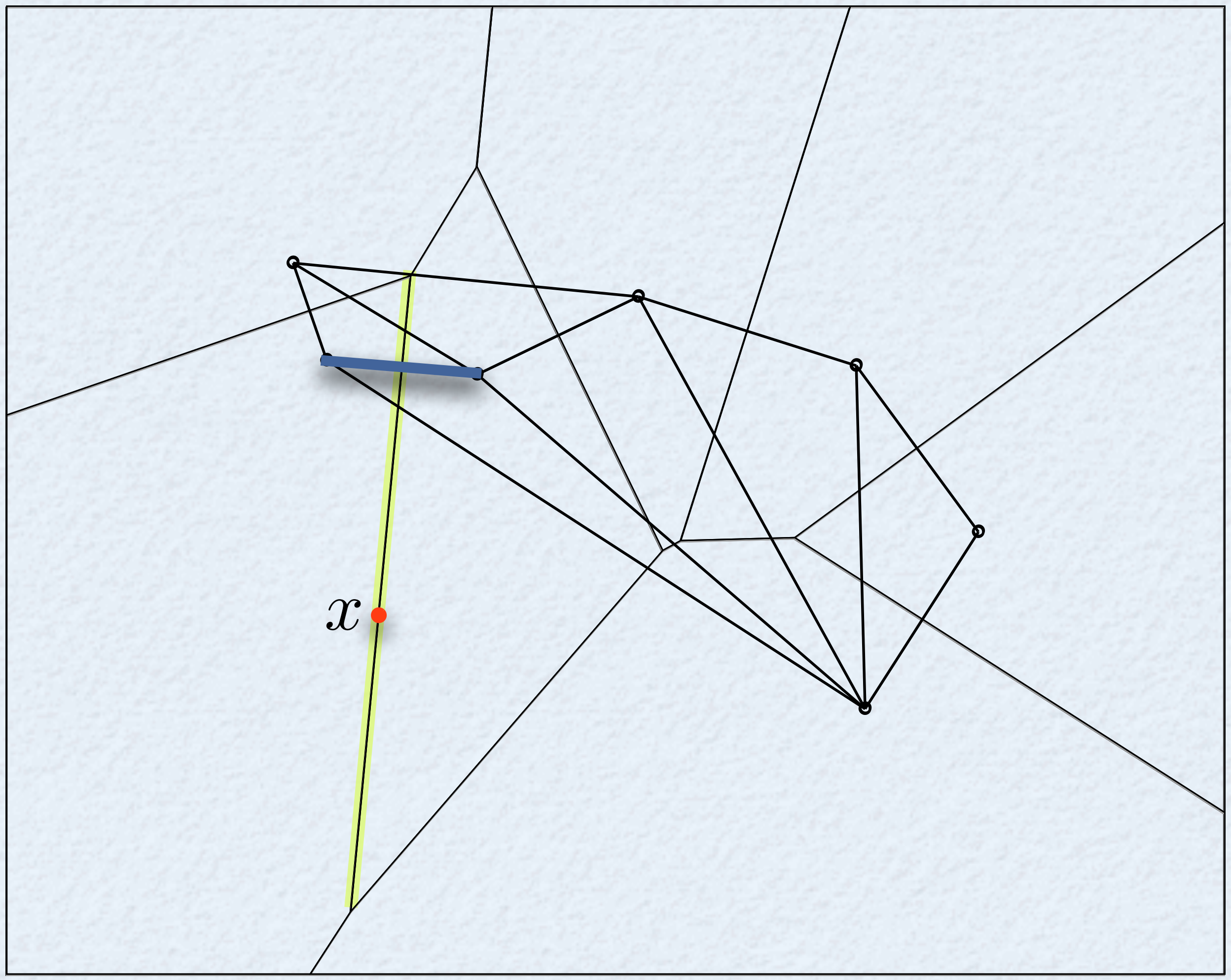

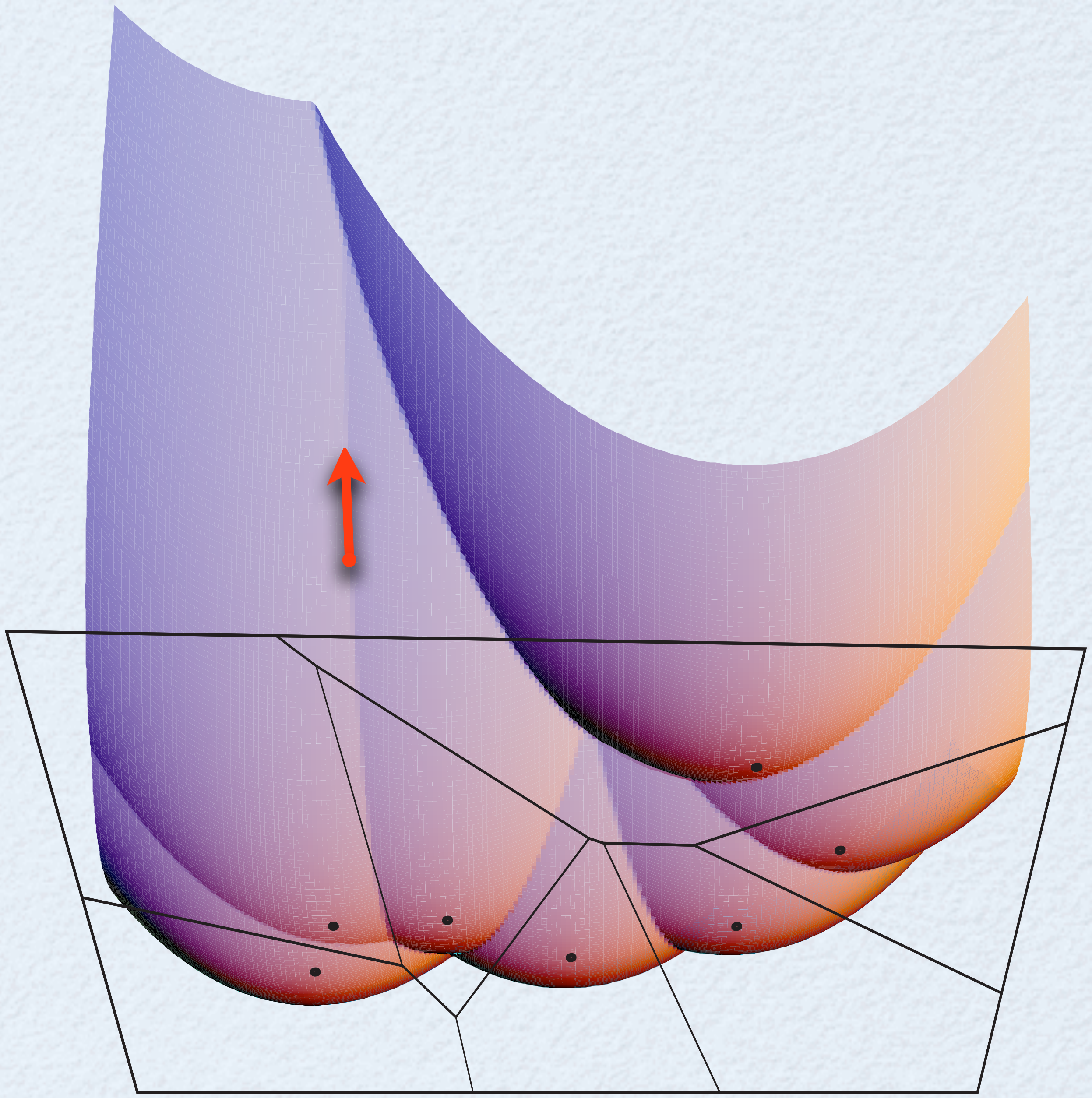

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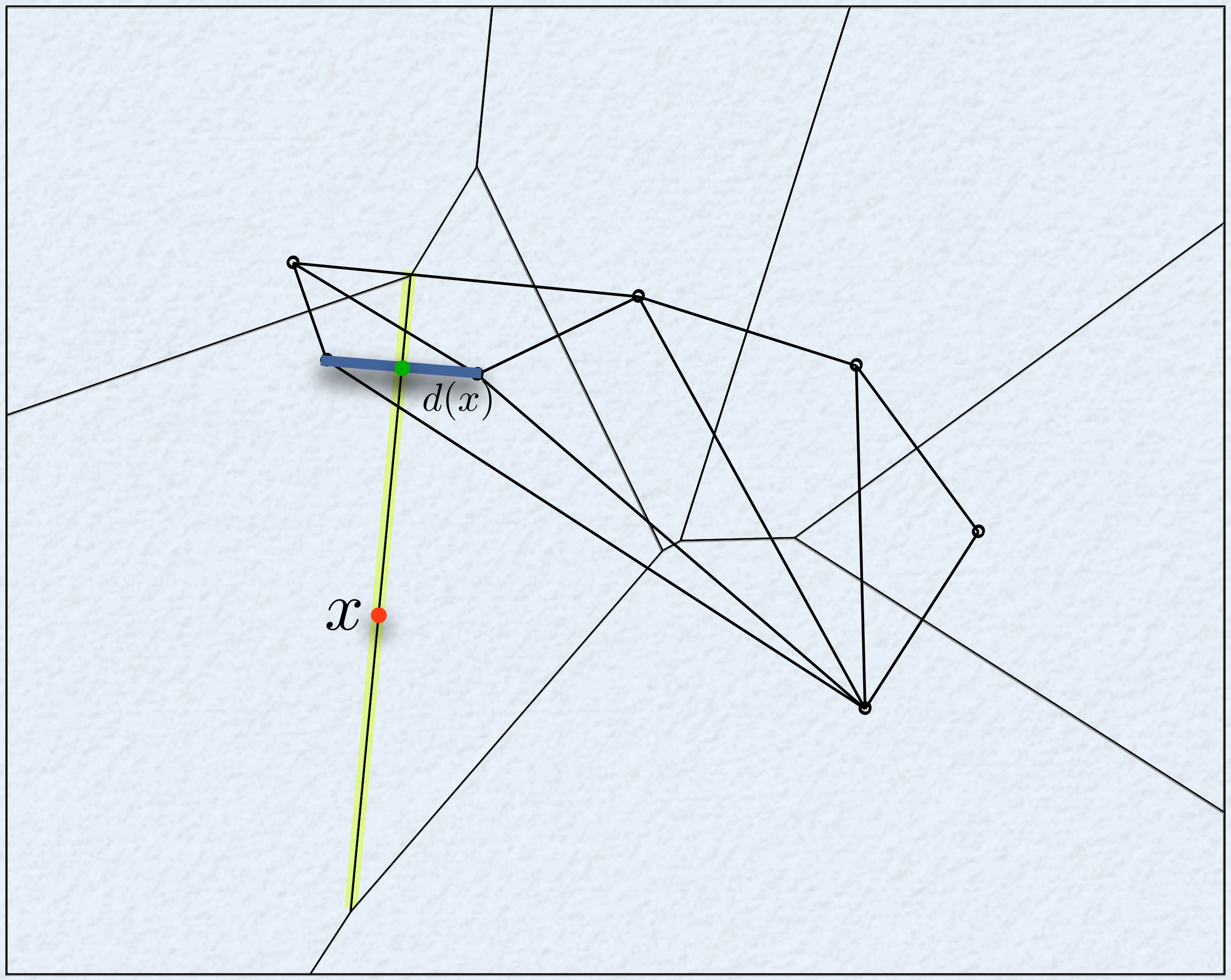

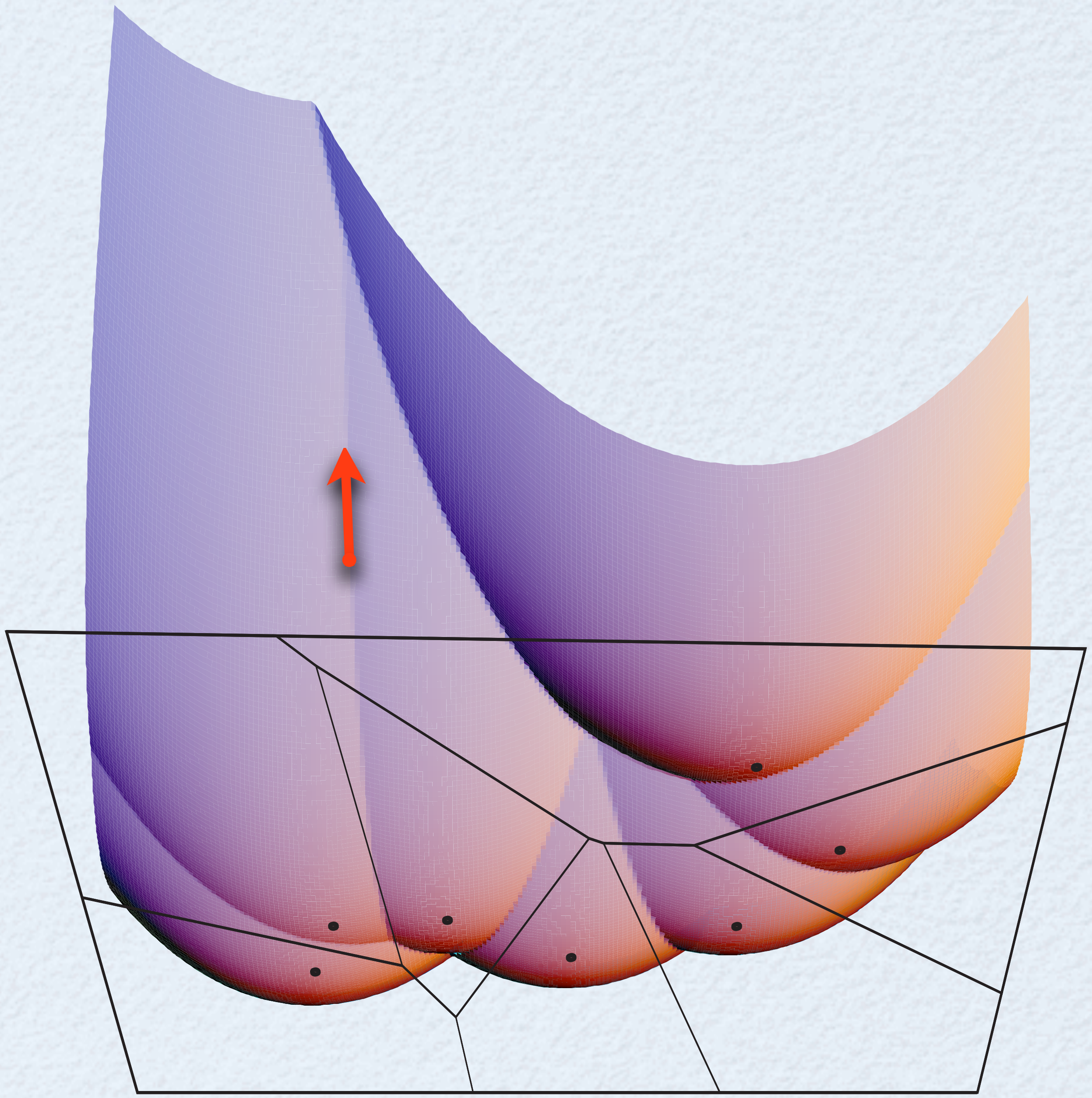

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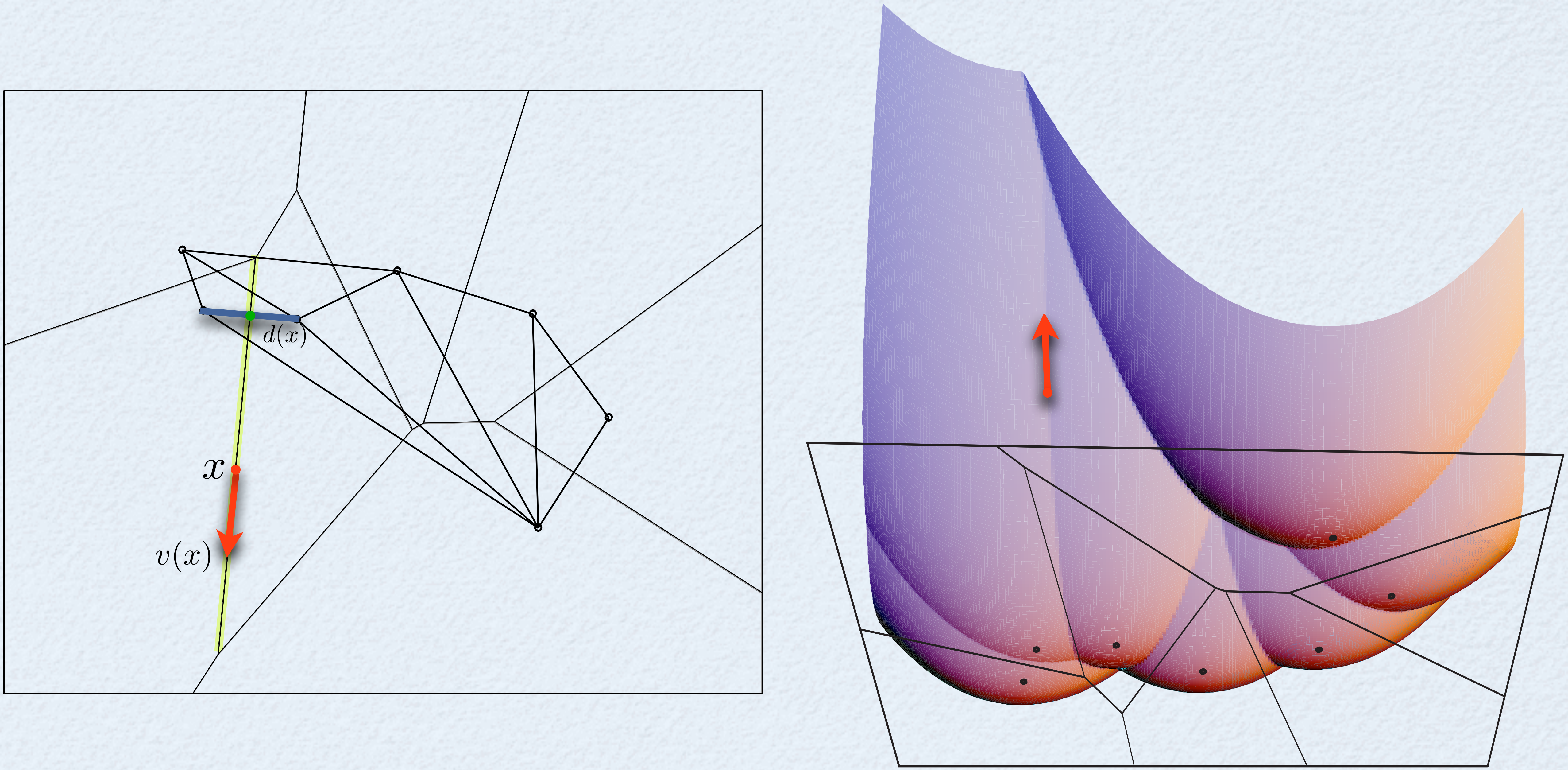

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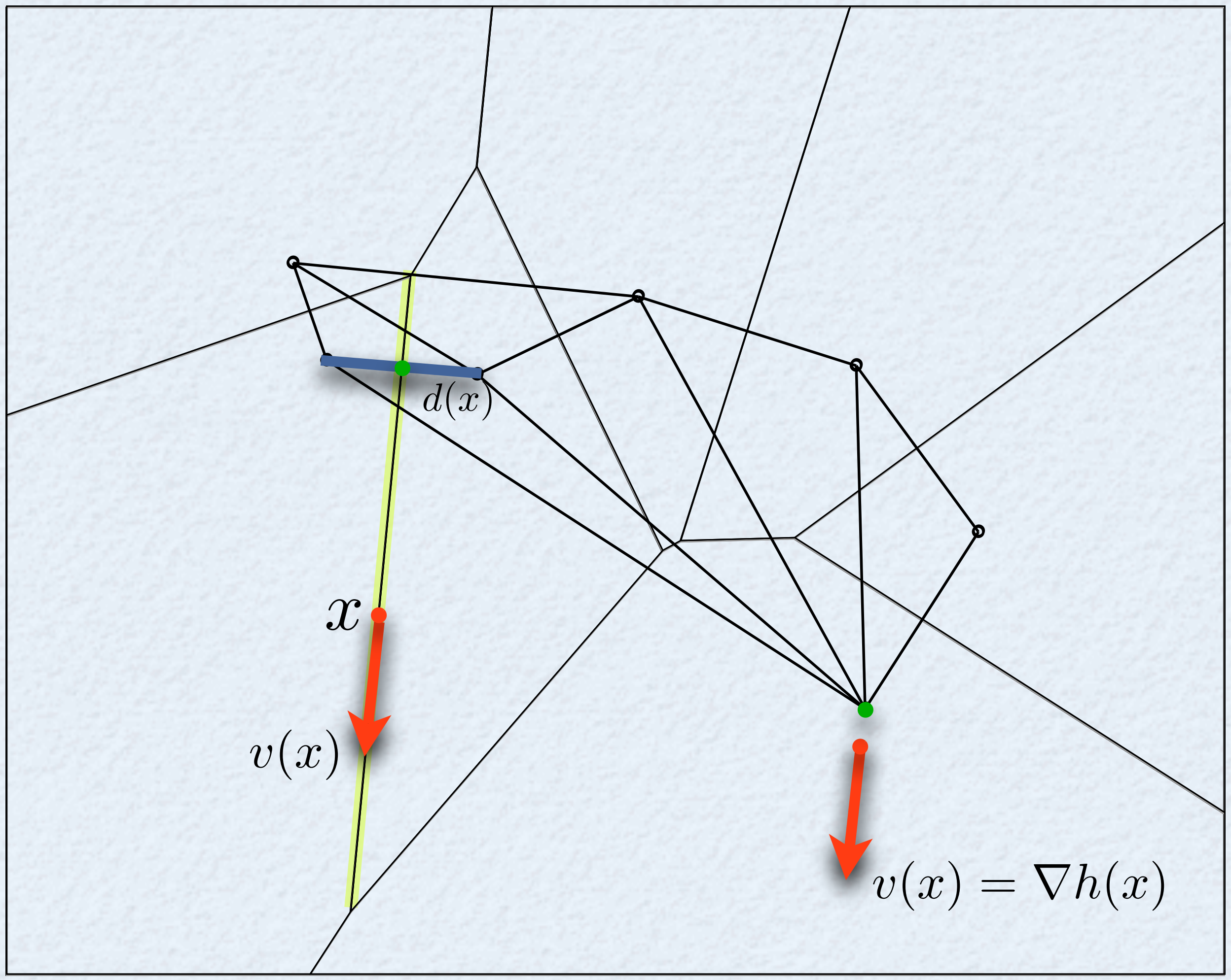

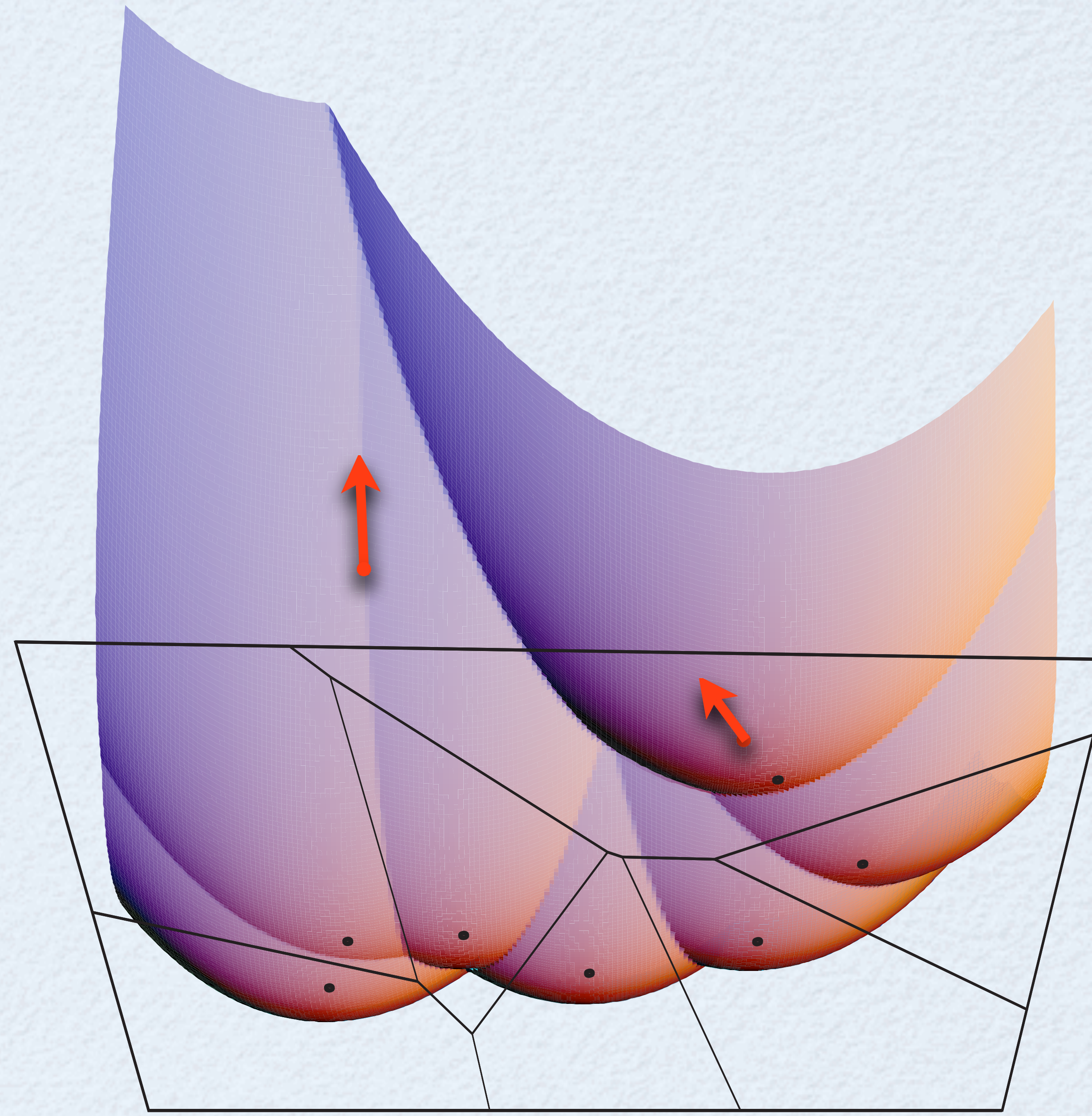

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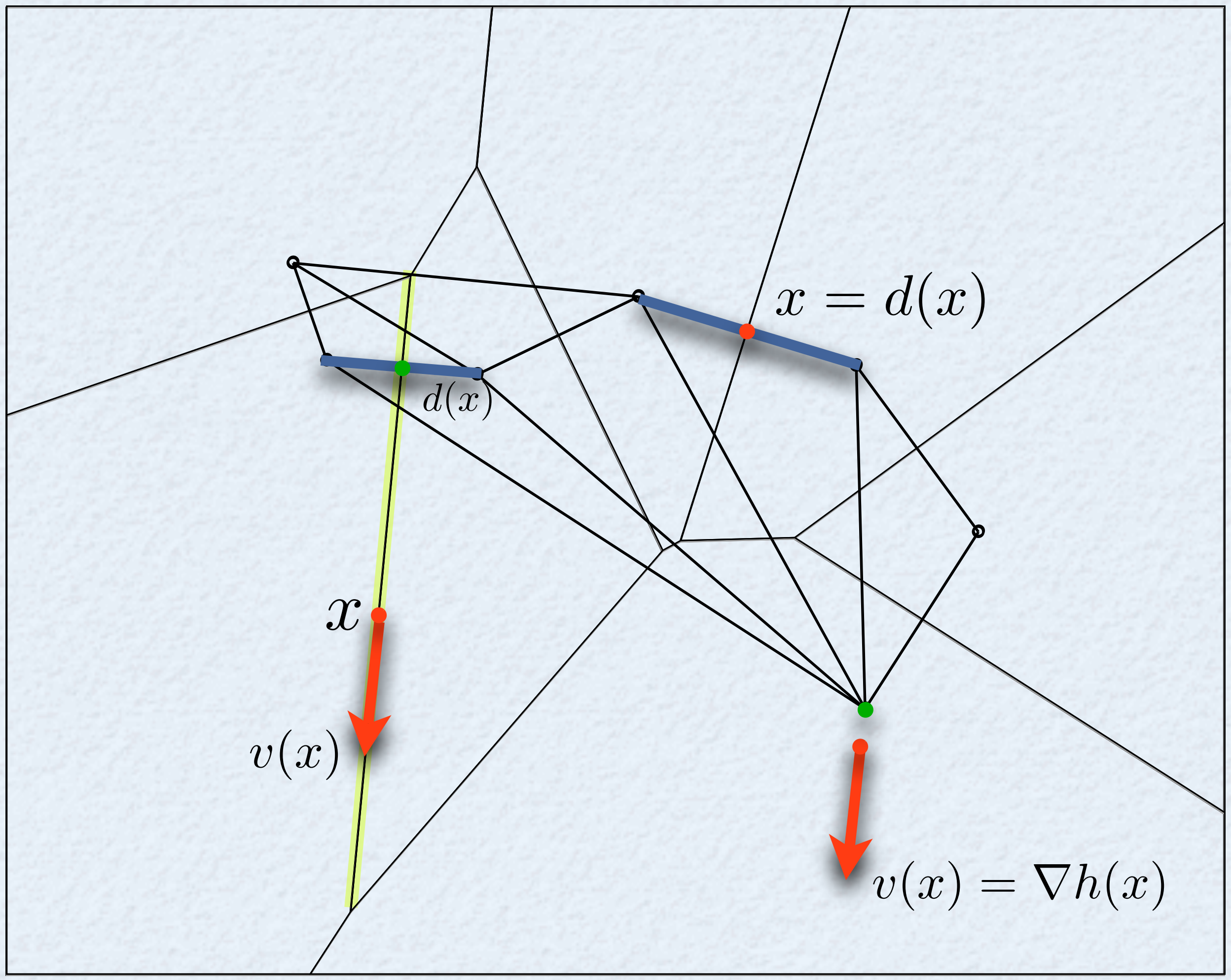

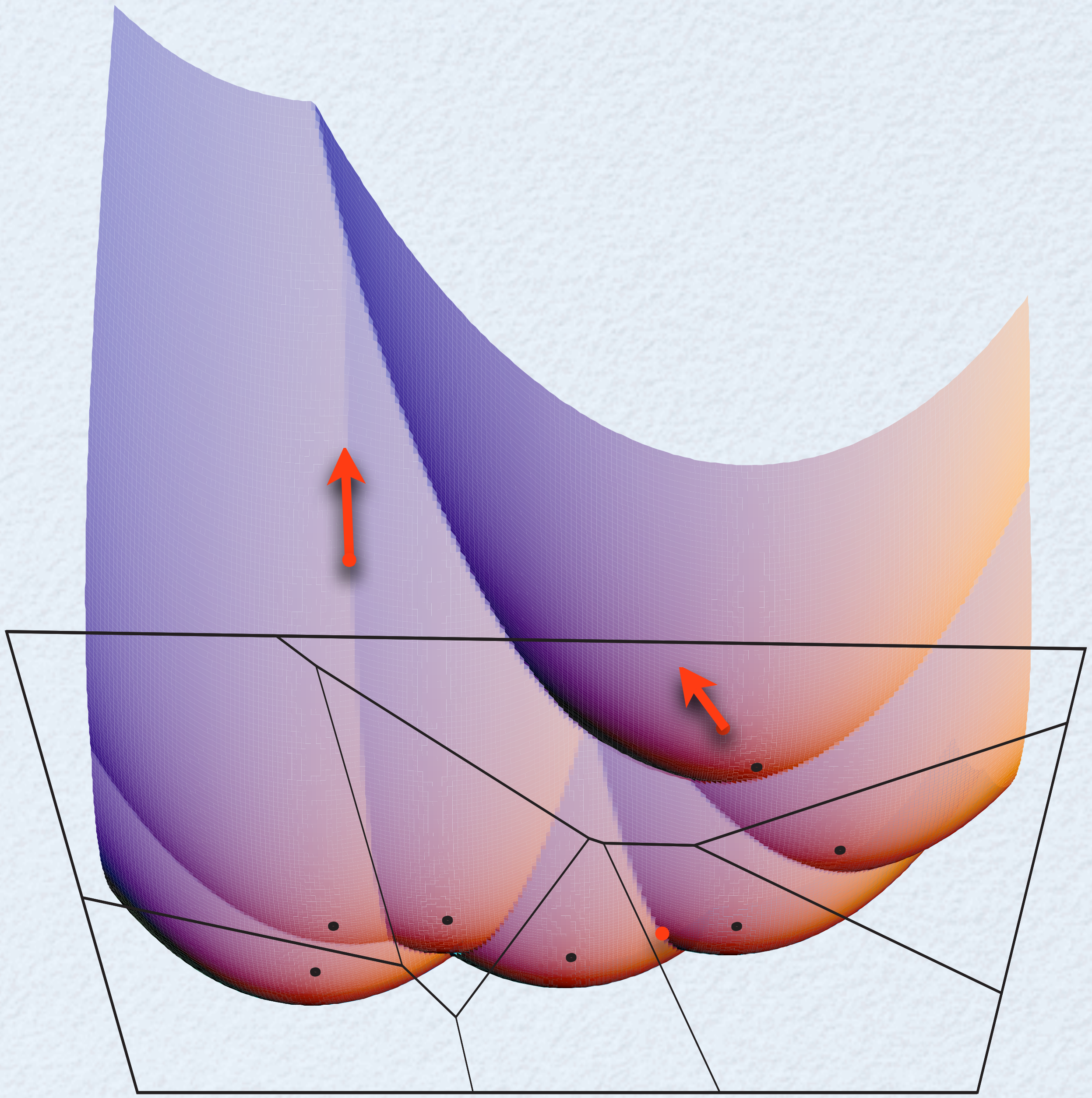

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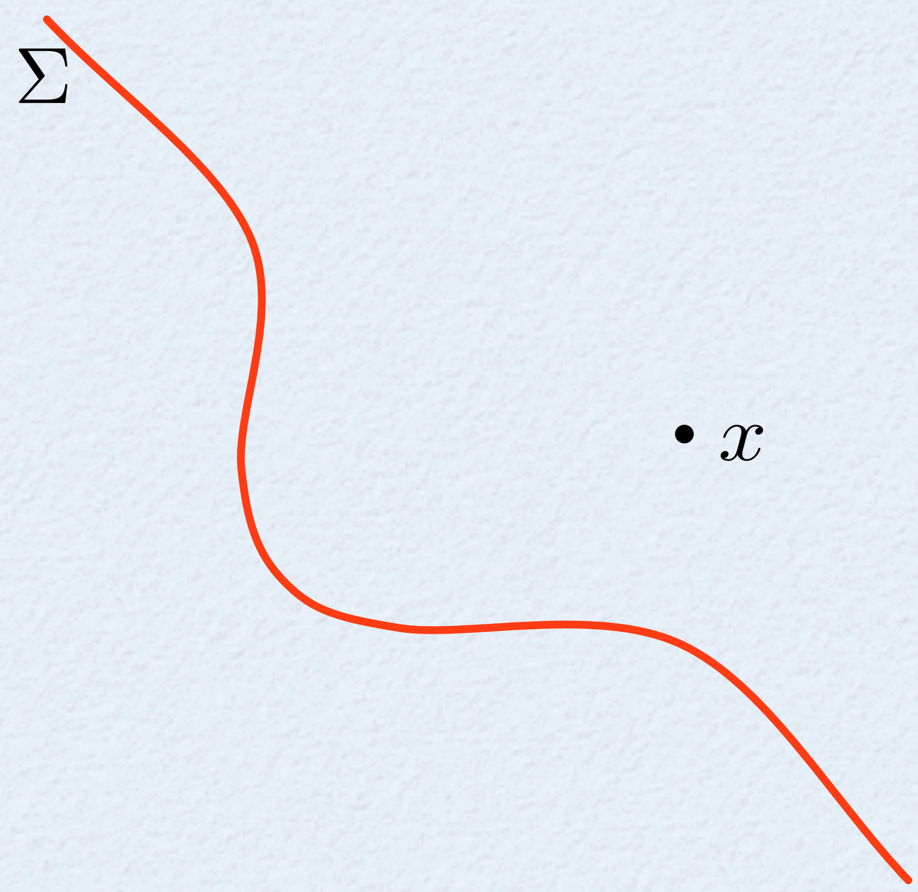

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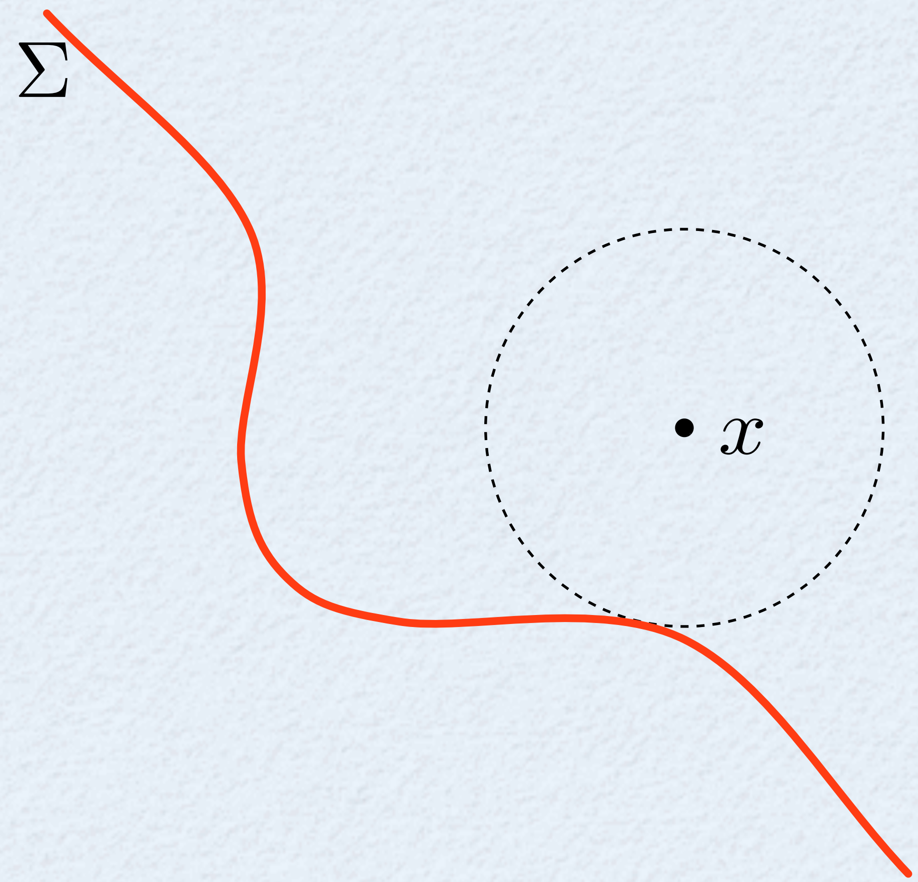

