I/O-Efficient Algorithms for Computing Contours on a Terrain

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University of Toronto

joint work with:

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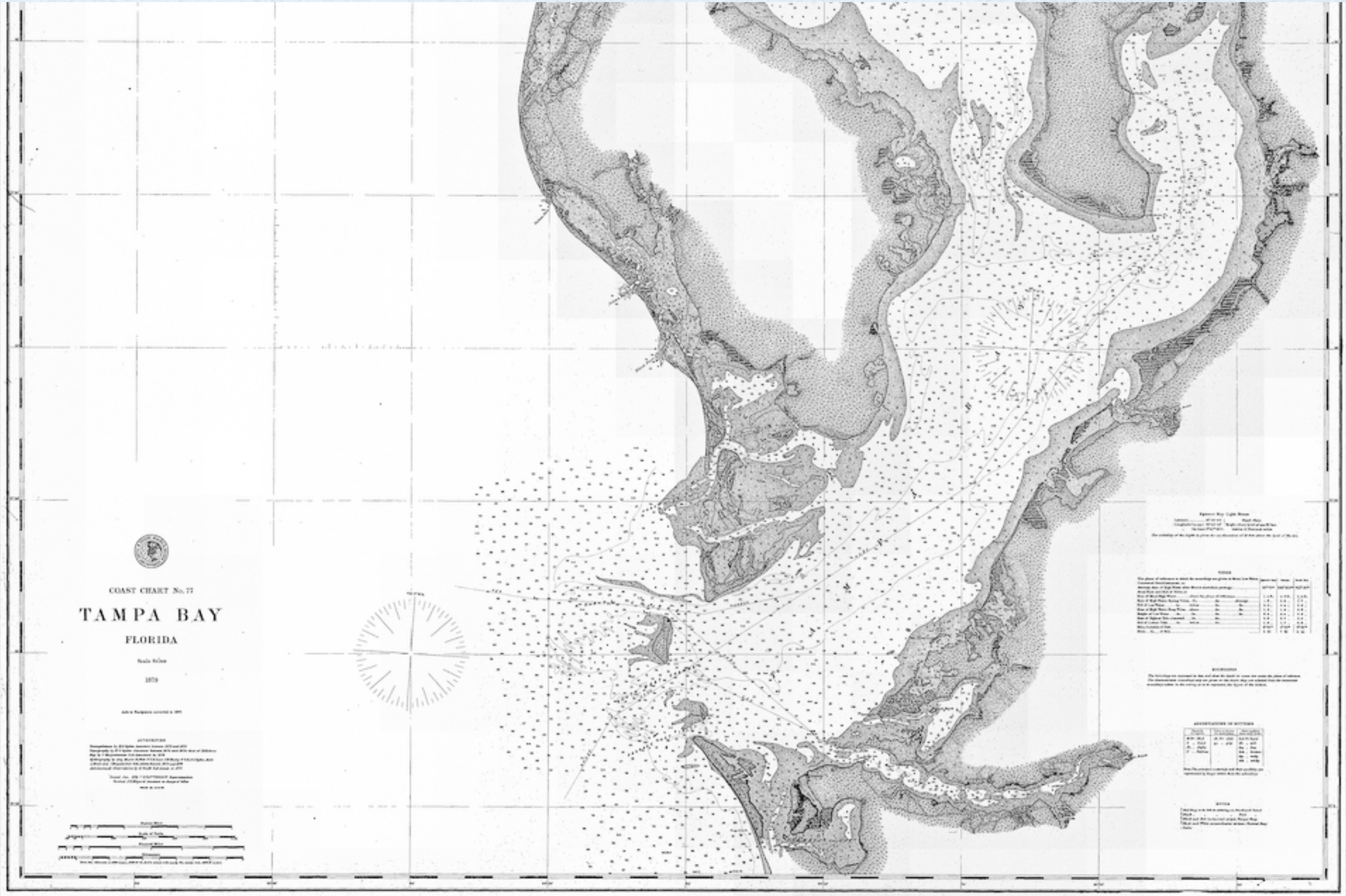
Duke University

Lars Arge
MADALGO

Thomas Mølhave
MADALGO

Pretty Old Stuff!



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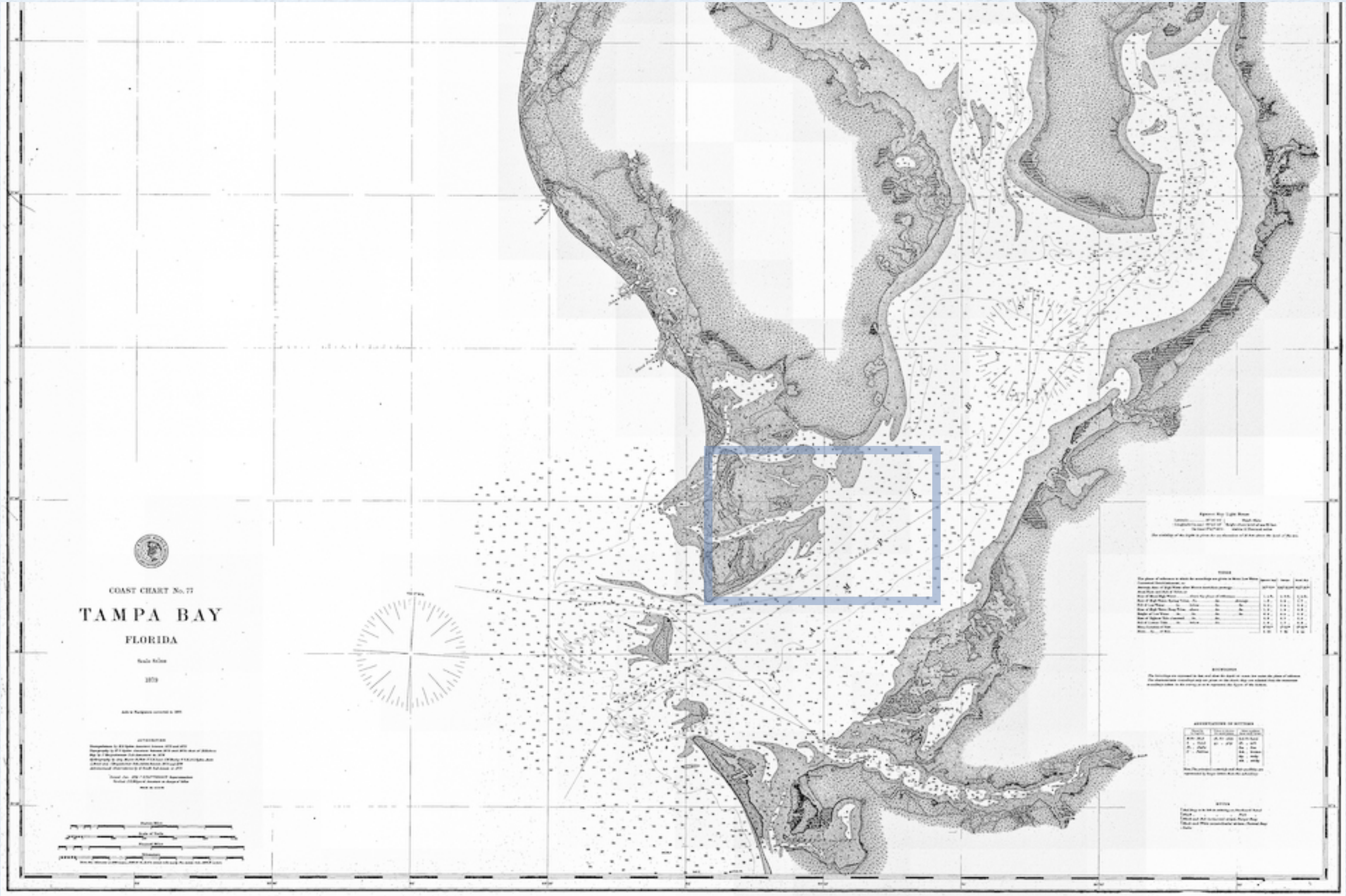

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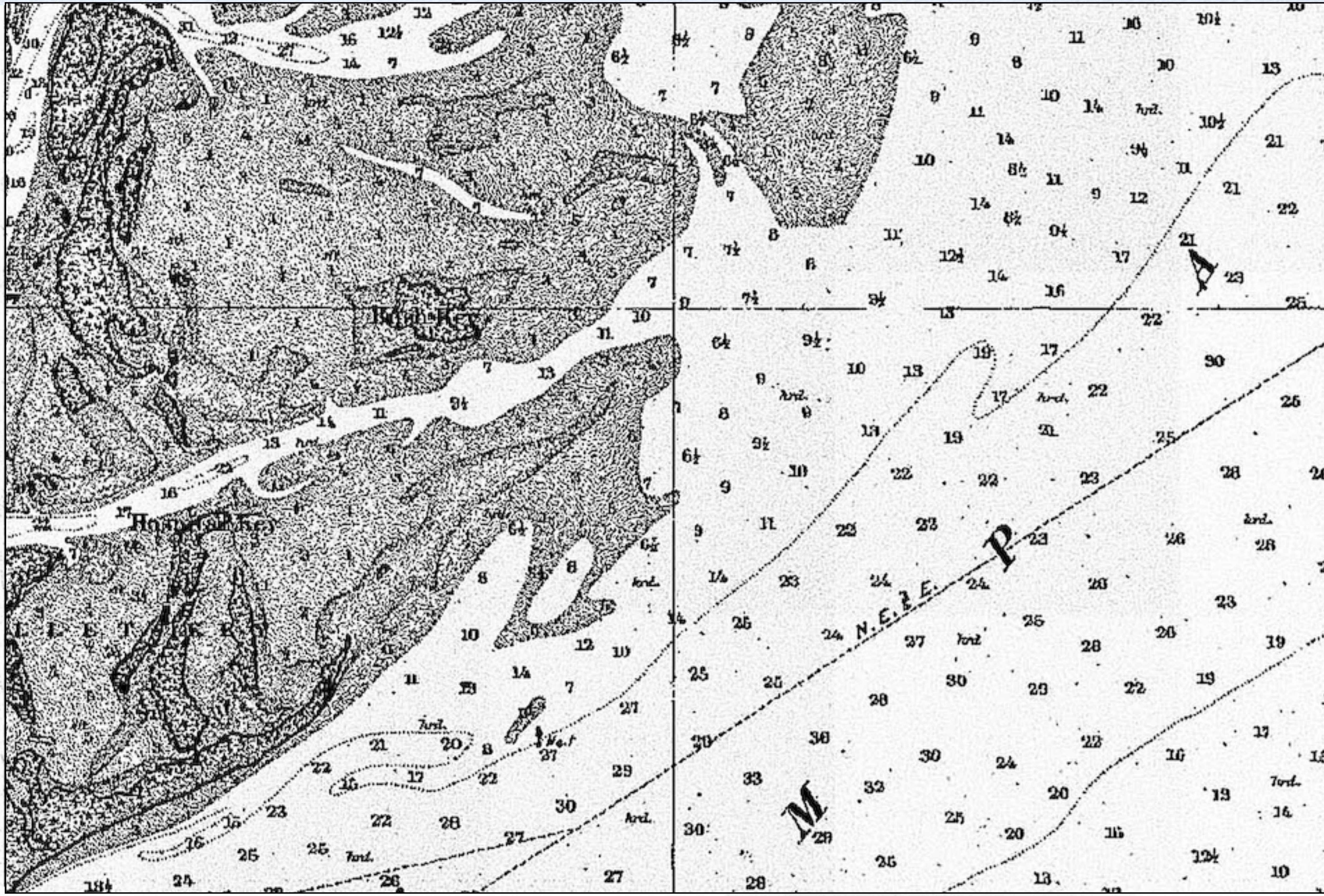
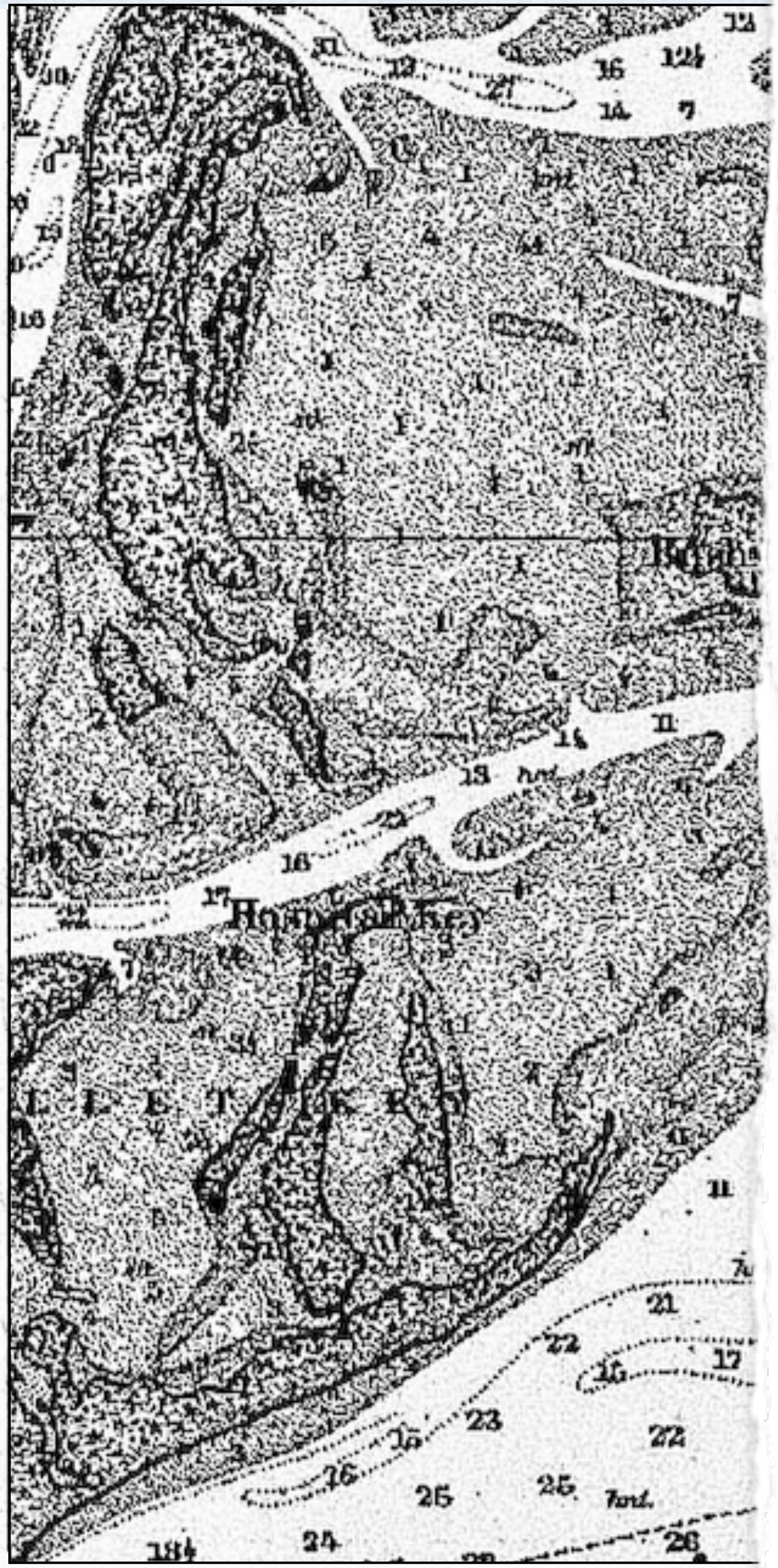