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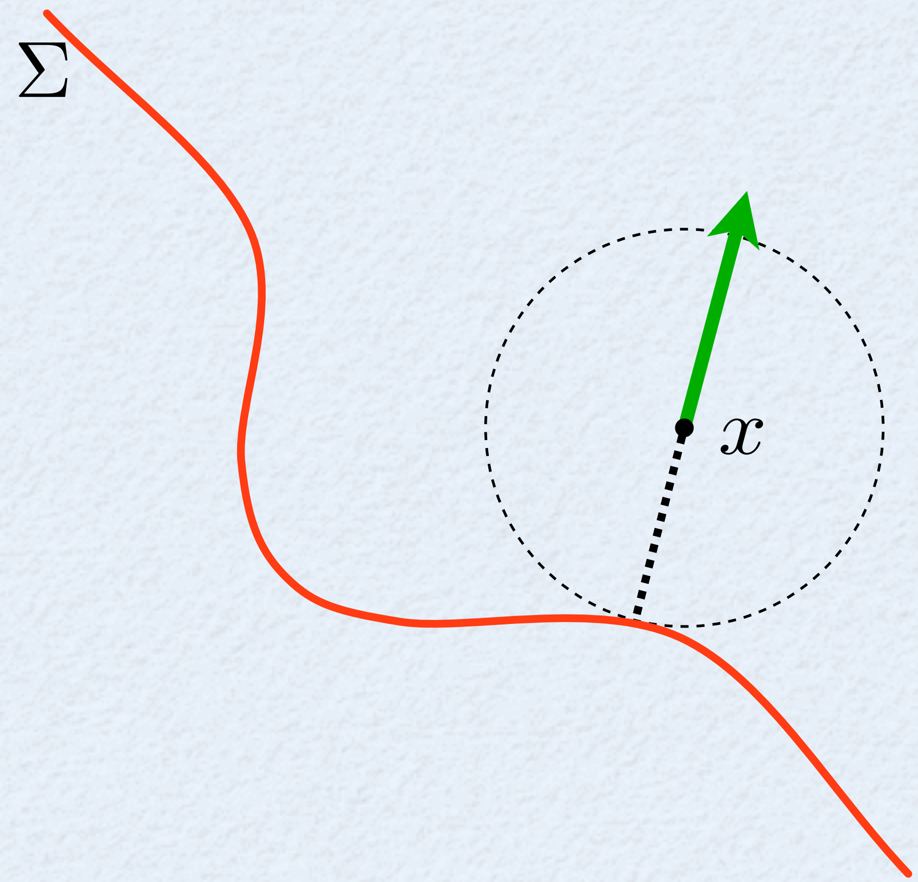

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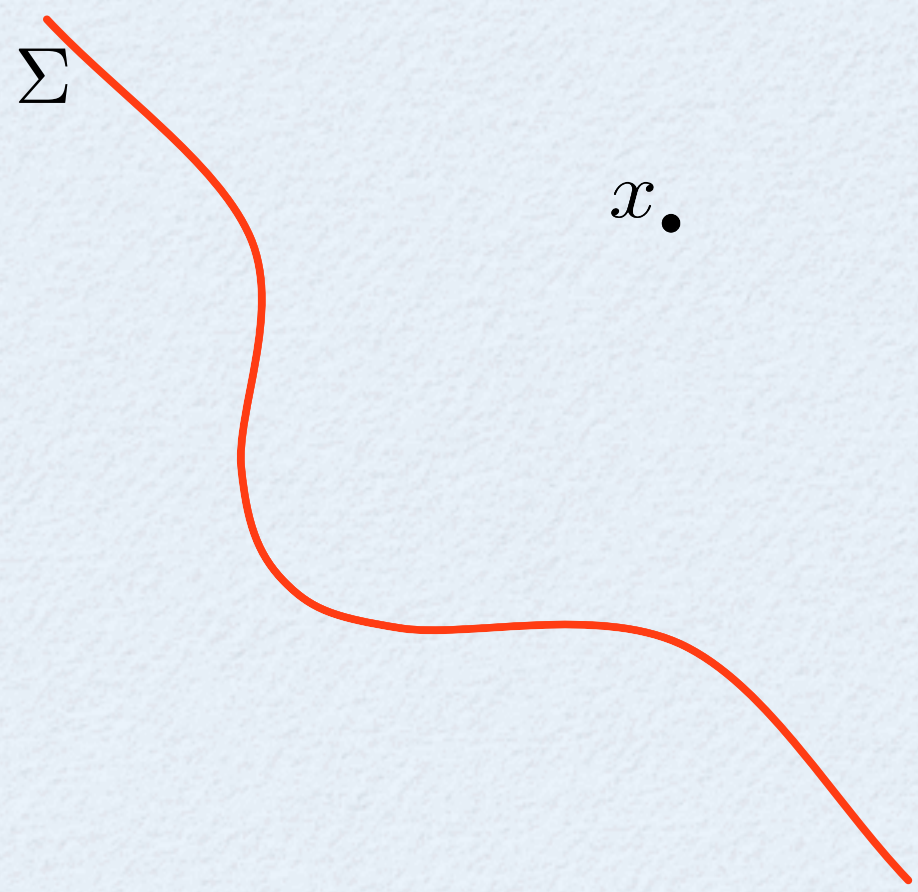

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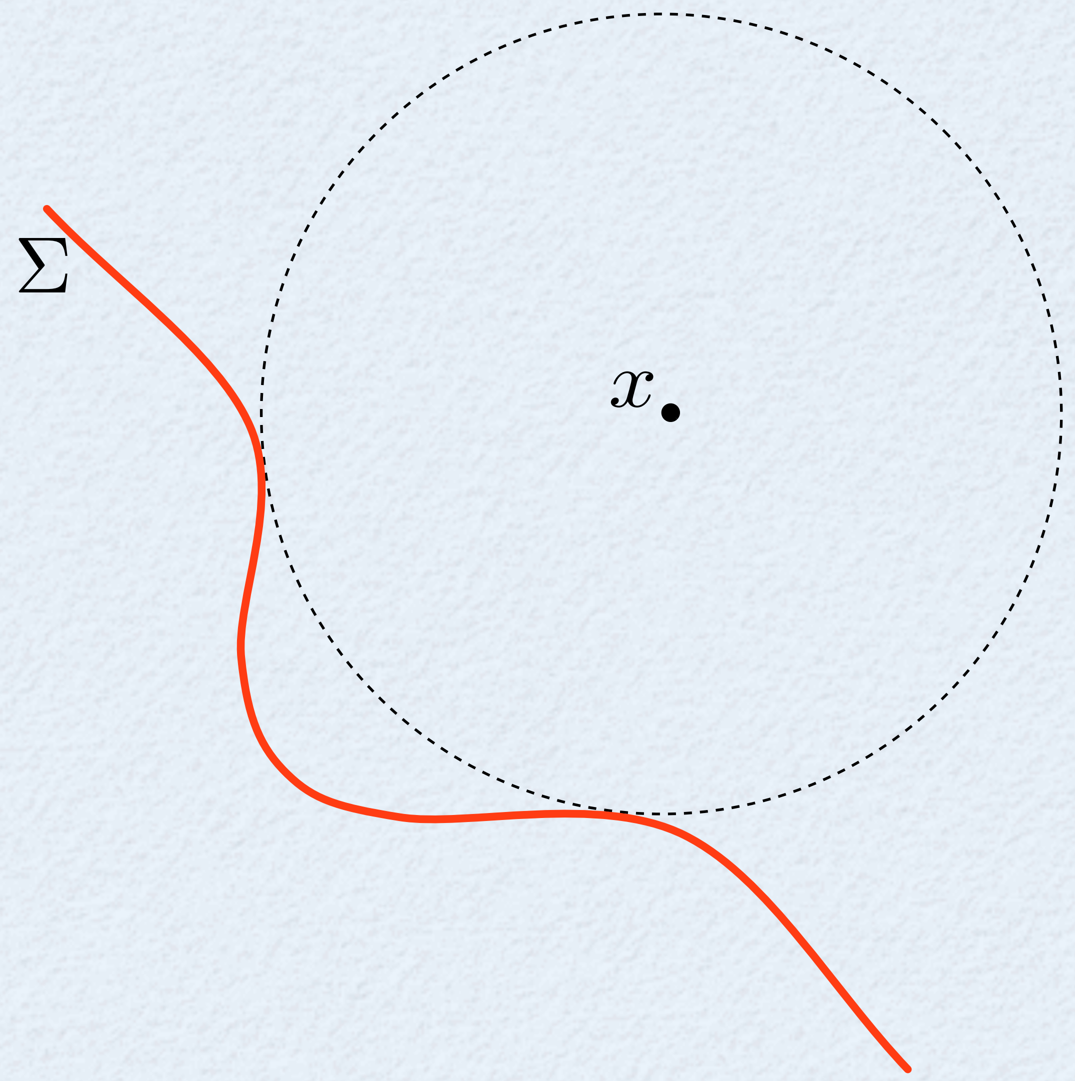

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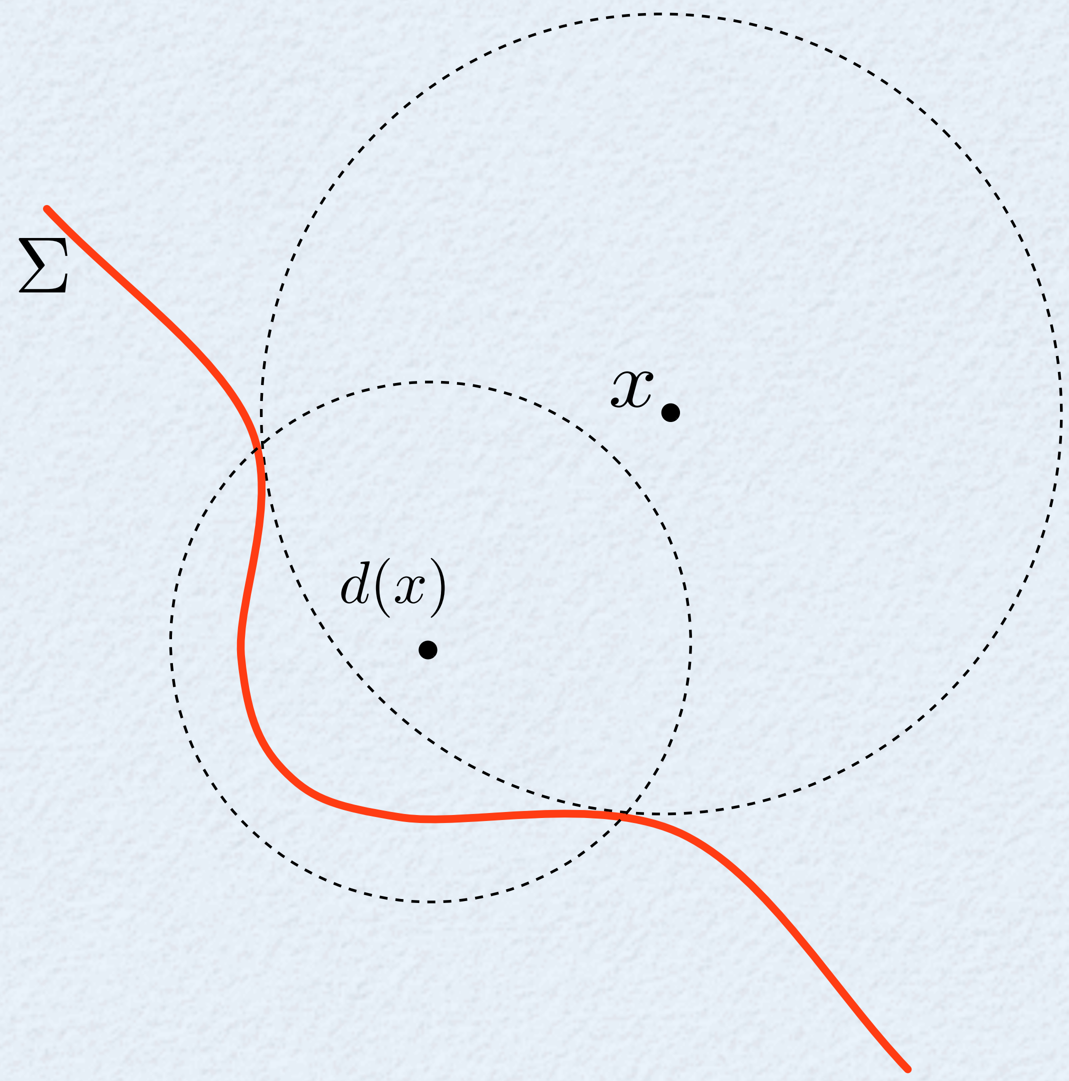

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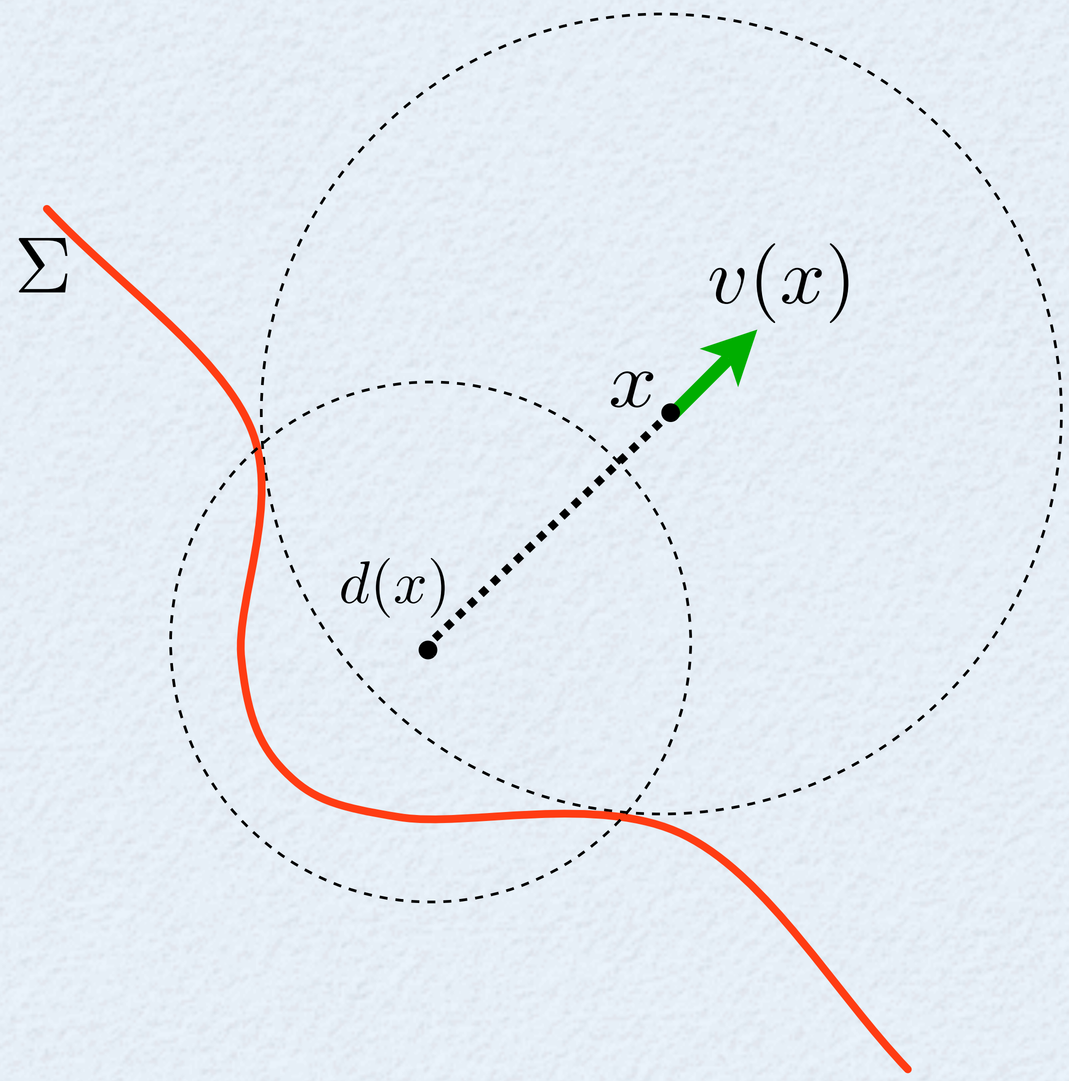

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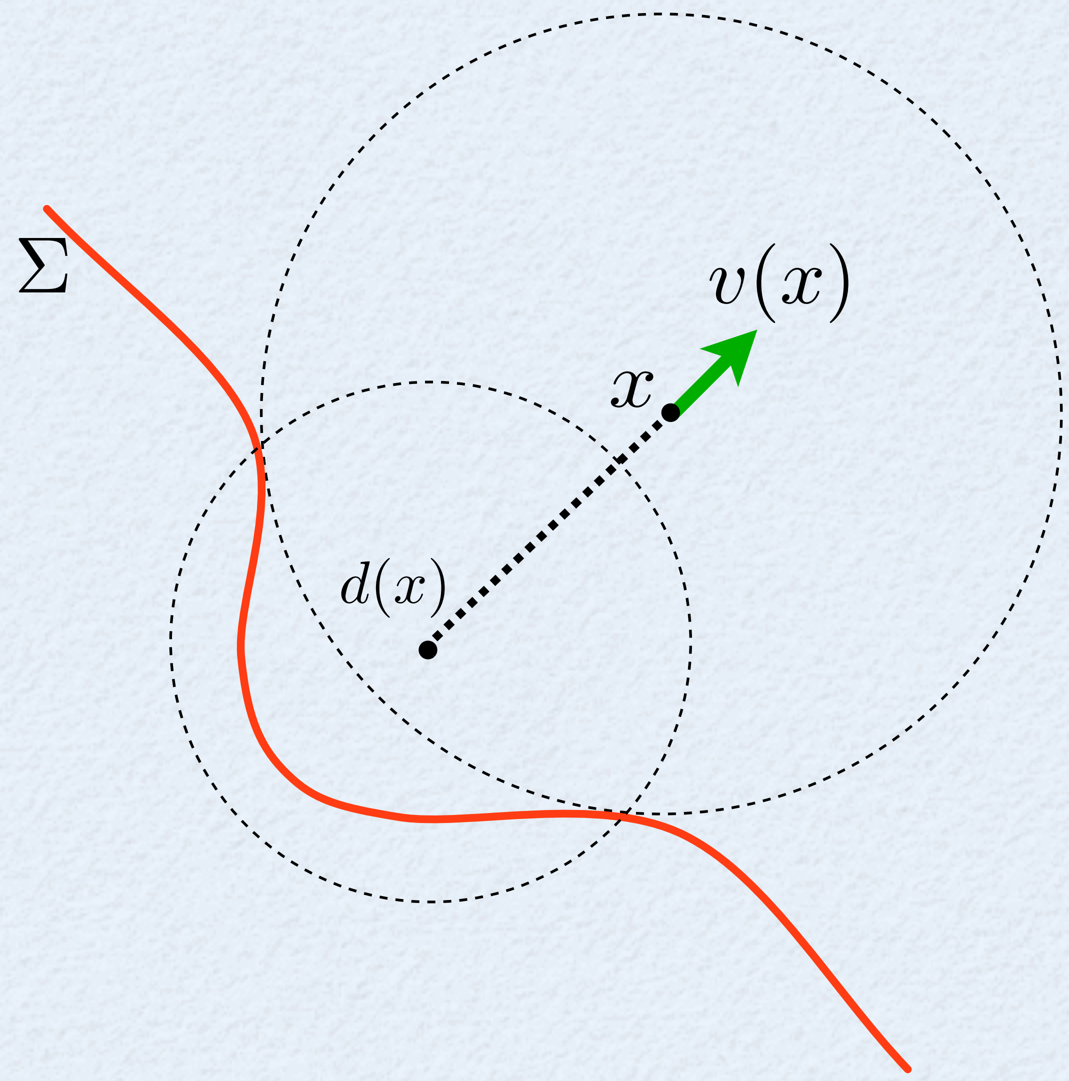
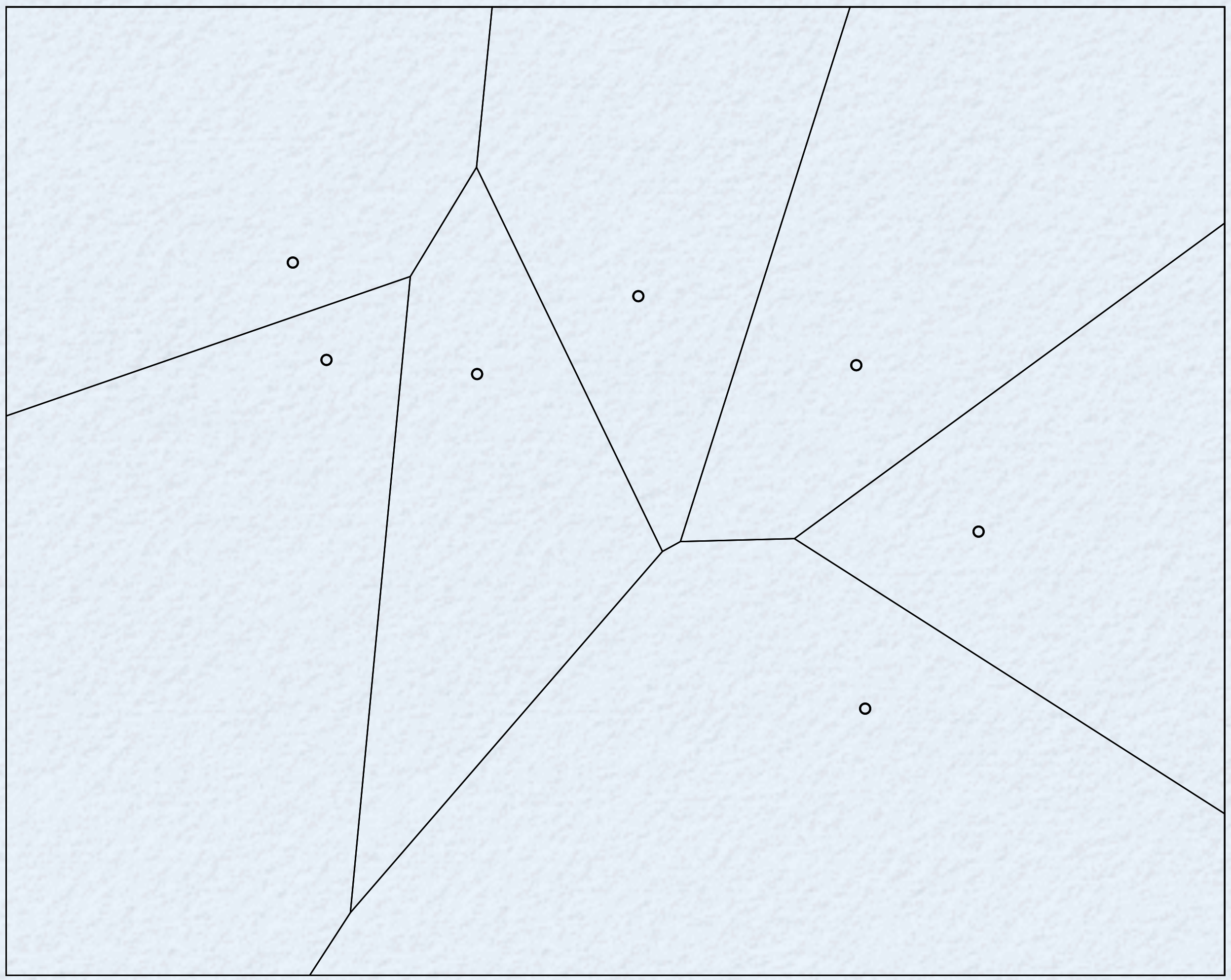
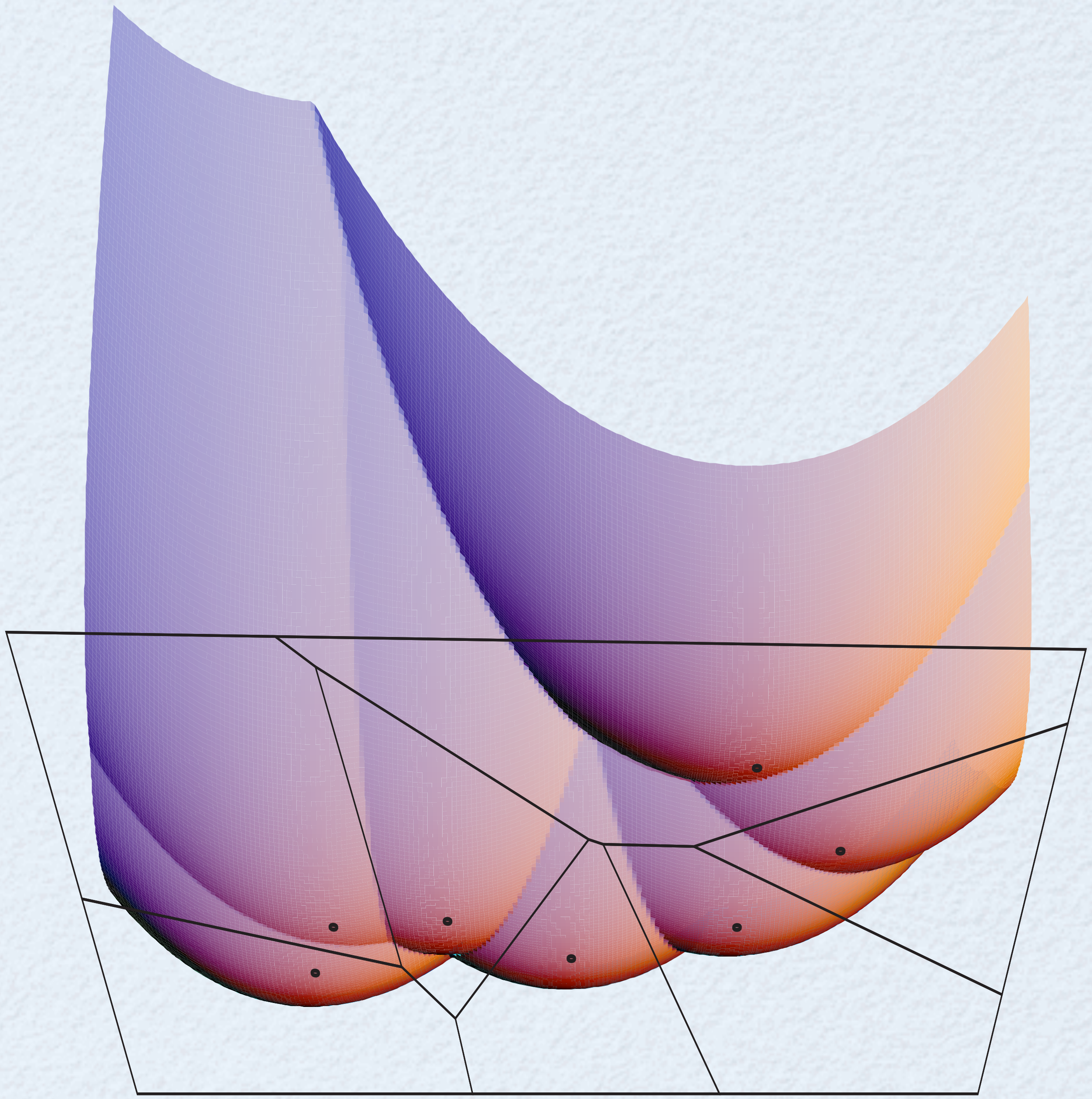


... or even for (geodesic) distances relative to a compact subset of a Reimannian manifold. [Grove' 93]

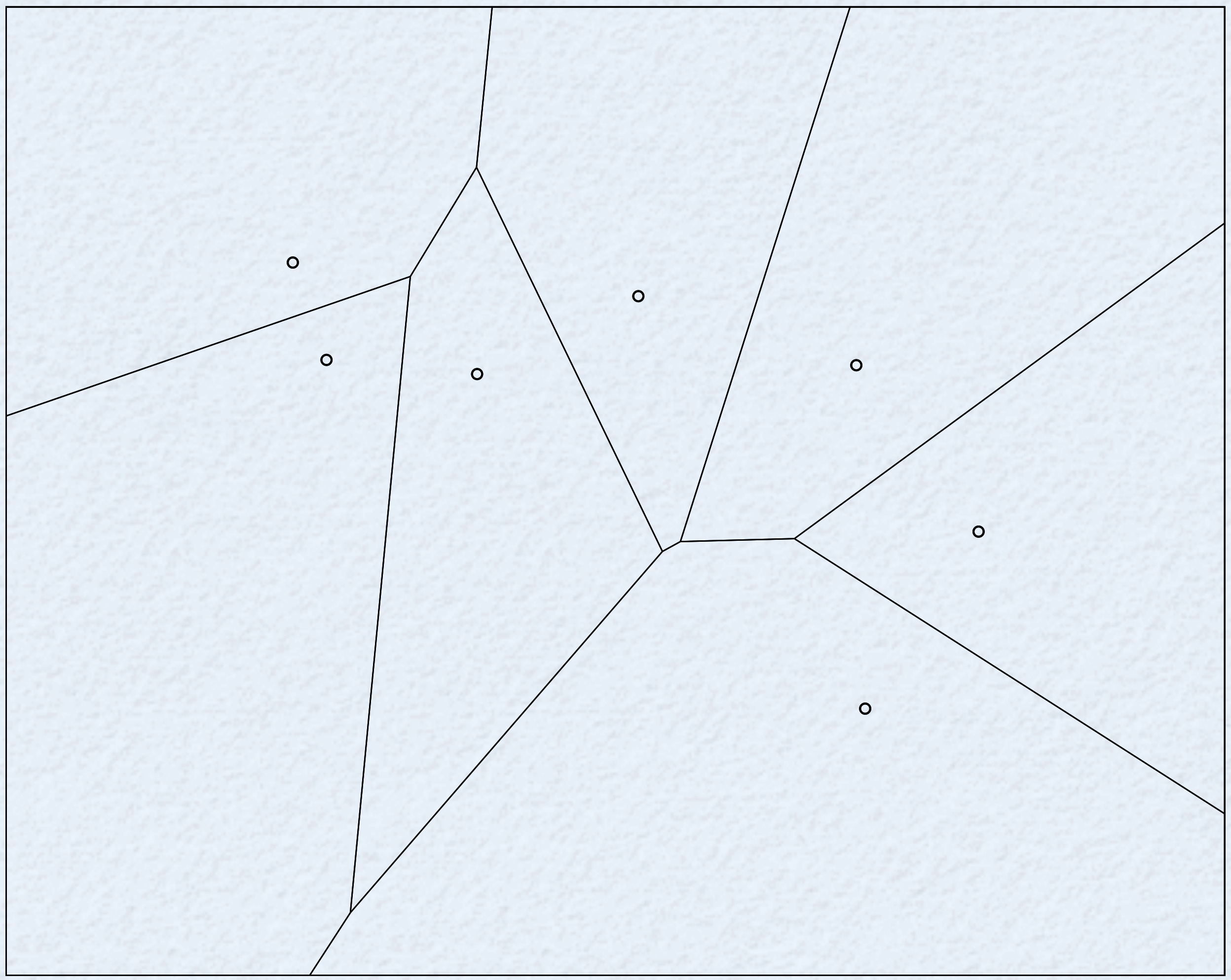
Integrating v

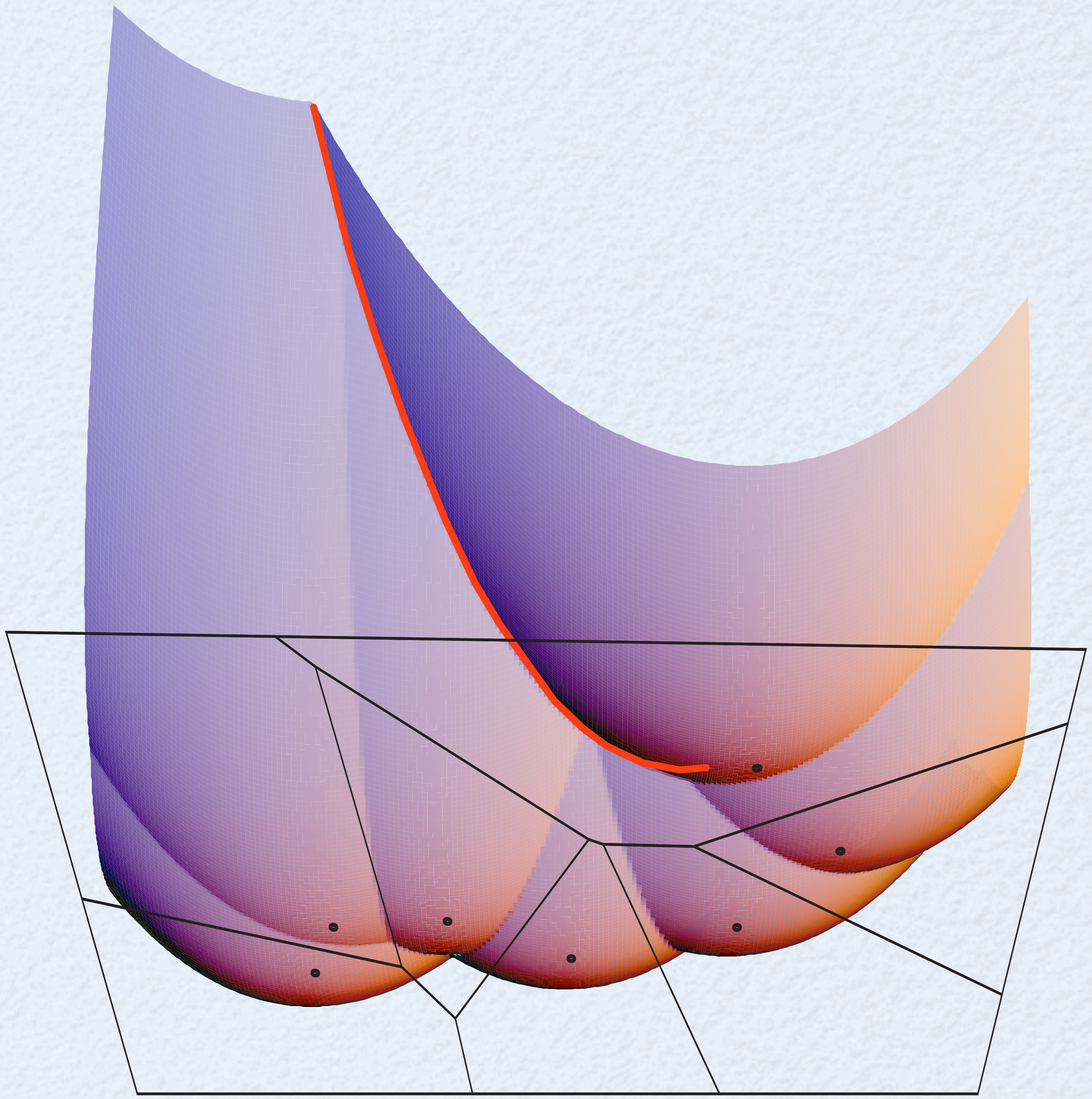




Moving at point x with speed $v(x)$ results a flow map $\phi : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^n$.

Integrating v





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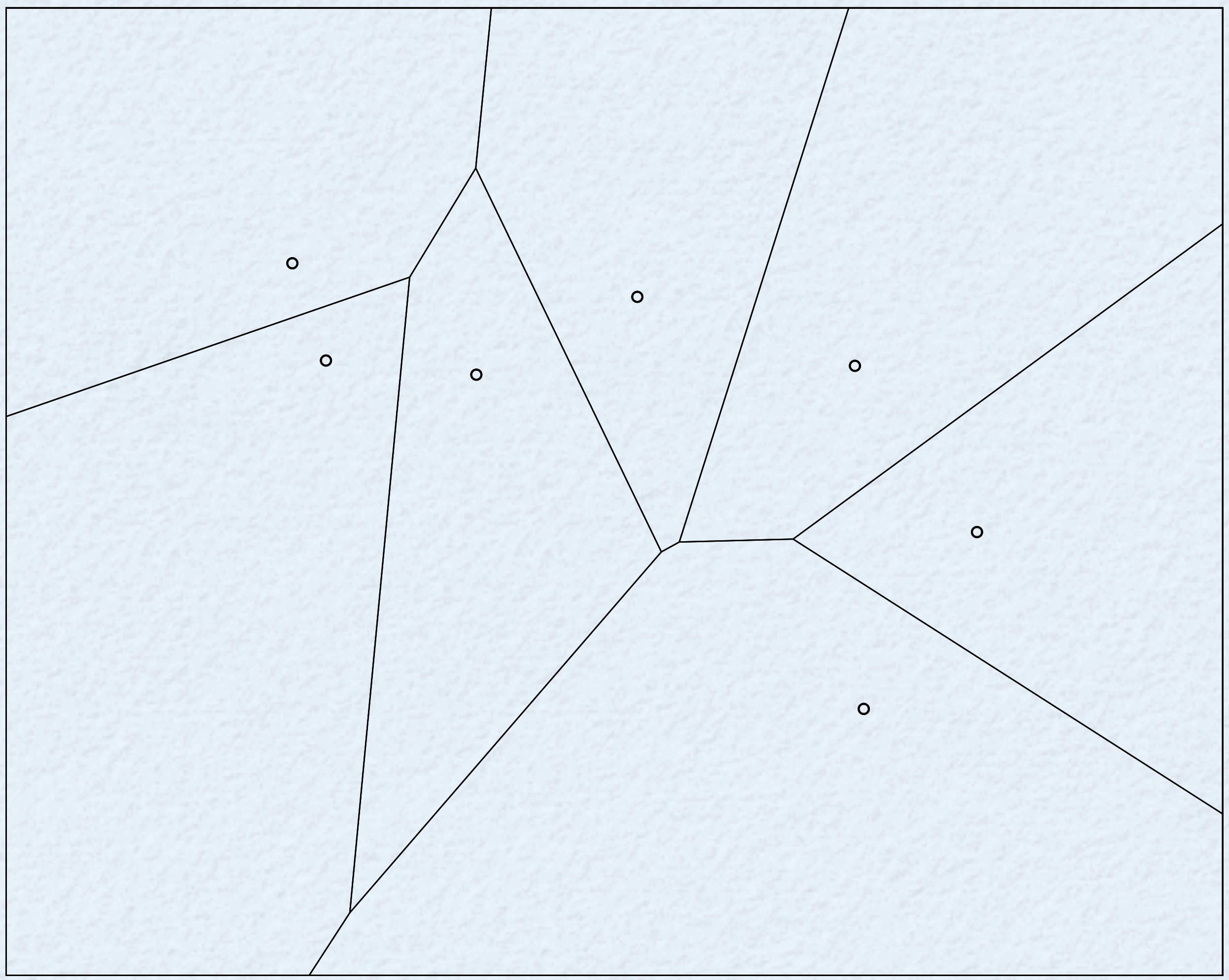

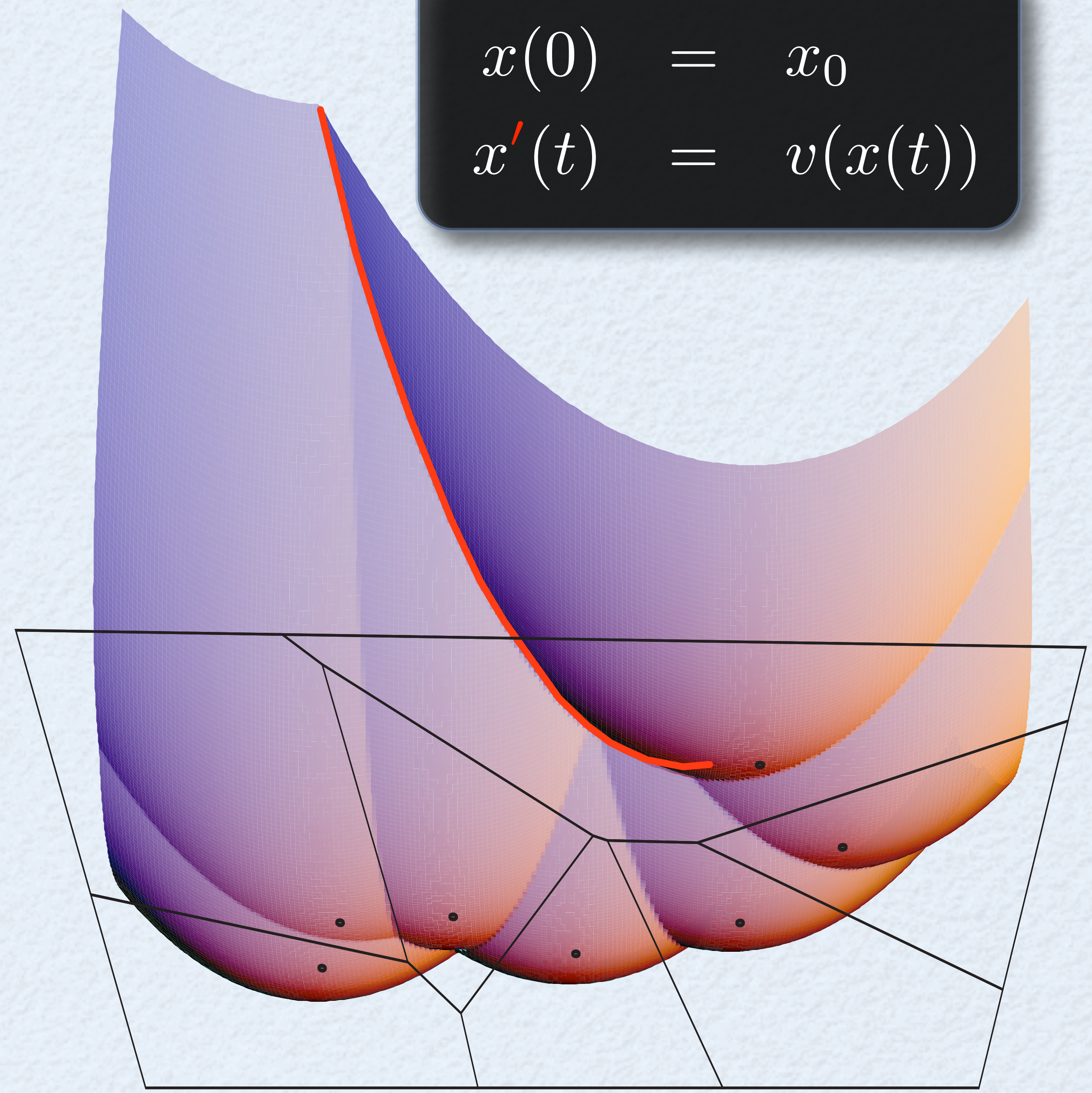

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\begin{aligned}
x(0) &= x_0 \\
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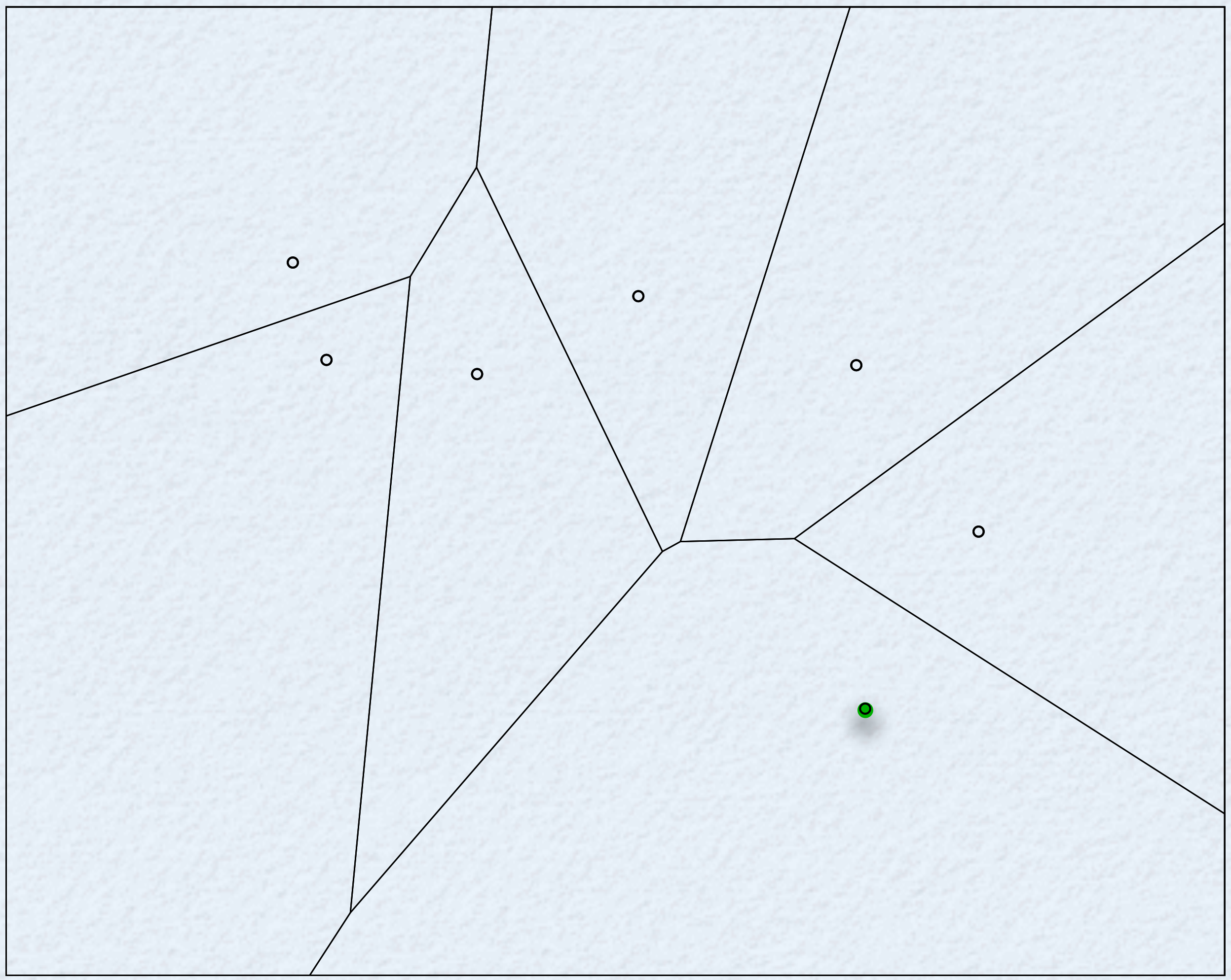

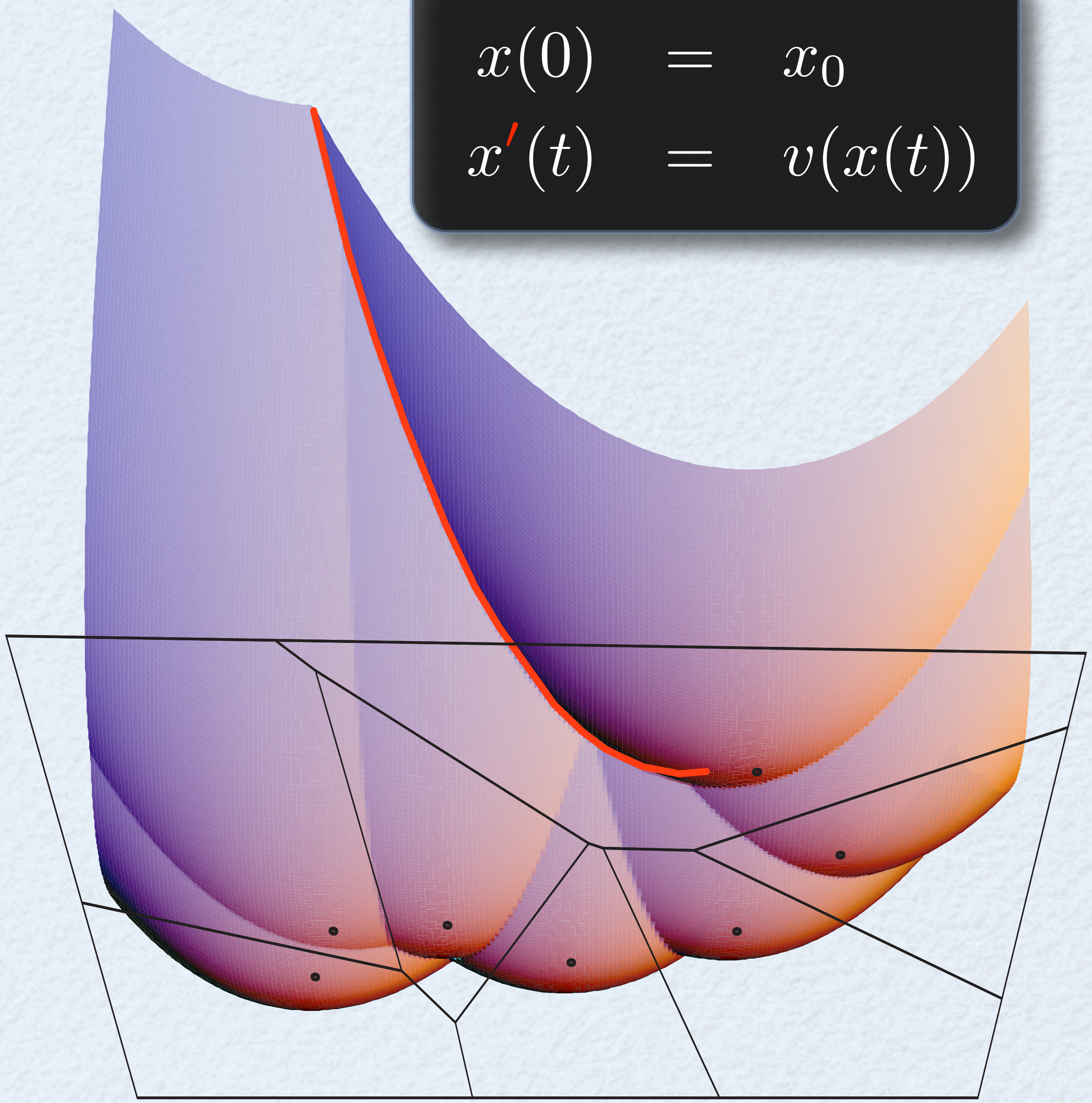

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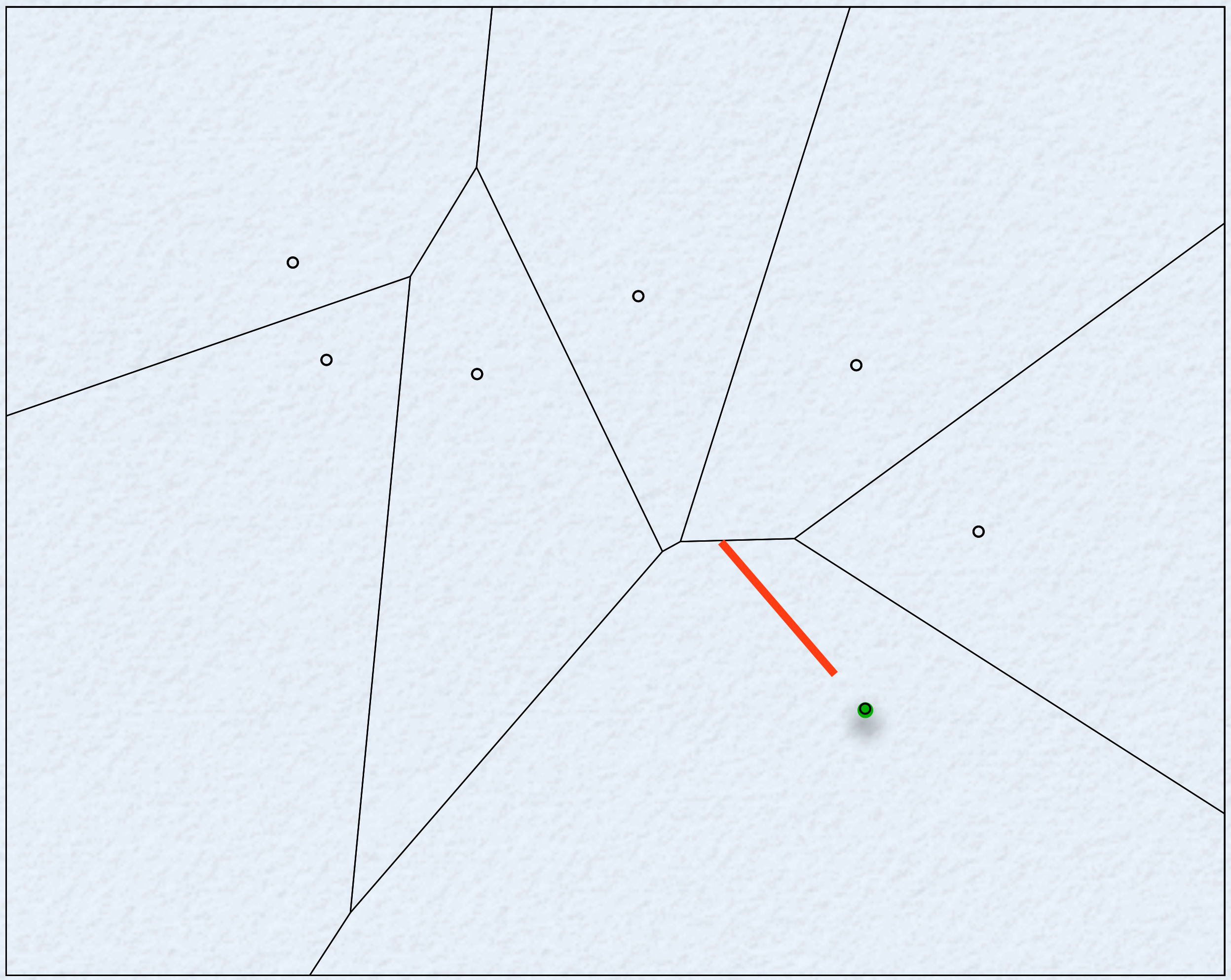

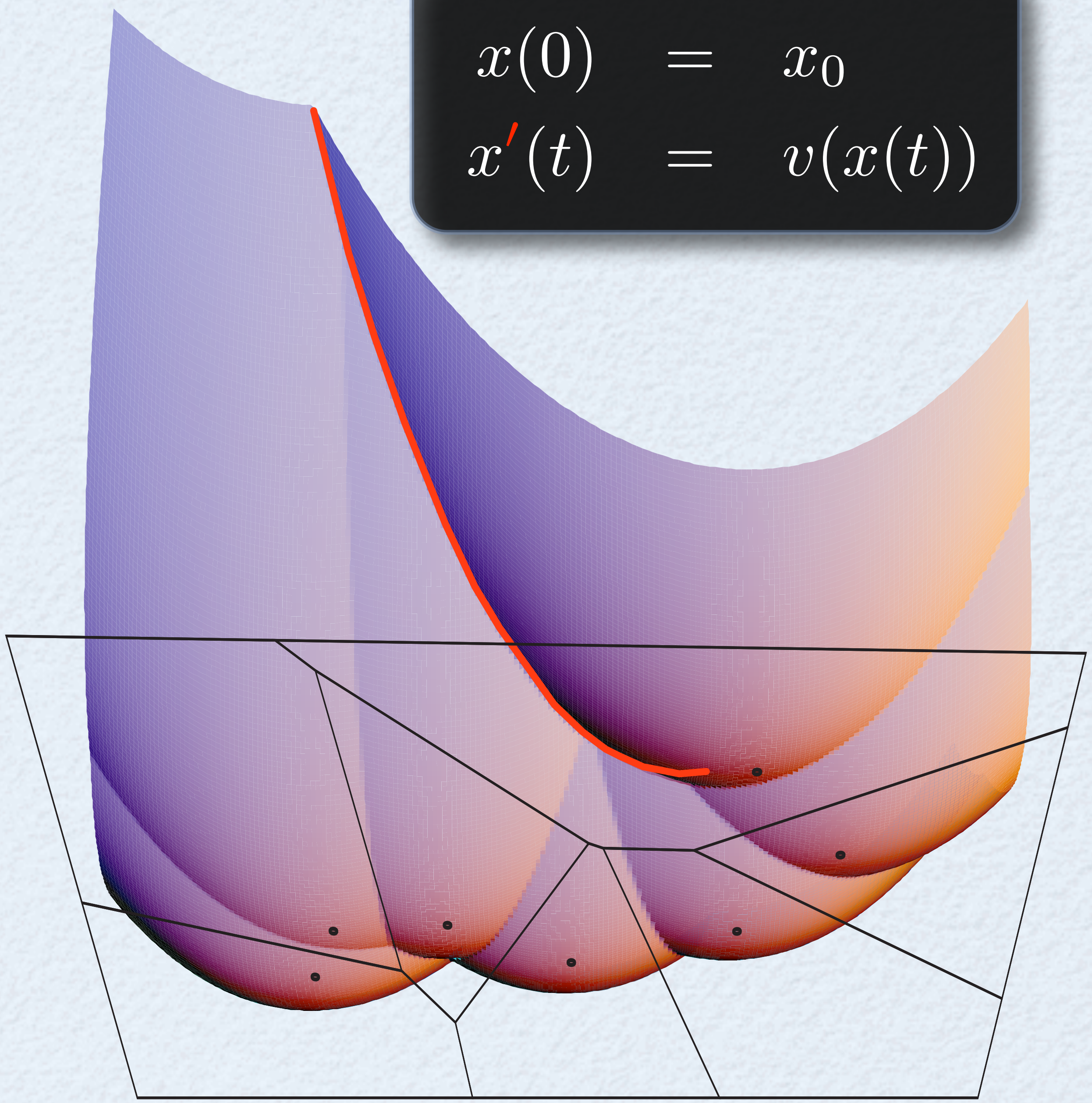

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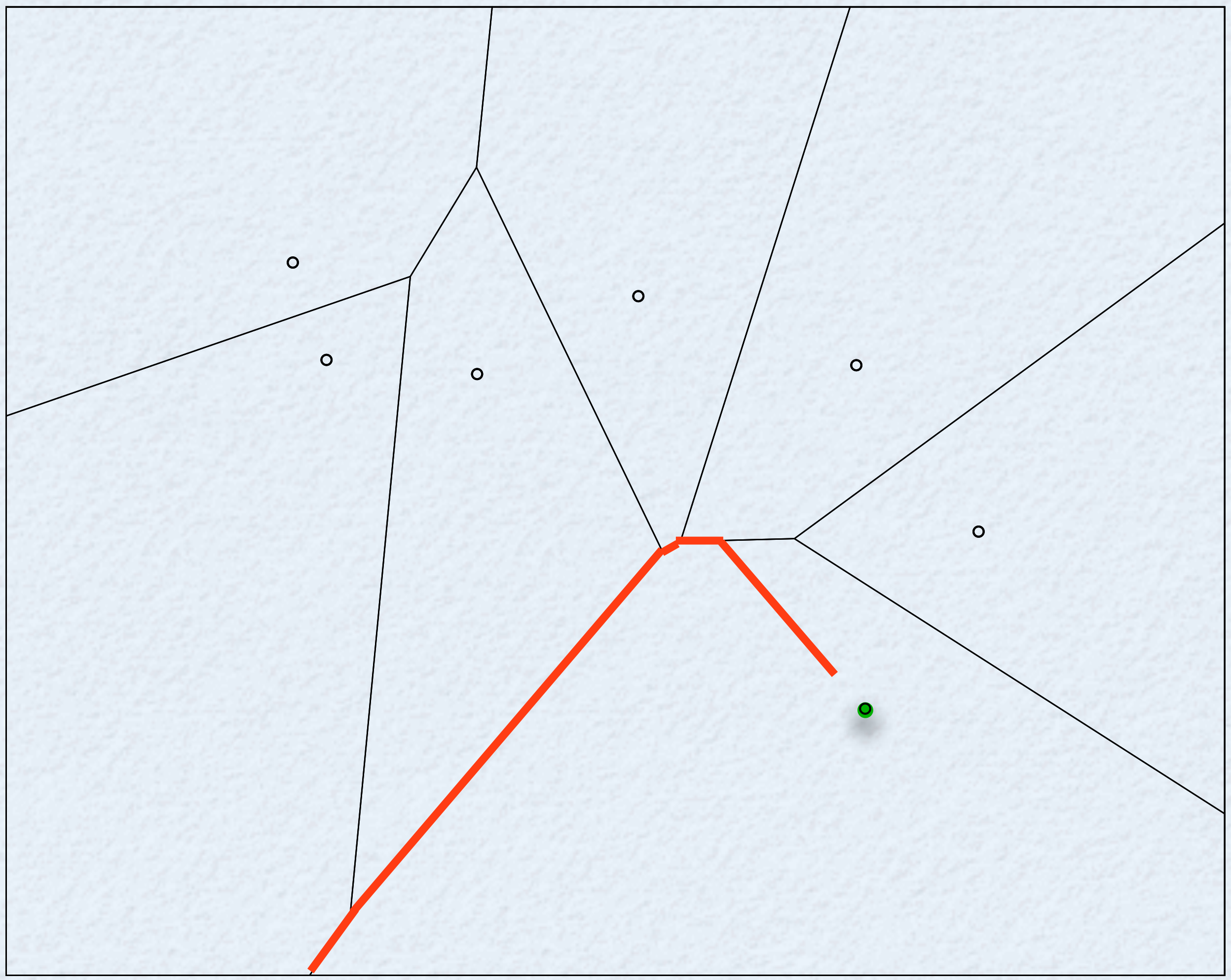

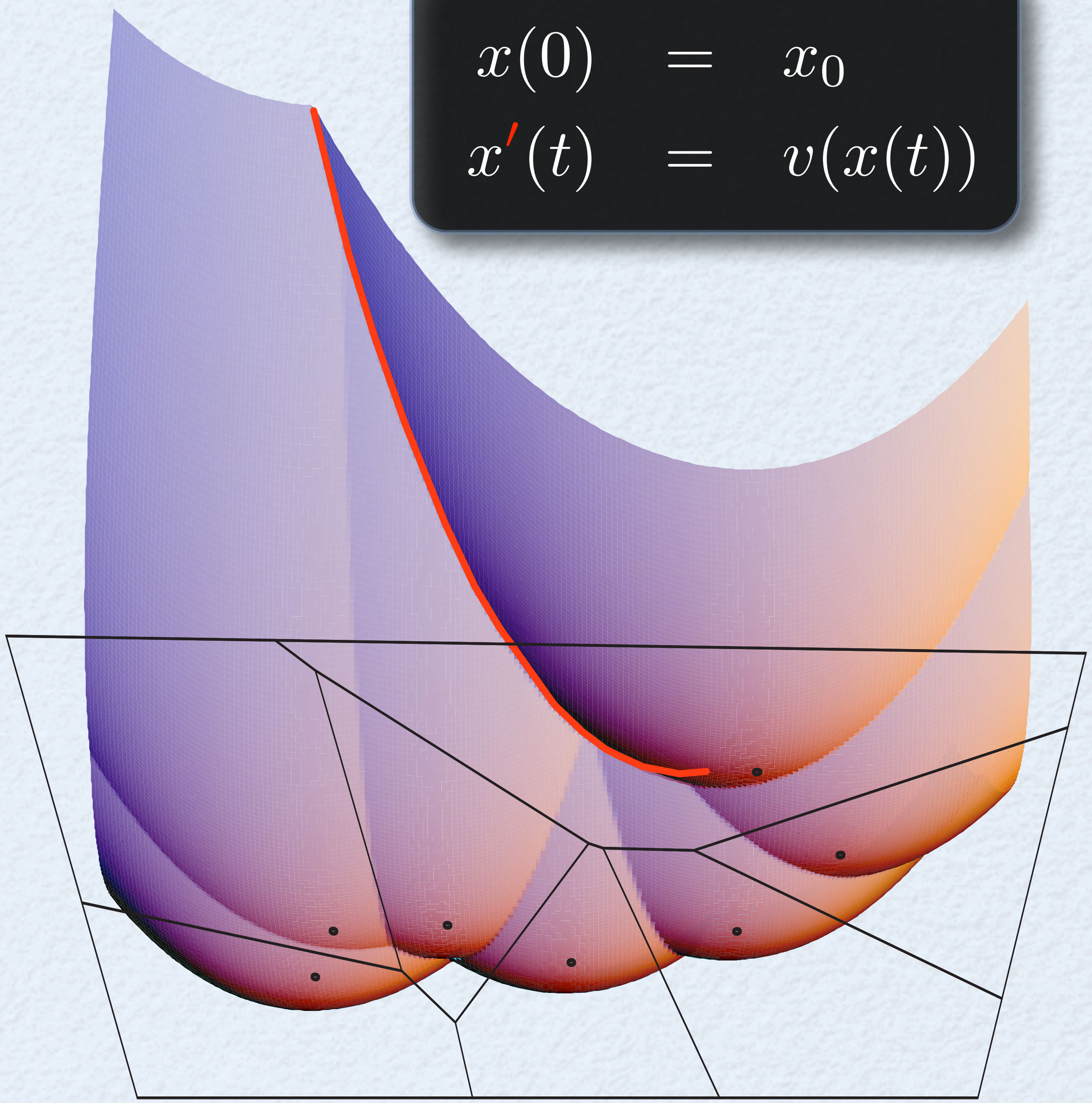

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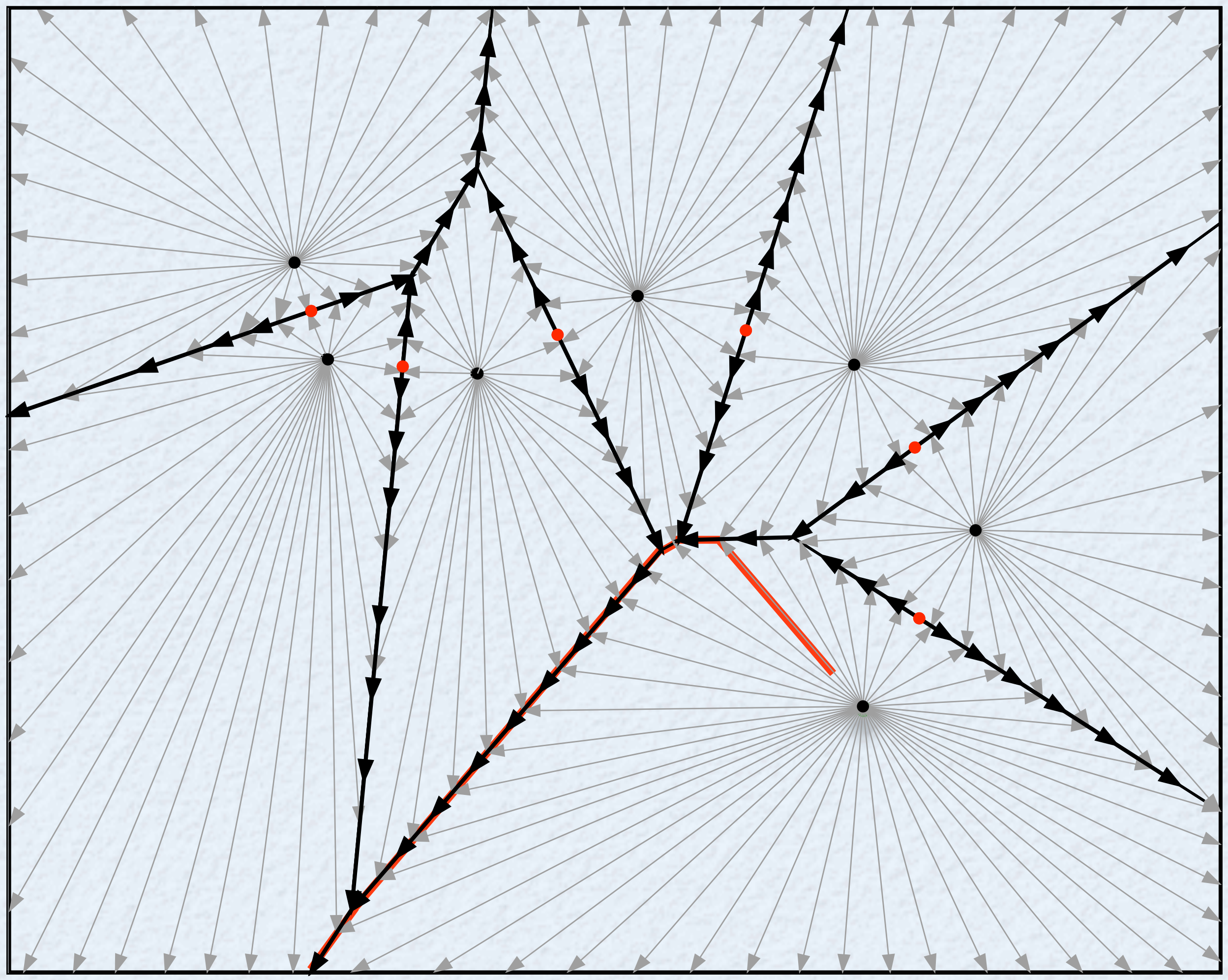

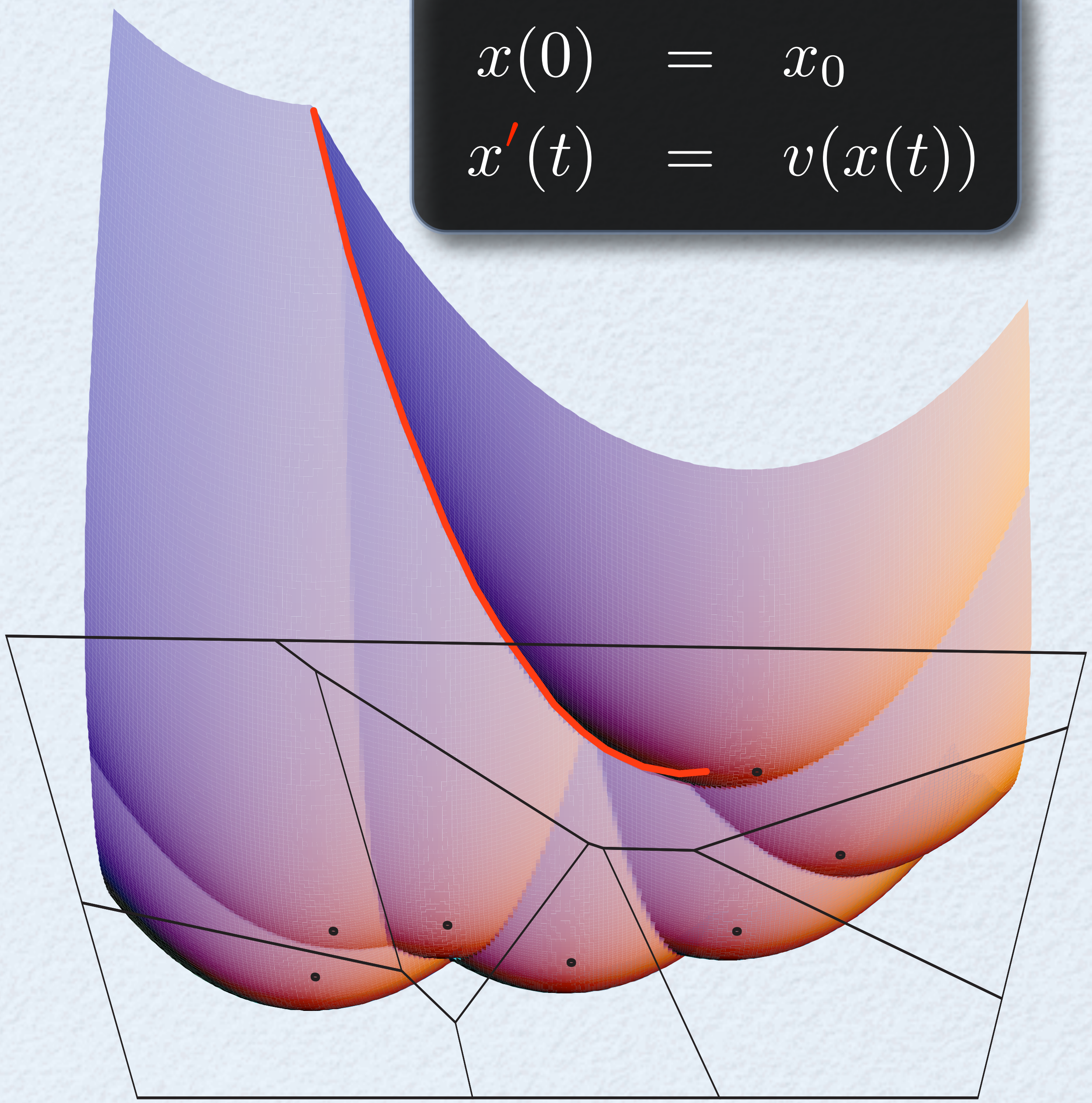

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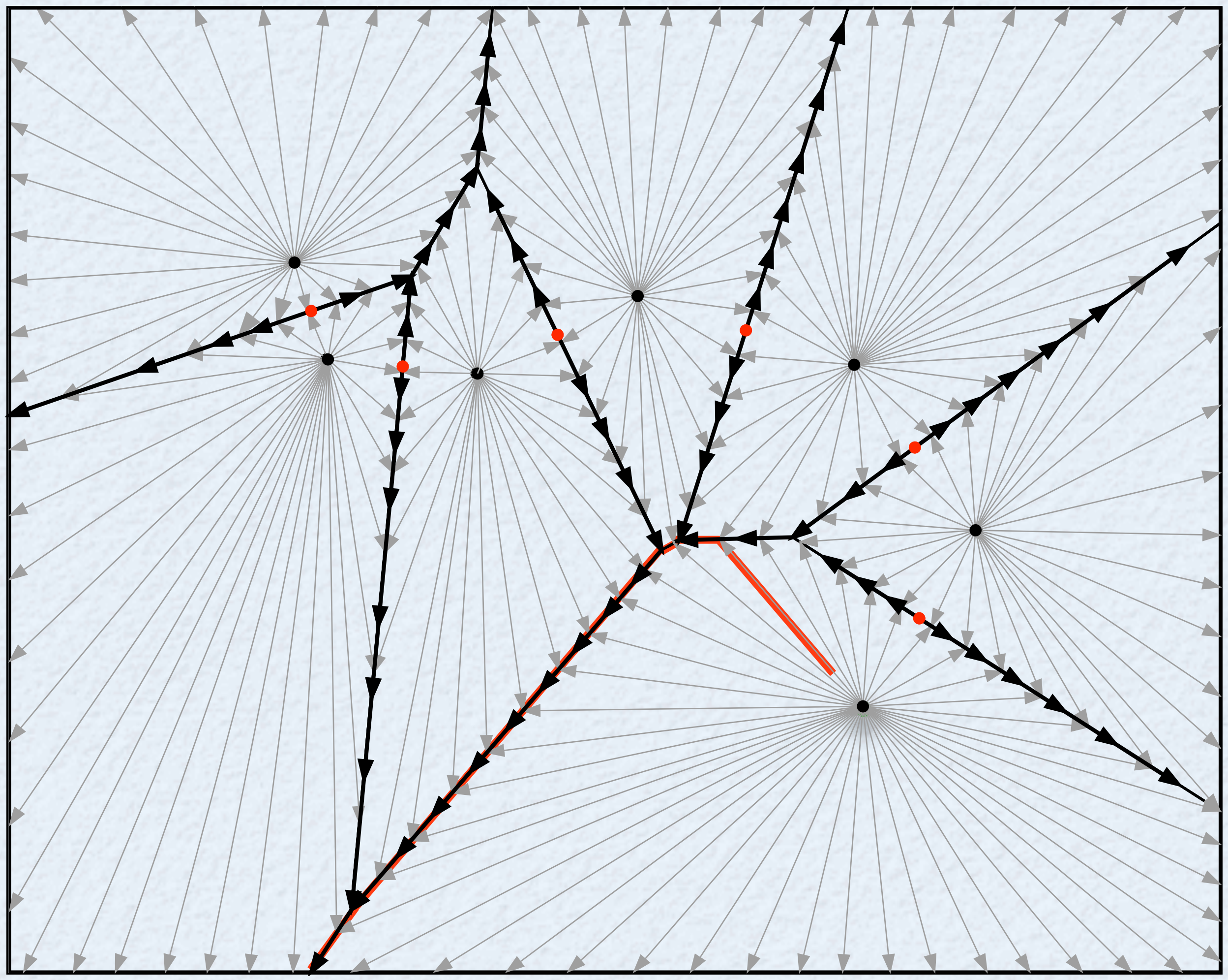