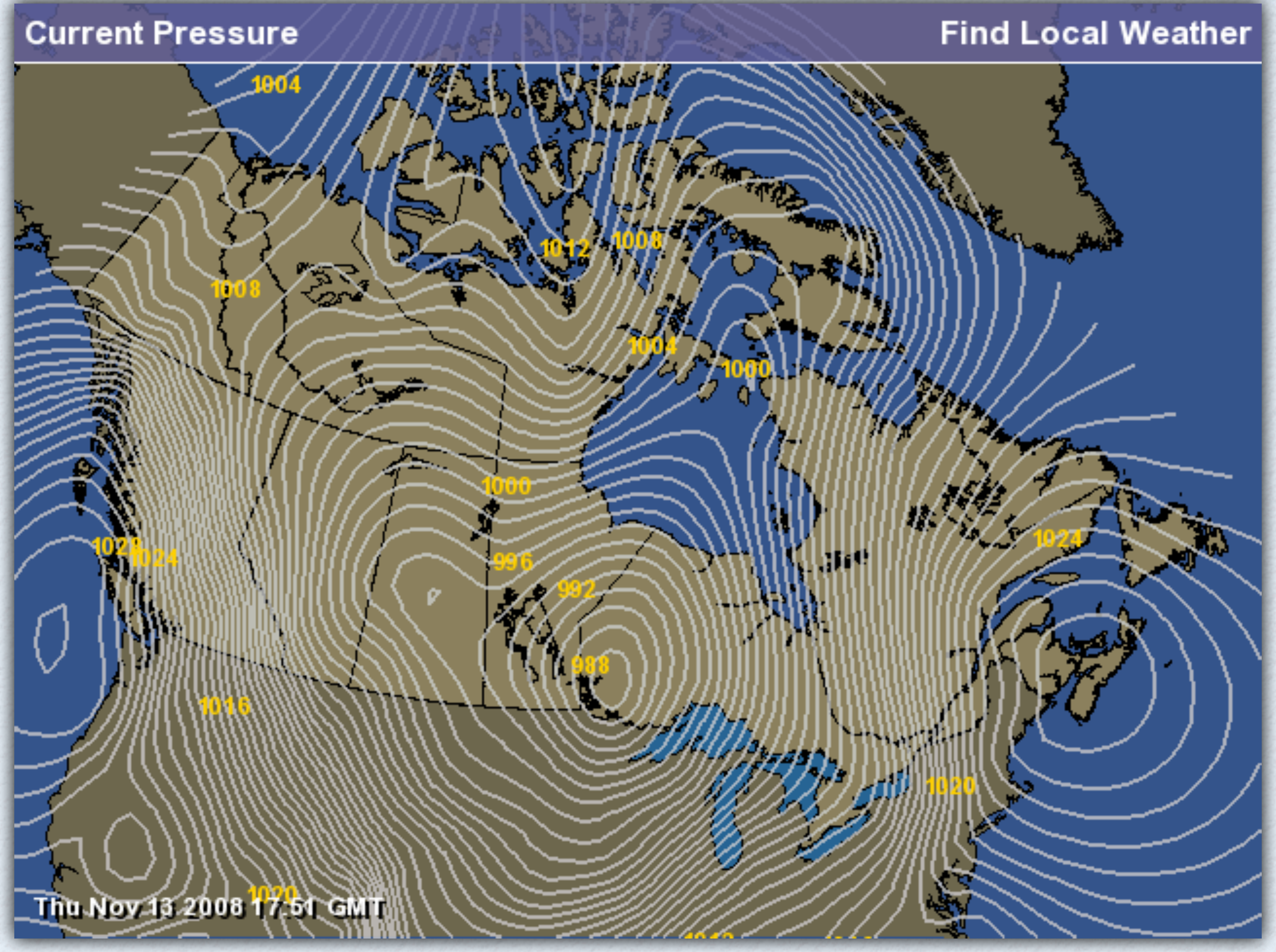


XXXIII. *An Account of the Calculations made from the Survey and Measures taken at* Schehallien, *in order to ascertain the mean Density of the Earth.* *By* Charles Hutton, *Esq. F. R. S.*

This circumstance at first gave me much trouble and dissatisfaction, till I fell upon the following method by which the defect was in a great measure supplied, and by which I was enabled to proceed in the estimation of the altitudes both with much expedition and a considerable degree of accuracy. This method was the connecting together by a faint line all the points which were of the same relative altitude: by so doing, I obtained a great number of irregular polygons lying within, and at some distance from, one another, and bearing a considerable degree of resemblance to each other: these polygons were the figures of so many level or horizontal sections of the hills, the relative altitudes of all the parts of them being known; and as every base or little space had

Philosophical Transactions of Royal Society of London, 1779

Not Just for Altitude



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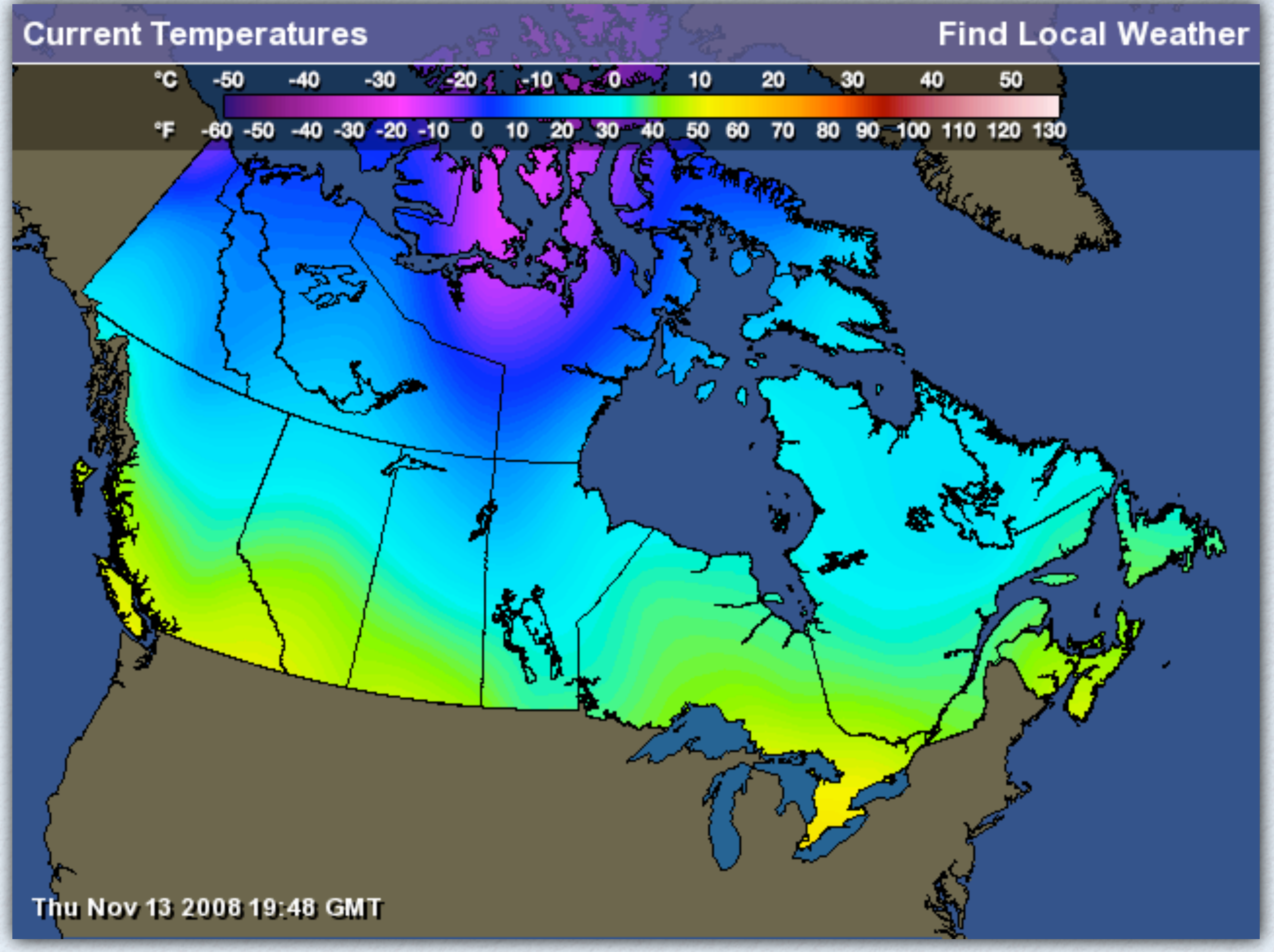
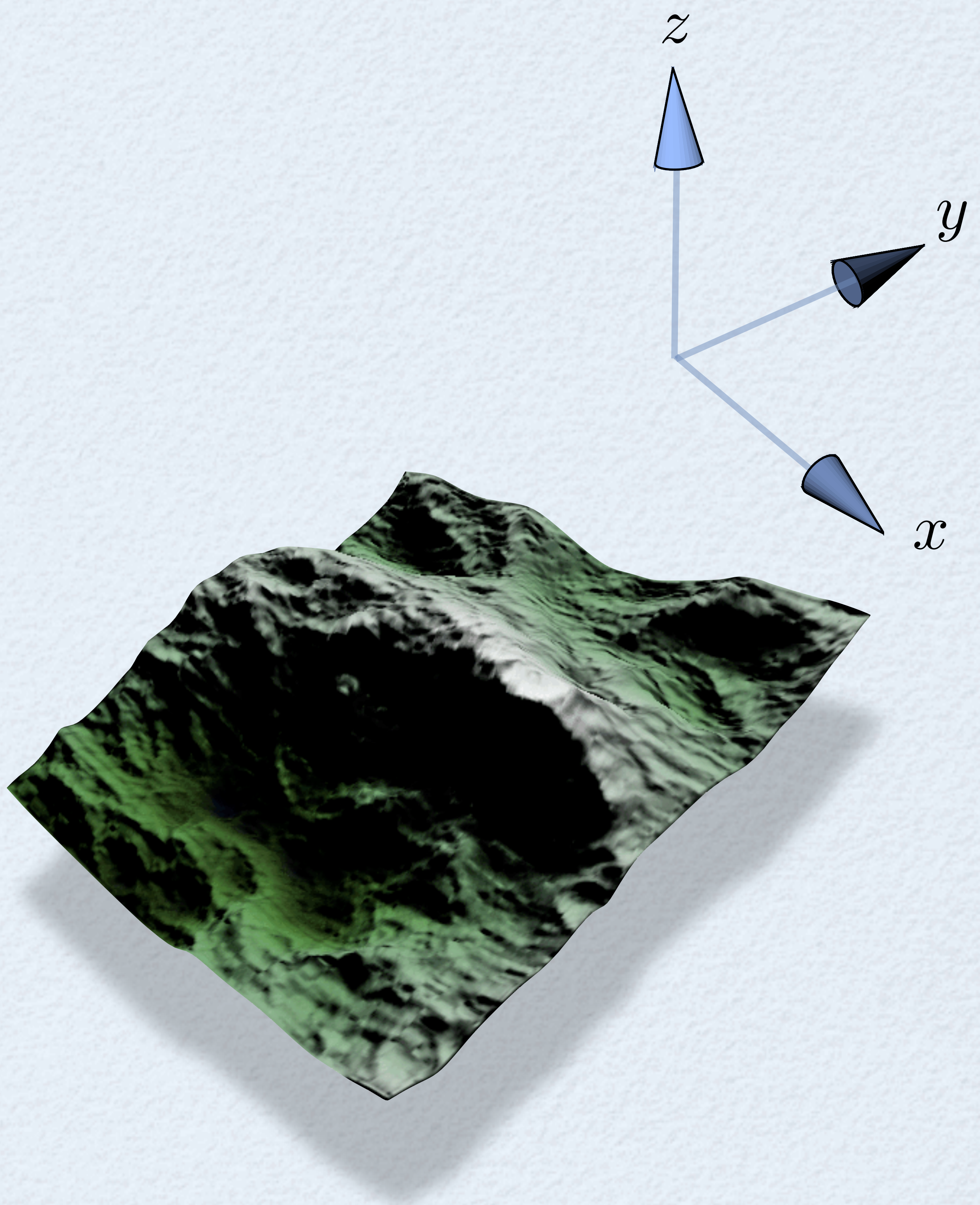


Terrains

A terrain is the graph of a continuous bivariate function.



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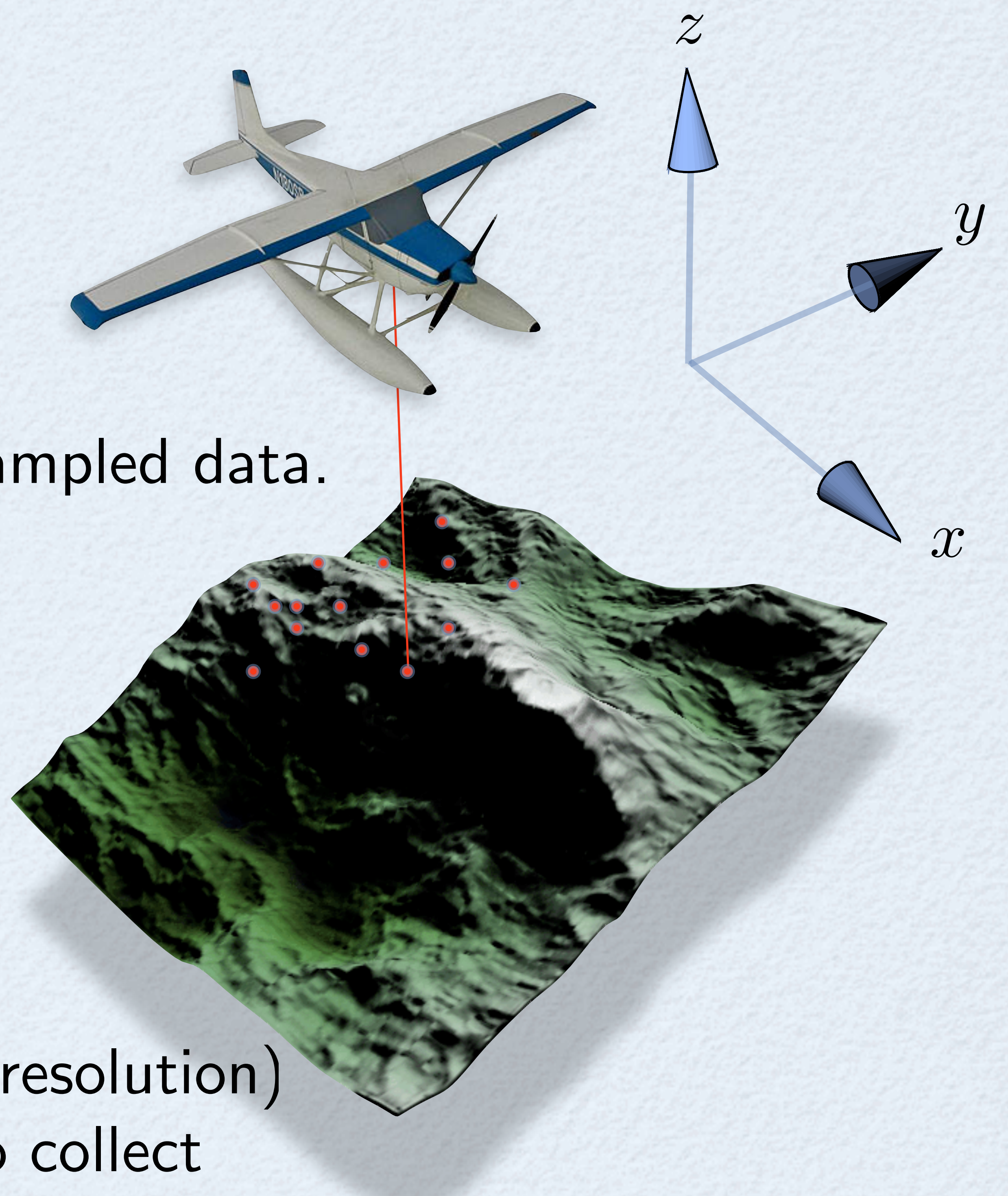


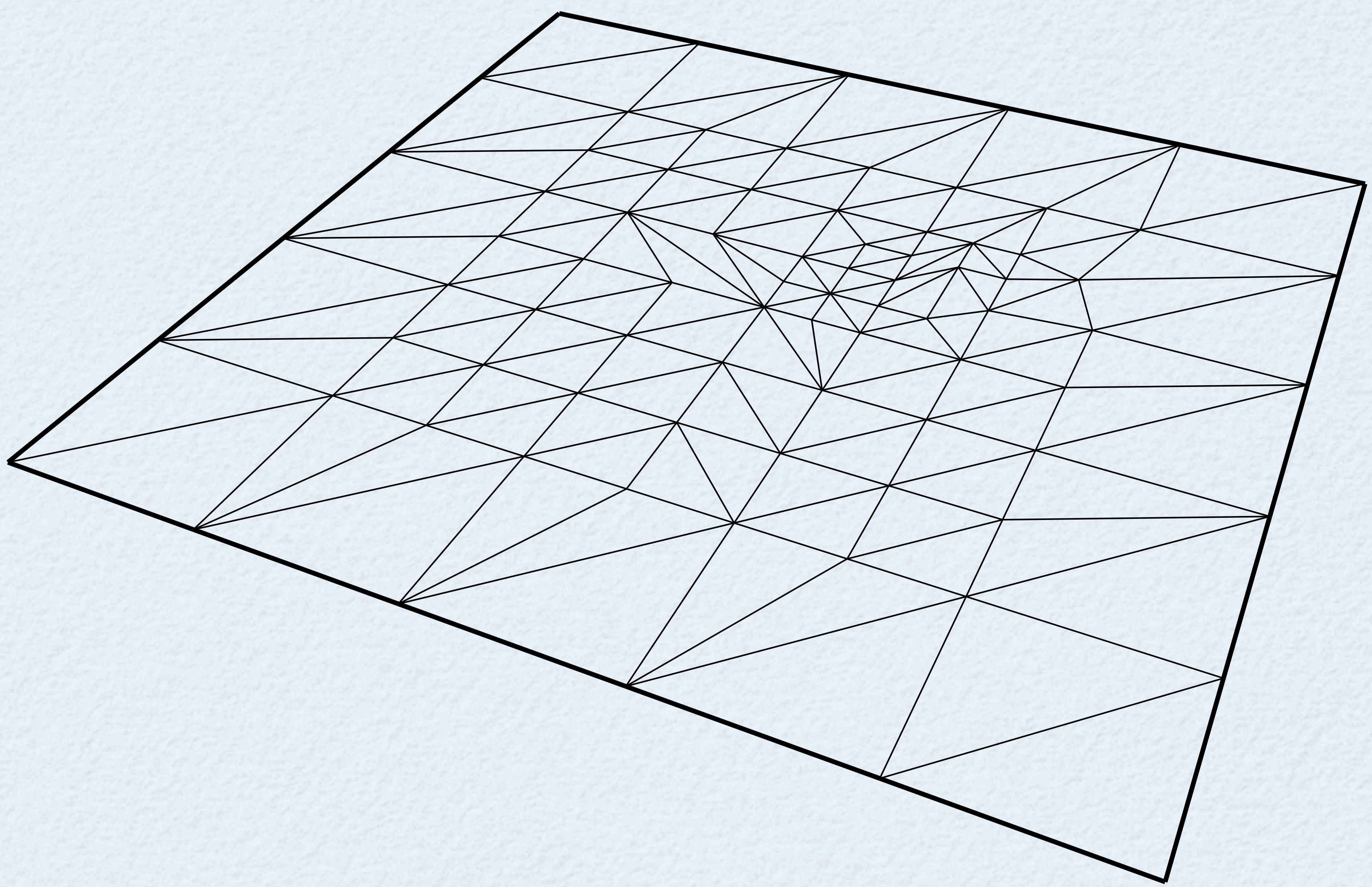
In practice terrains often interpolated sampled data.

LIDAR (Light Detection and Ranging)

- Massive (irregular) point sets (1-10m resolution)
- Becoming relatively cheap and easy to collect
- Appalachian mountains between 50GB to 5TB

Representation: Triangulated Irregular Network (TIN)

Given a plane triangulation $\mathbb{M}$ with a height $h(v)$ for each vertex v, one can linearly interpolate h in the interior of every face of $\mathbb{M}$.



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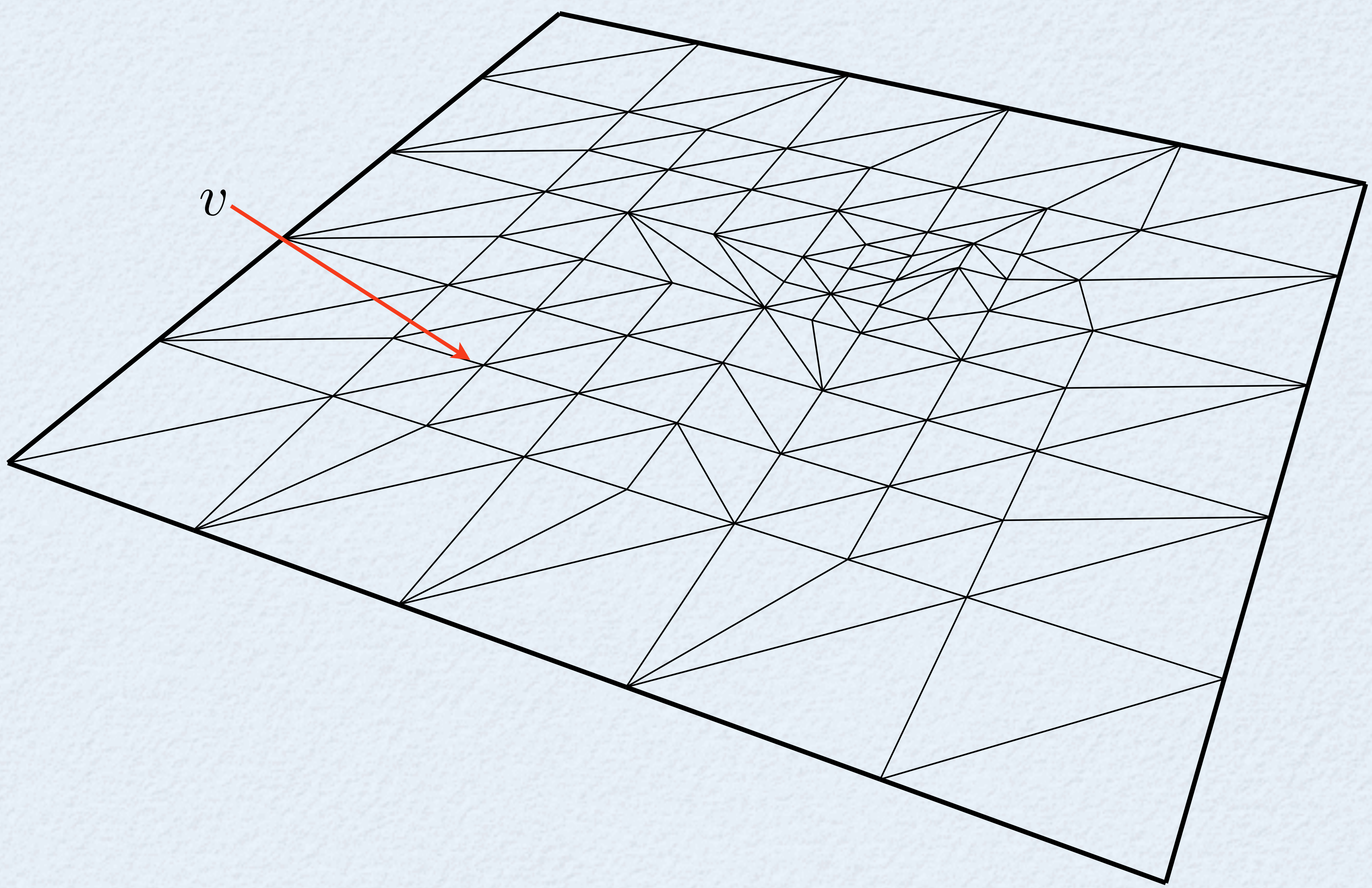

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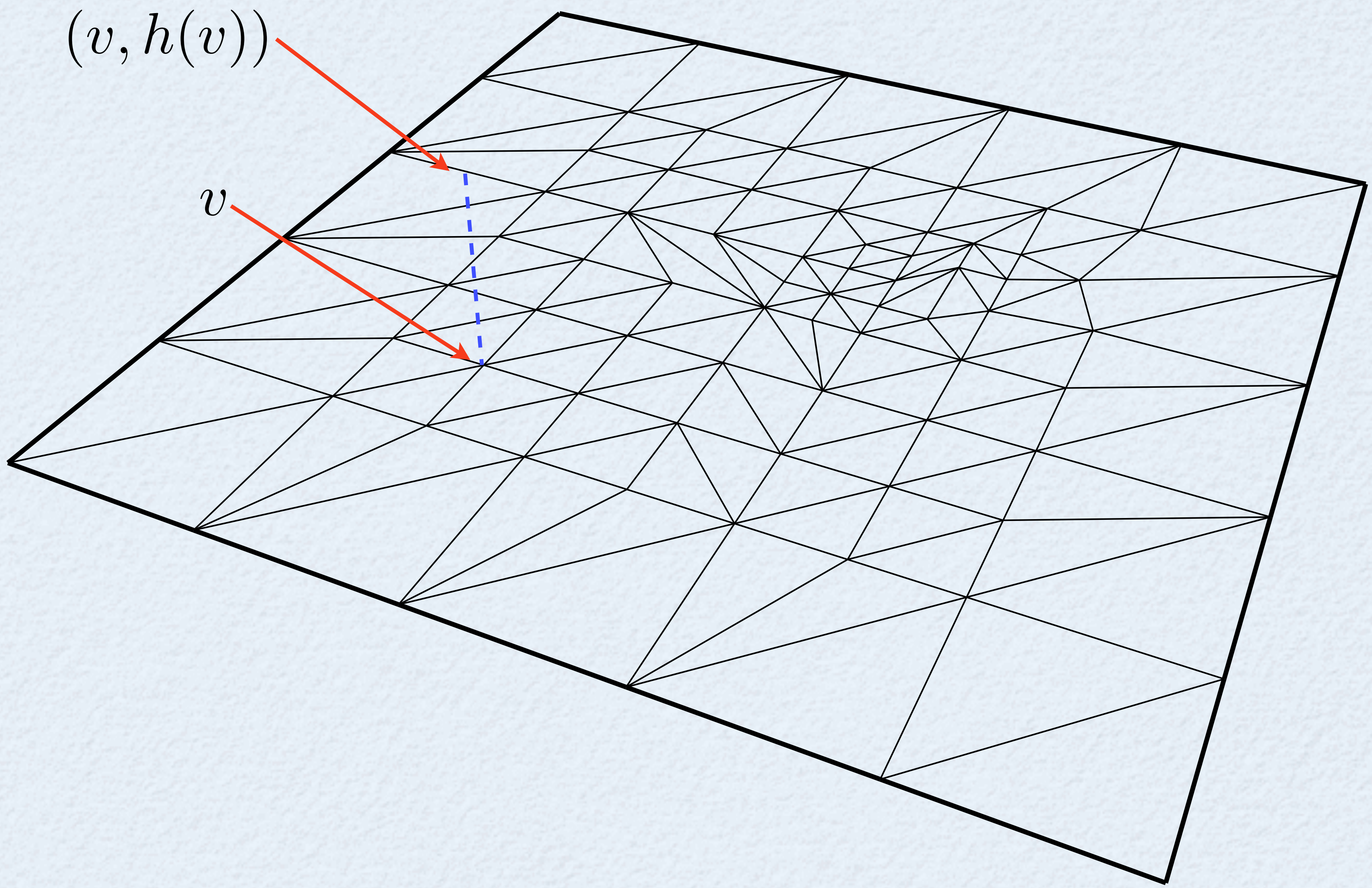

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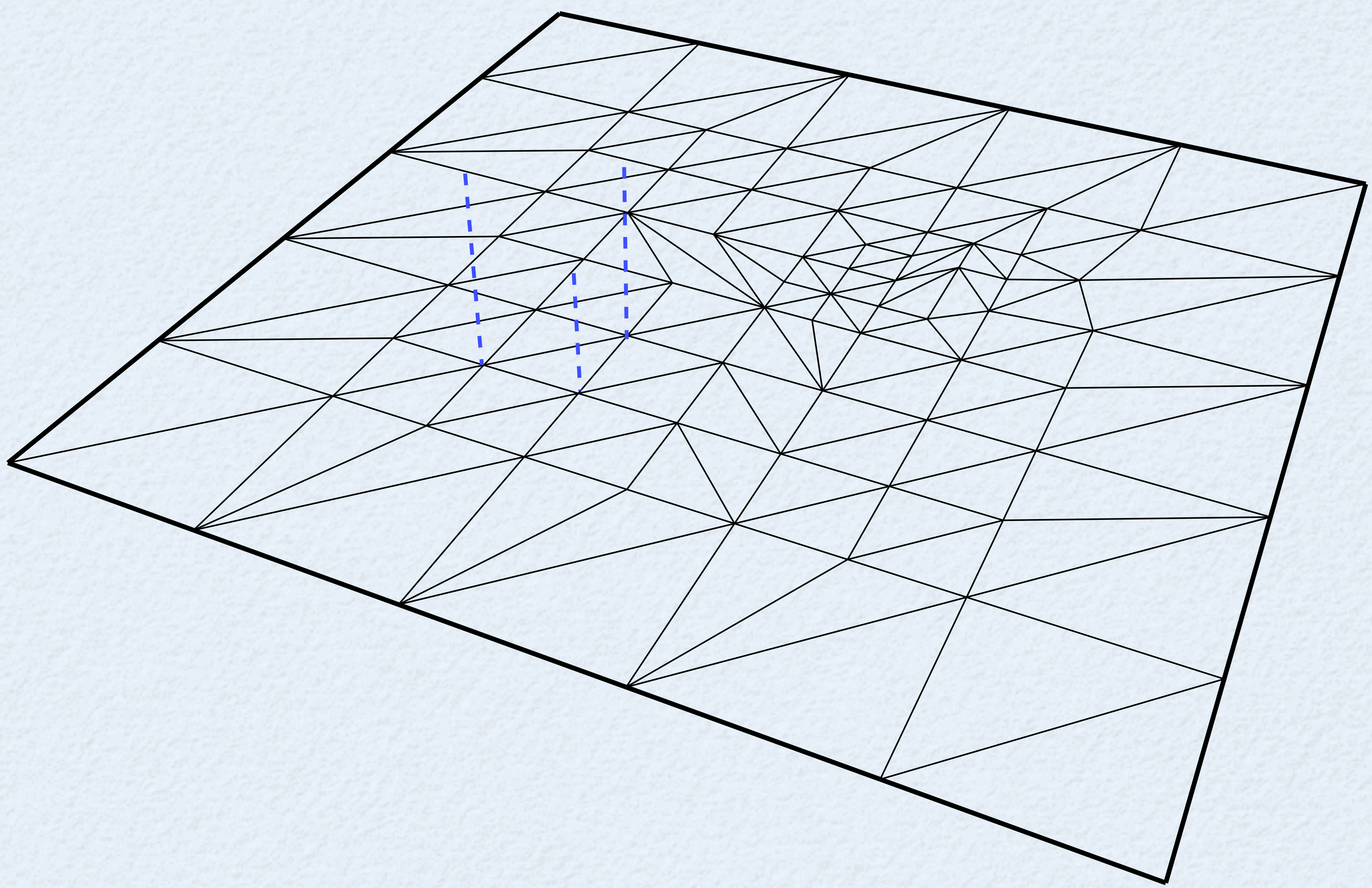

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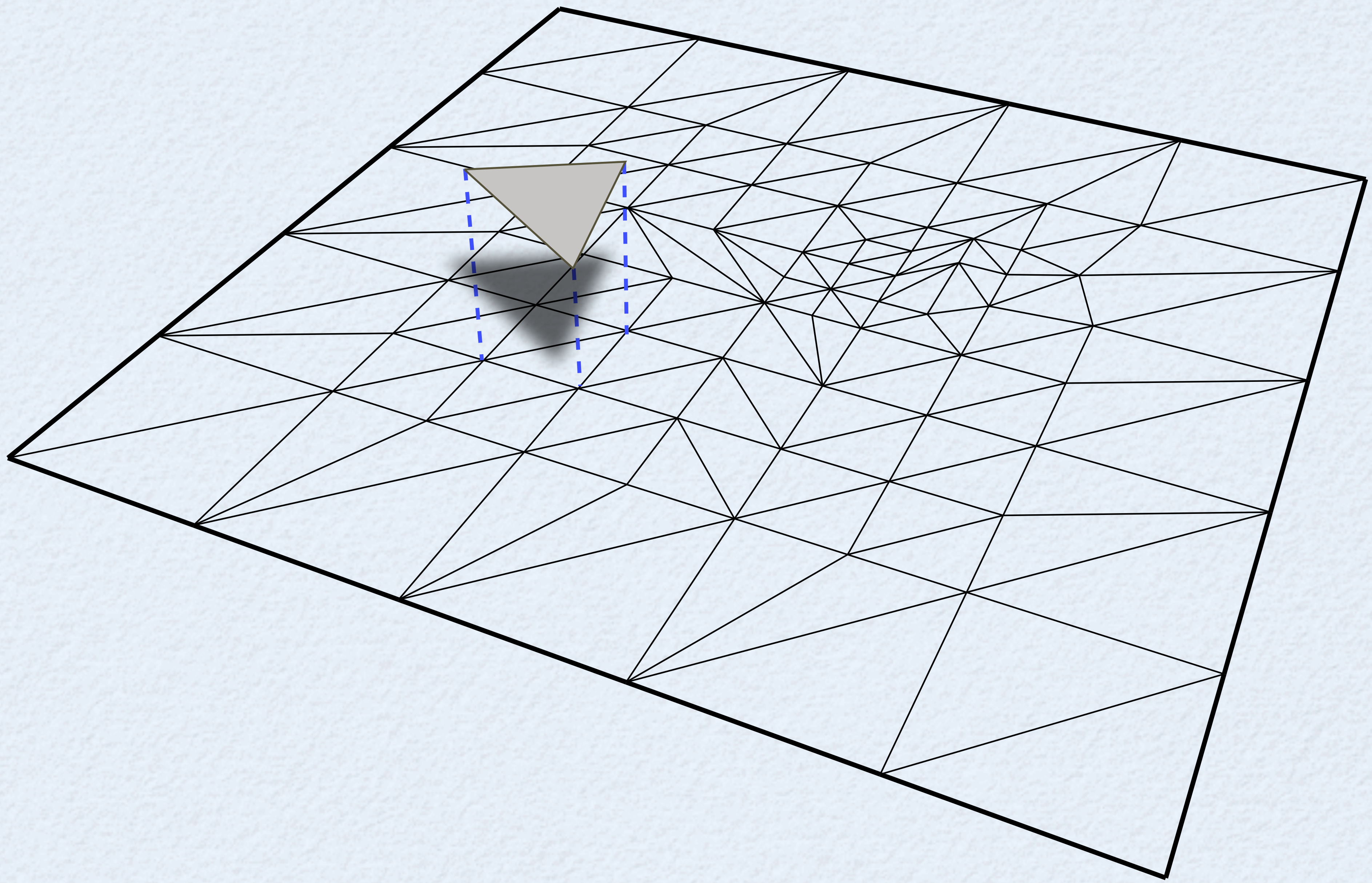