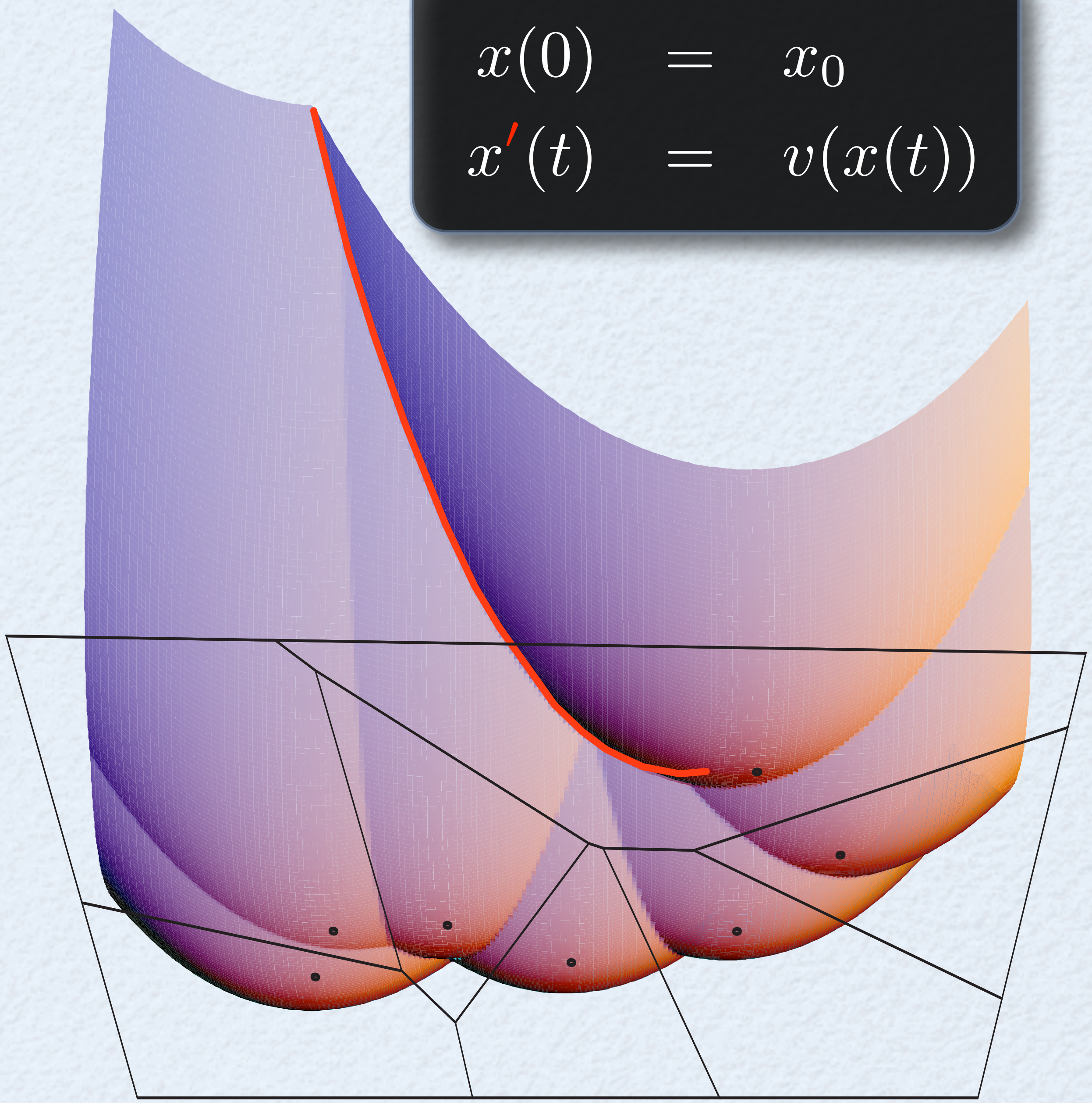

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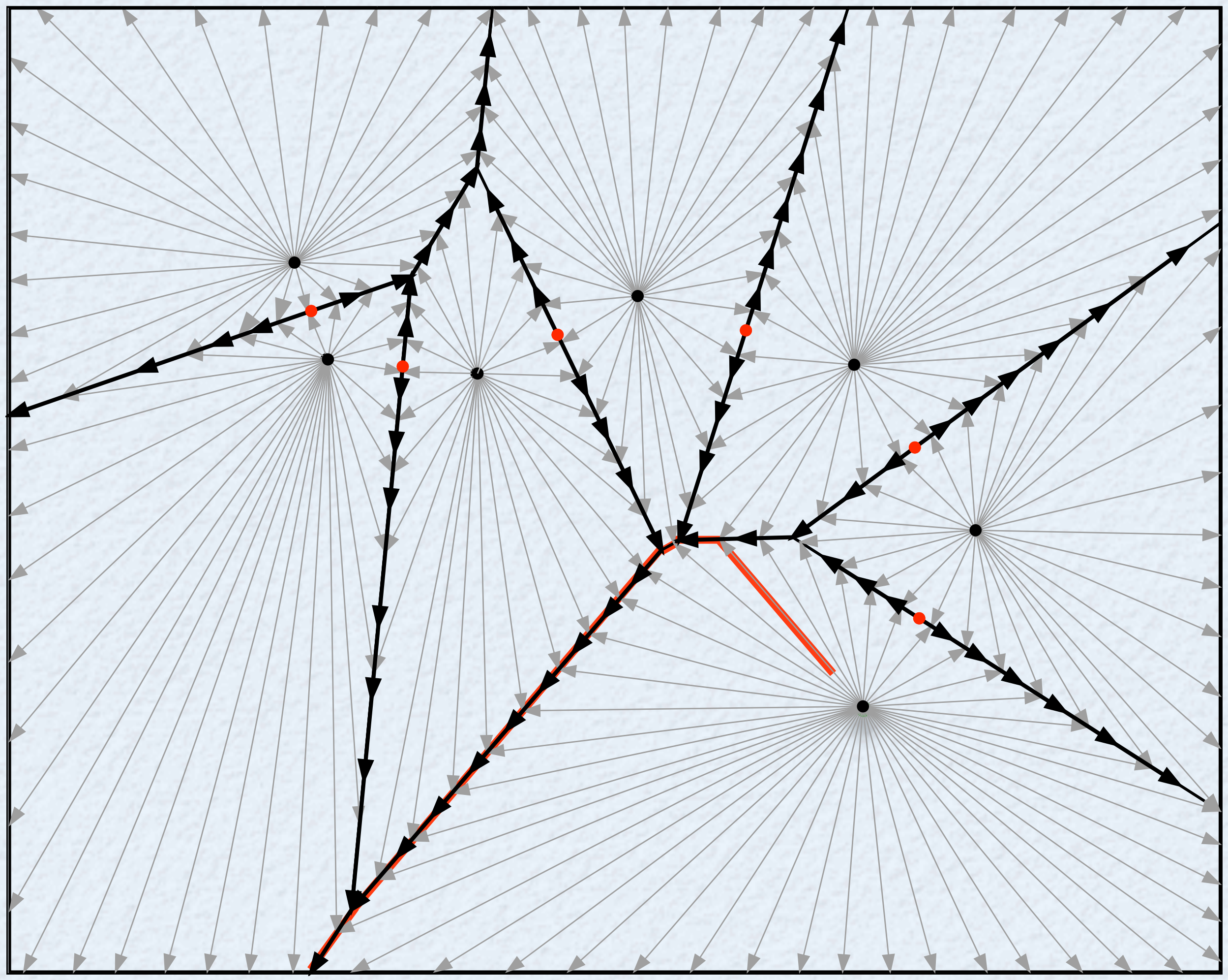

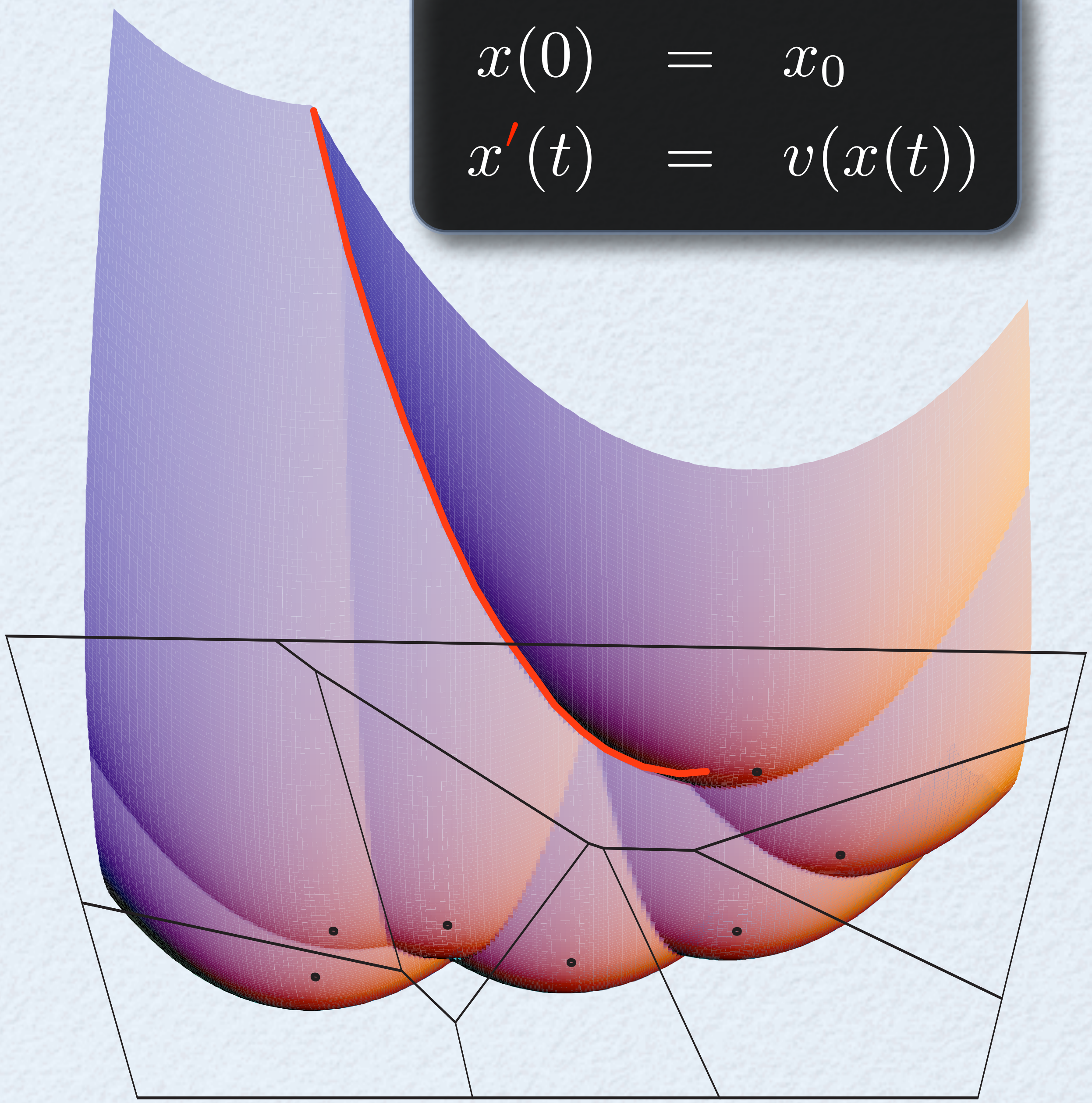

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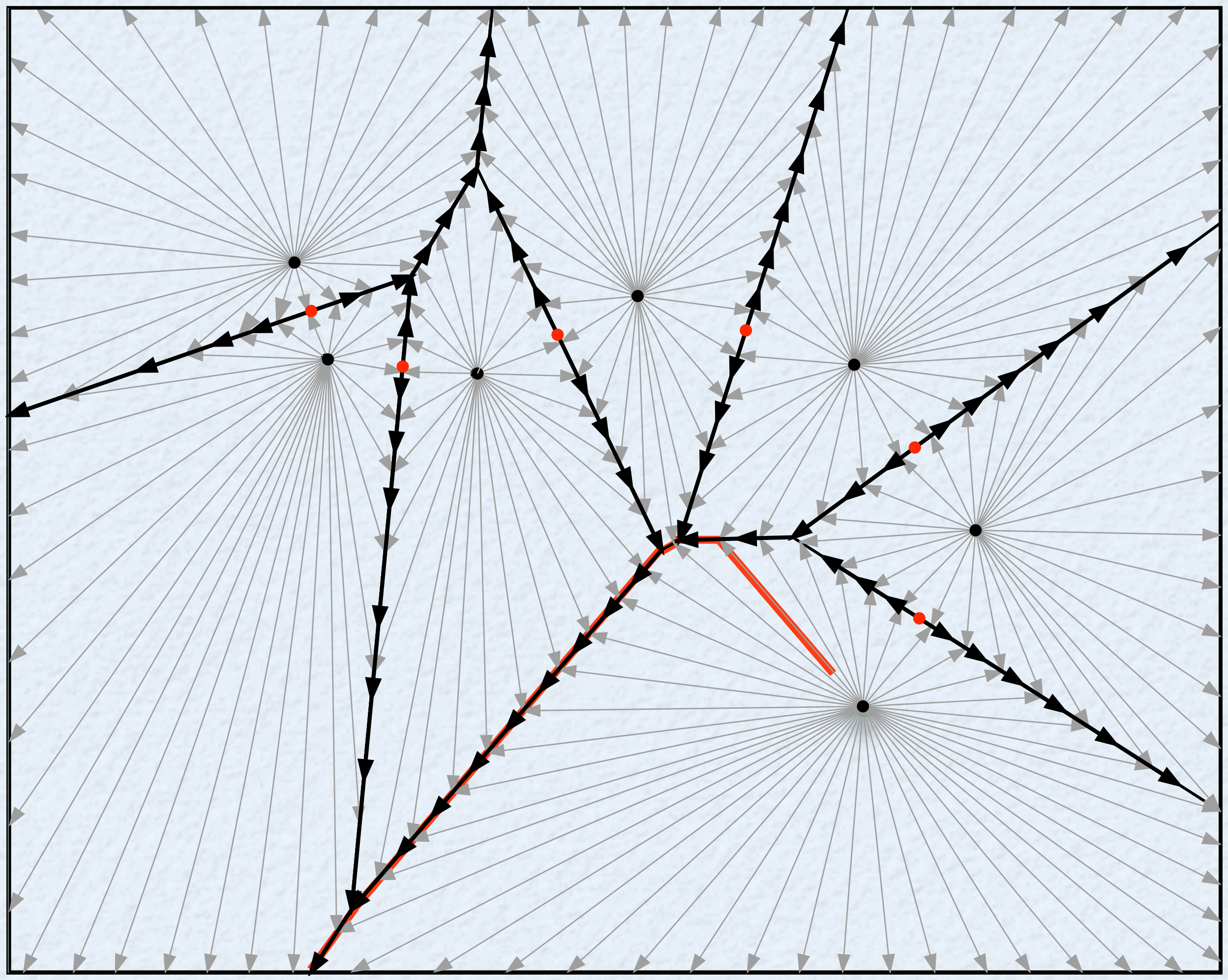

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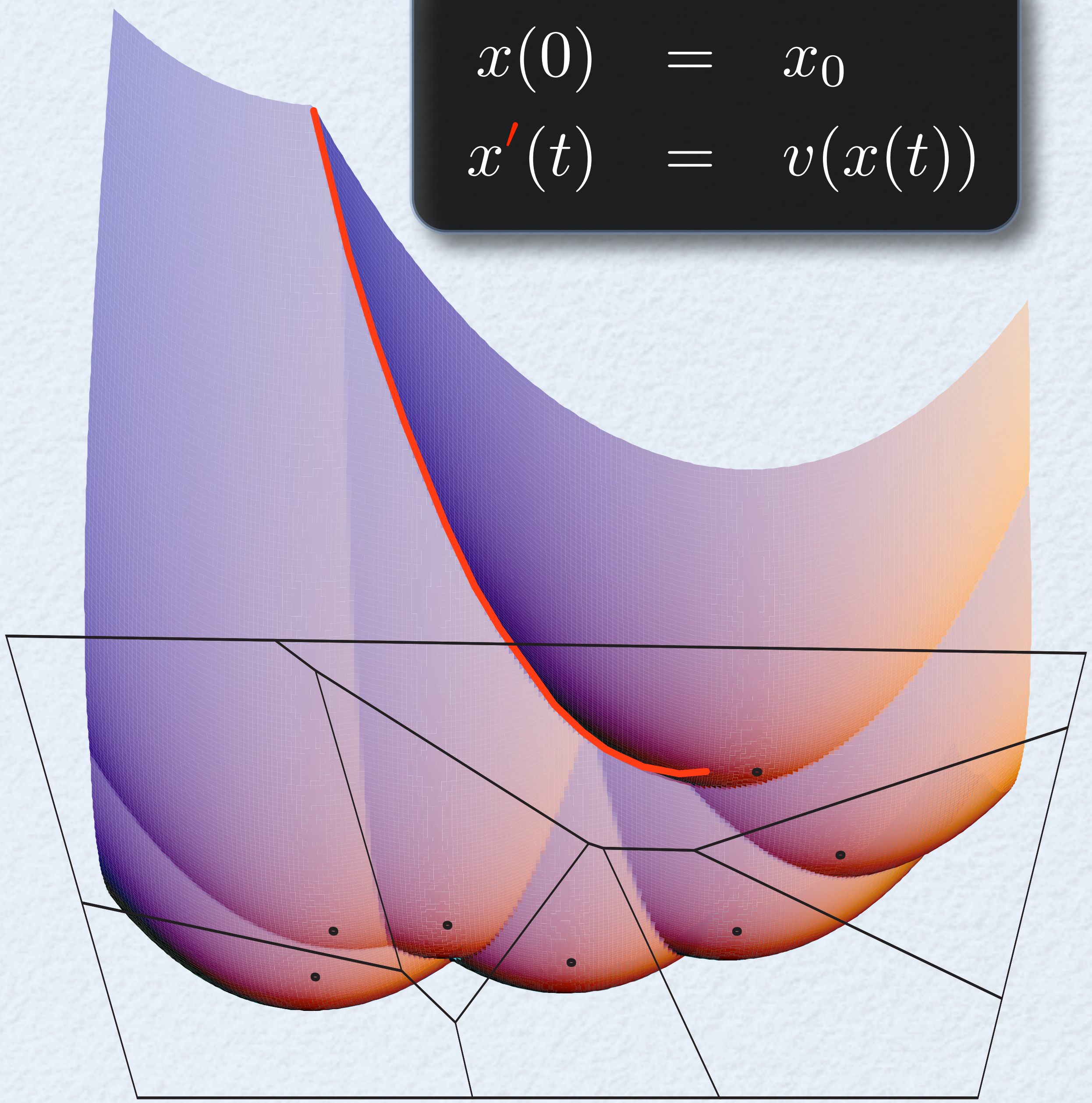

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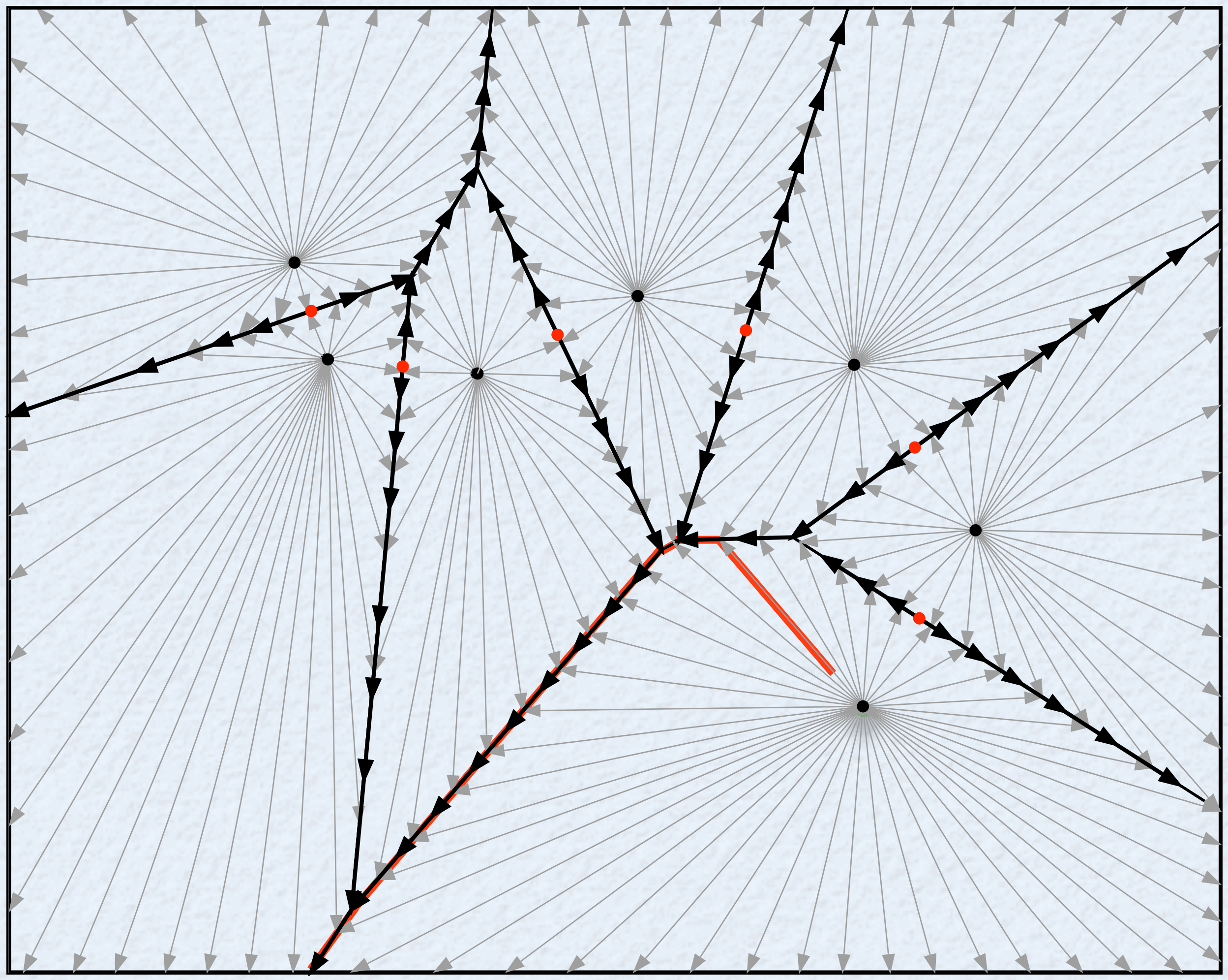

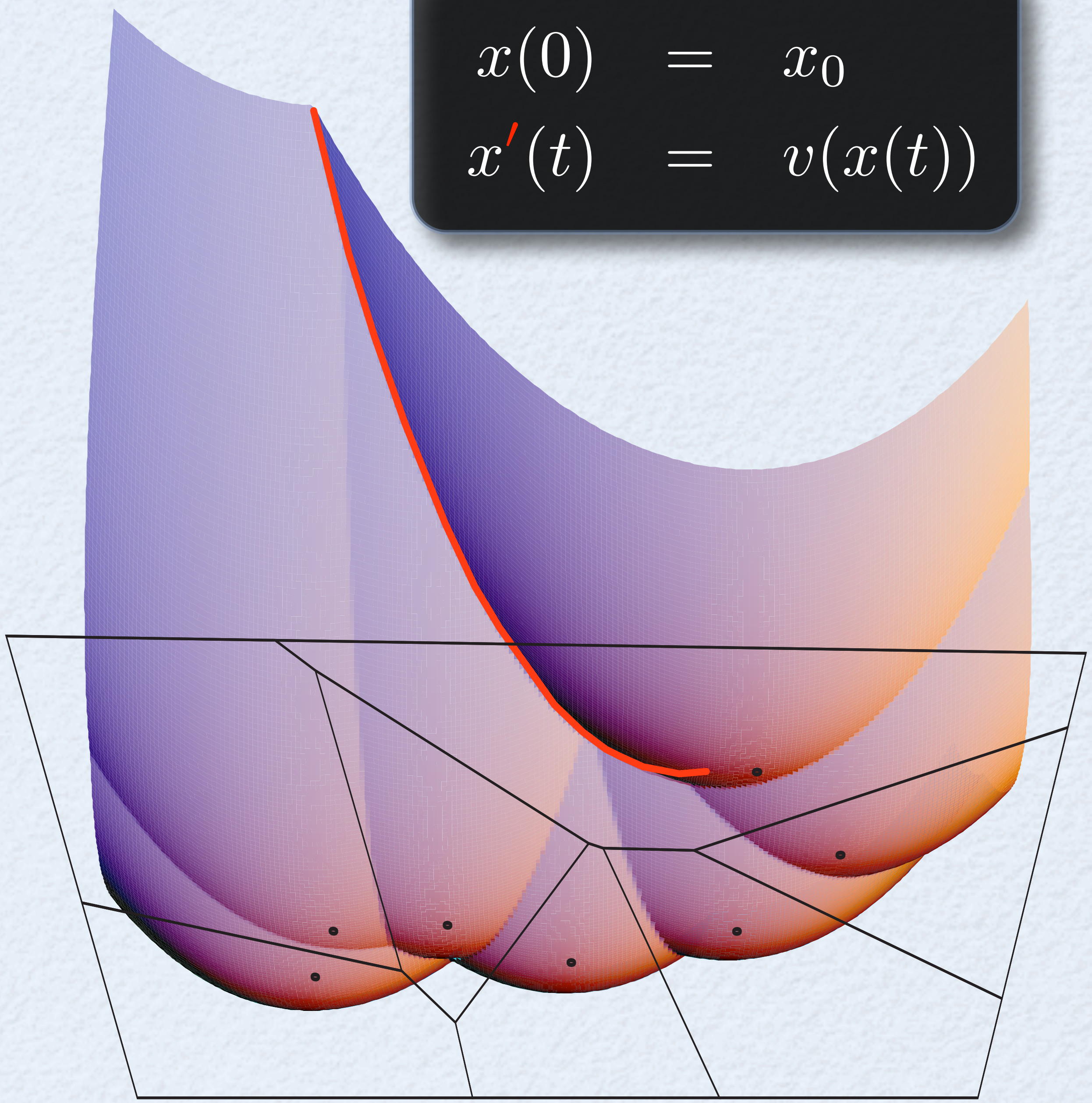

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Theorem [Lieutier'04]. ϕ is continuous.

A criterion for homotopy equivalence

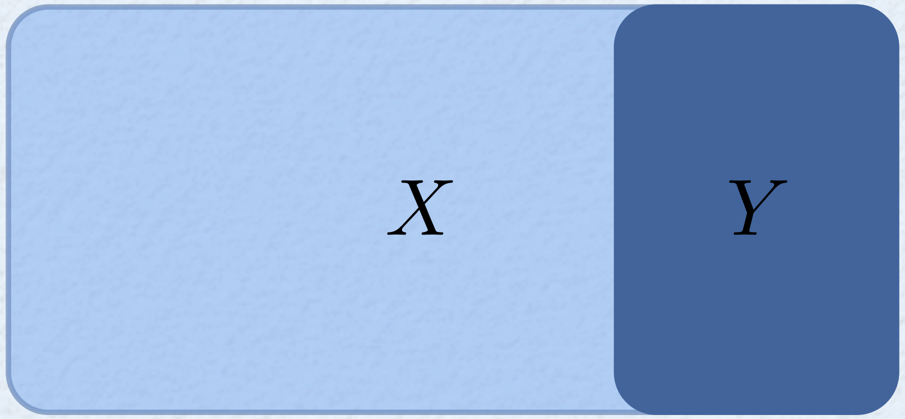
Proposition. Let X and $Y \subseteq X$ be arbitrary sets and

$$H : [0,1] \times X \to X$$

be a continuous function (on both variables) satisfying

1. $\forall x \in X : H(0,x) = x$
2. $\forall y \in Y, \forall t \in [0,1] : H(t,y) \in Y$
3. $\forall x \in X : H(1,x) \in Y$

Then X and Y have the same homotopy type.



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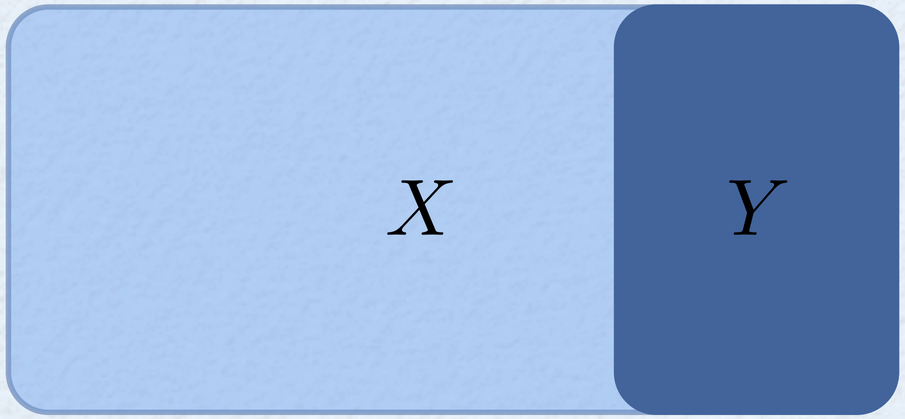

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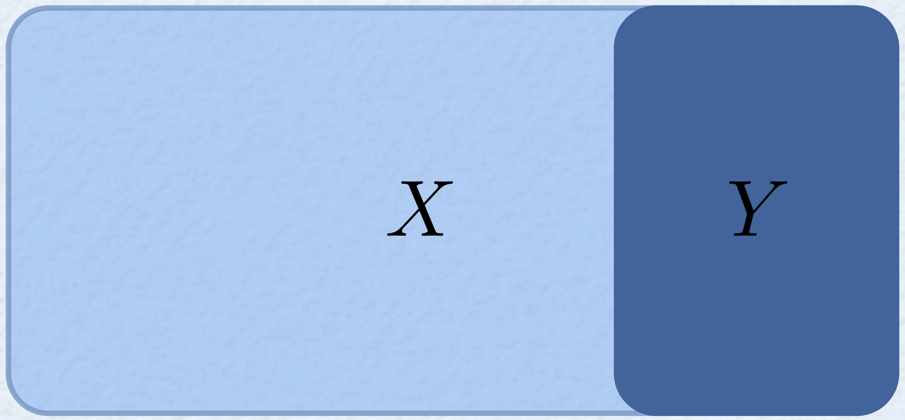

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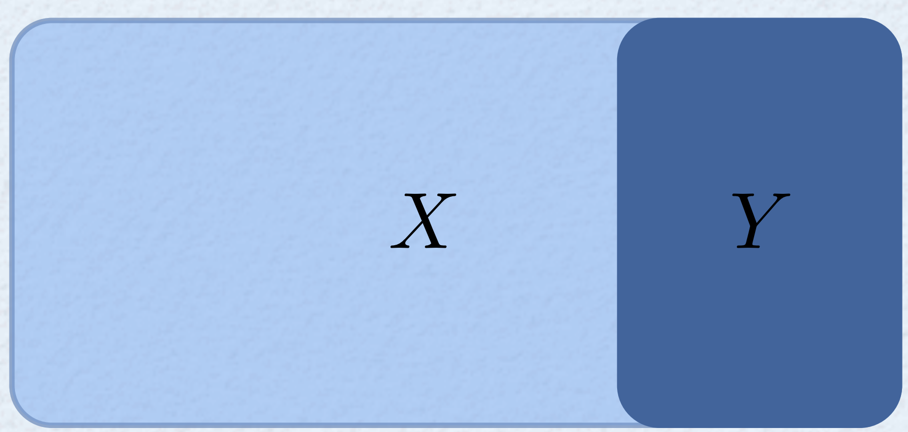

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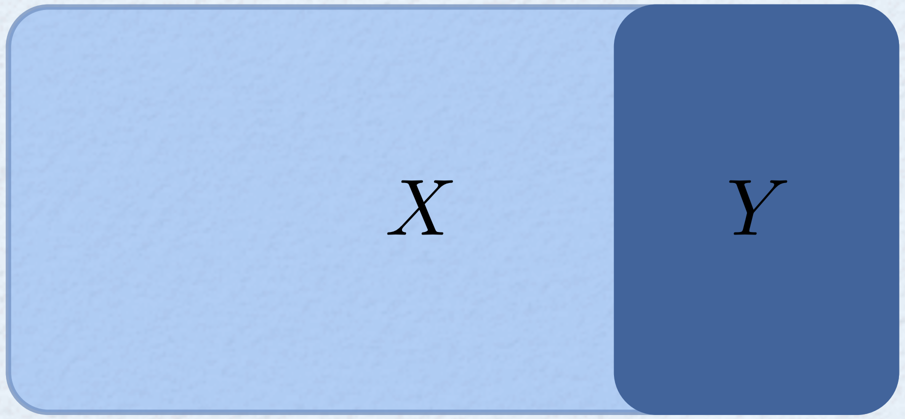

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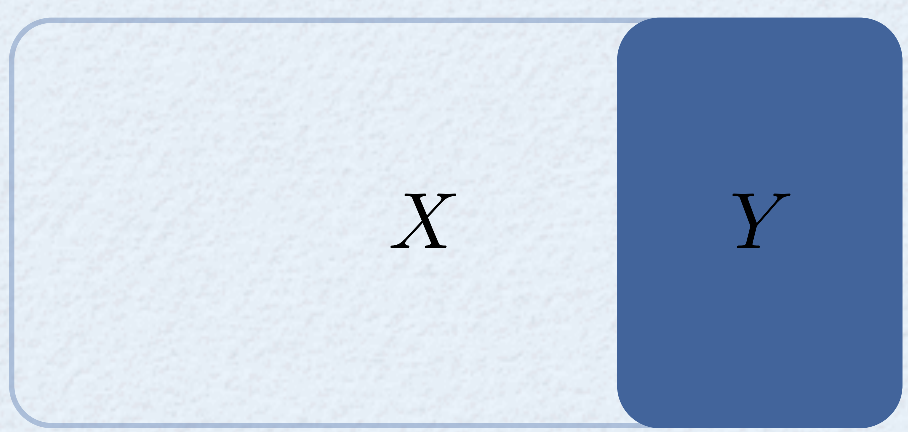

A criterion for homotopy equivalence

Proposition. Let X and $Y \subseteq X$ be arbitrary sets and

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time

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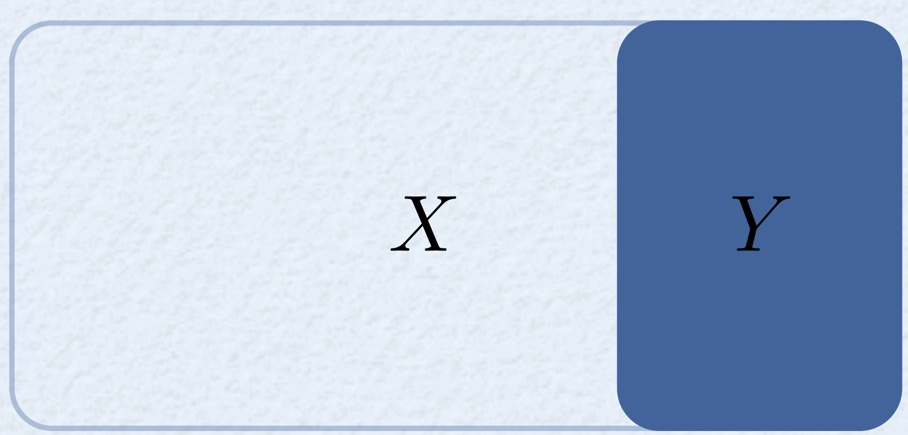

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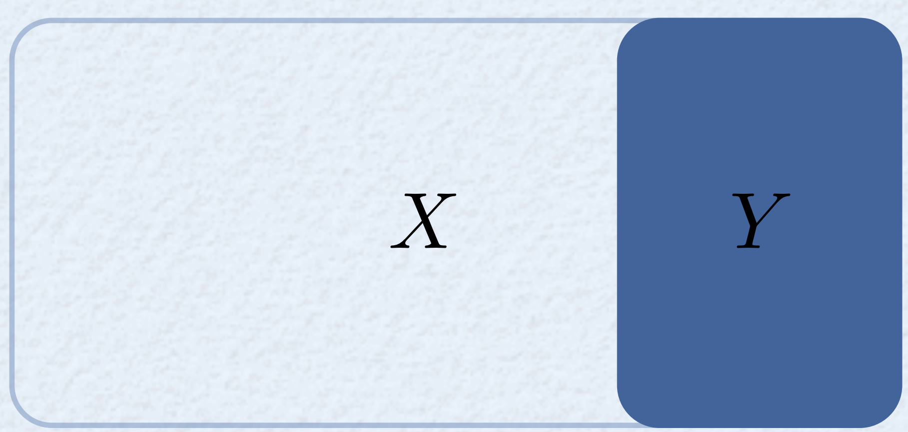

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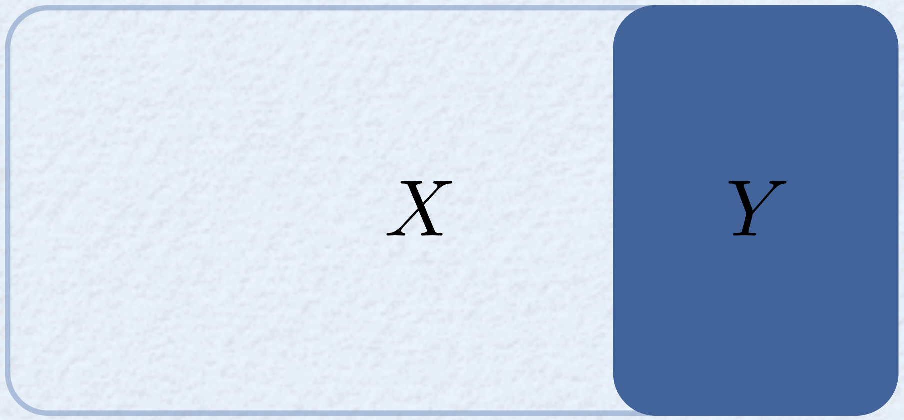

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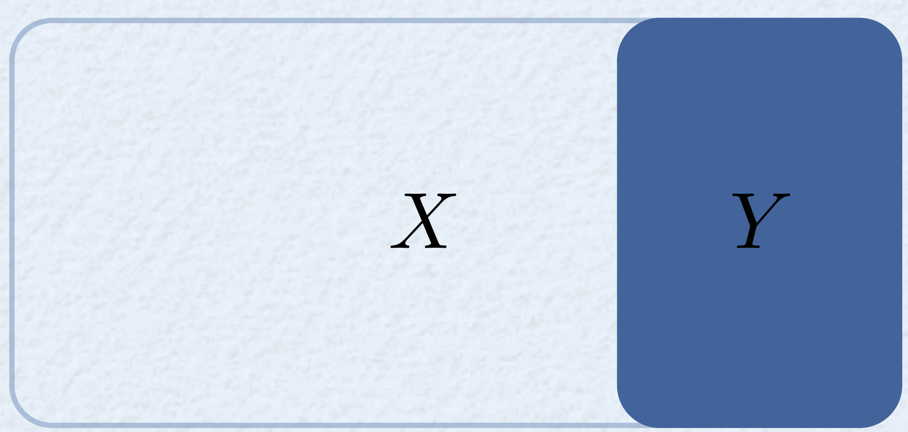

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$\phi(X) = X$

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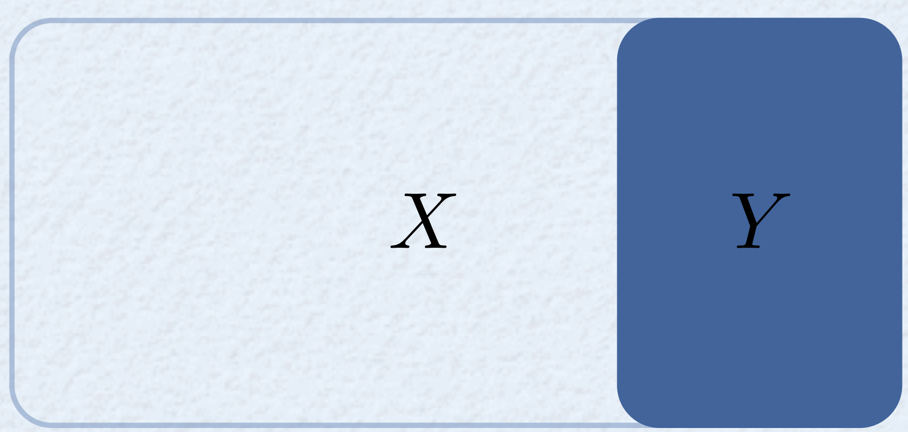
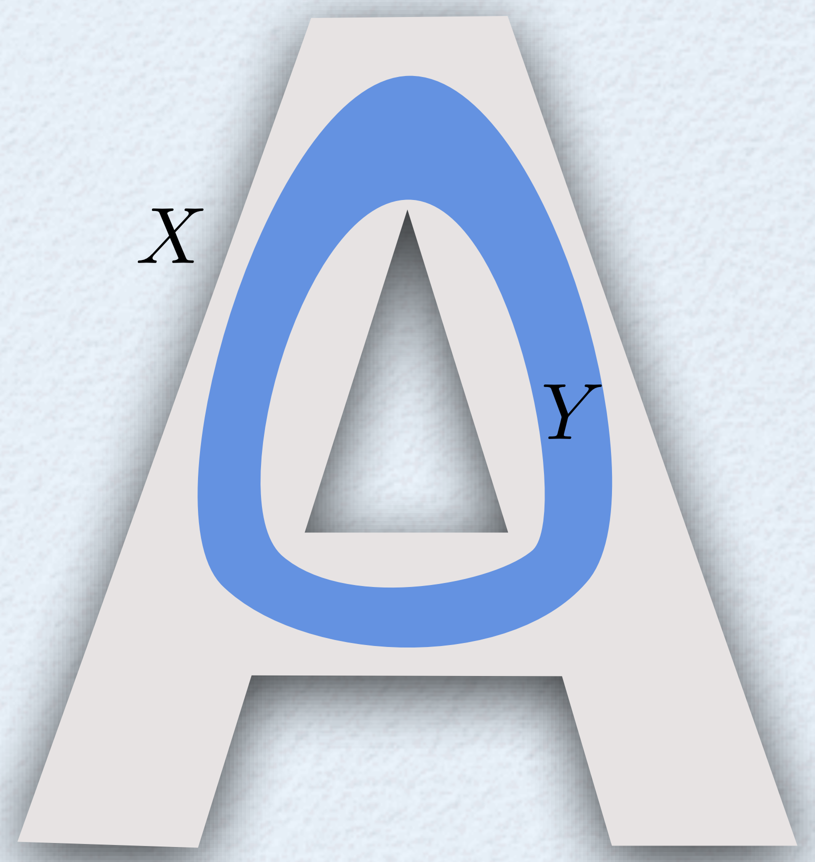


Using flow for proving homotopy equivalence

Lemma [Lieutier'04]. If $Y \subset X$ are bounded and

1. $\phi(X) = X$ and $\phi(Y) = Y$, and
2. $\|v(x)\| \geq c > 0$ for $x \in X \setminus Y$,

then X and Y are homotopy equivalent.