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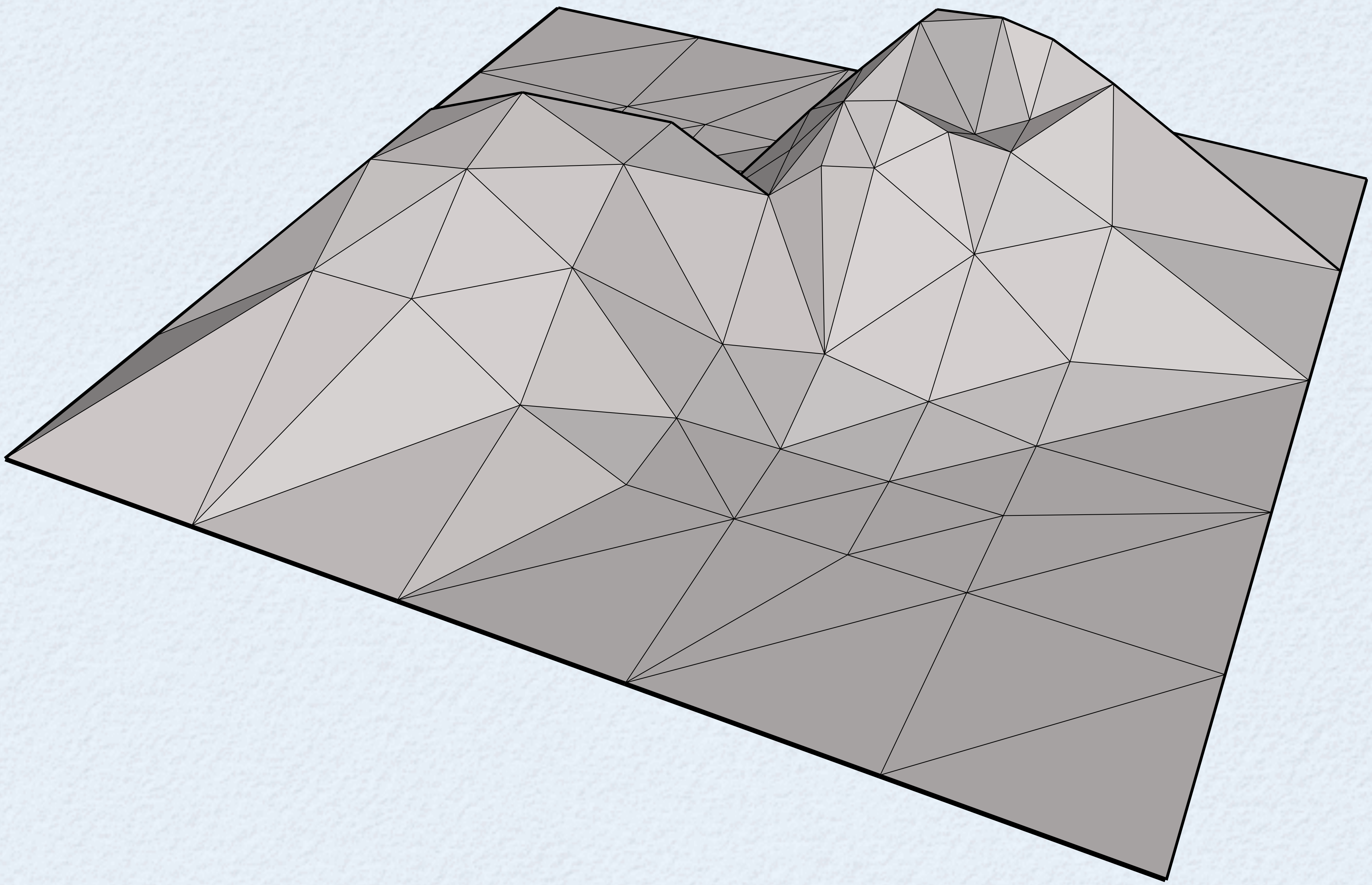

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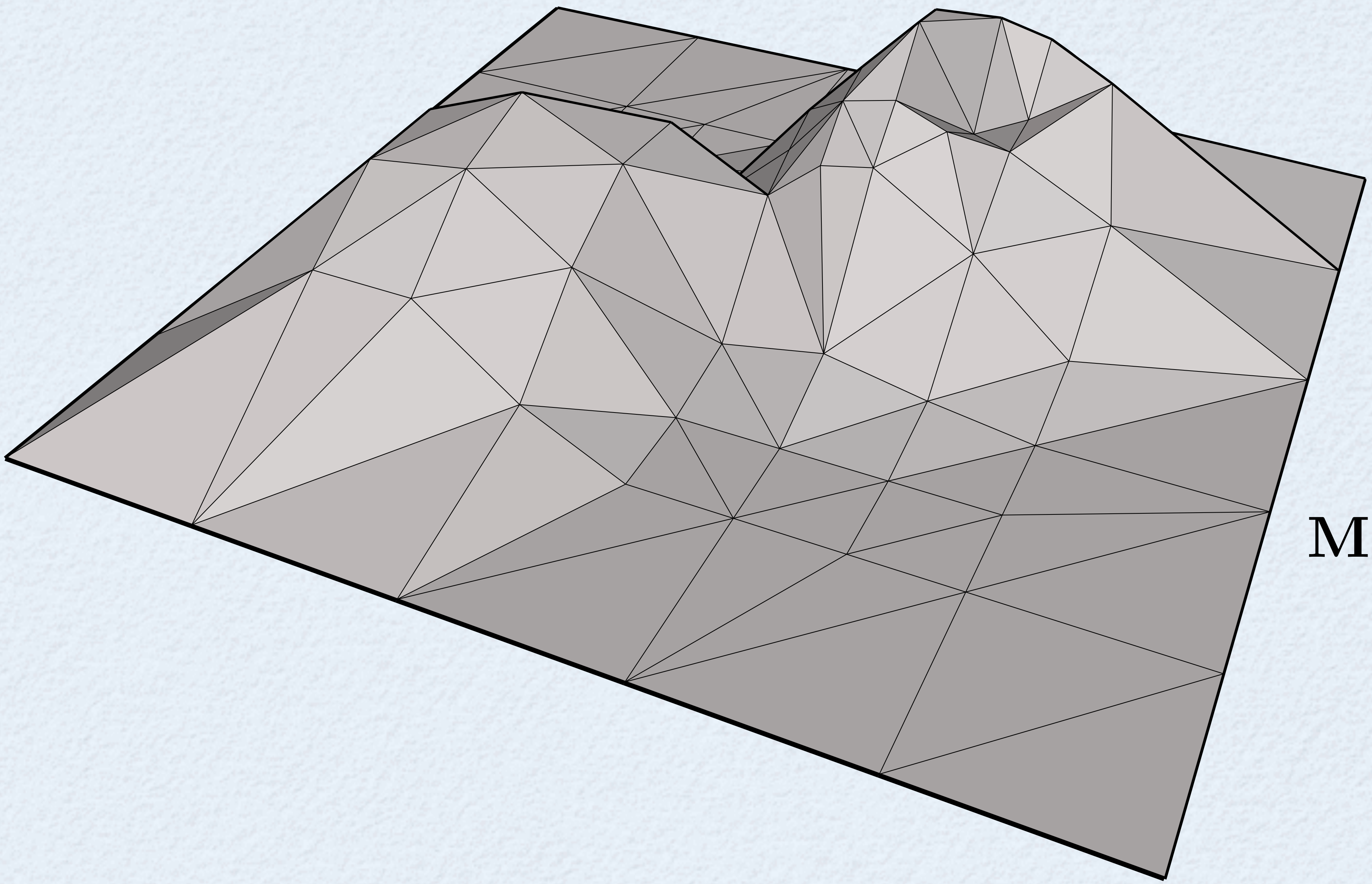
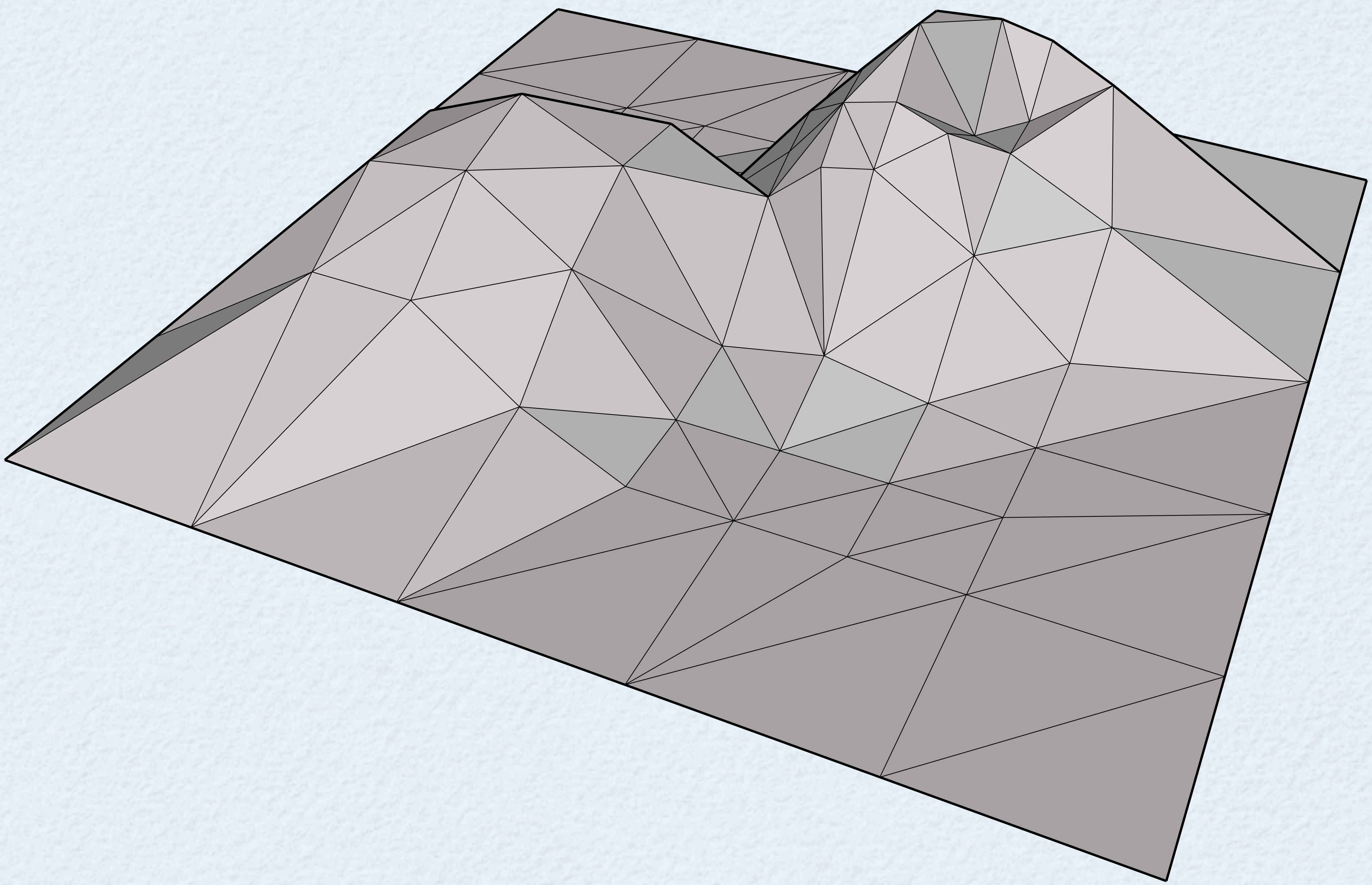


$\mathbf{M} = (\mathbb{M}, h)$

Level Sets, Contours, and Contour Maps

The level-set $\mathbb{M}_\ell$ at height ℓ is $h^{-1}(\ell)$.



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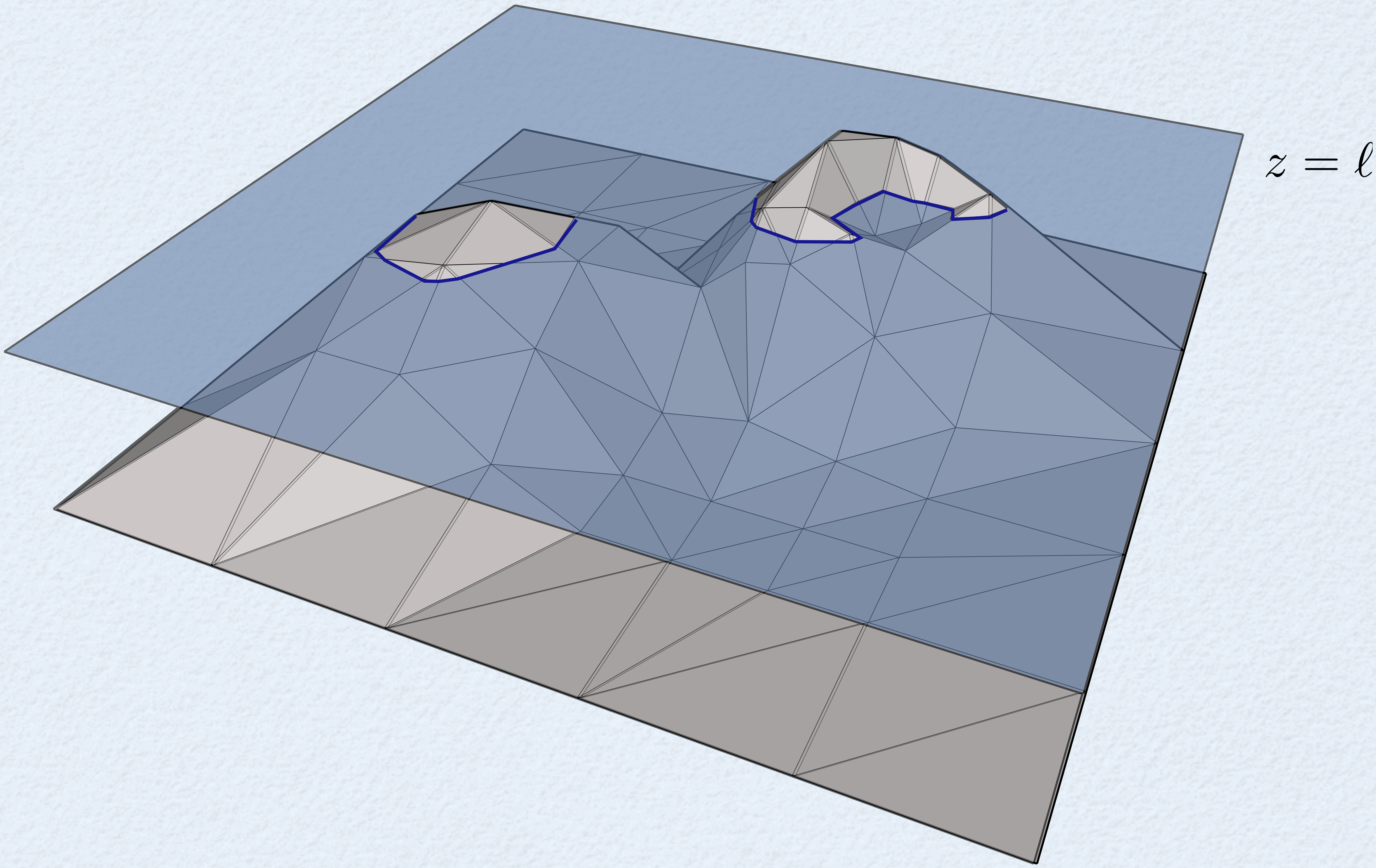