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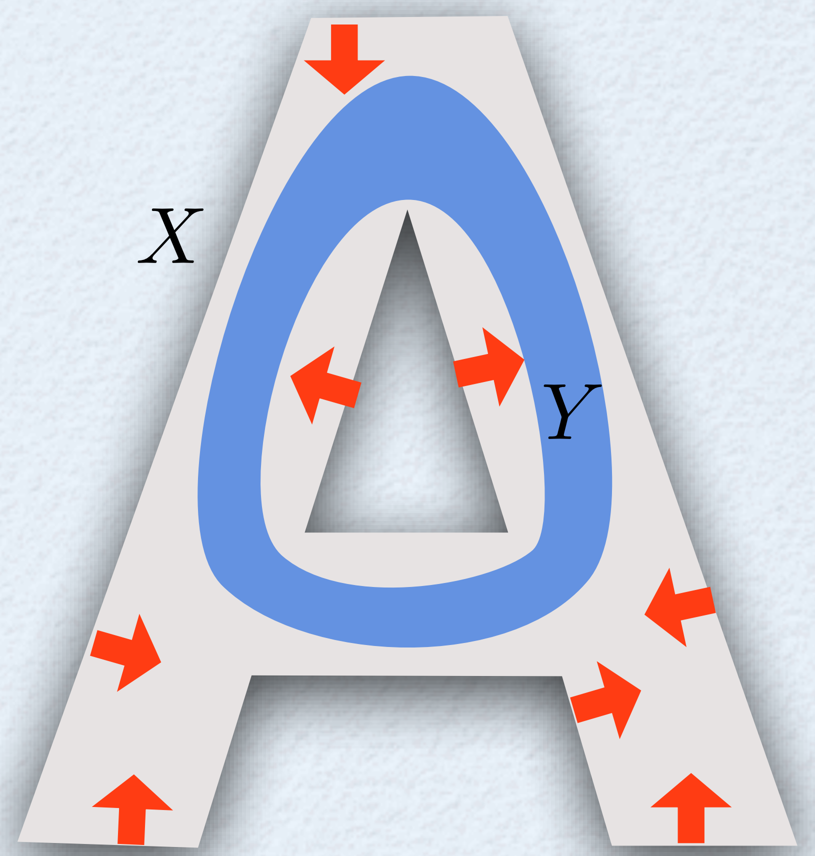

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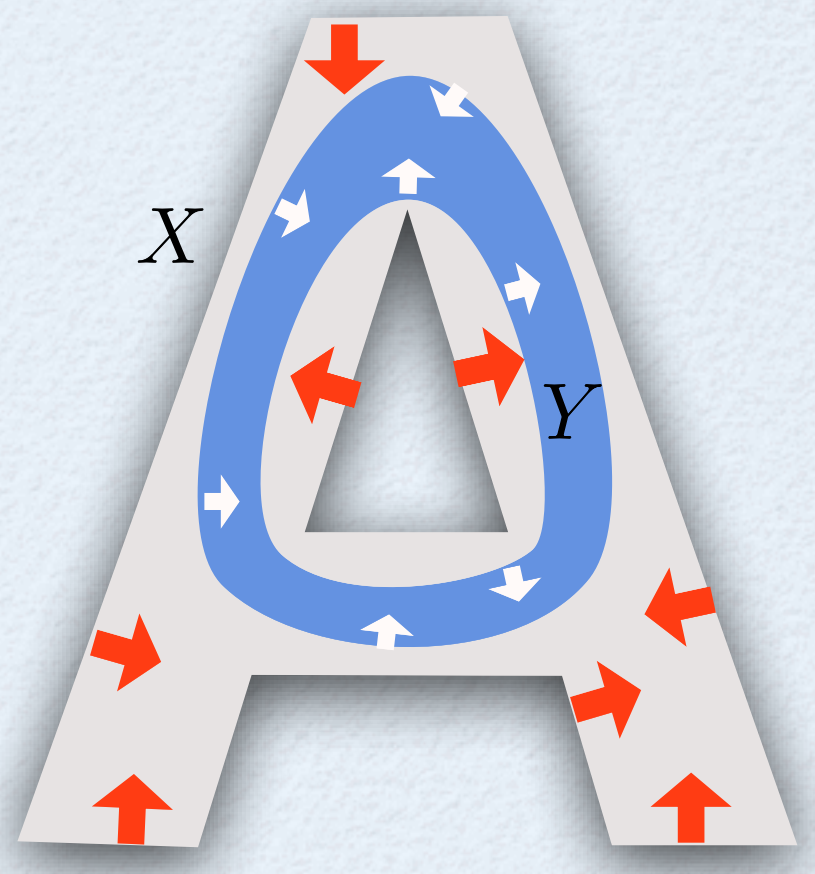

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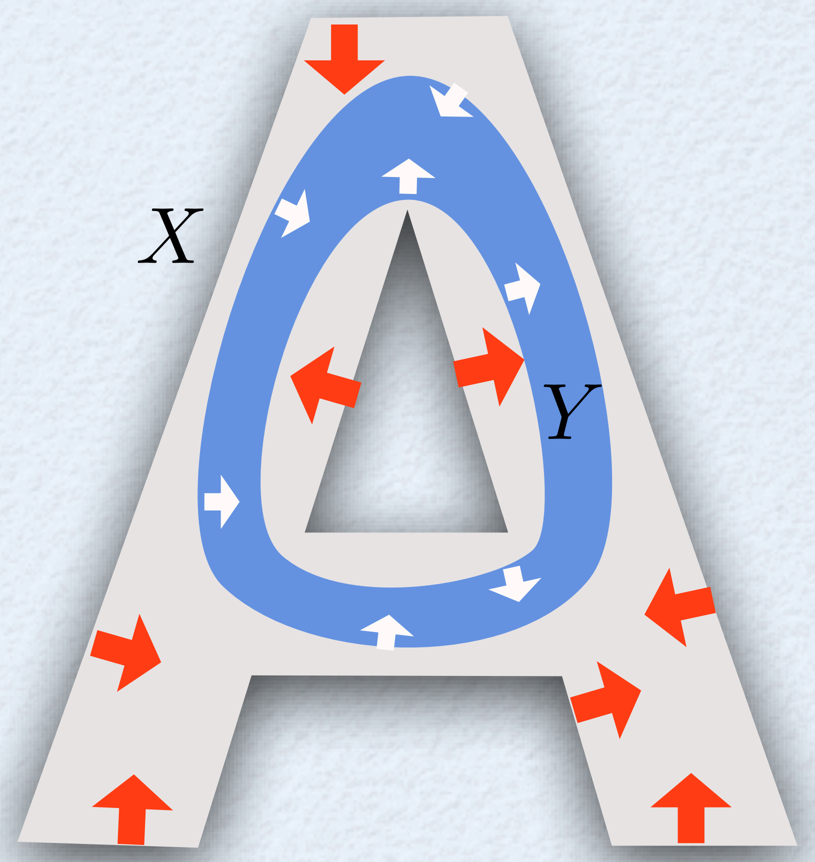

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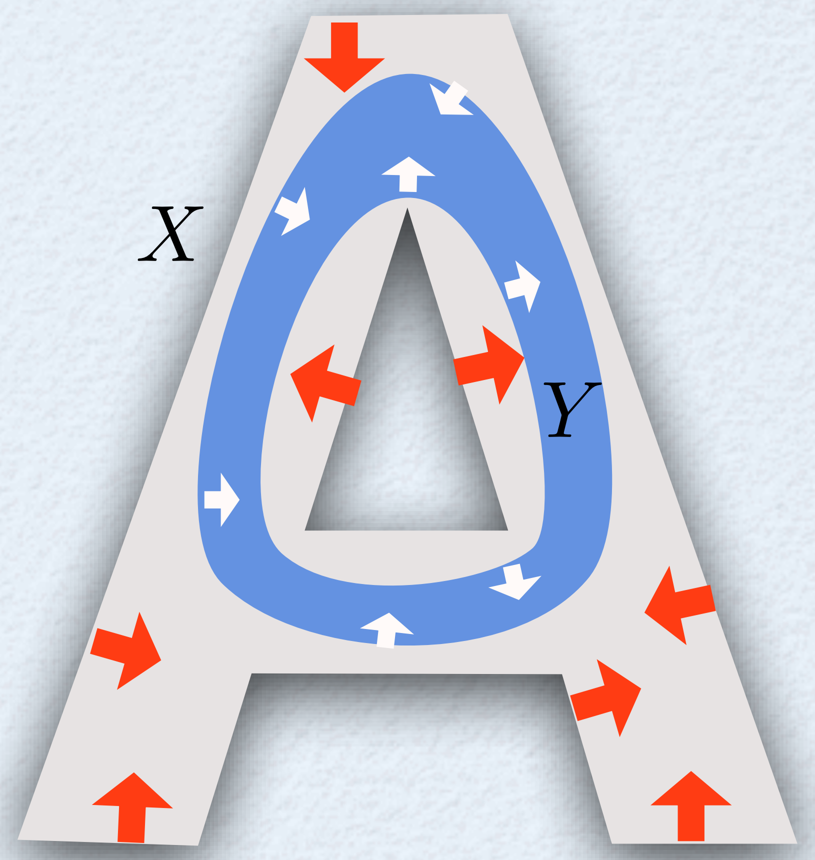
"speed" lower bound



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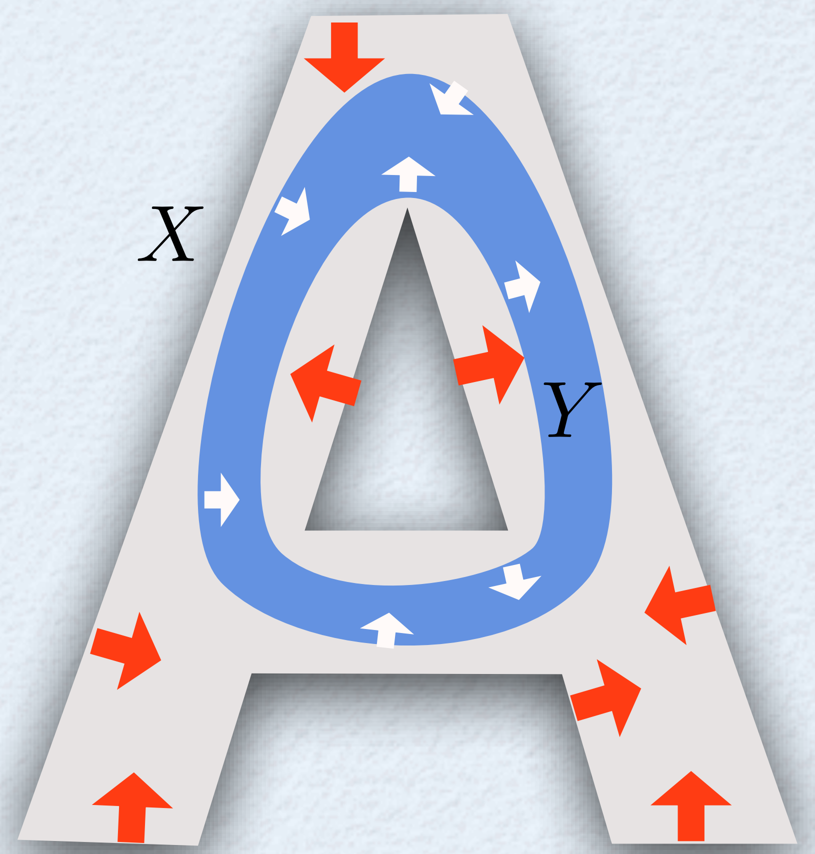
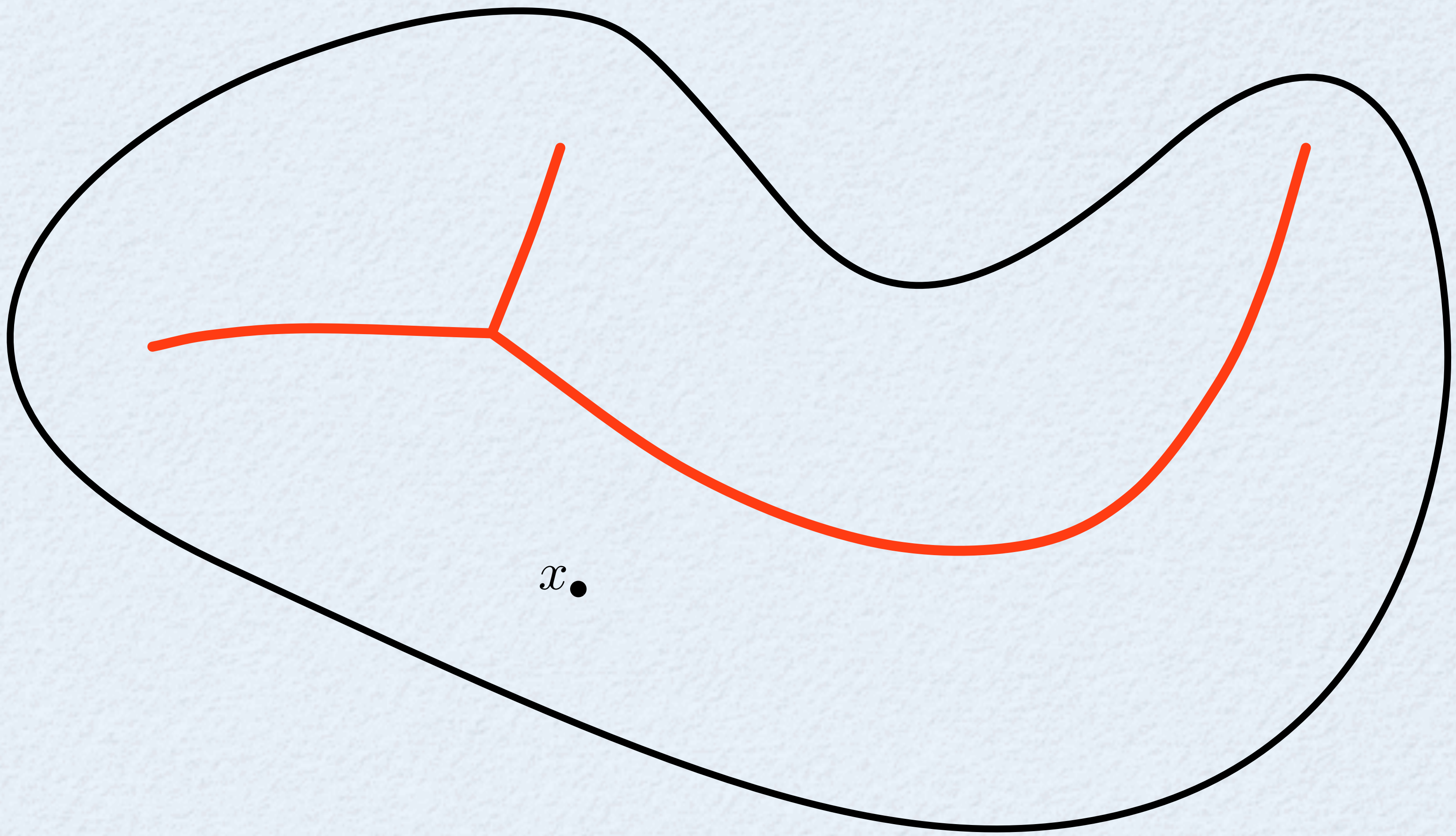
then X and Y are homotopy equivalent.

So, we "push X into Y" at speed > 0.



Important flow-tight sets

complements of tubular neighborhoods of the manifold, and union of balls placed at samples are flow-tight, for the right range of parameters.



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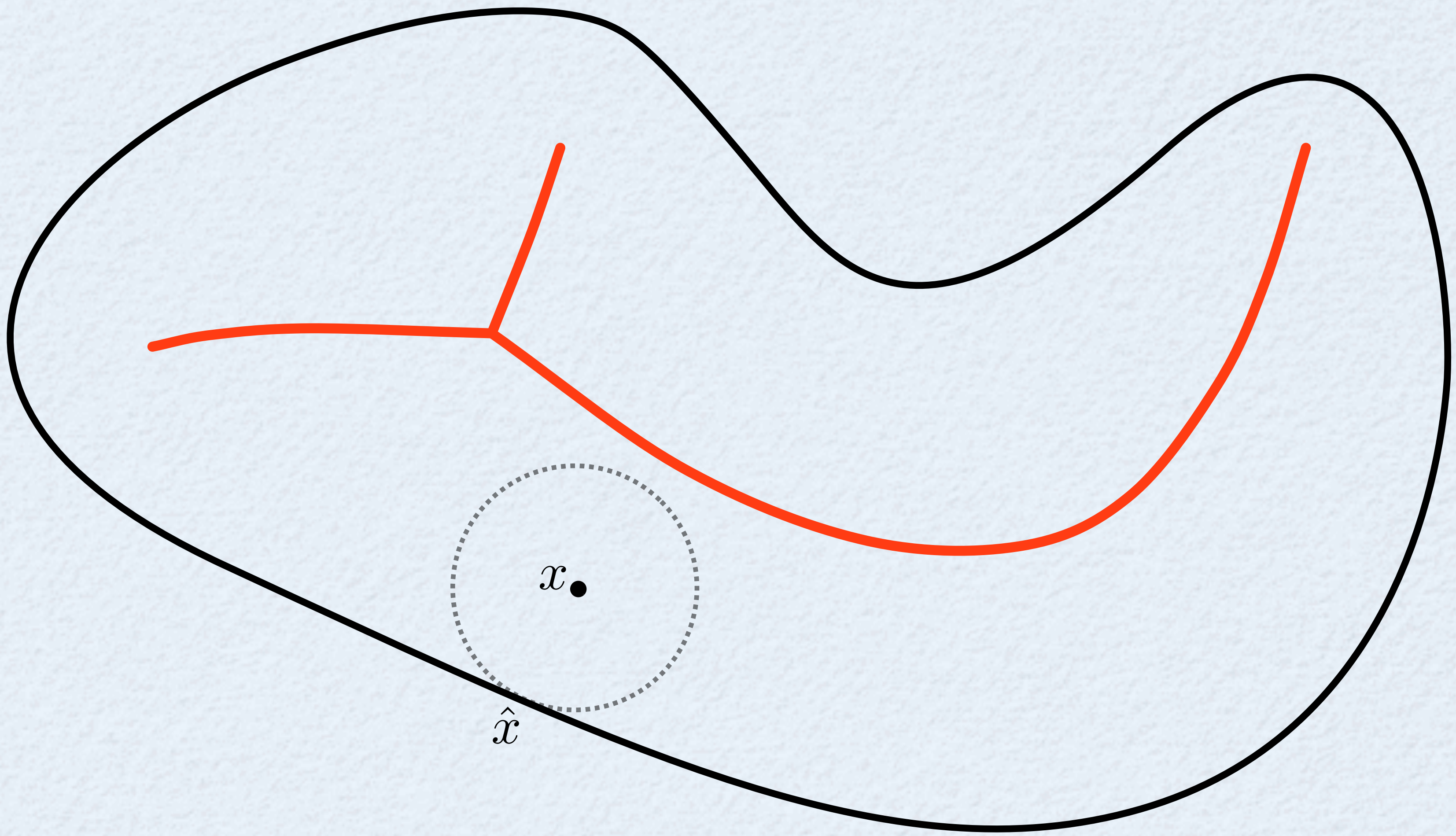

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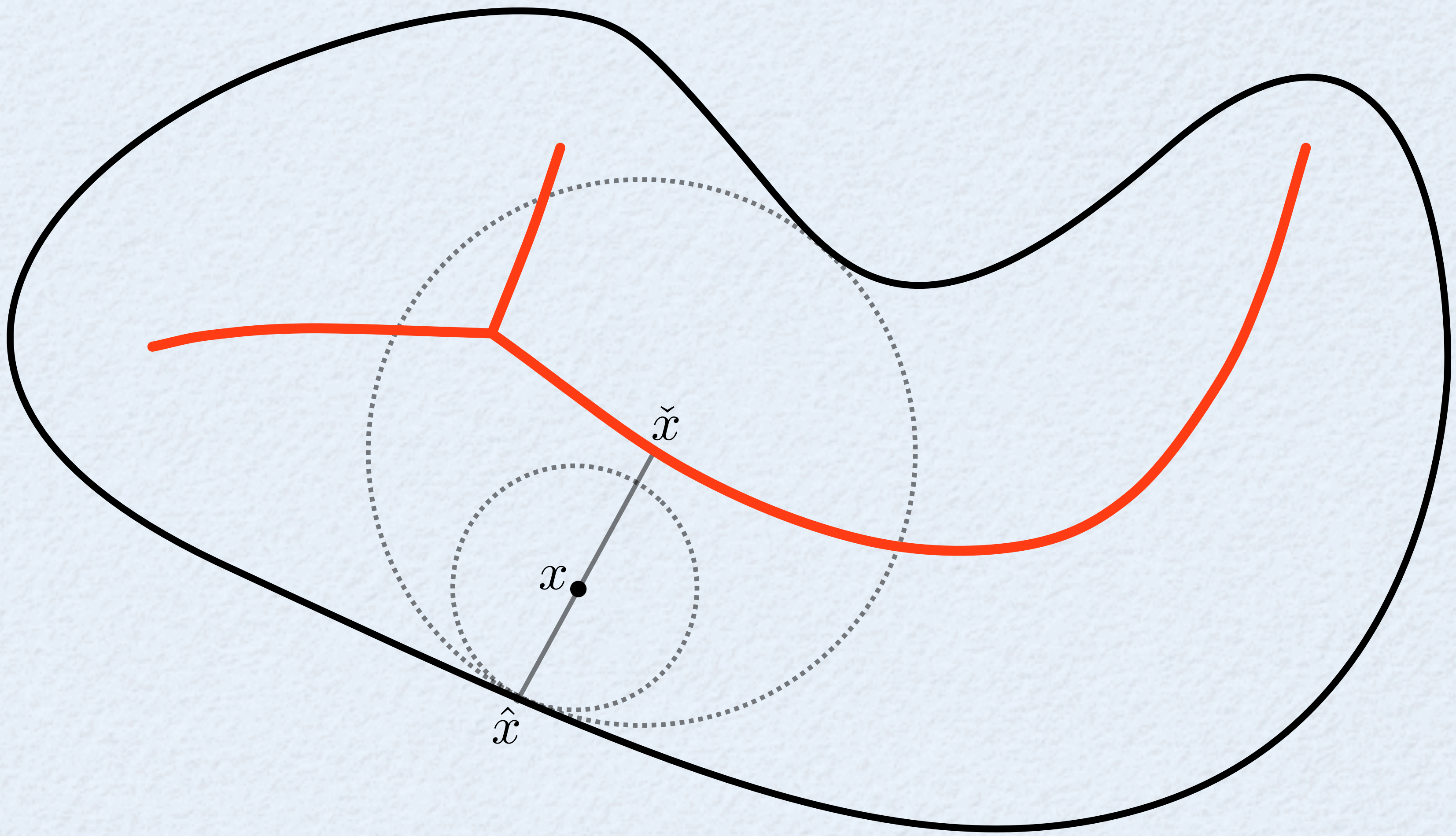

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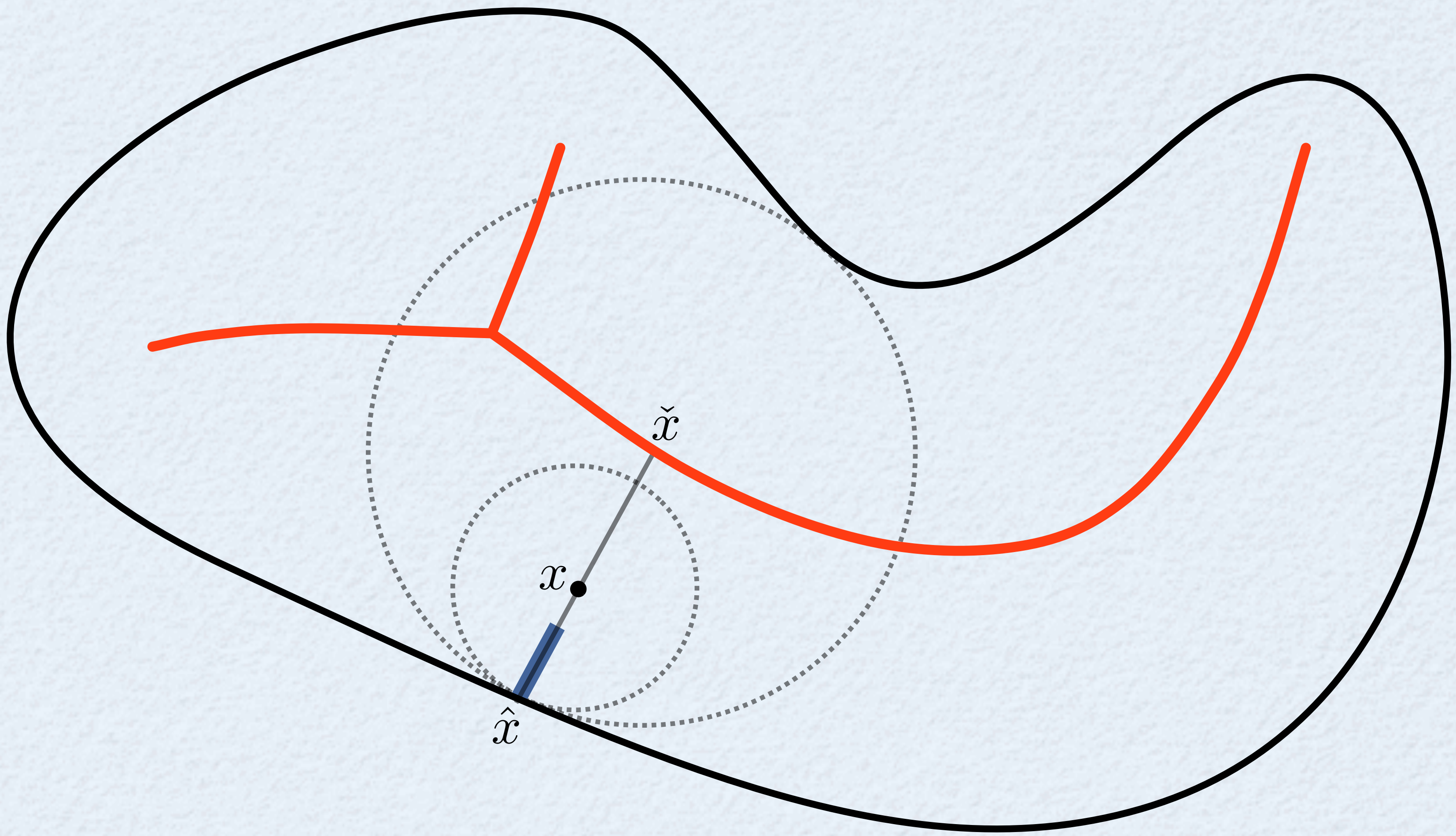

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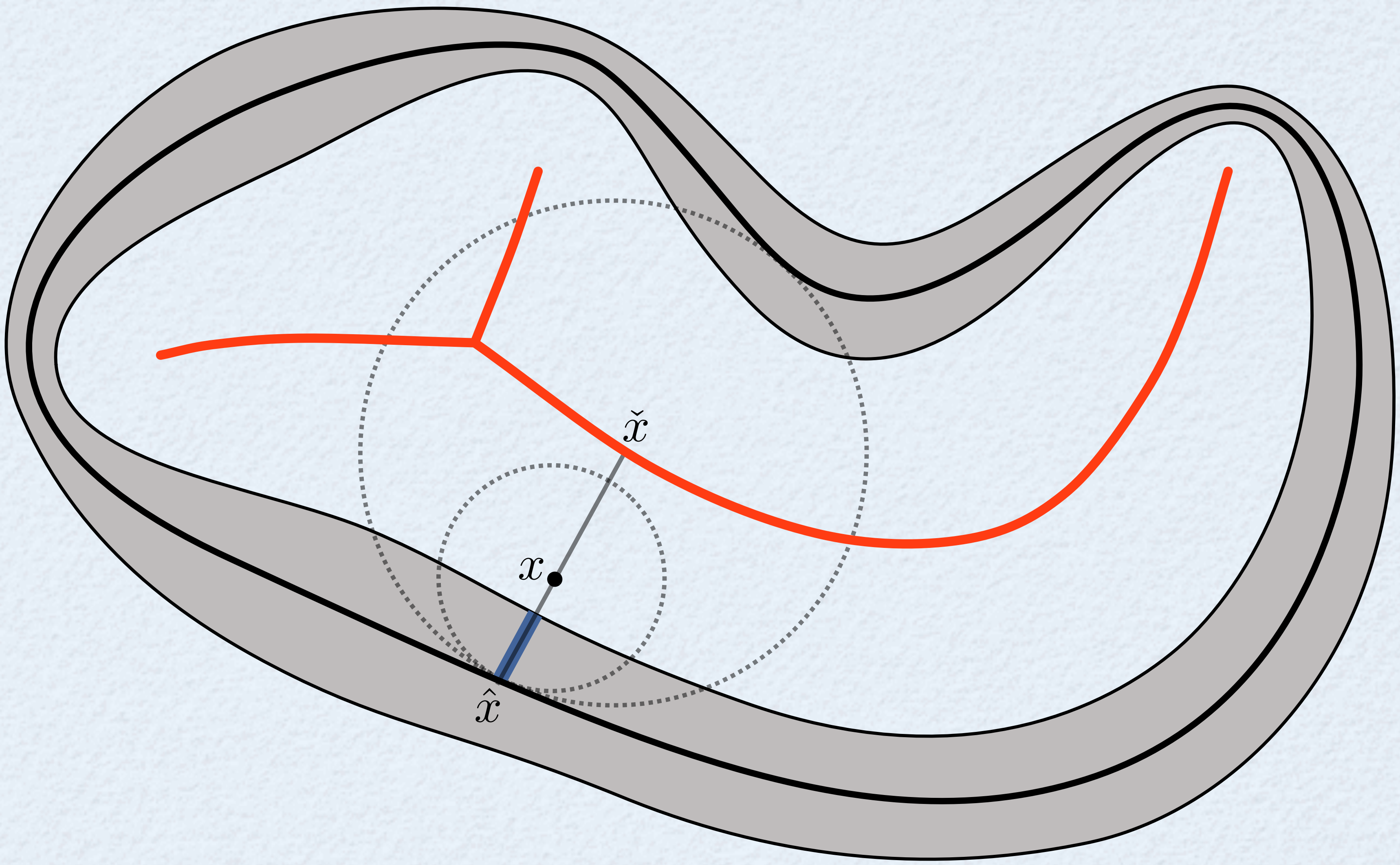

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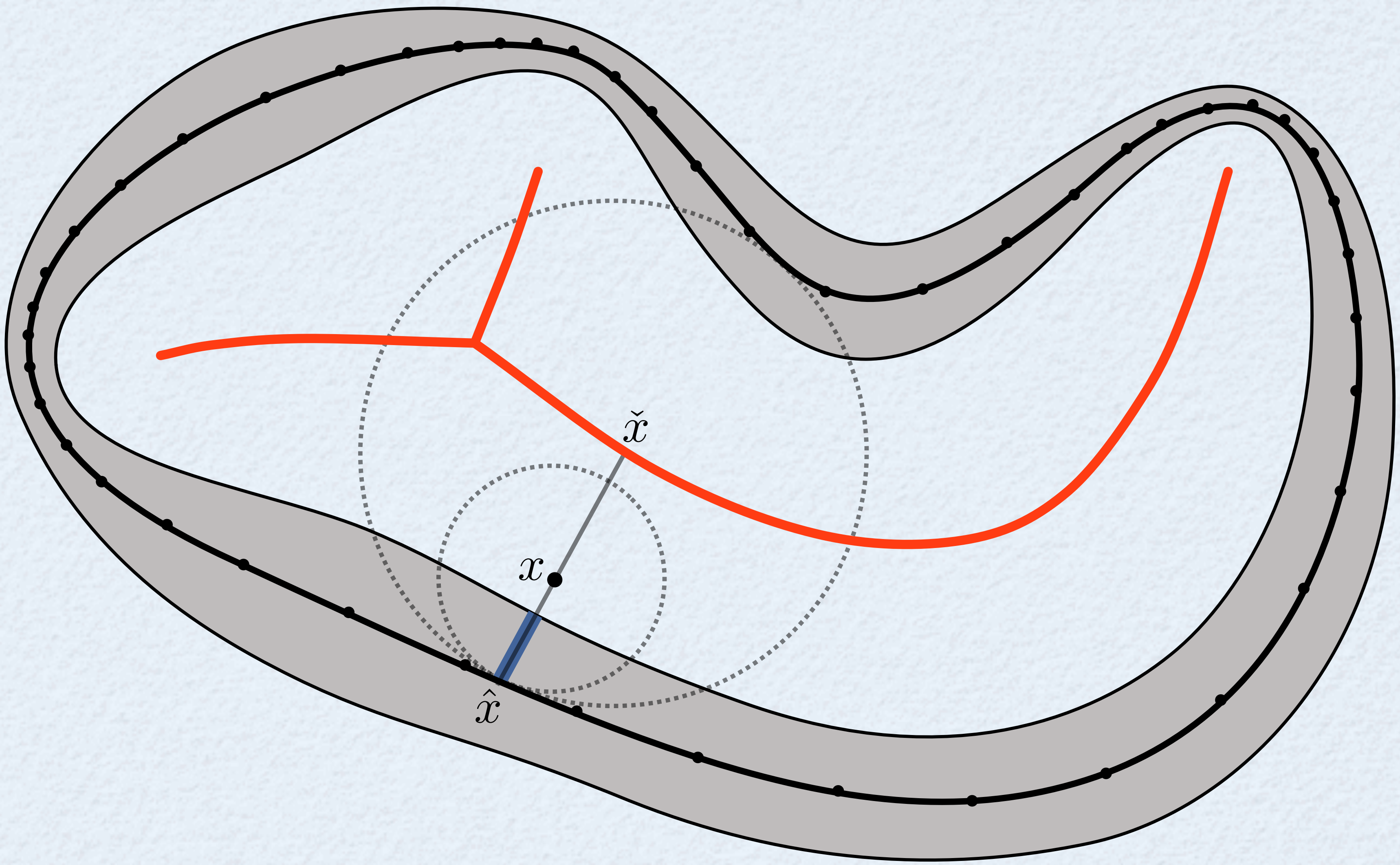