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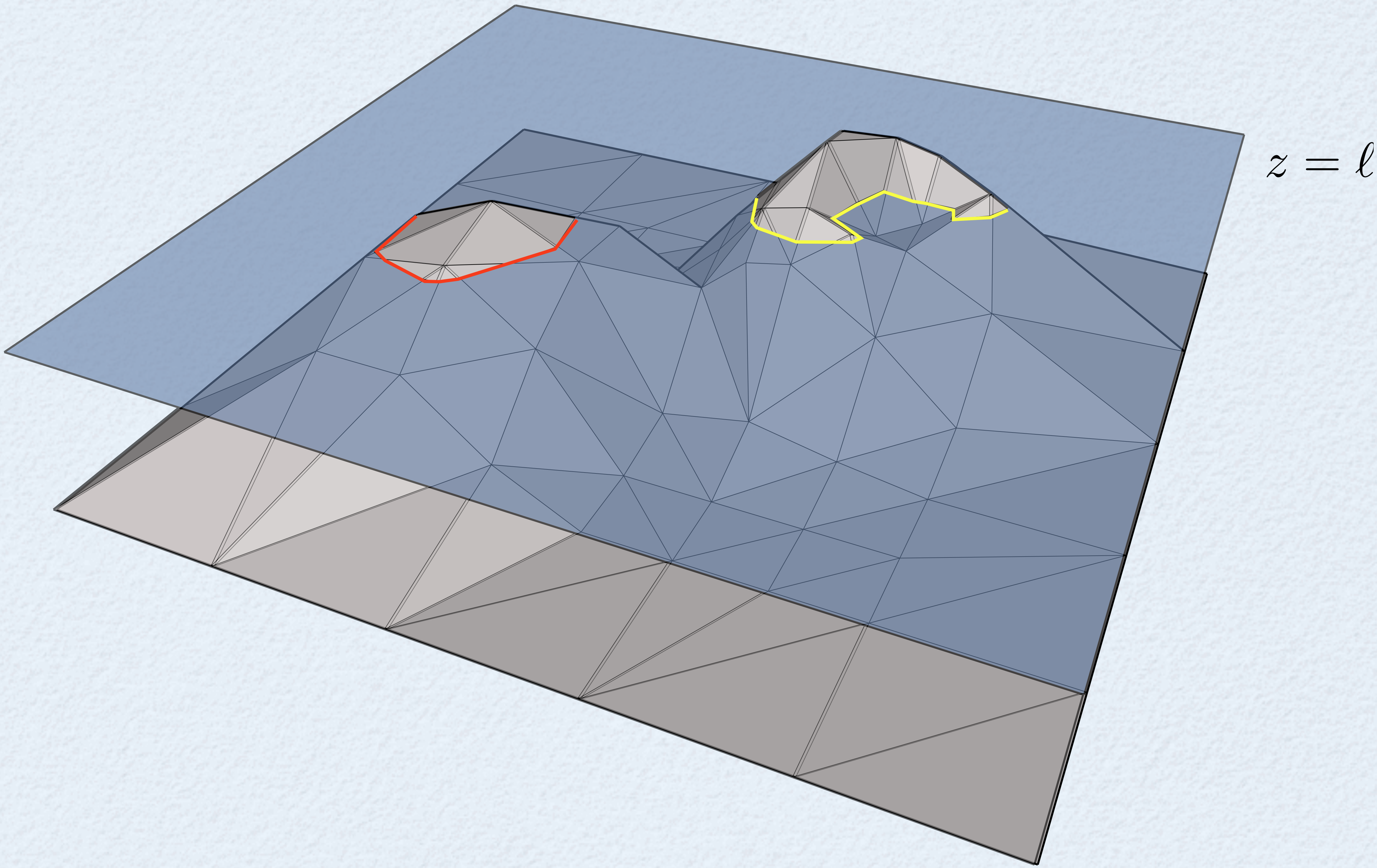

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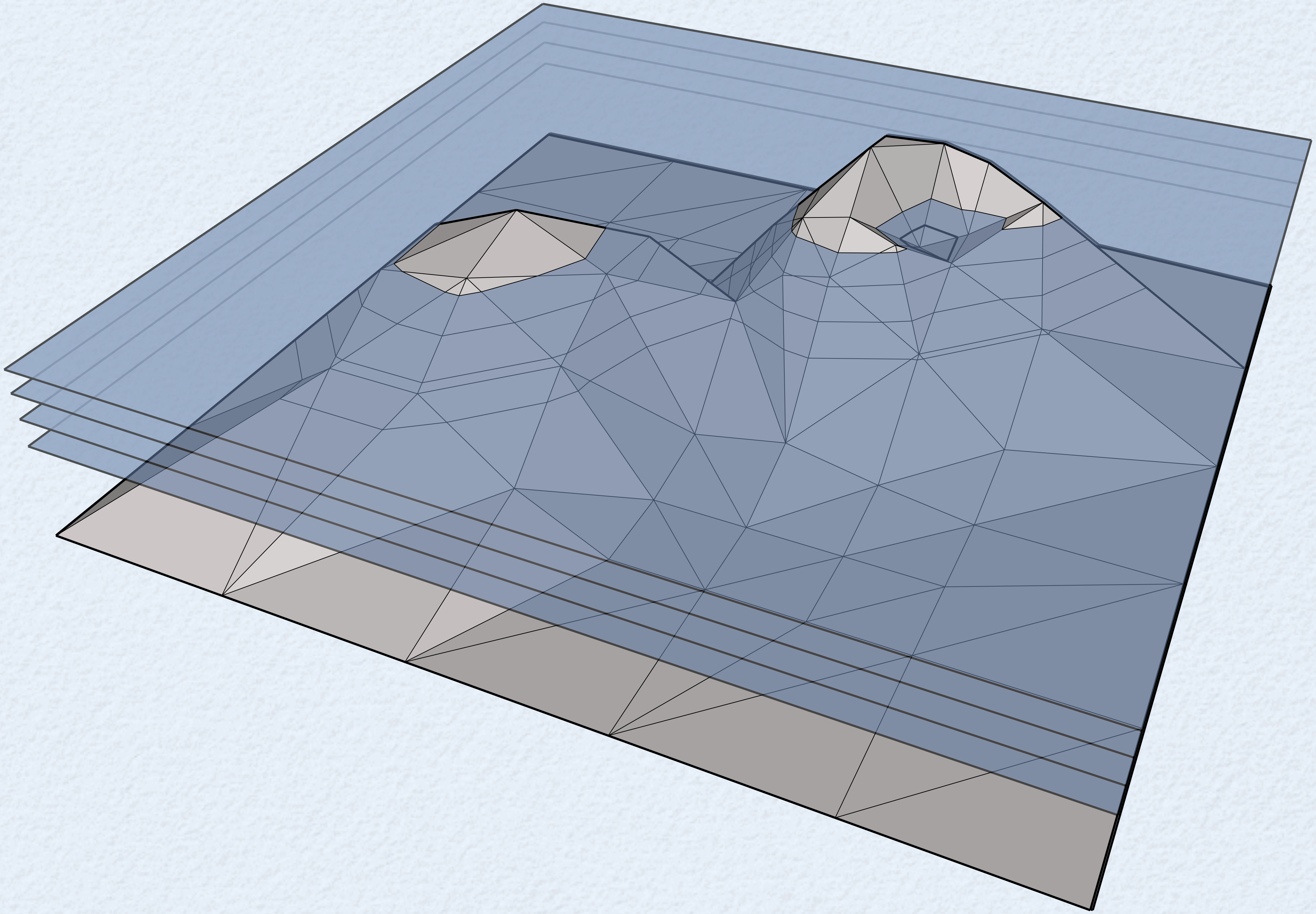

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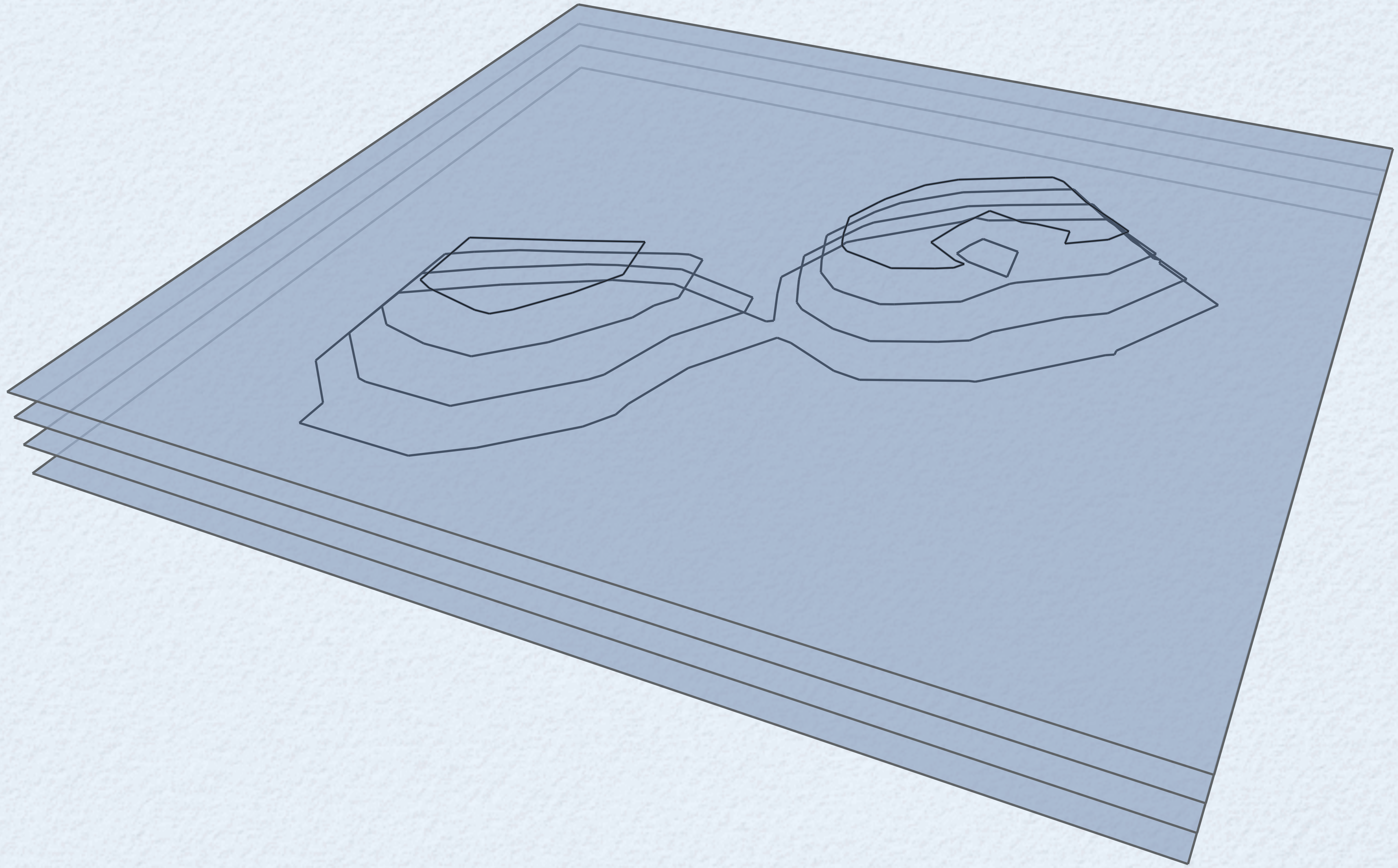

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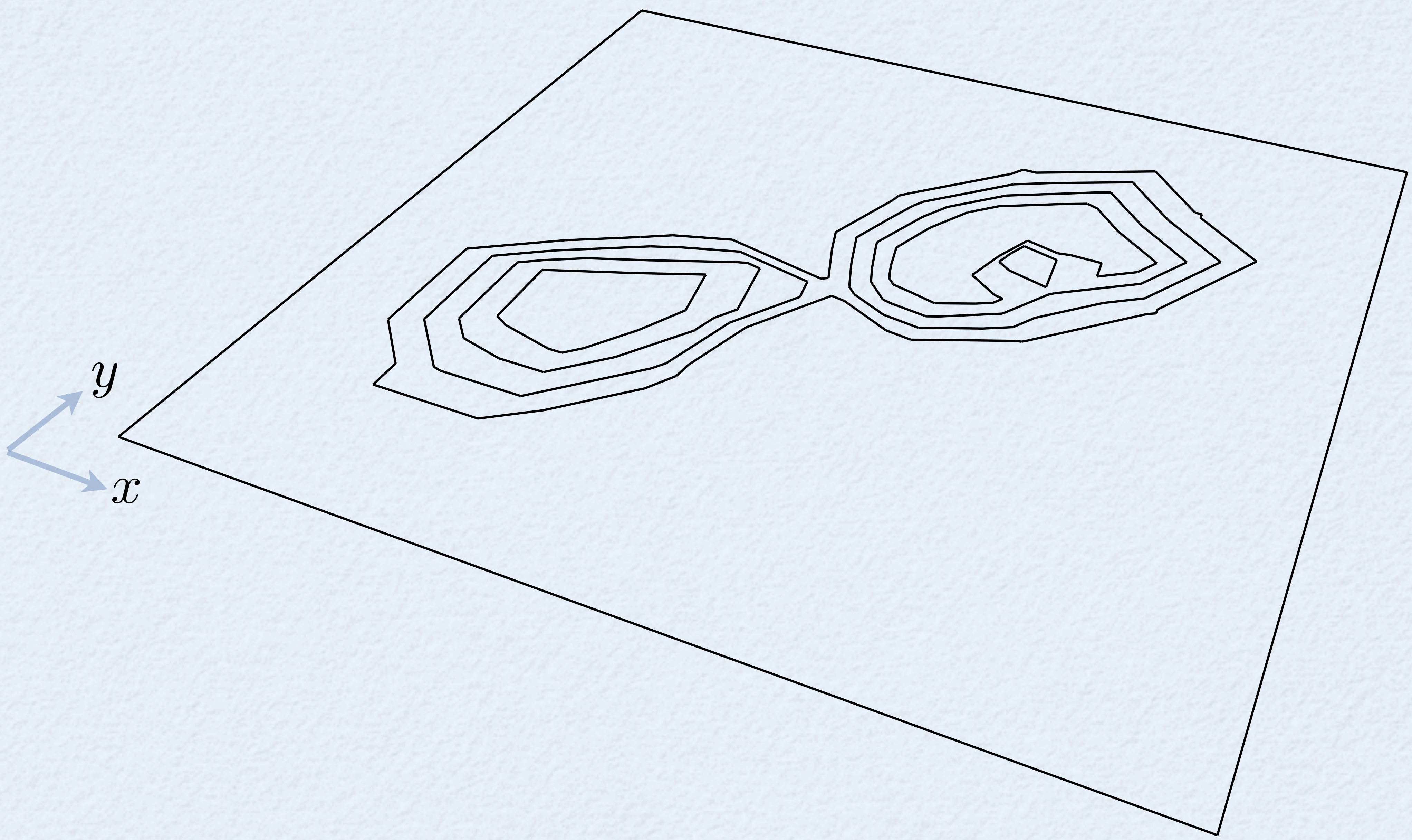
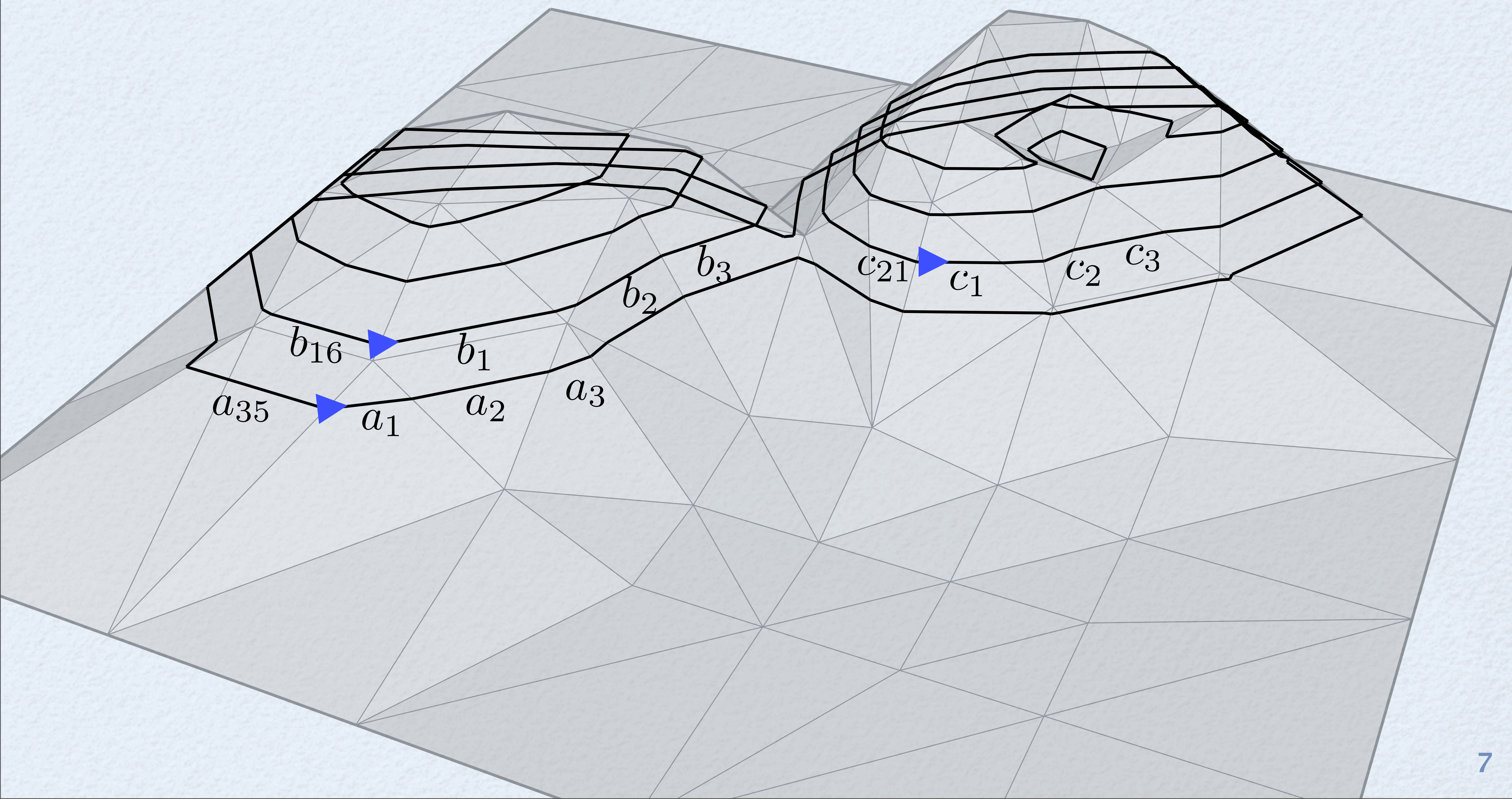


Computing Contour Maps

Given a set of levels $L = \{\ell_1, \ldots, \ell_k\}$, compute the contour map $h^{-1}(L)$ such that each contour is reported separately and in sorted (circular) order.