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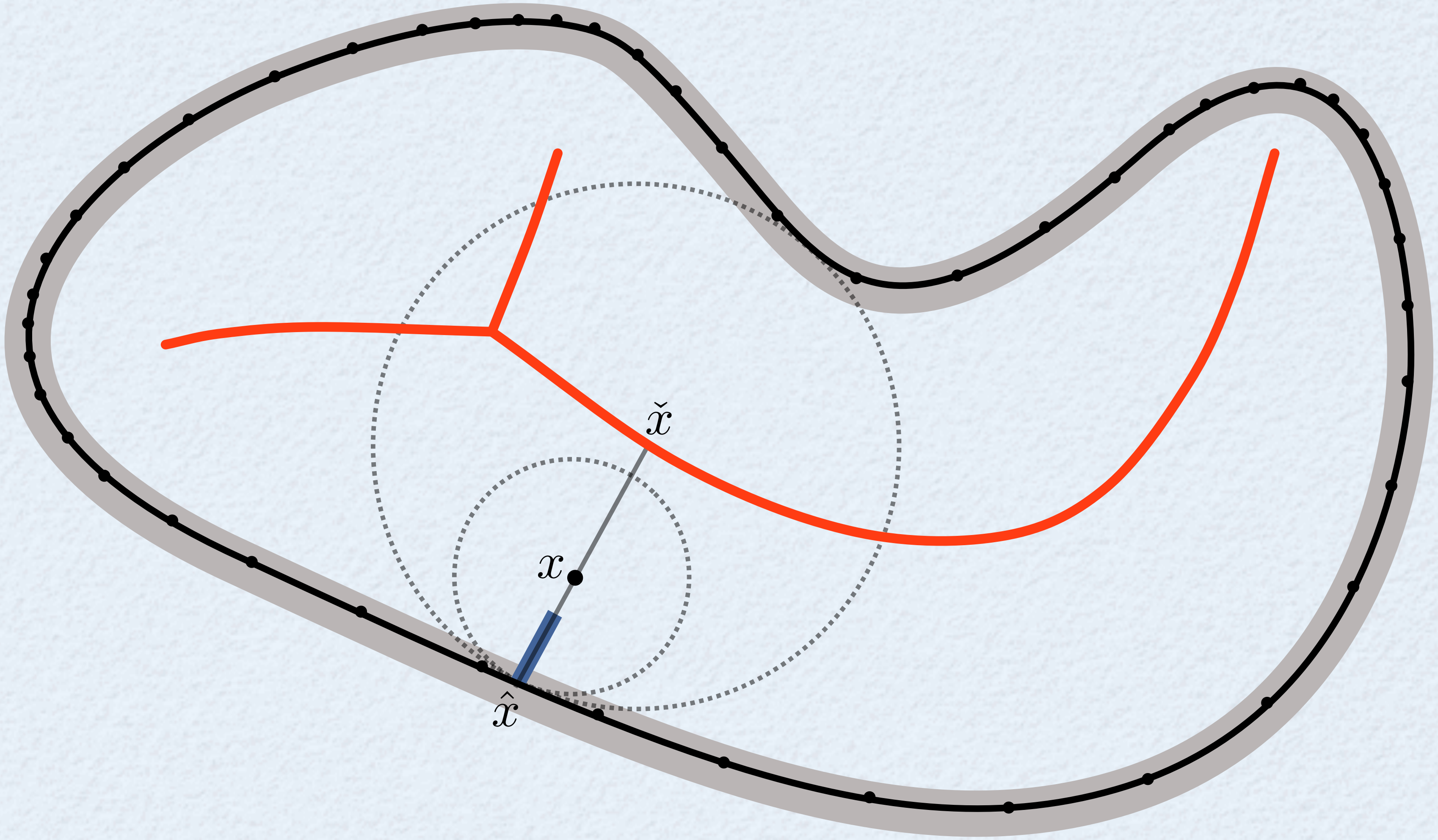

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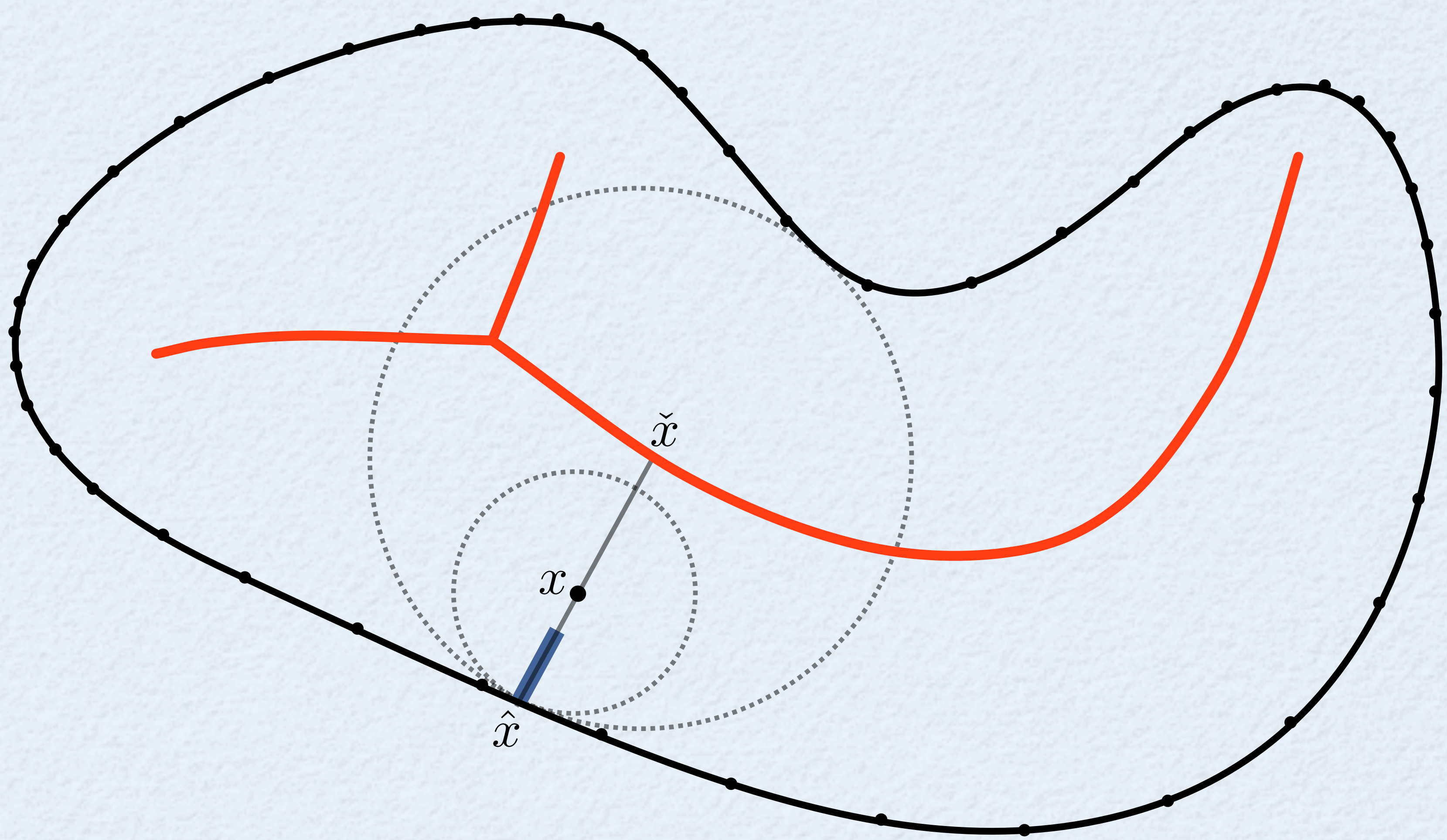

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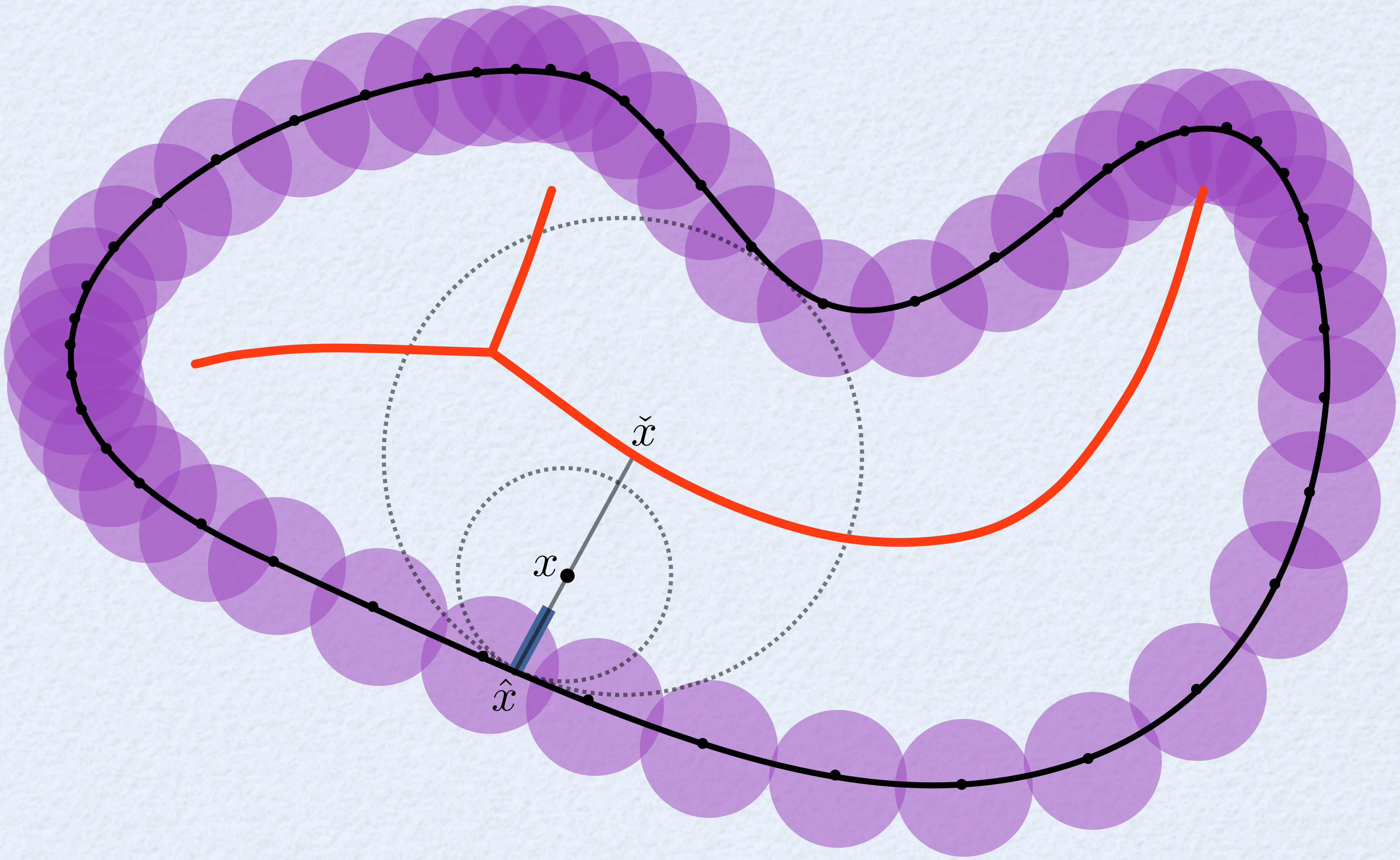

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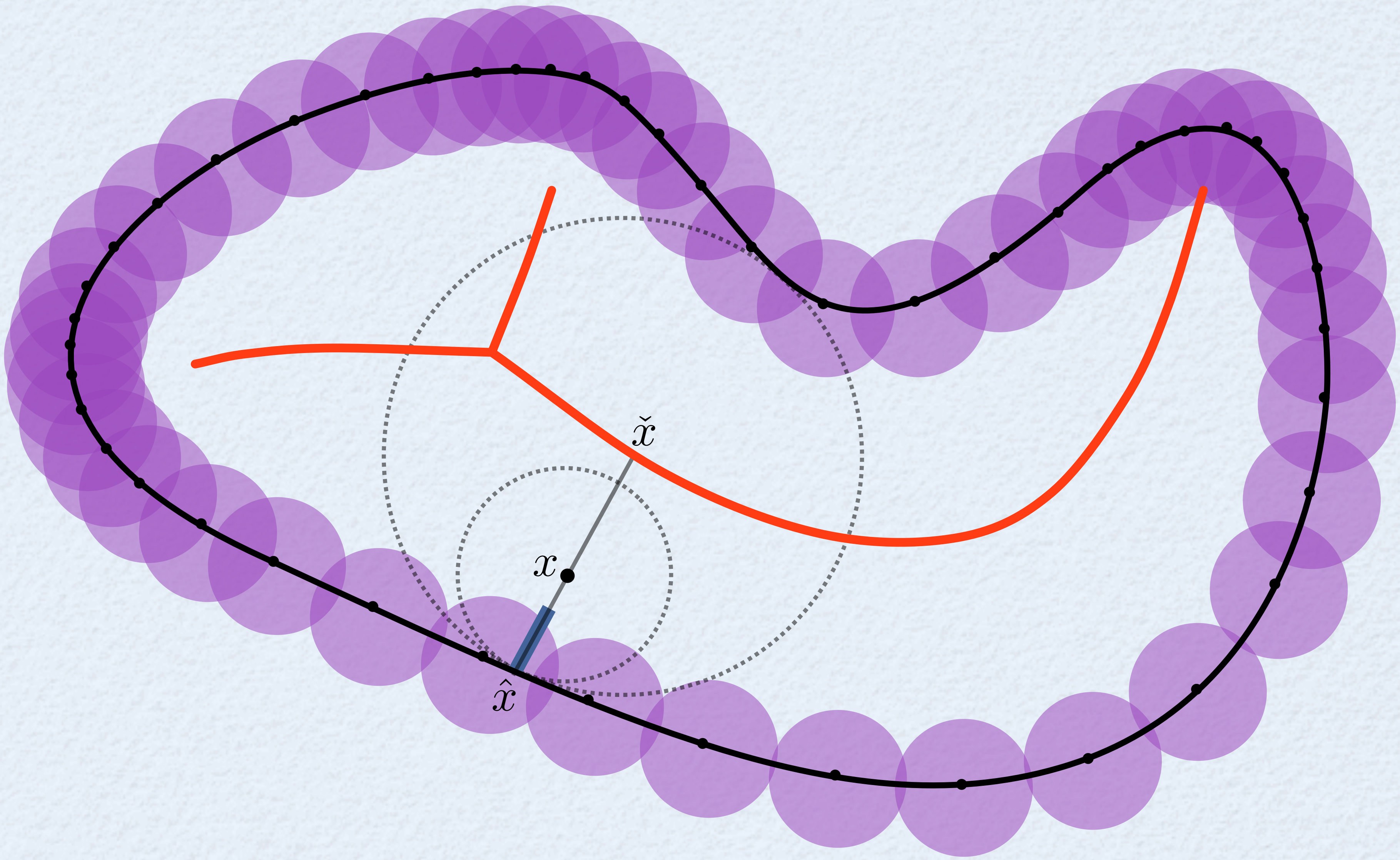
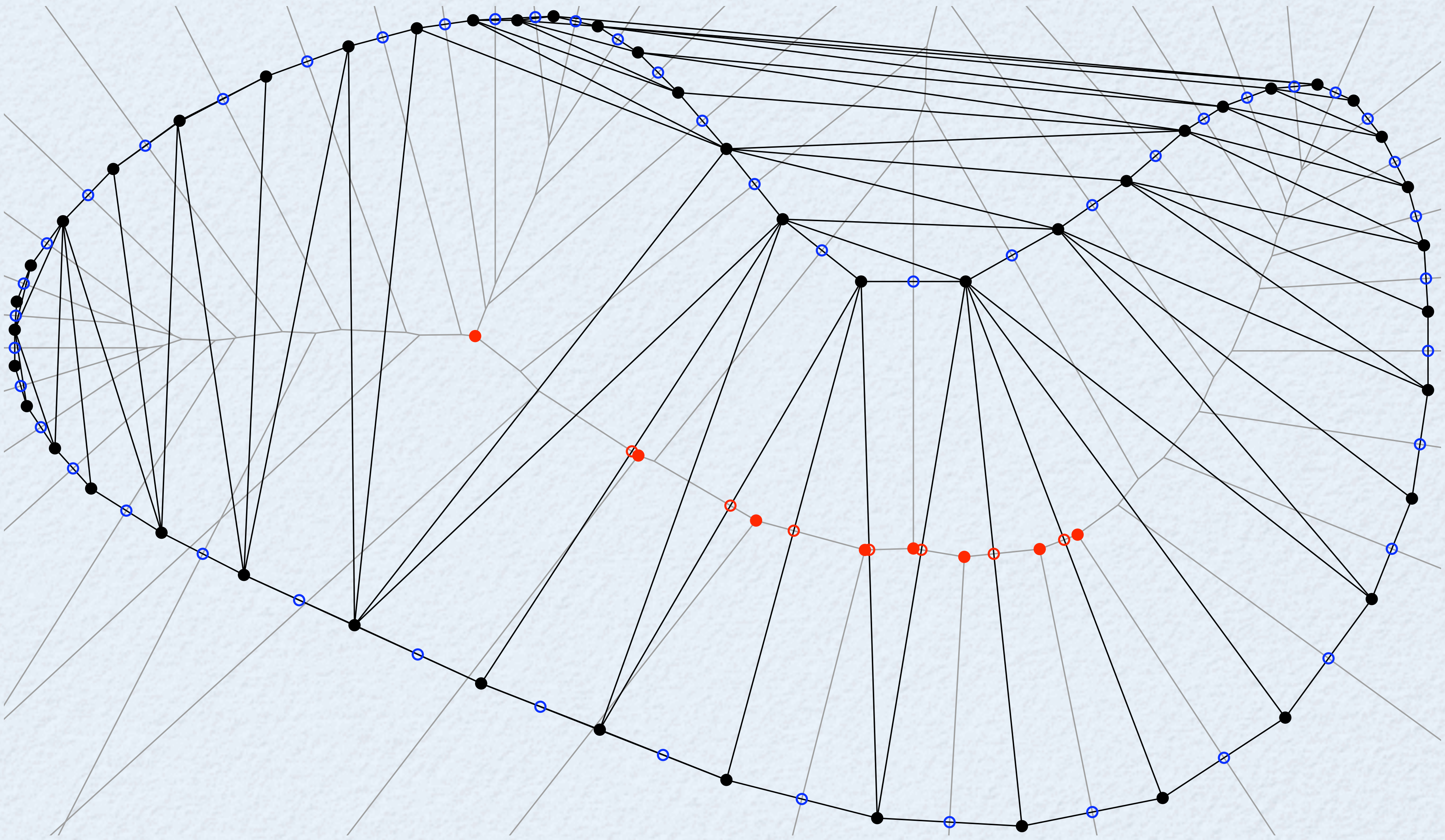


Finite unions and intersections of flow-tight sets are flow-tight.

Critical Points of Distance Function

A point c with $v(c) = 0$ is called critical.



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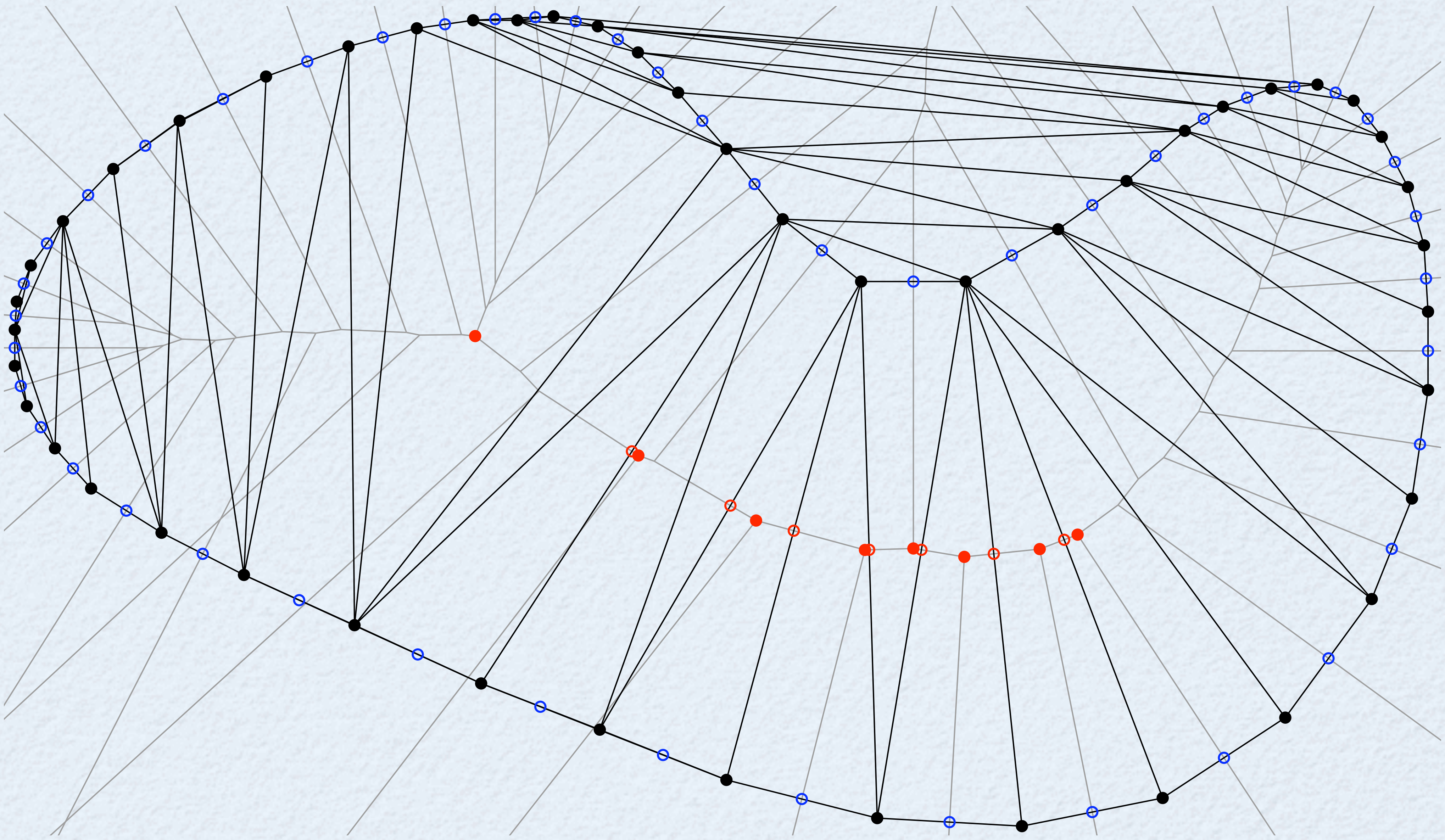

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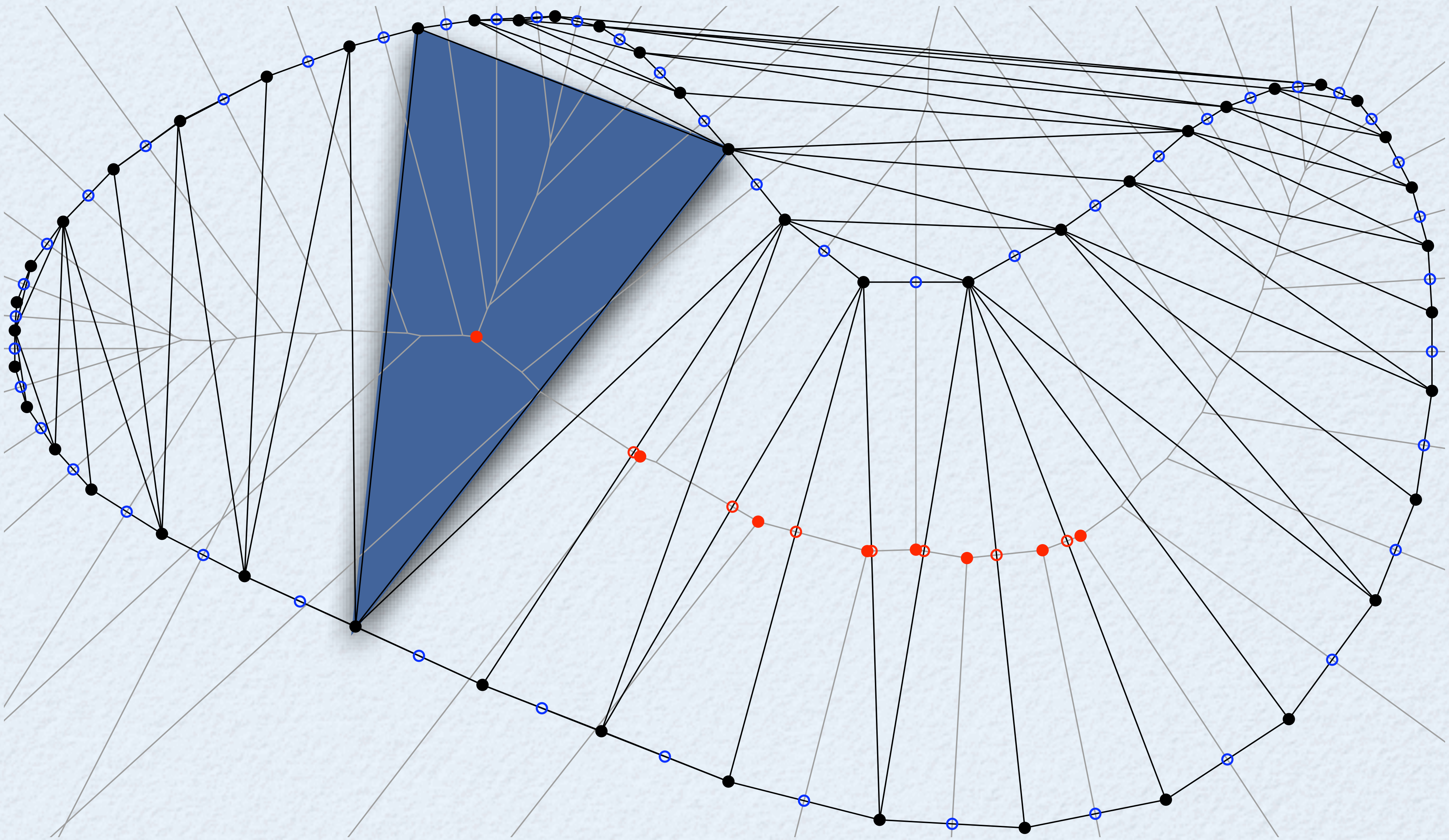

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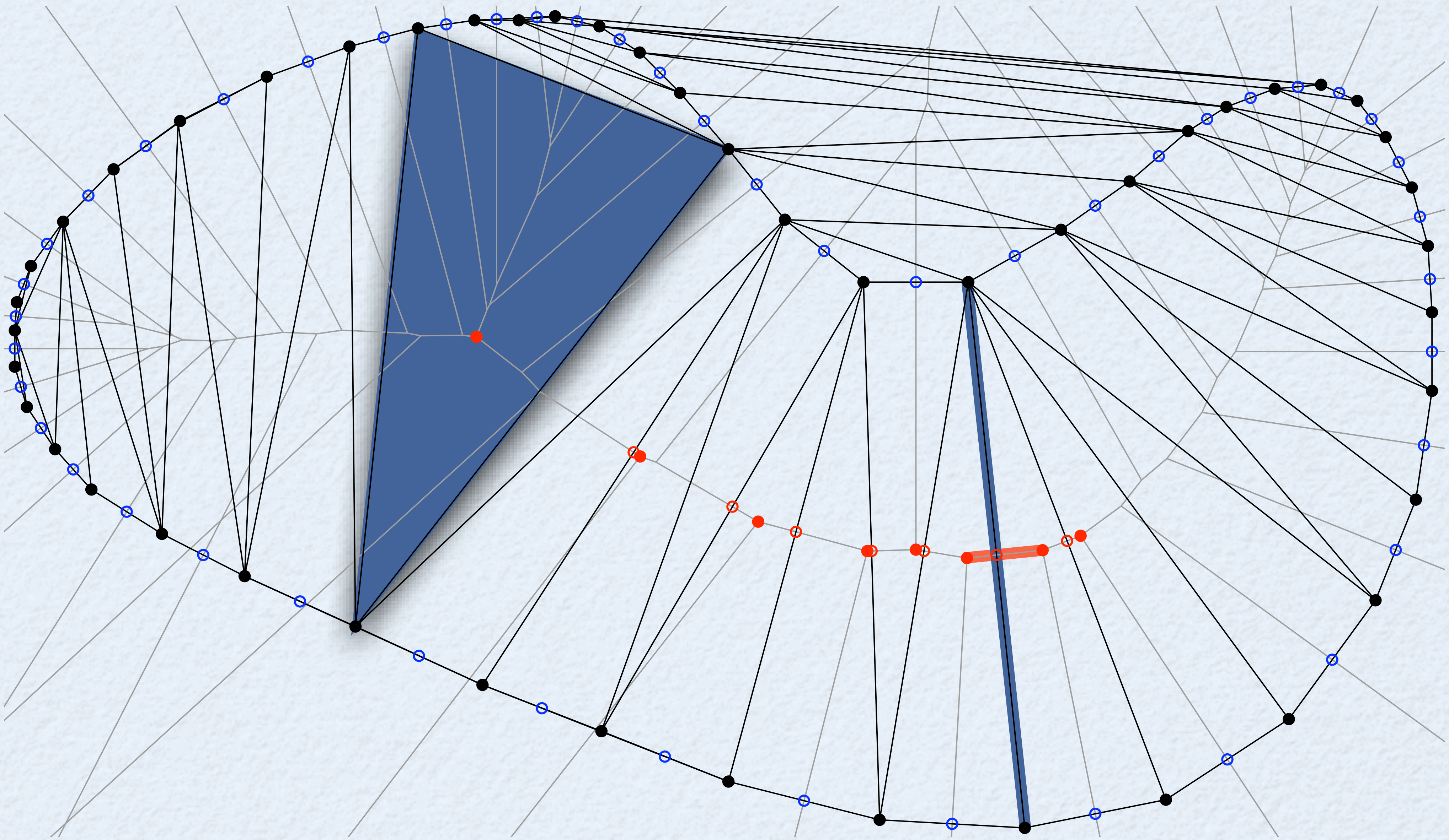

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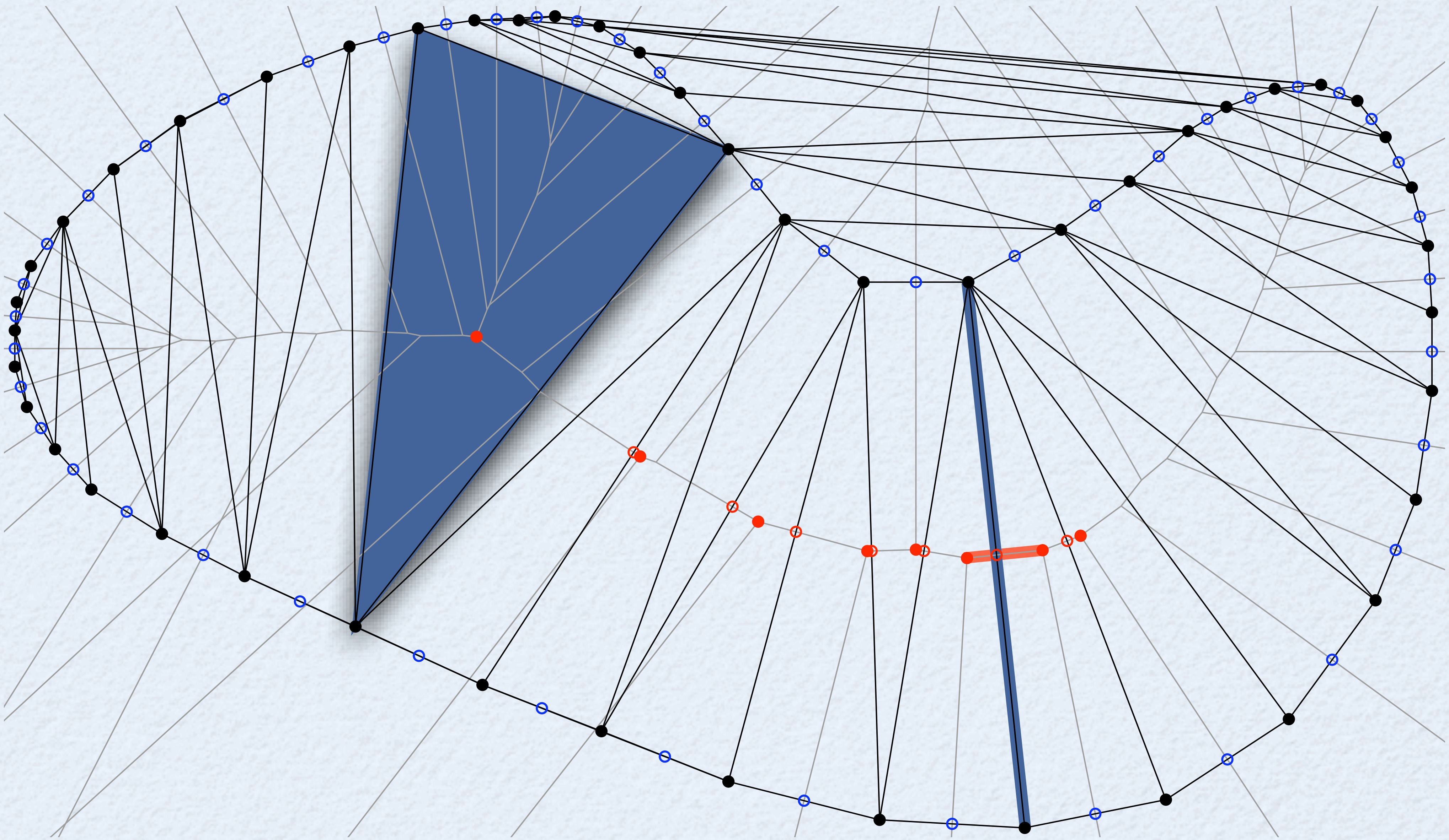

The index of c is the dimension of $D(c)$.