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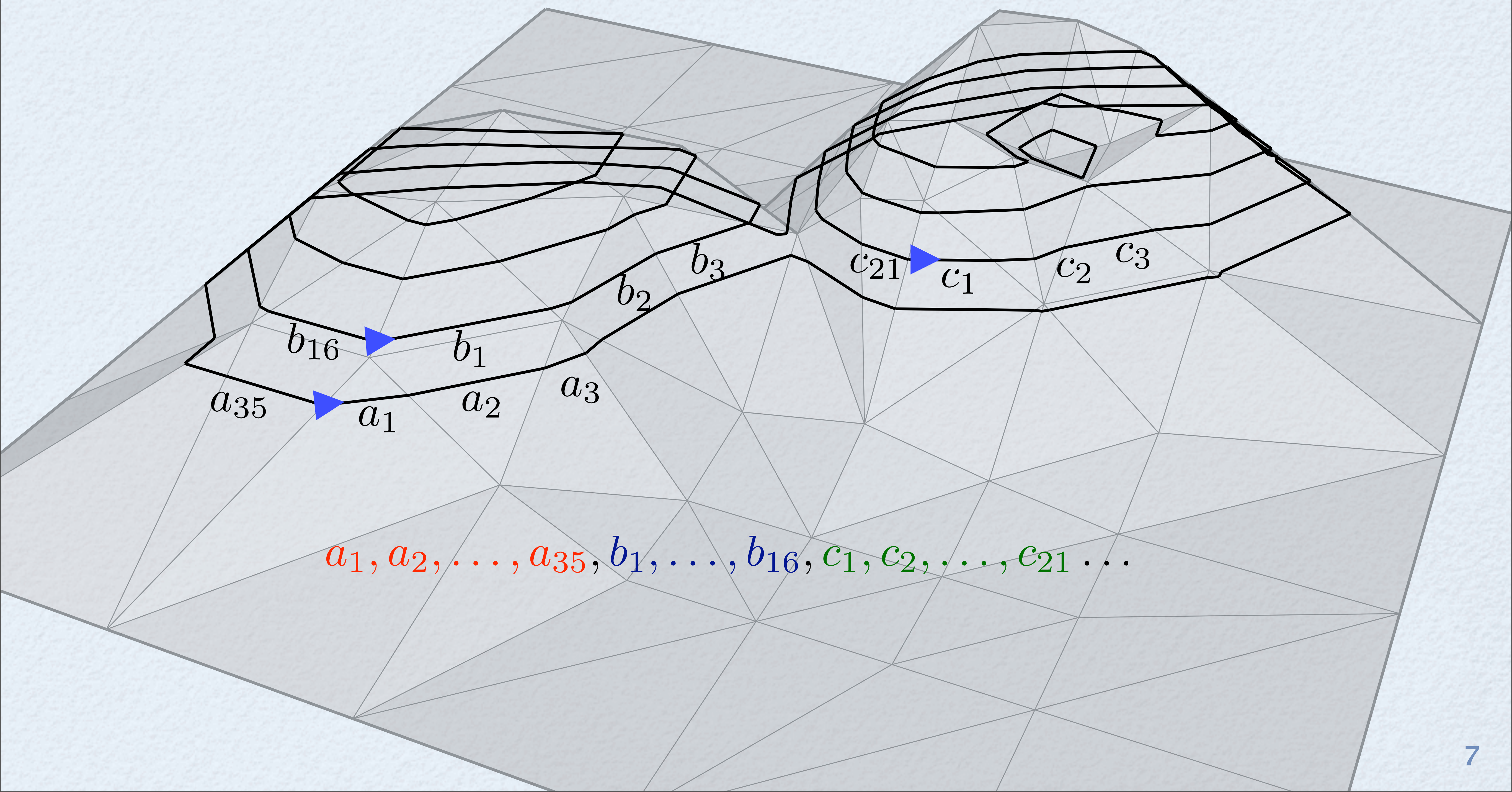

$a_1, a_2, \ldots, a_{35}, b_1, \ldots, b_{16}, c_1, c_2, \ldots, c_{21} \ldots$

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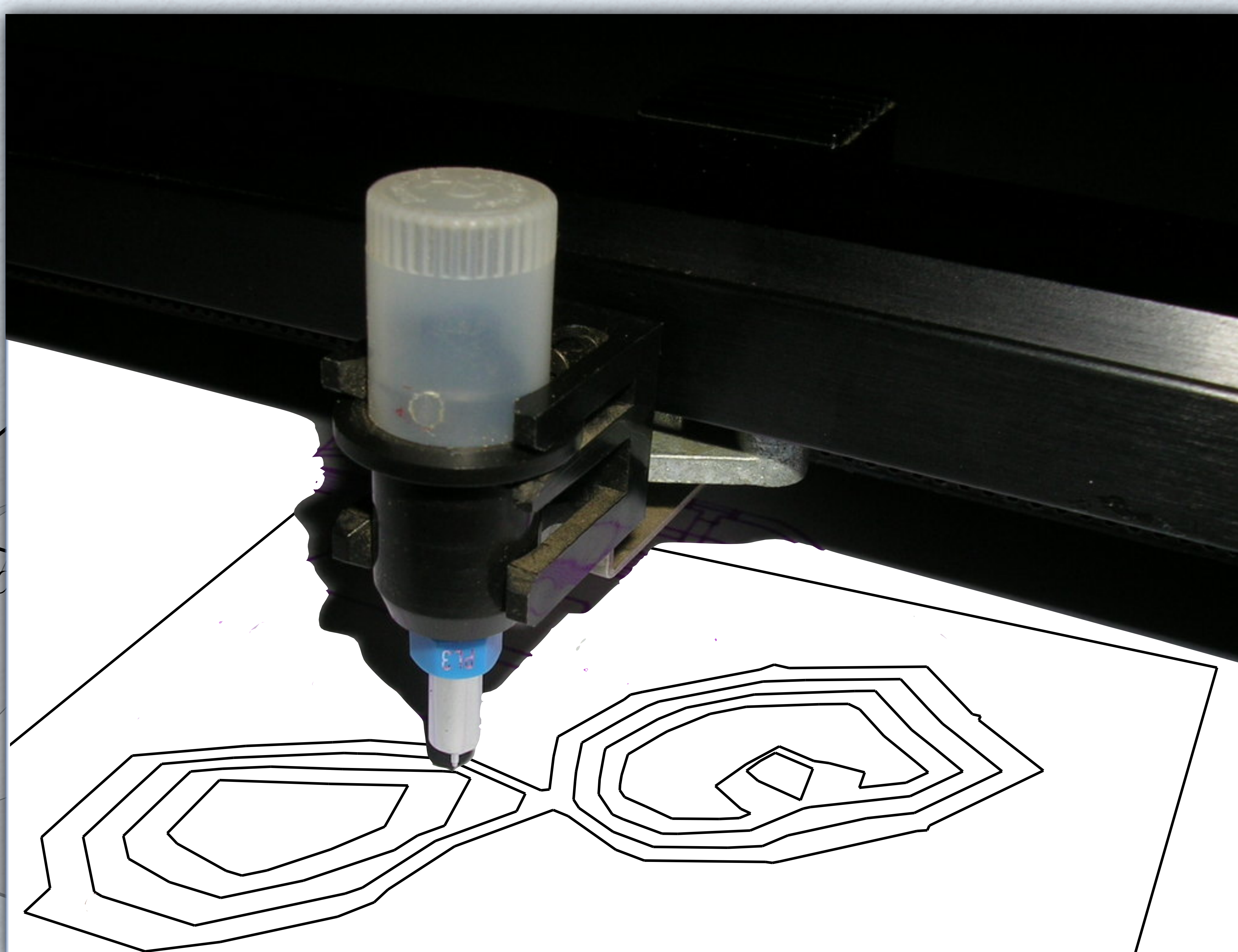

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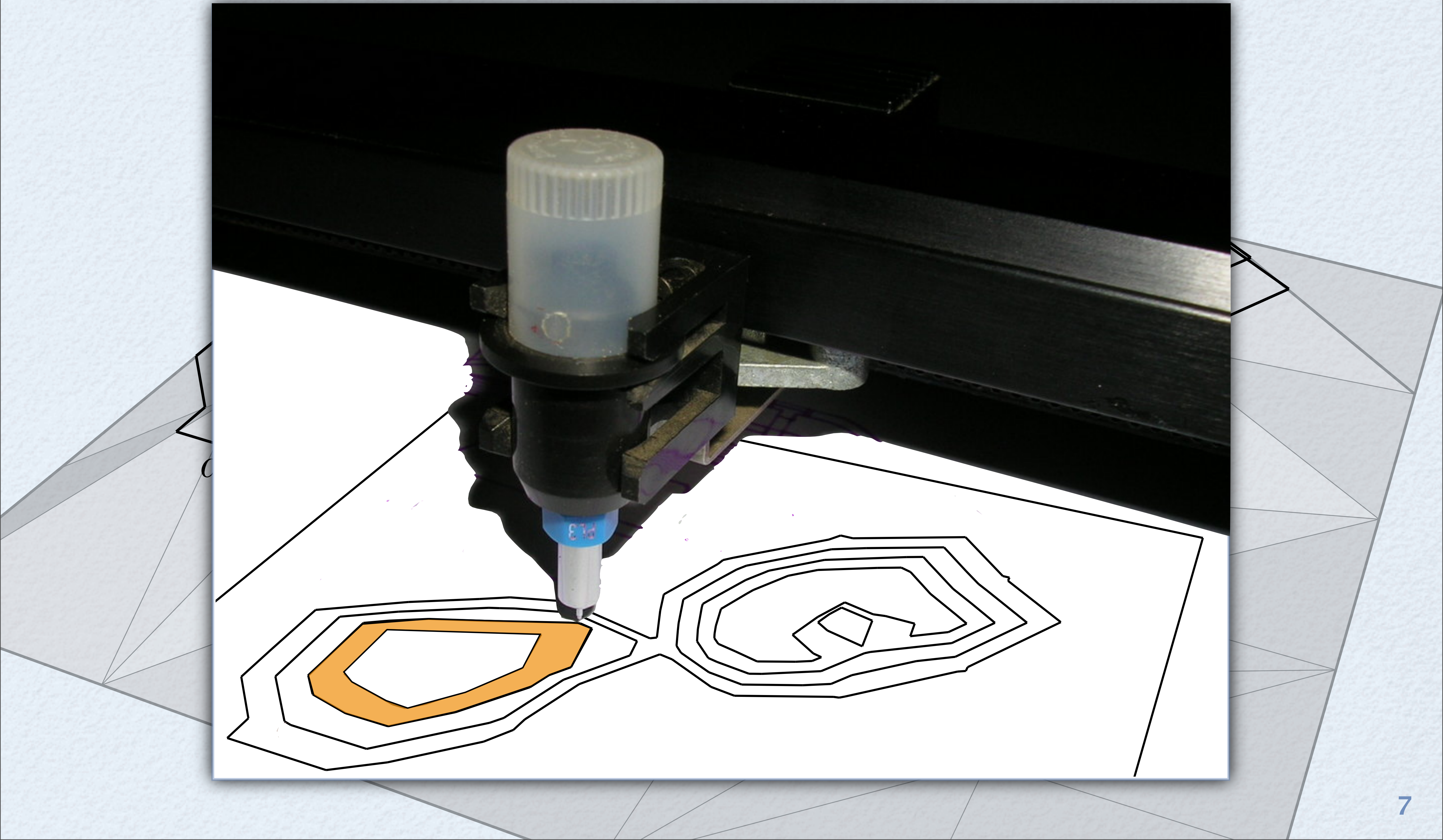
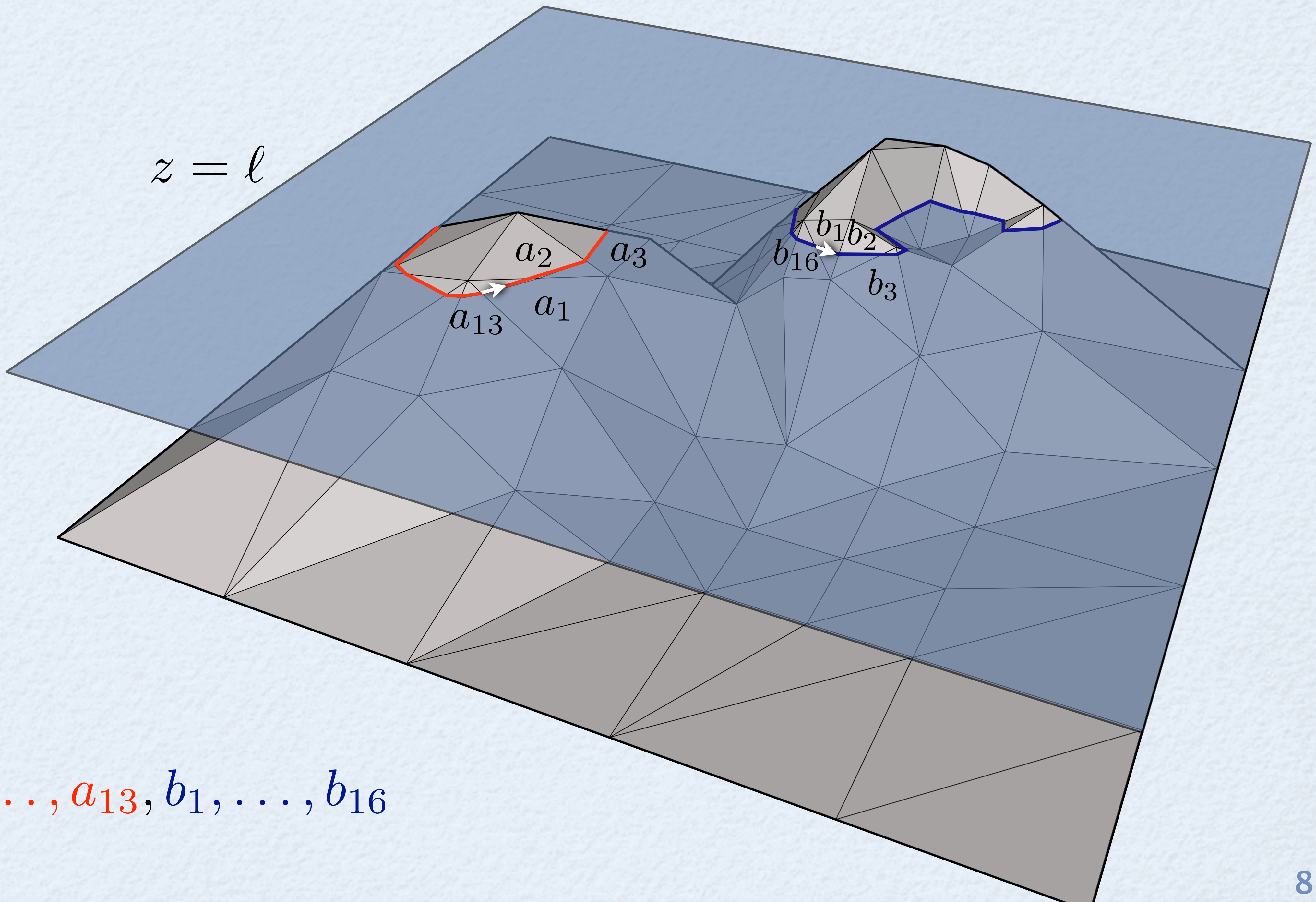