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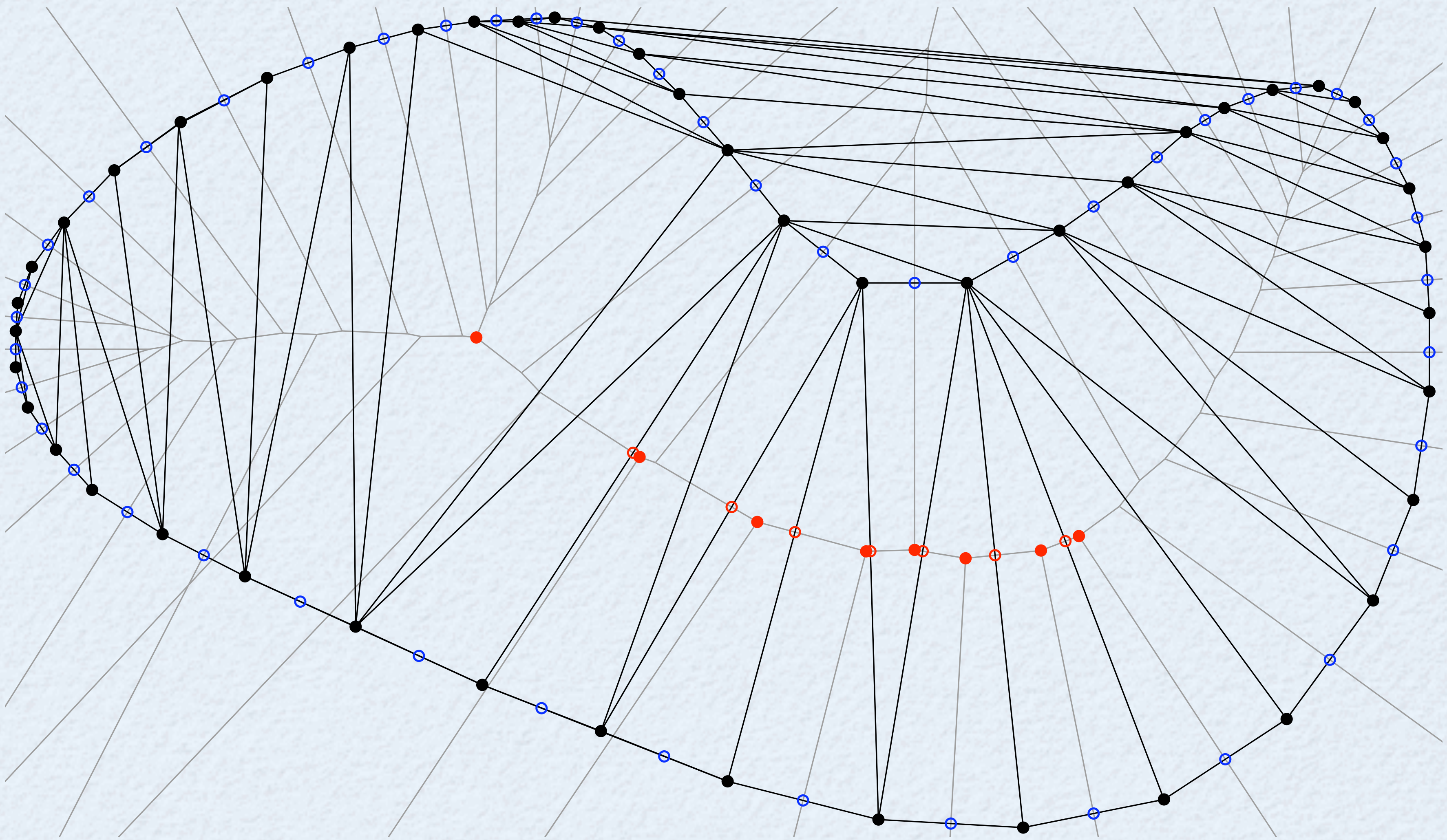
Stable manifold of a critical point c is the set of all points that flow to c.

$$\mathsf{Sm}(c) = \{x : \phi(\infty, x) = c\}.$$



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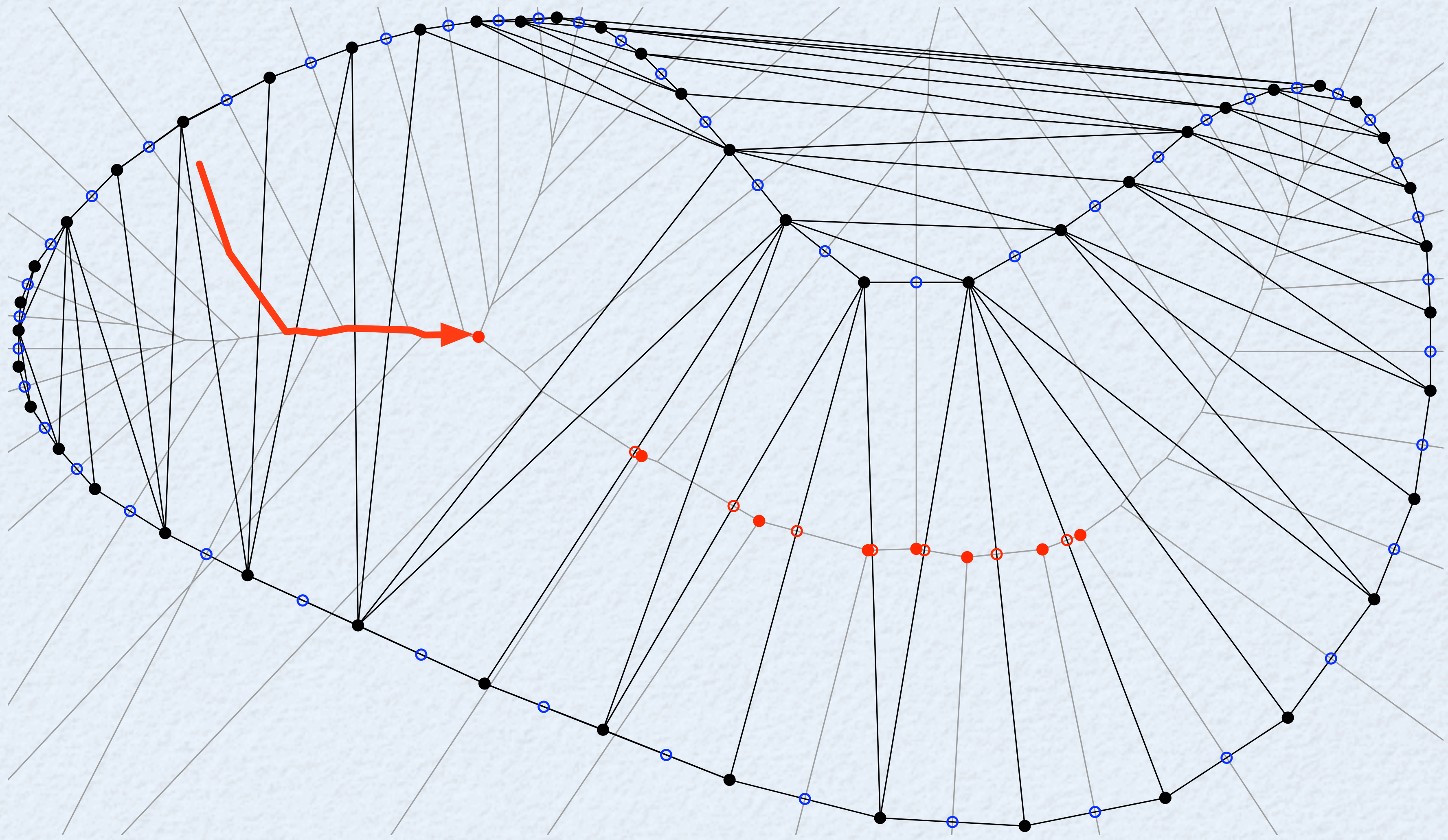

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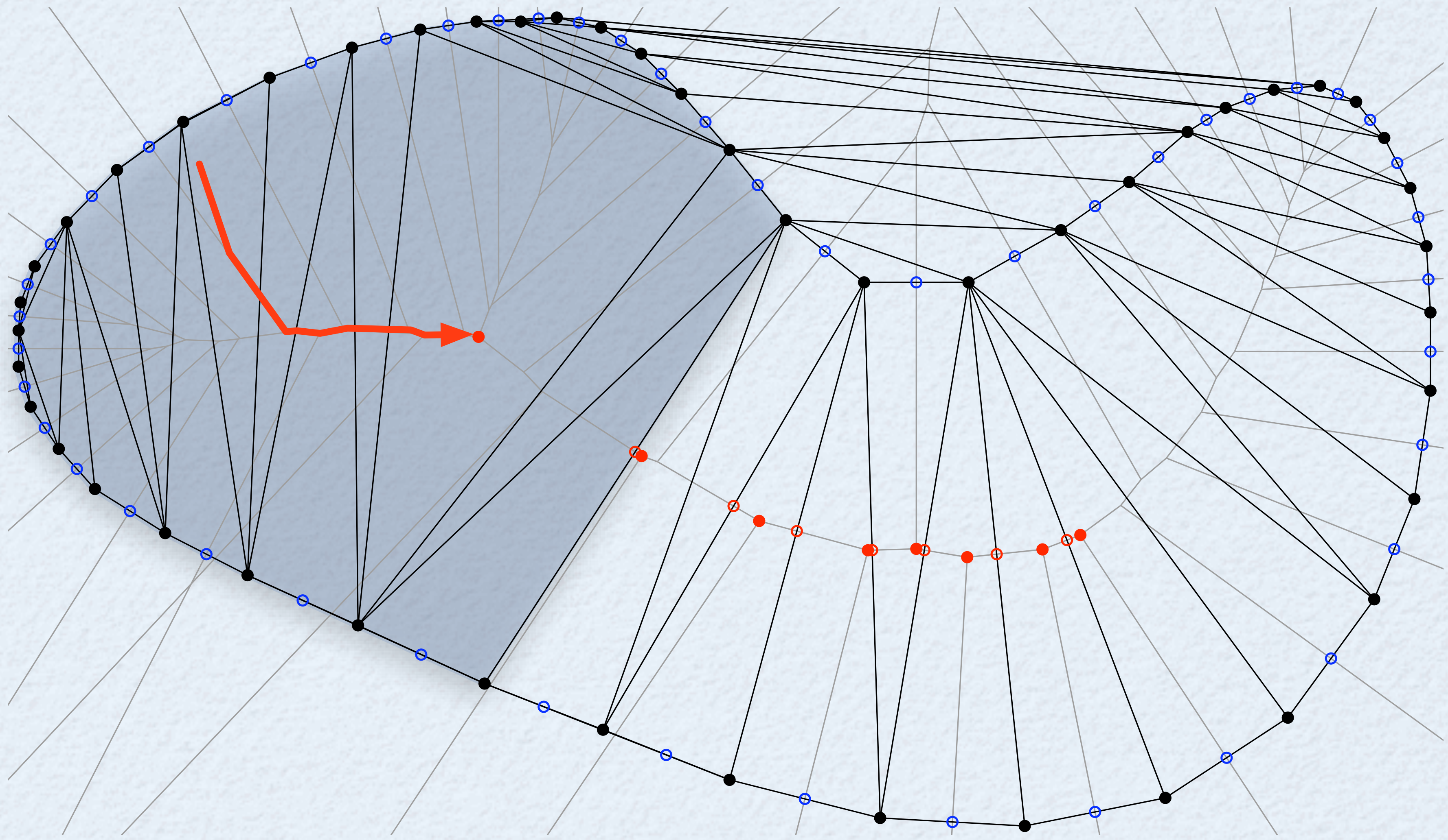

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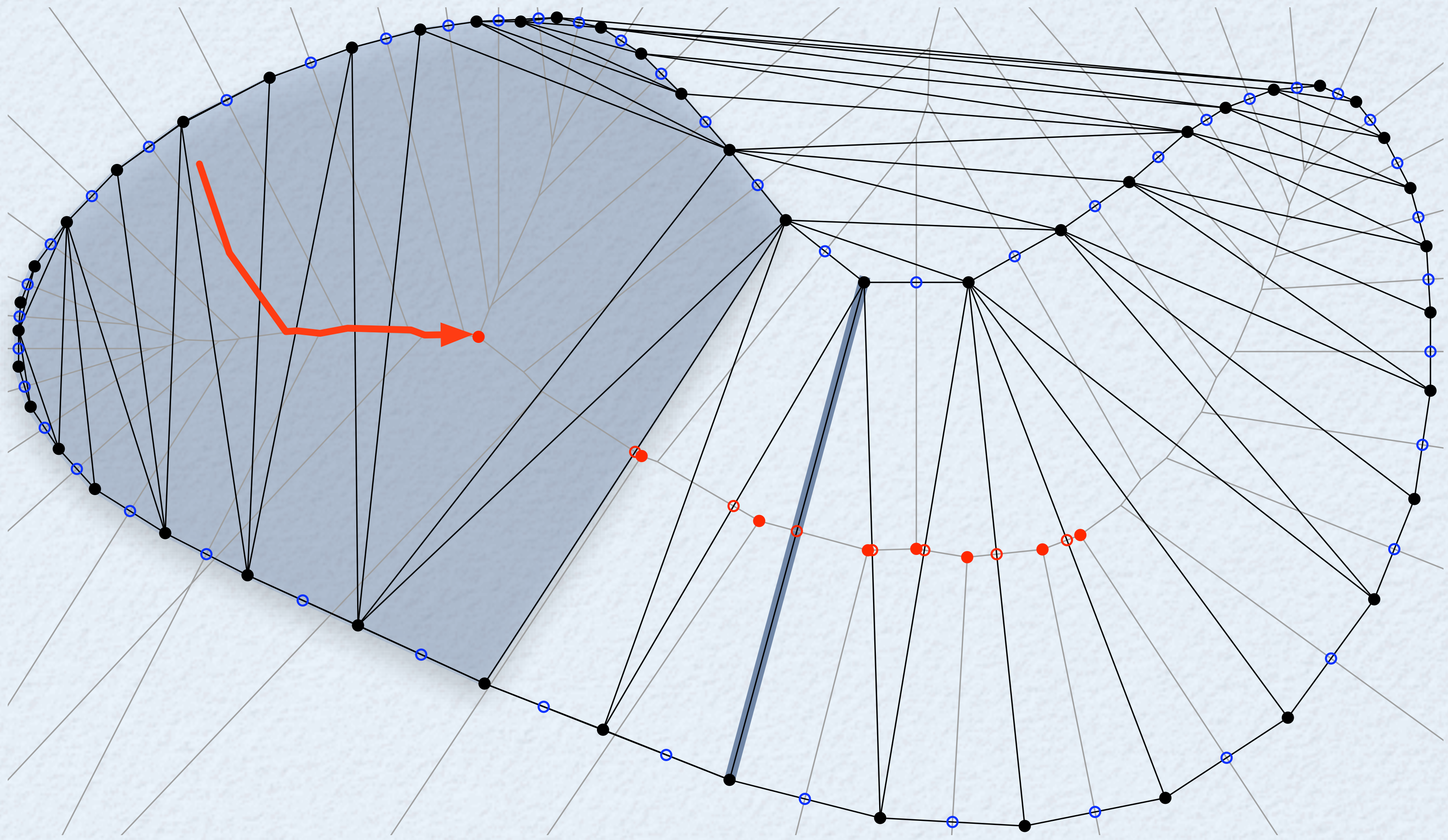

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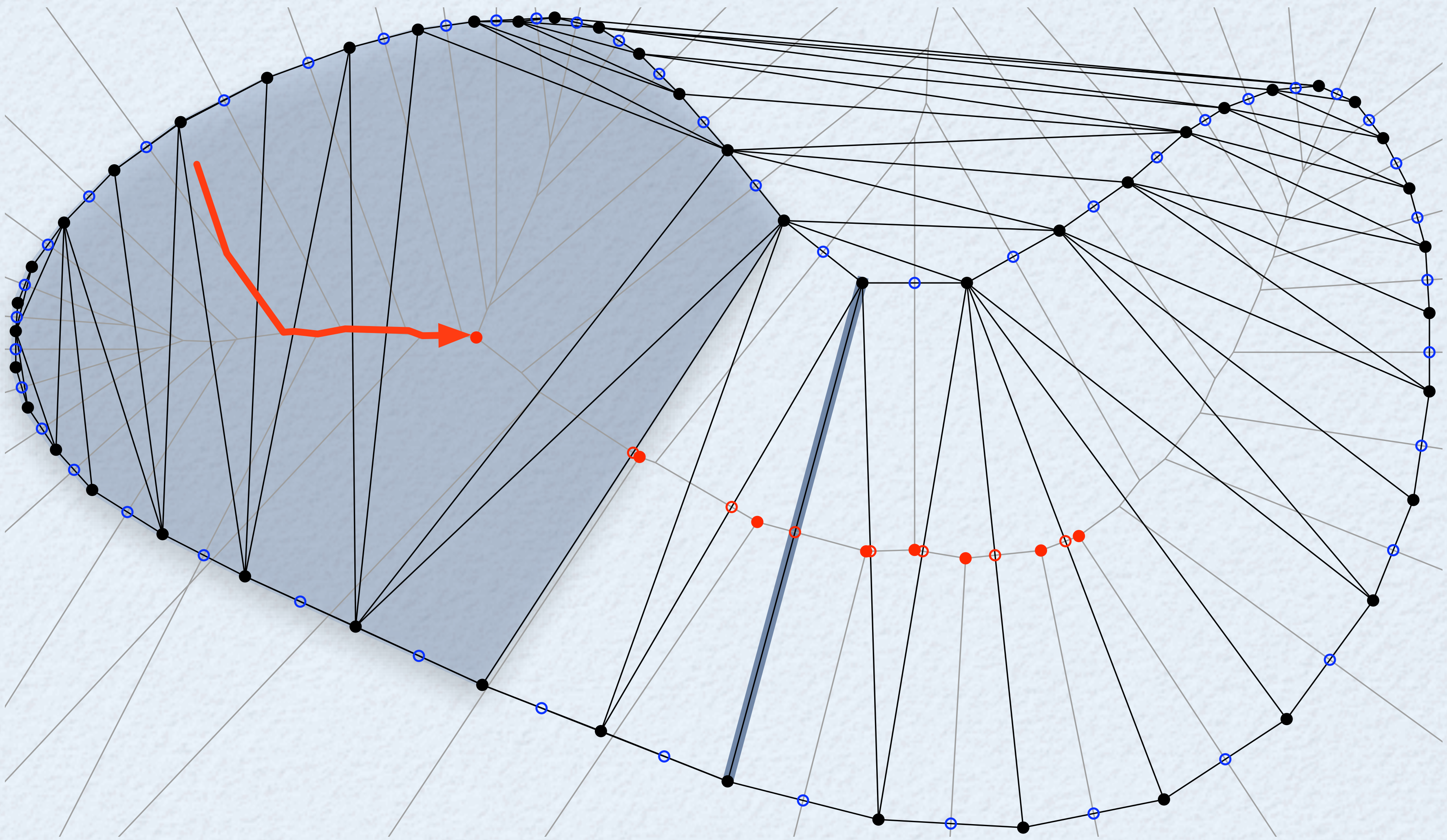

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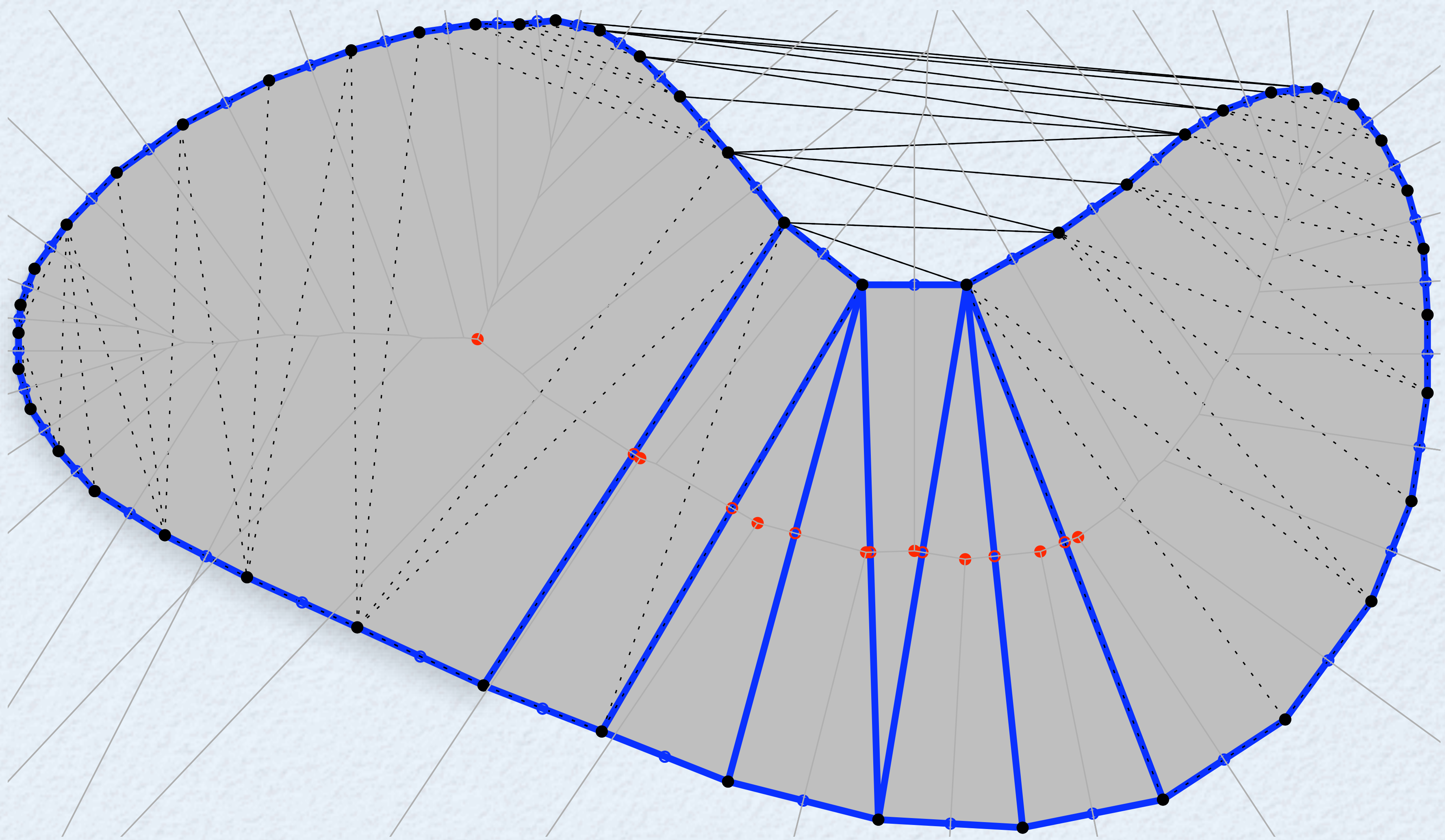

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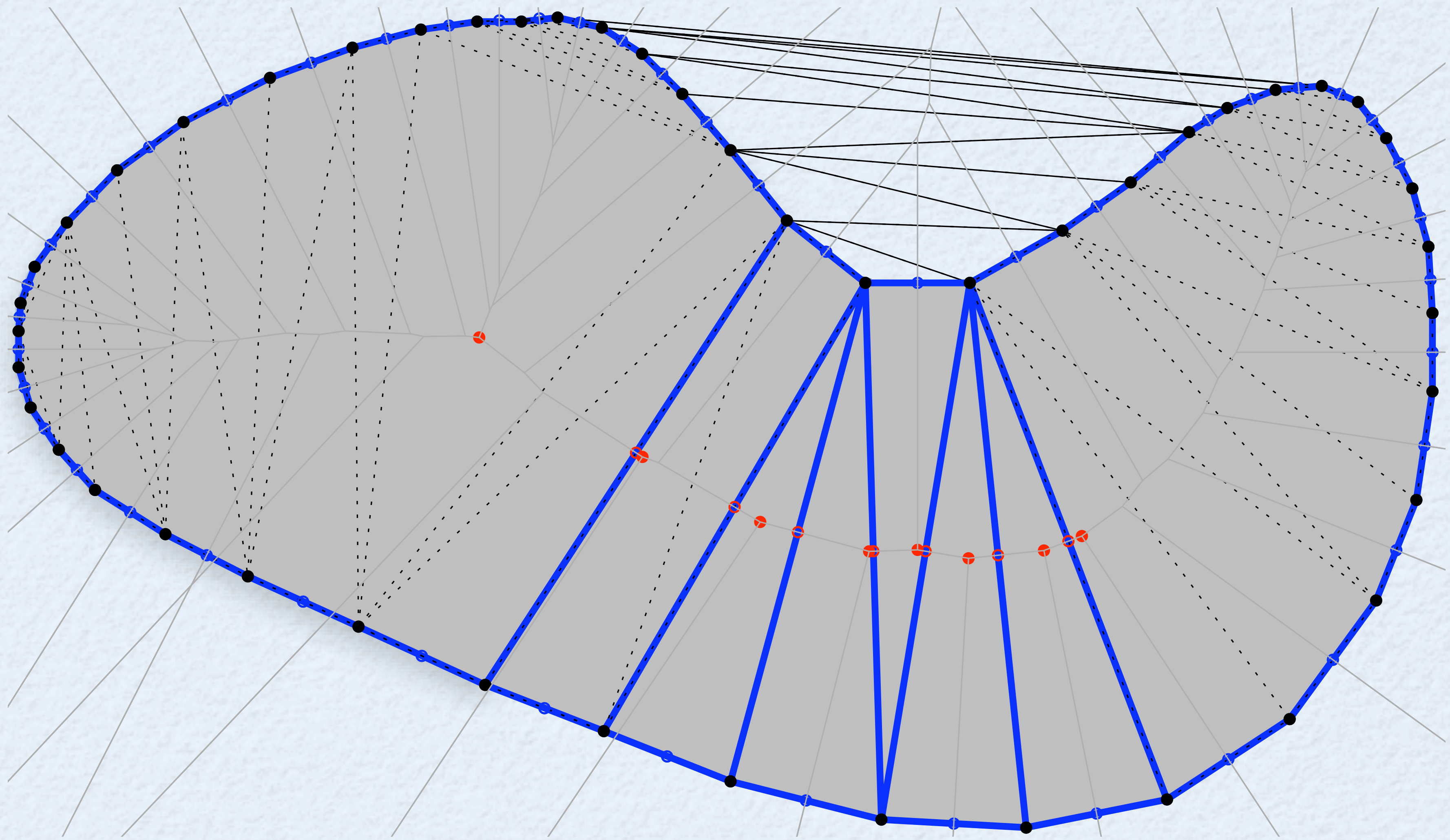

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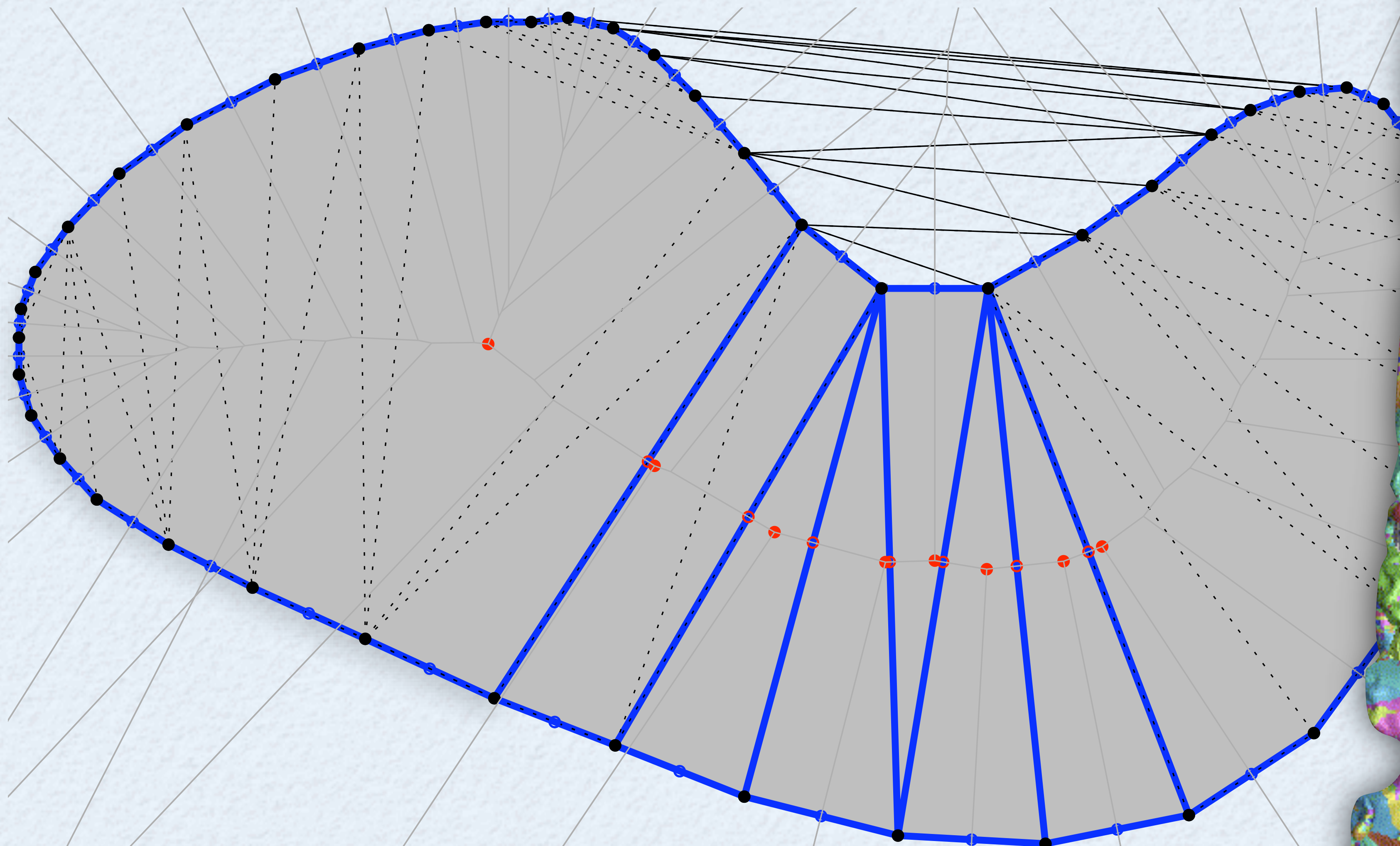

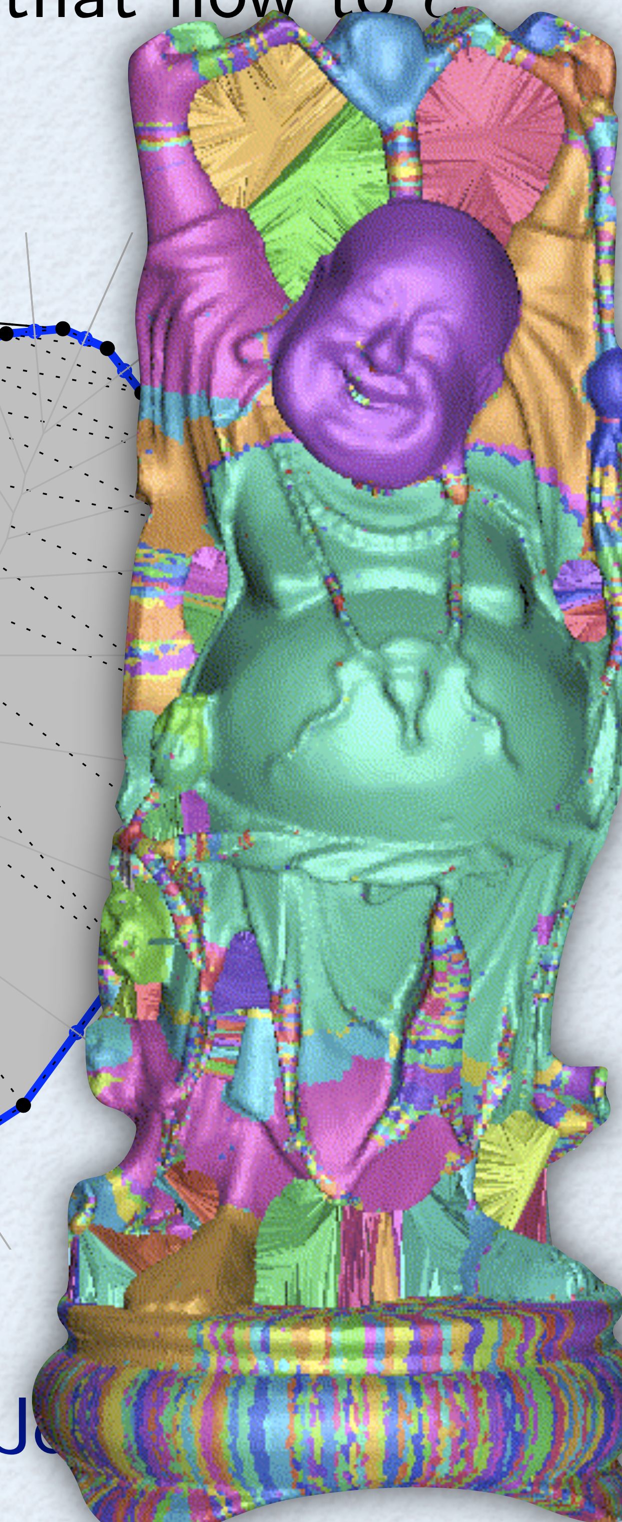

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[Giesen-John'03] found out that a subcomplex of FC reconstructs manifold.

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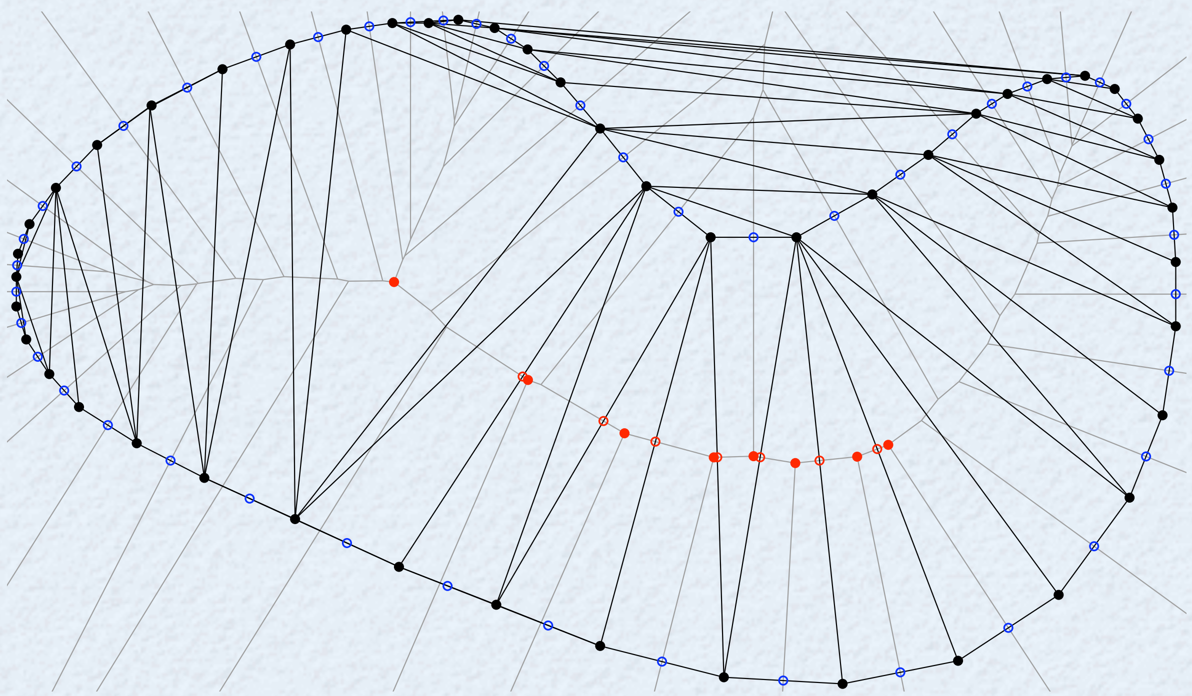

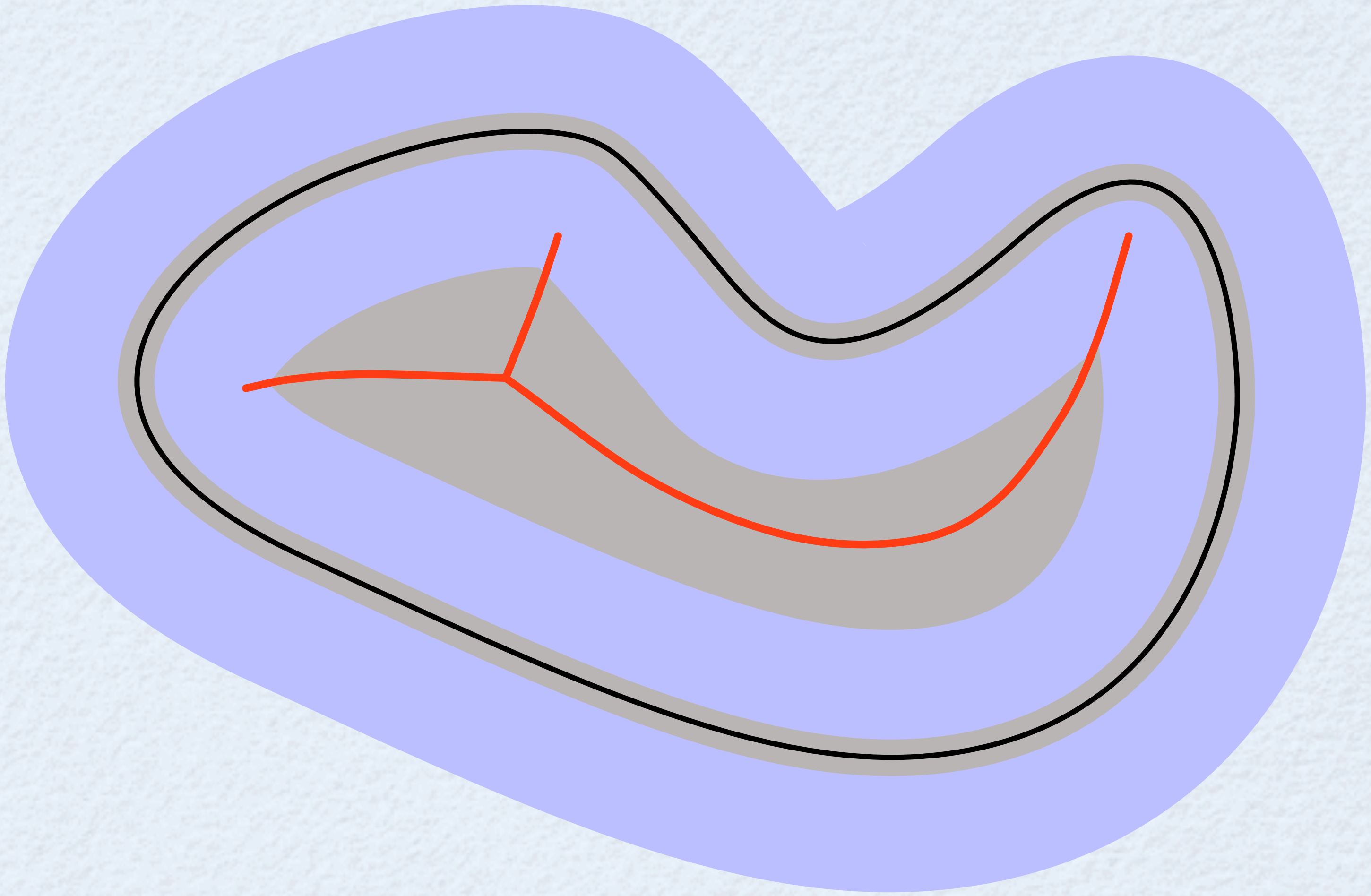

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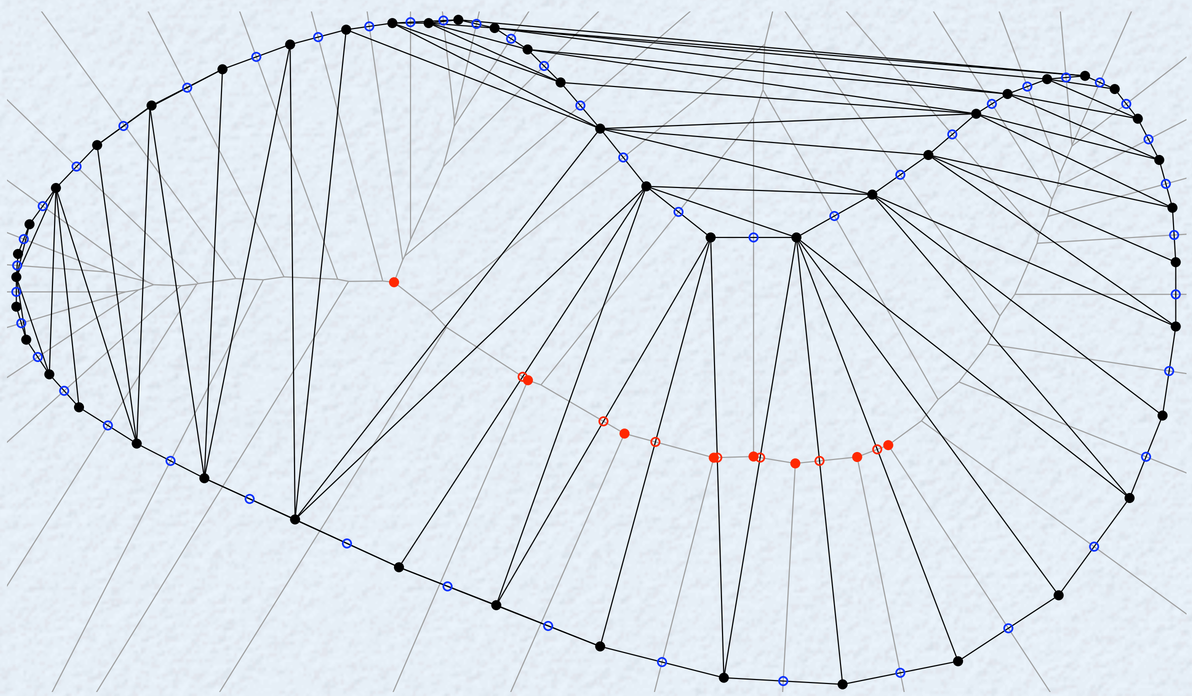