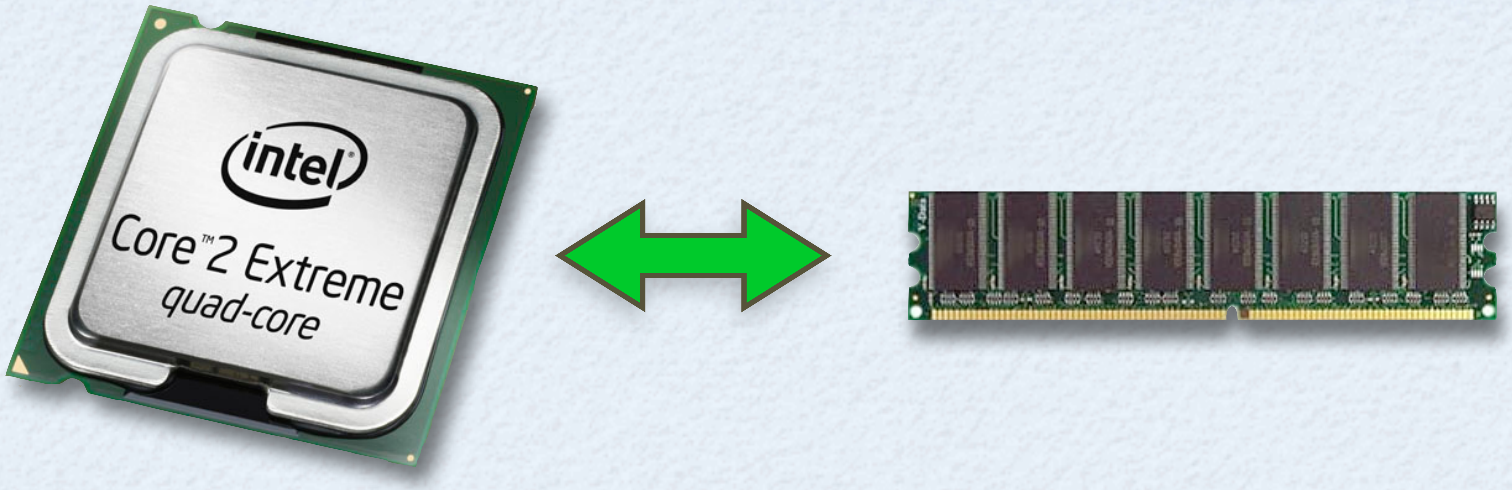
Answering Contour Queries

Preprocess the terrain to answer contour queries efficiently:
Given a level $\ell \in \mathbb{R}$*, return the level set* $h^{-1}(\ell)$ *such that each contour is reported separately and in sorted (circular) order.*



Output: $a_1, a_2, \ldots, a_{13}, b_1, \ldots, b_{16}$

The I/O Model



Classical Complexity: Number of basic operations as a function of N.

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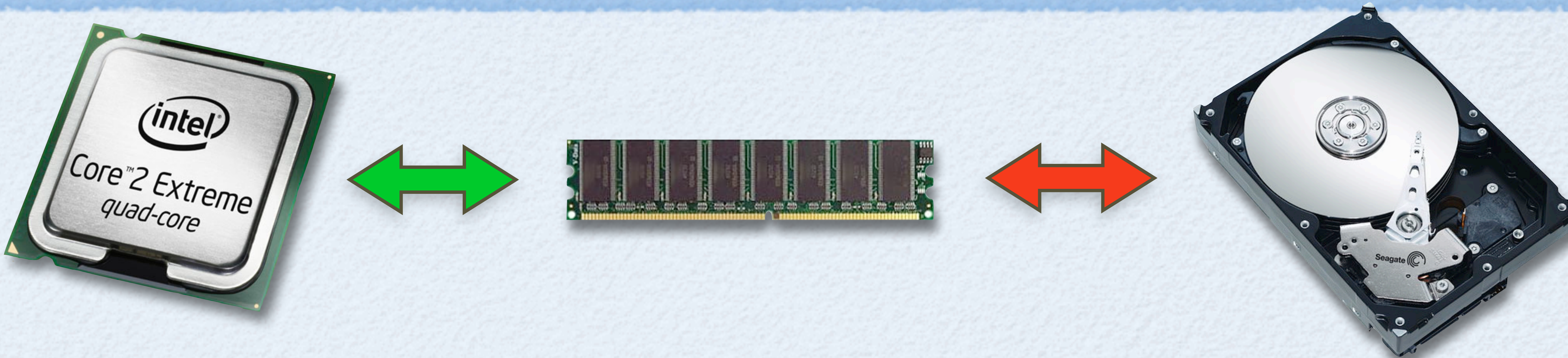

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Disk access is about 10^6 times slower than main memory access.

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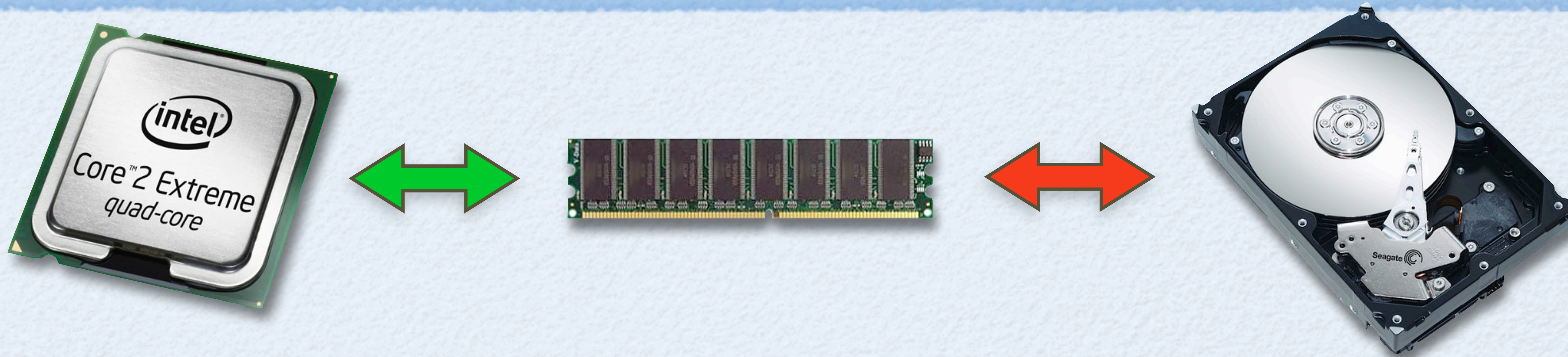

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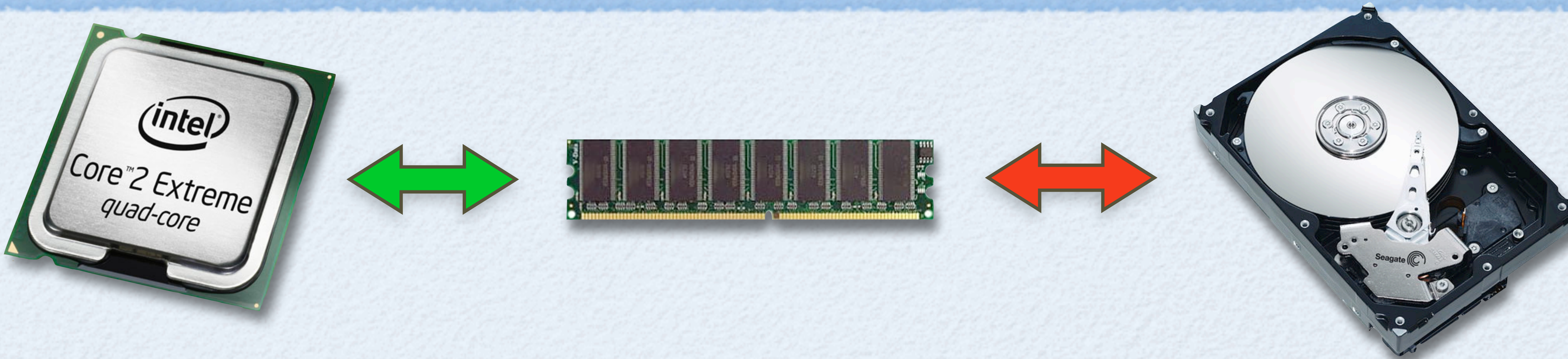

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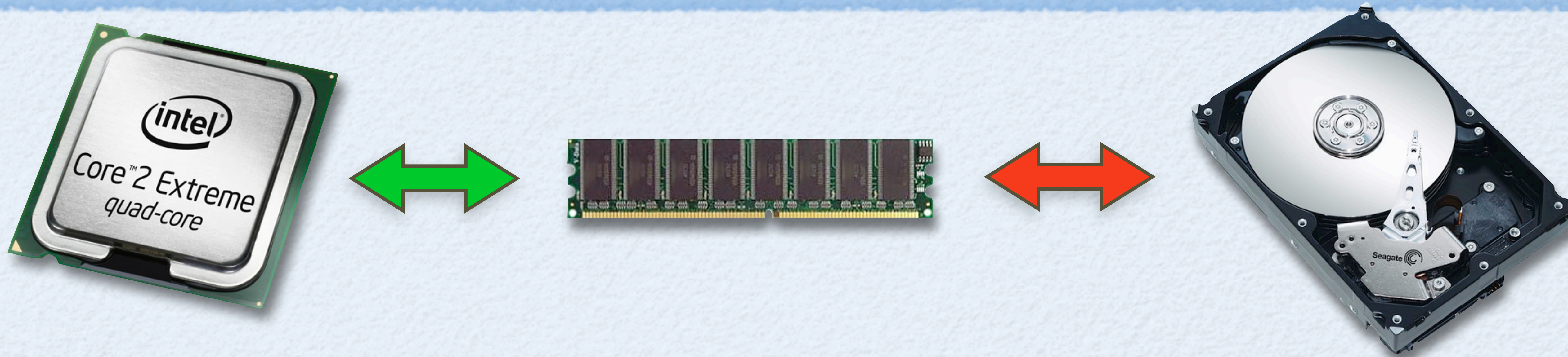

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I/O-Complexity: Number of I/Os as a function of N, B, M, and T.

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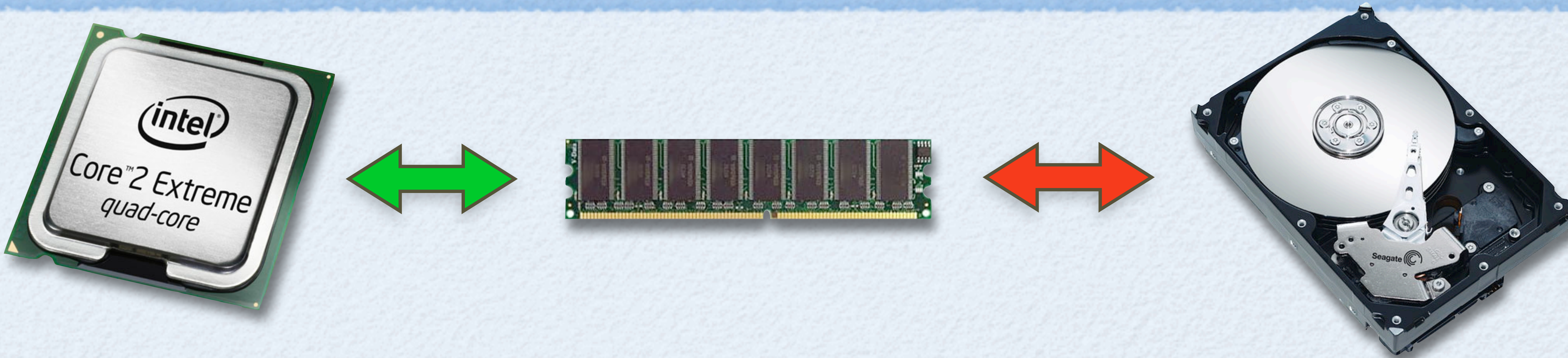

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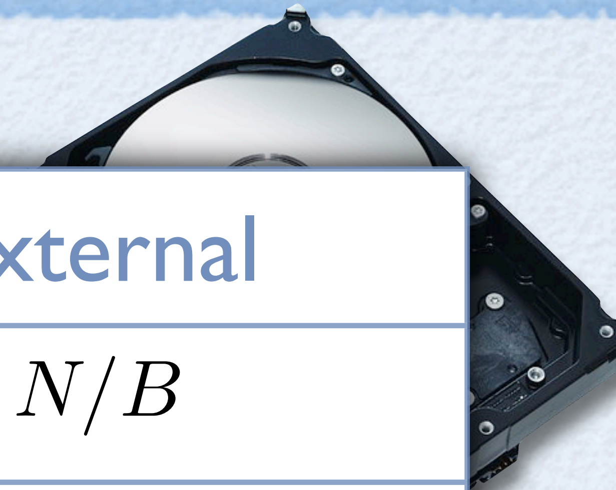

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Sorting	$N \log N$	$\frac{N}{B} \log_{\frac{M}{B}} \frac{N}{B}$
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Previous Work

Answering contour queries I/O-efficiently:

	Preprocessing I/Os	Structure Size	Query I/Os
Chiang, Silva'97	$O(\mathsf{Sort}(N))$	$O(N)$	$O(\log_B N + T/B)$
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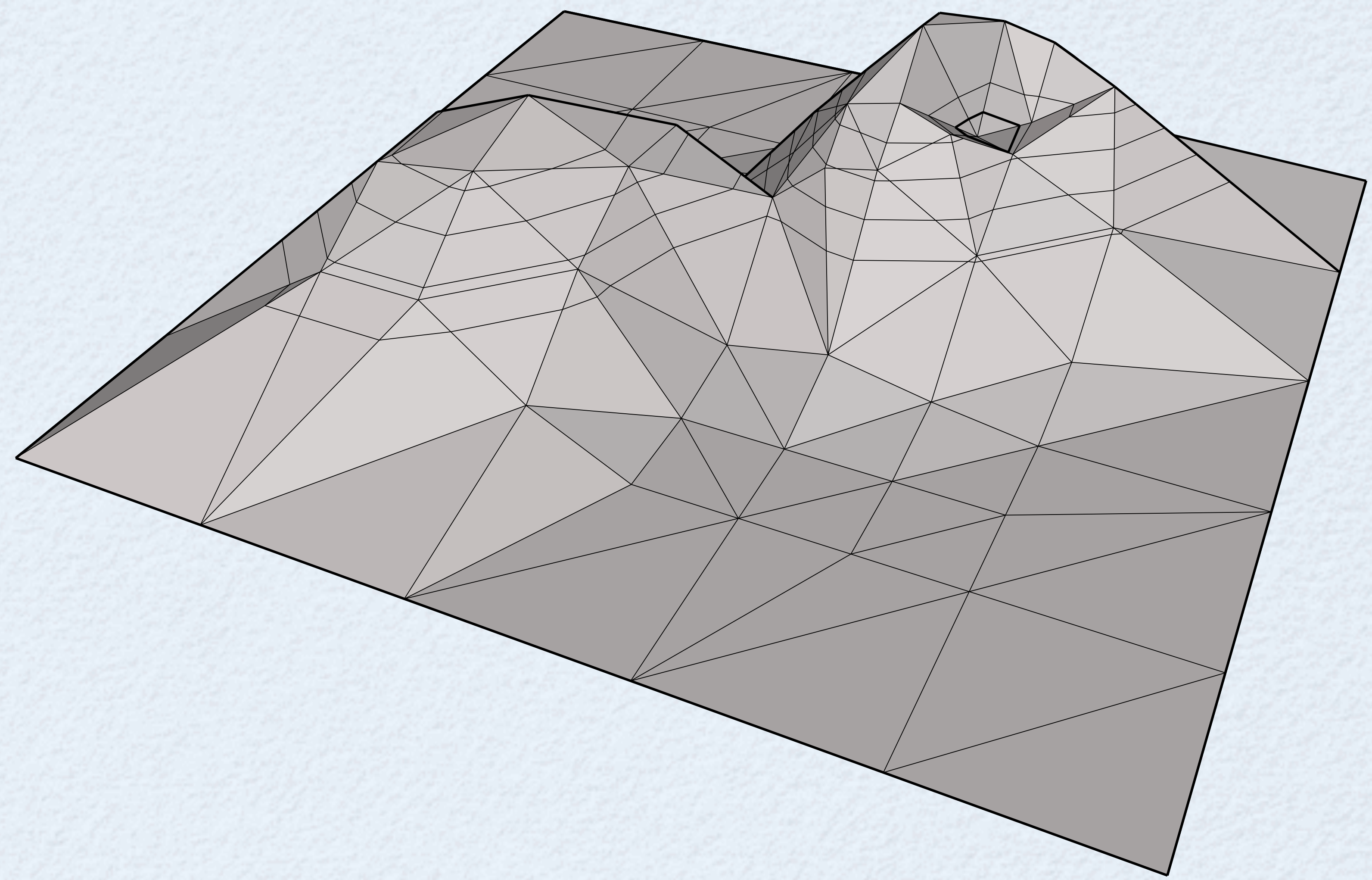
Unsorted Contours!