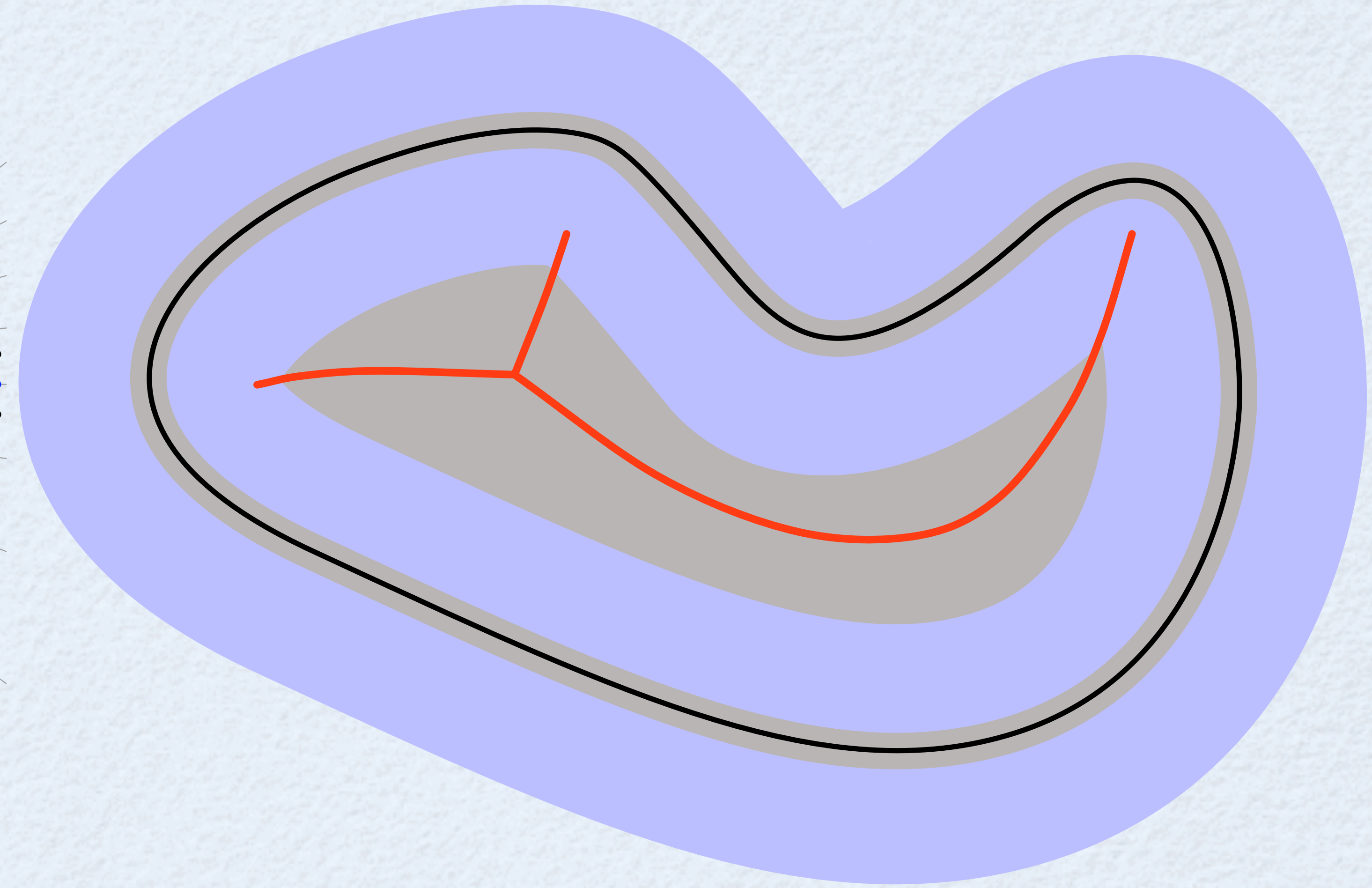

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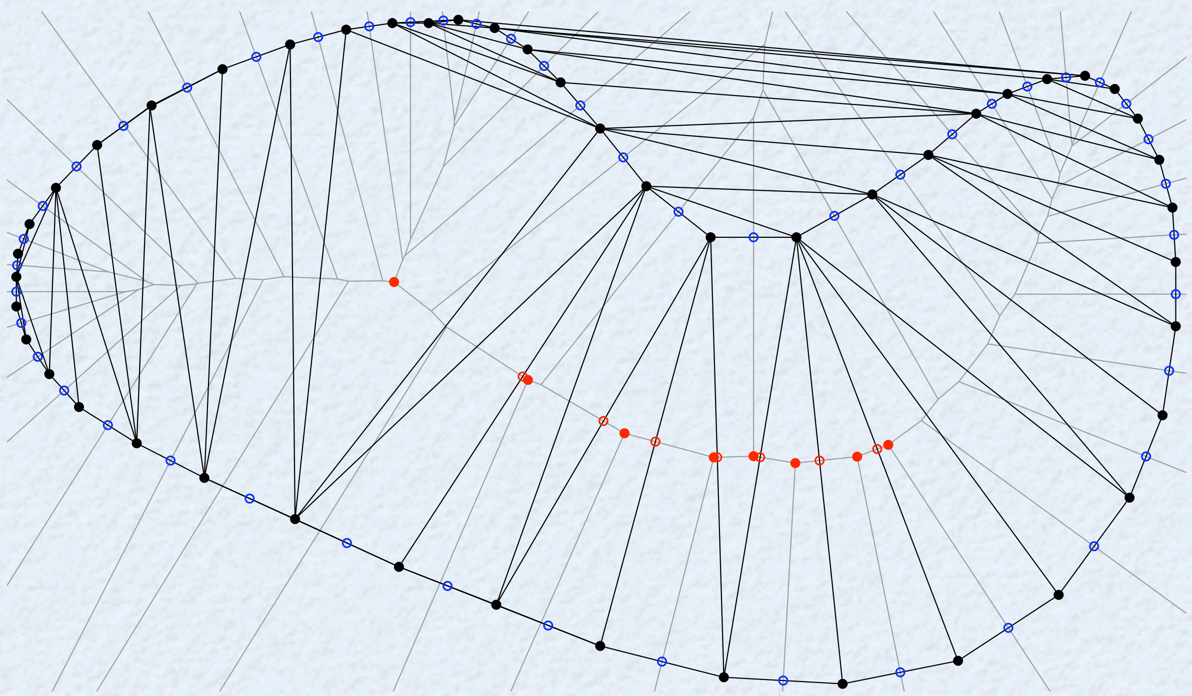

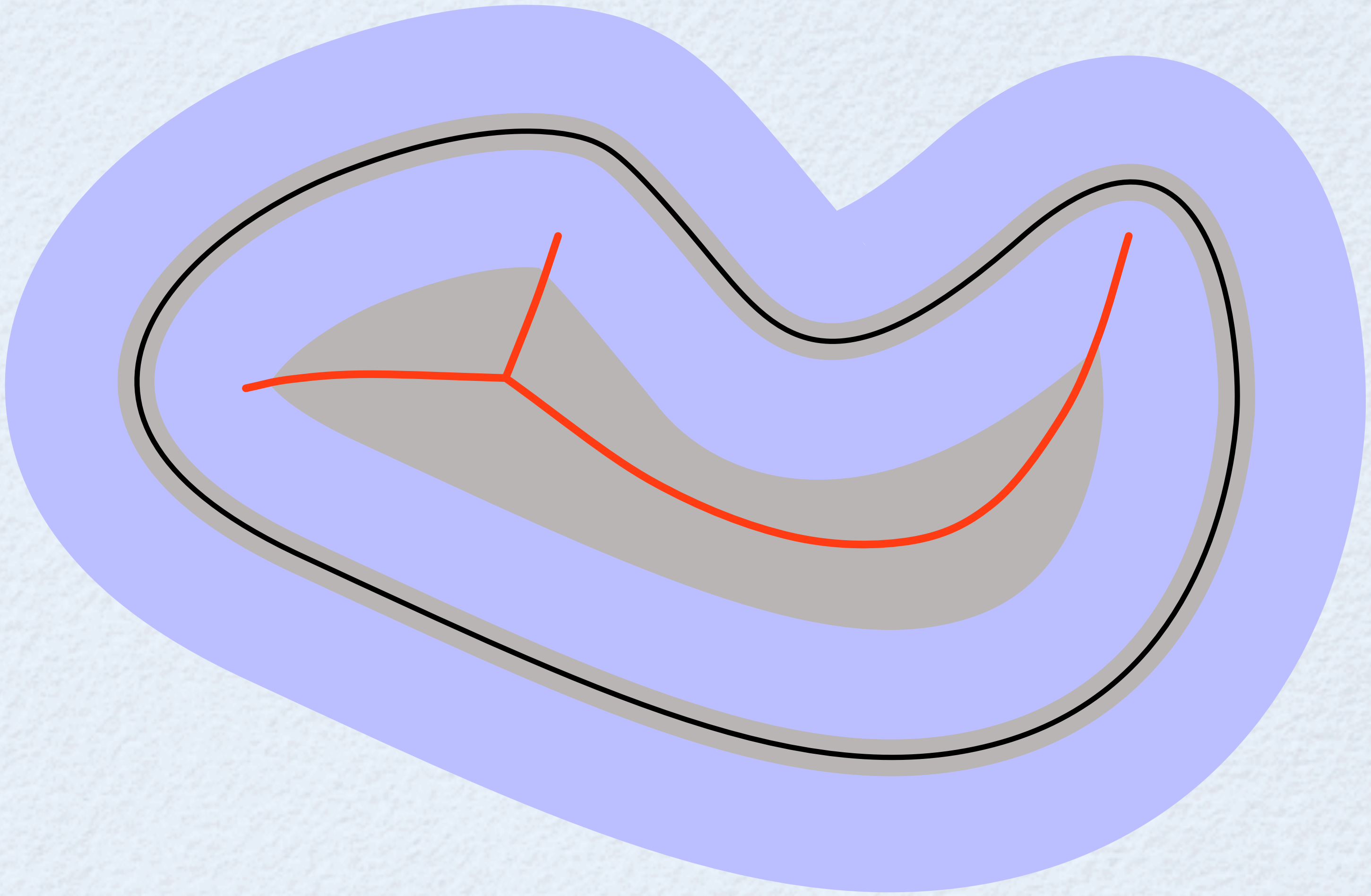

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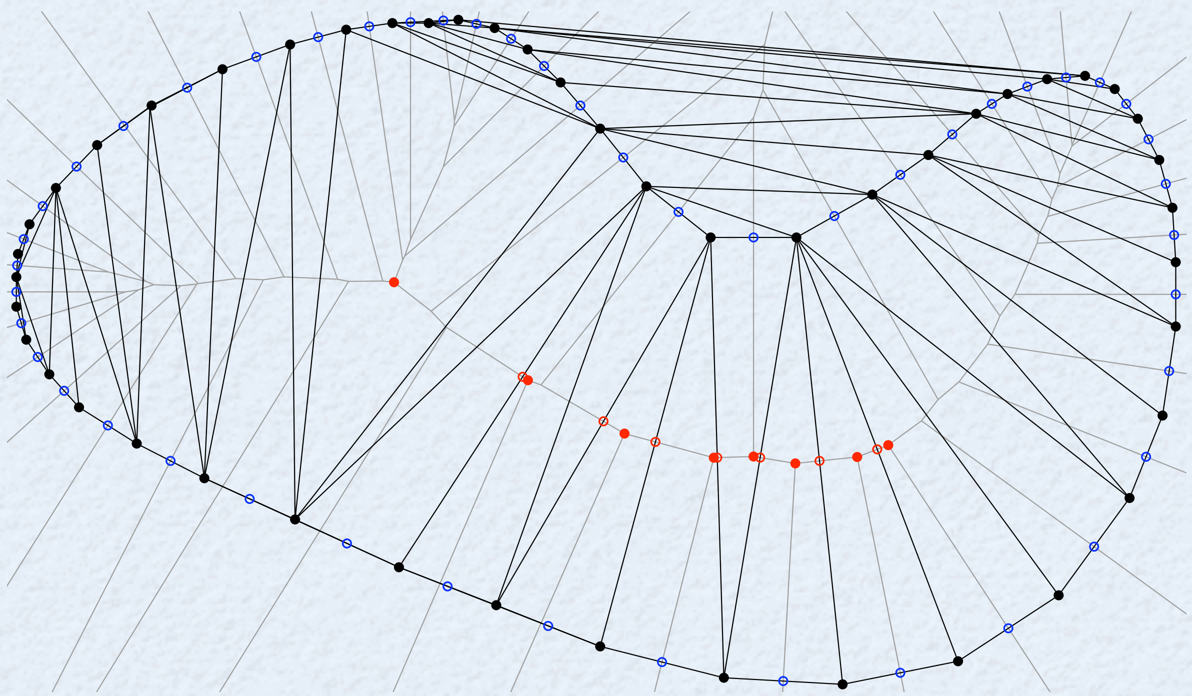

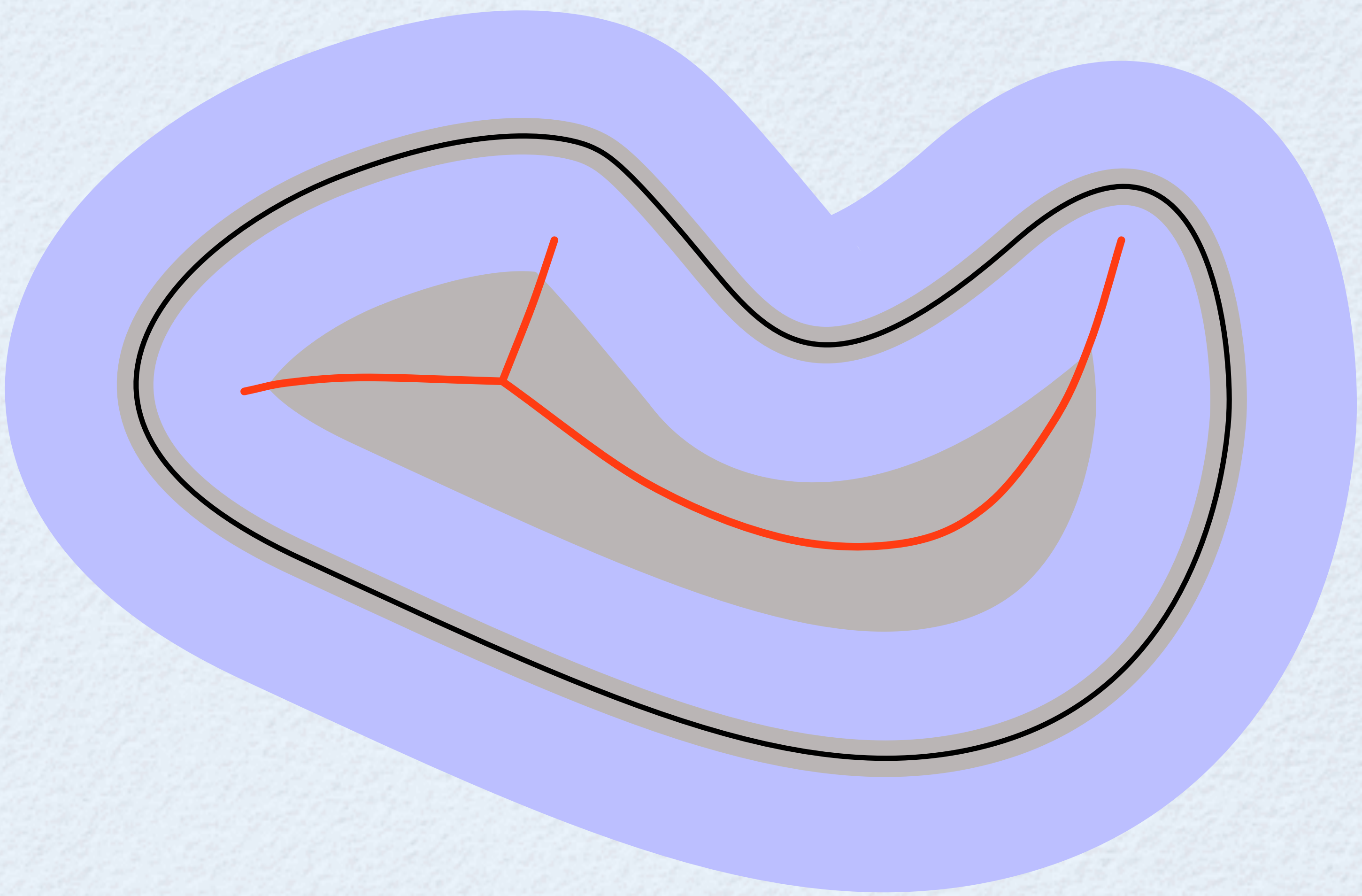

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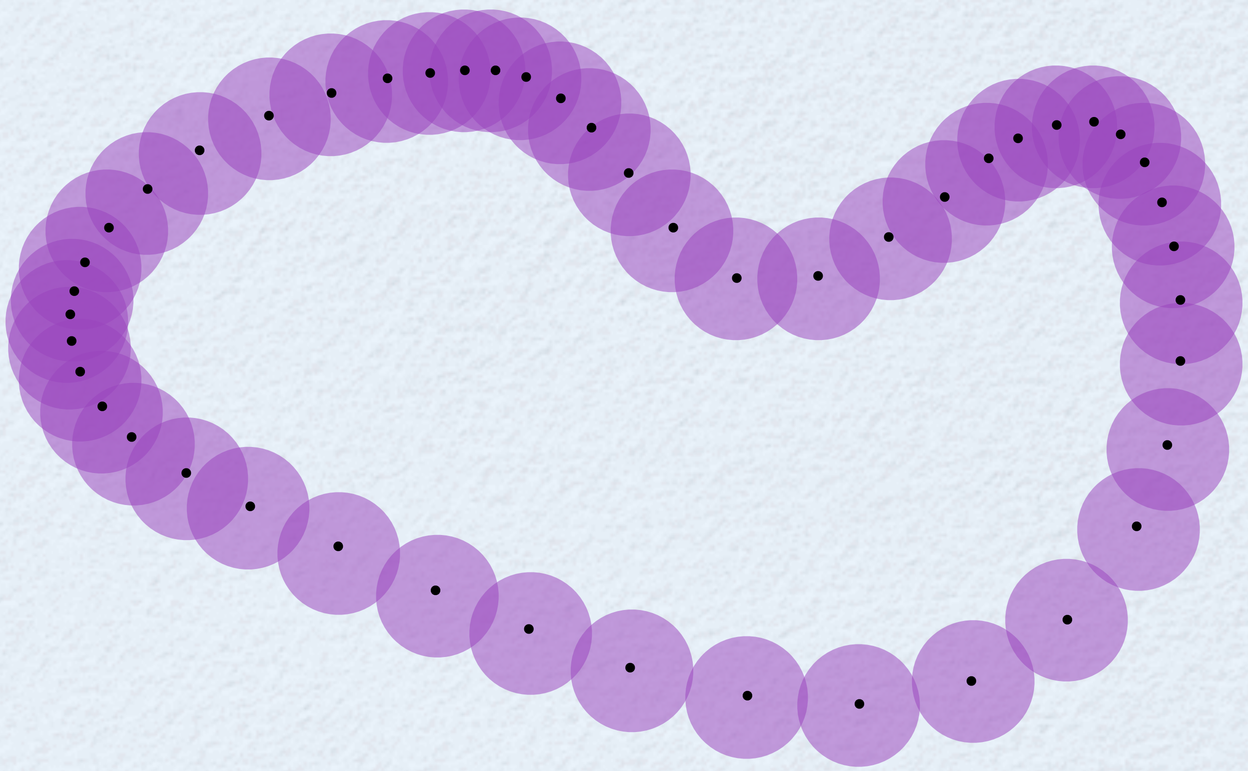

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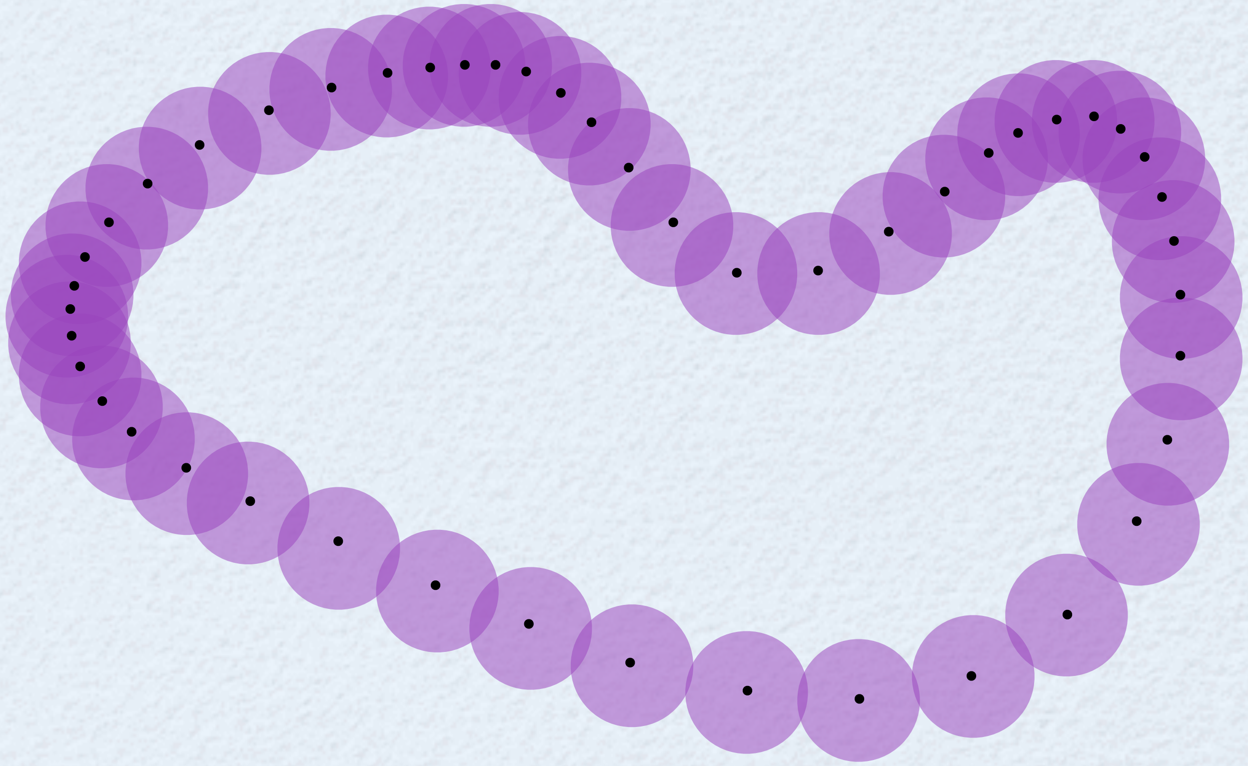

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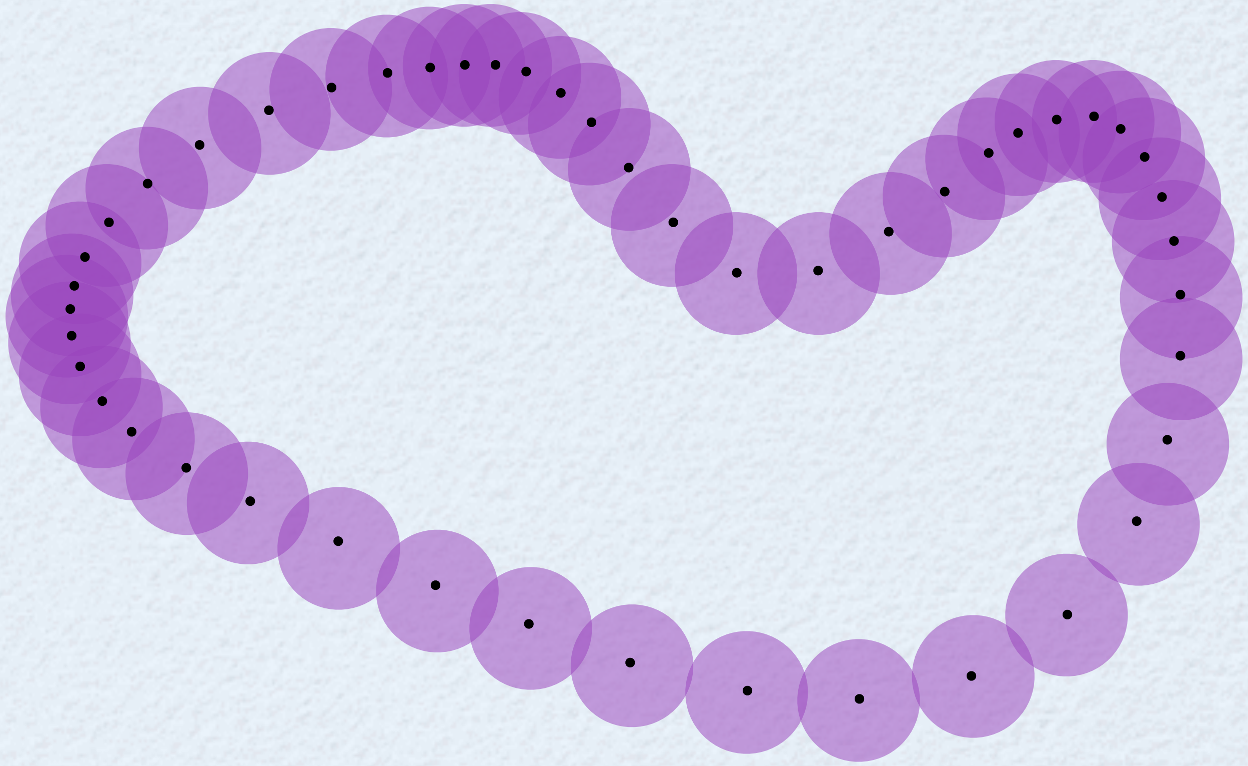

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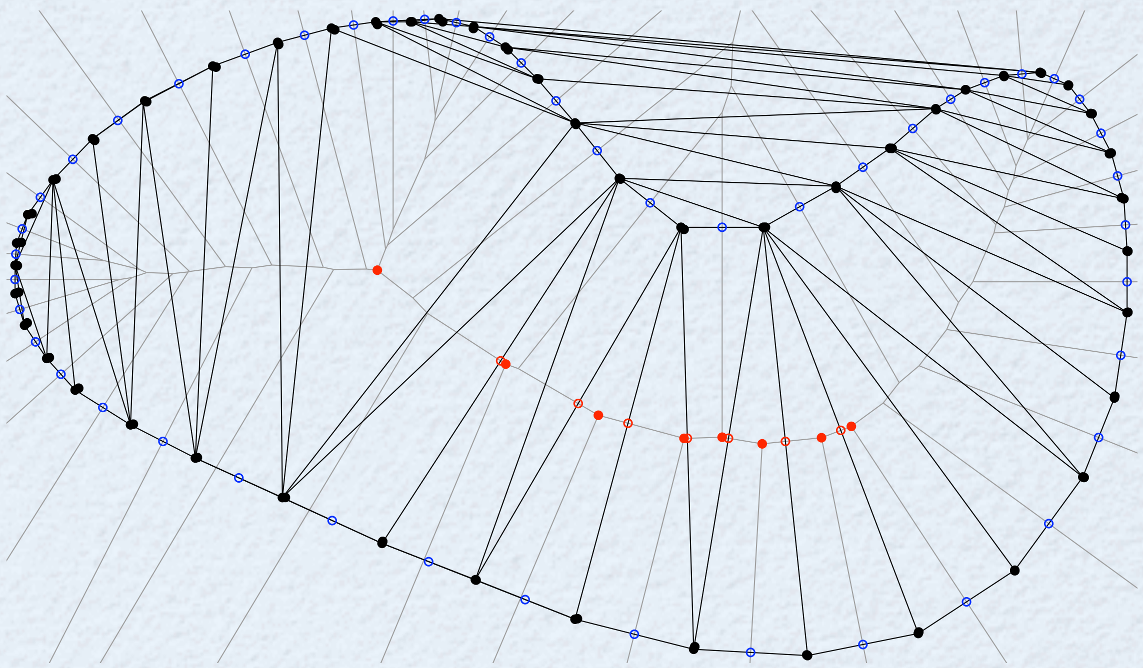

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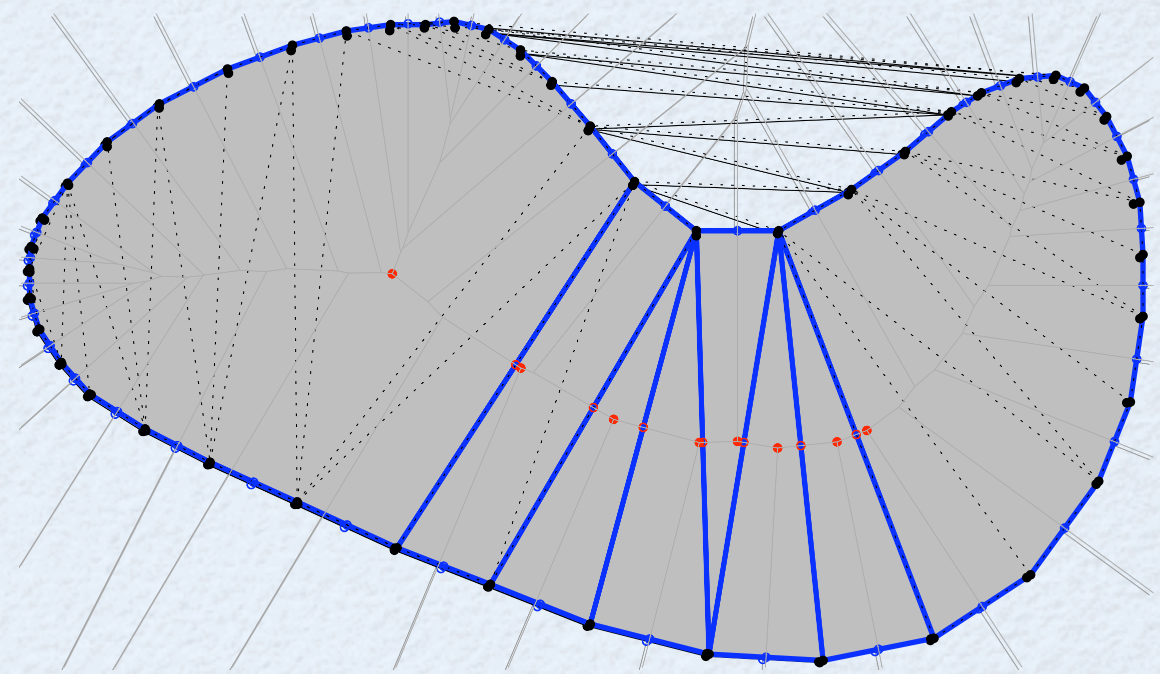

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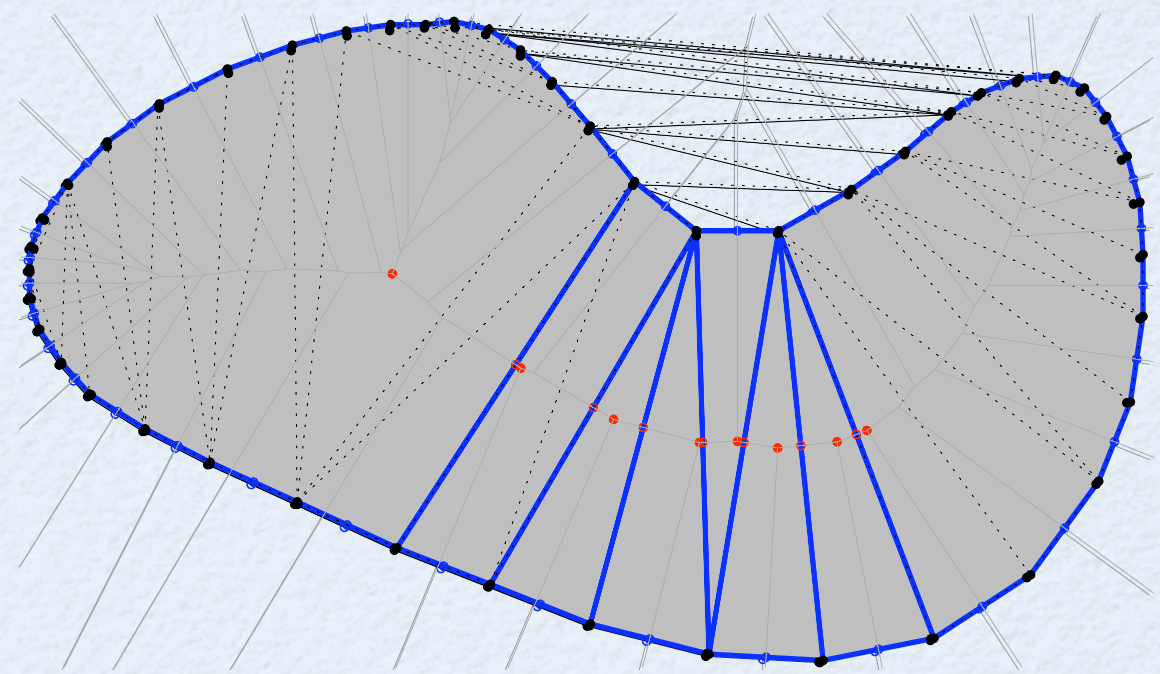

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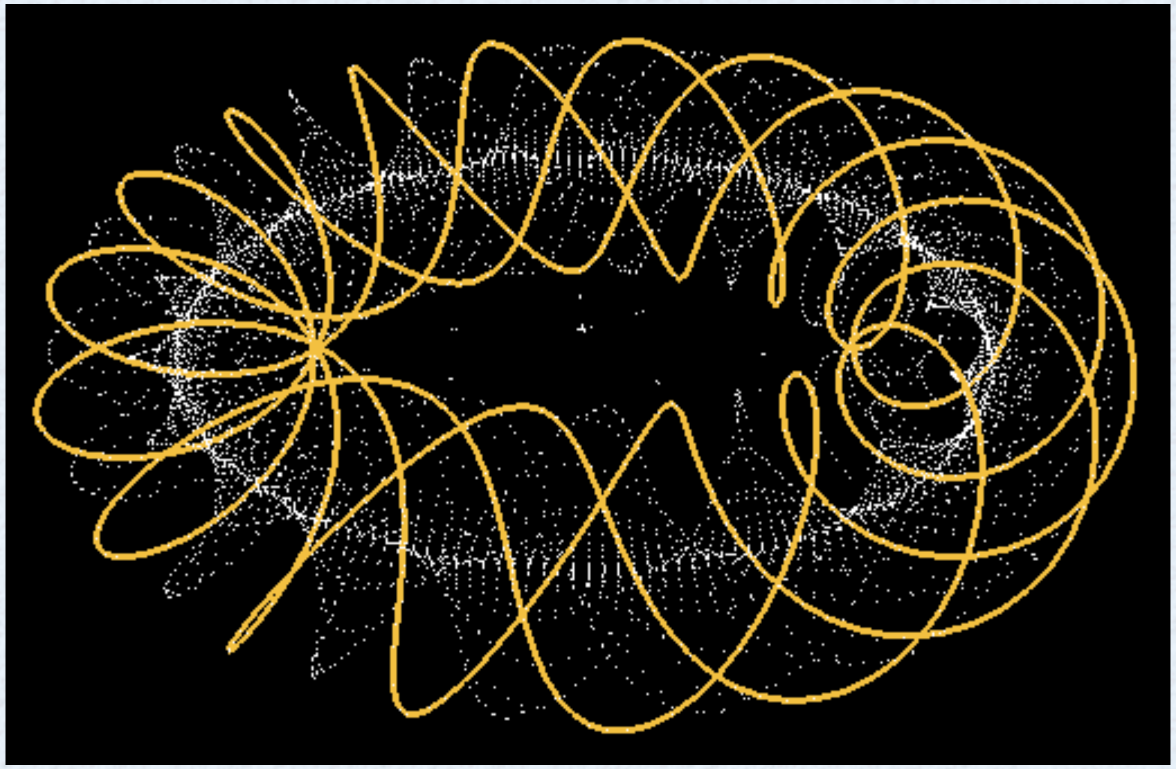
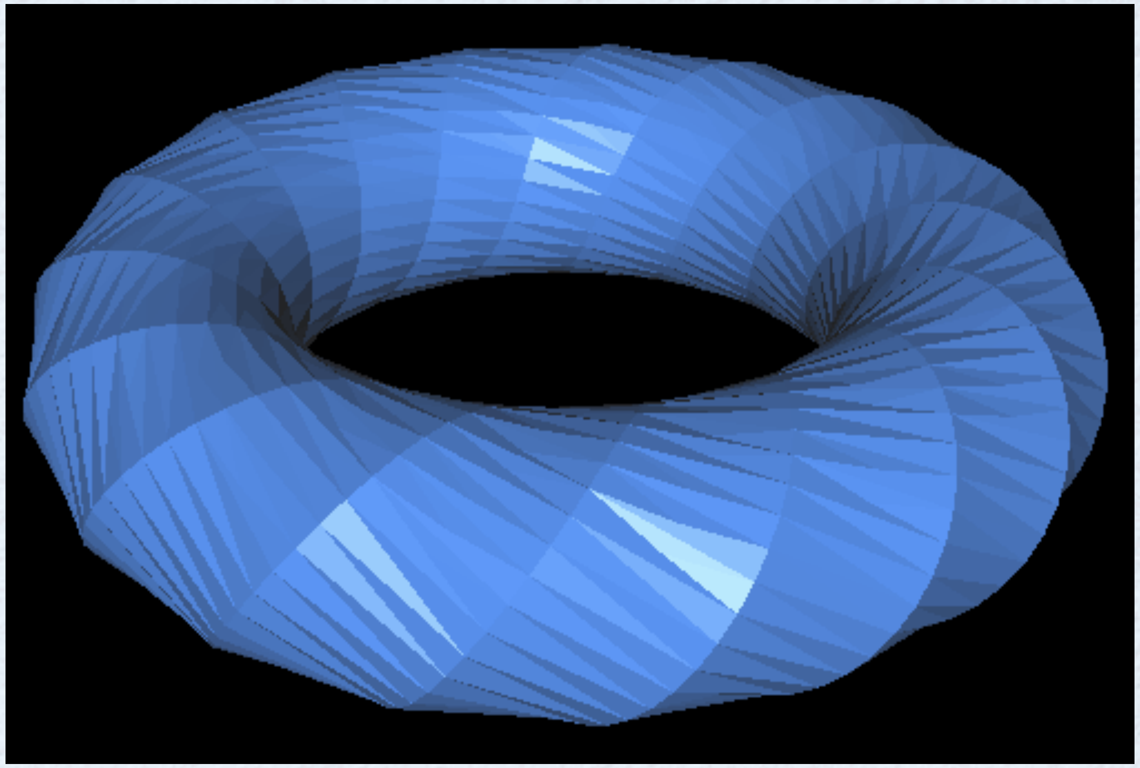
4. [S'08] Under uniform sampling, union of Sm's of shallow crit pts is homotopy equiv to the sampled manifold and that of deep ones is homotopy equiv to the complement of manifold.

Filtering the flow complex

Theorem.
Let $P \subset \mathbb{R}^n$ and h be the induced distance function. If $h(c_1) < \cdots < h(c_k)$ are critical points of h, then for any submanifold Σ of $\mathbb{R}^n$ densely sampled by P, there is a $1 < j < k$, such that:

$$\bigcup_{i=1}^{j} \mathsf{Sm}(c_i) \simeq \Sigma \quad \text{and} \quad \bigcup_{i=j+1}^{k} \mathsf{Sm}(c_i) \simeq \Sigma^c$$





Other results using distance induced flows

[Lieutier'04] The medial axis of any bounded open subset of $\mathbb{R}^n$ is homotopy equivalent to it.

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