"Very naïve" Algorithm: Trace one contour at a time

Find one "seed" triangle intersecting each contour and trace out the contour sequentially.



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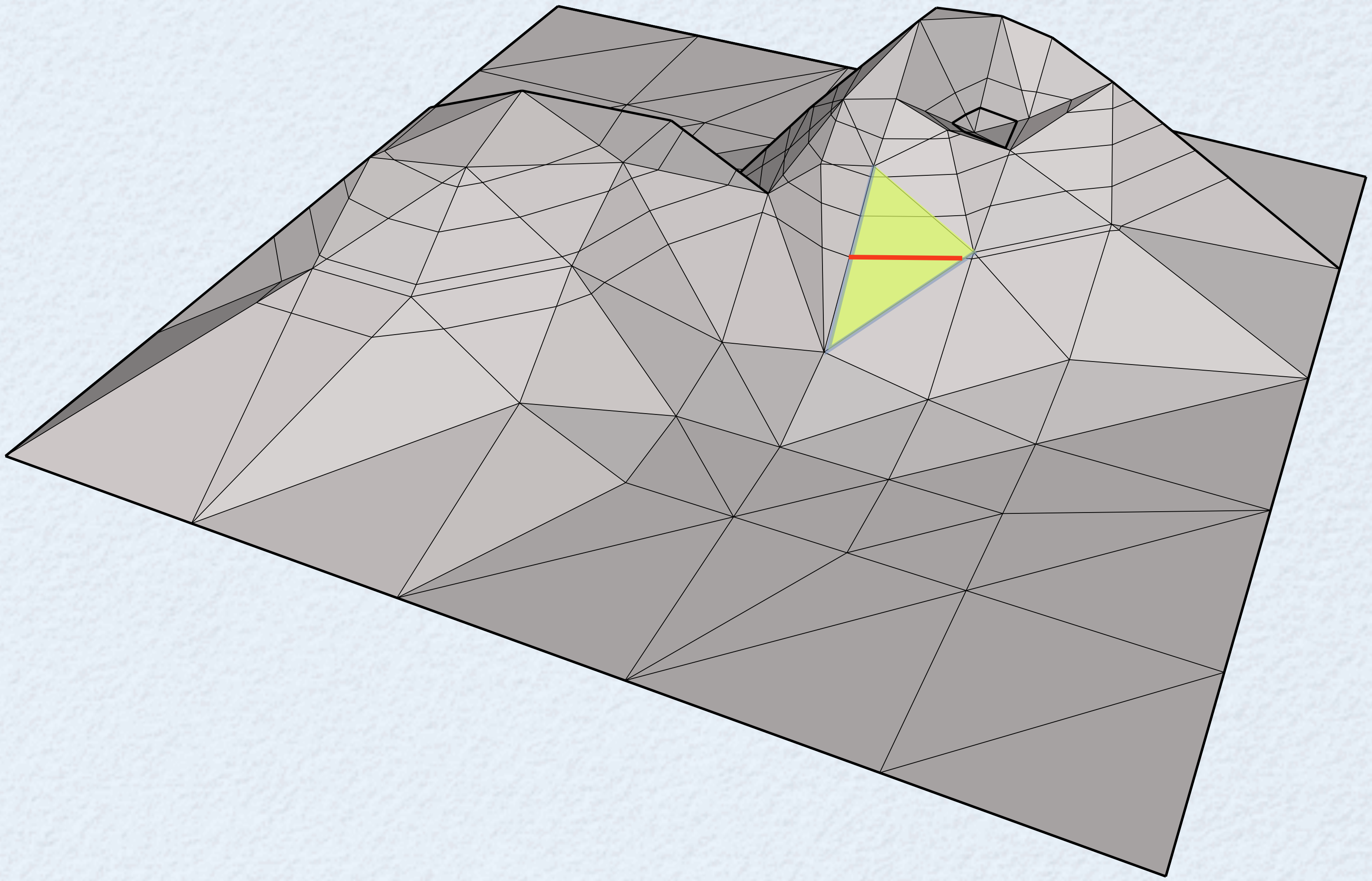

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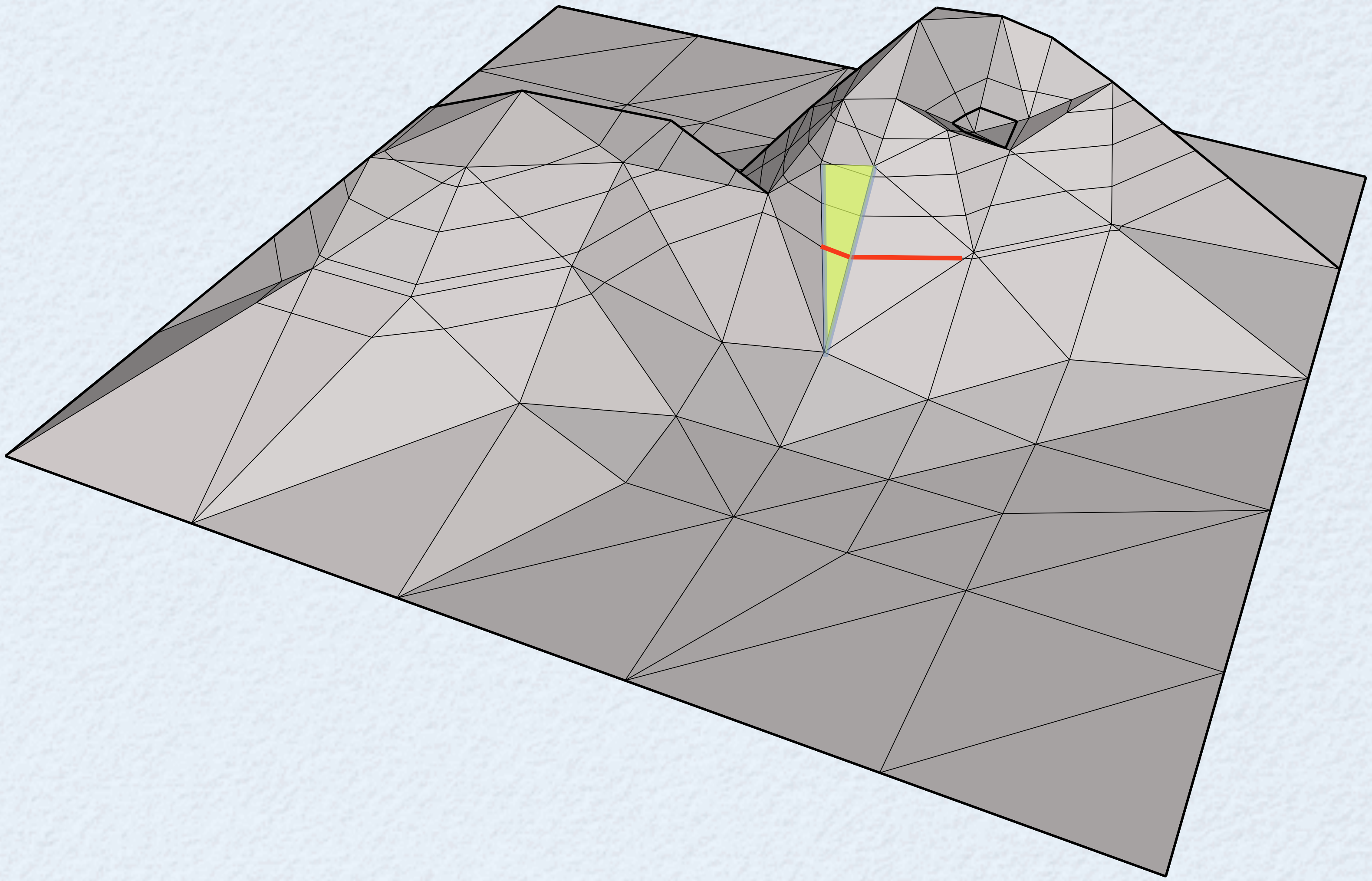

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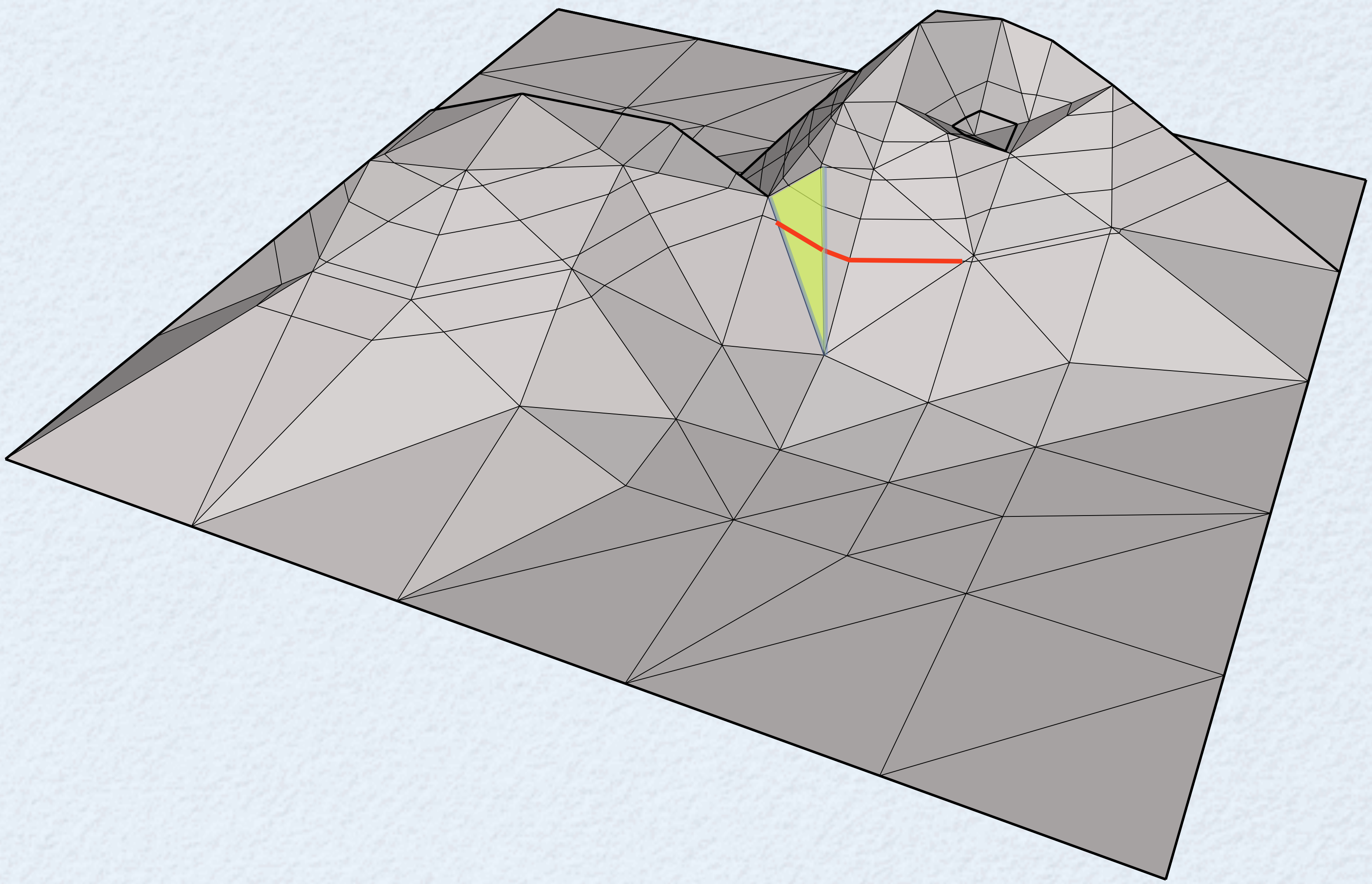

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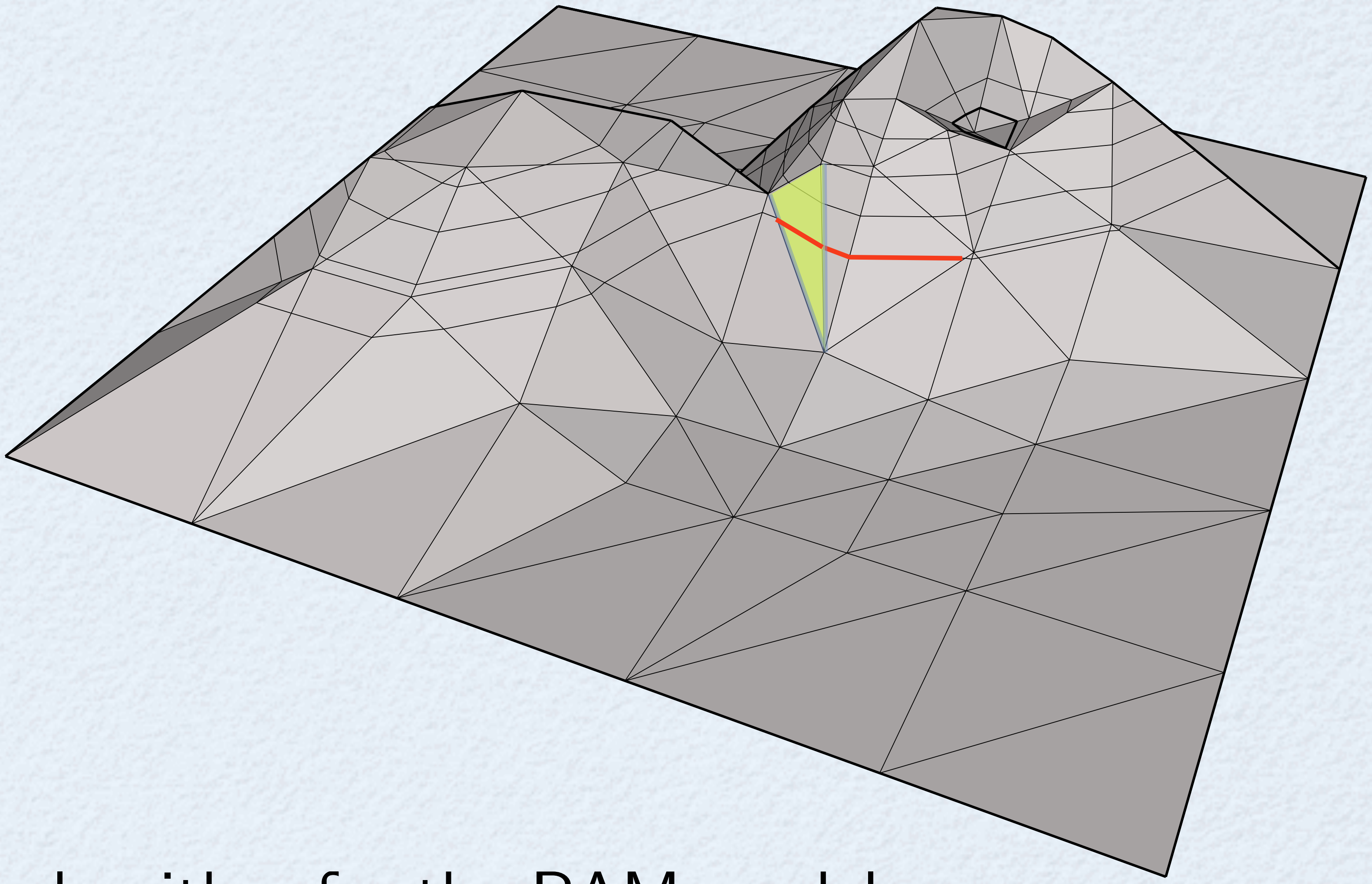

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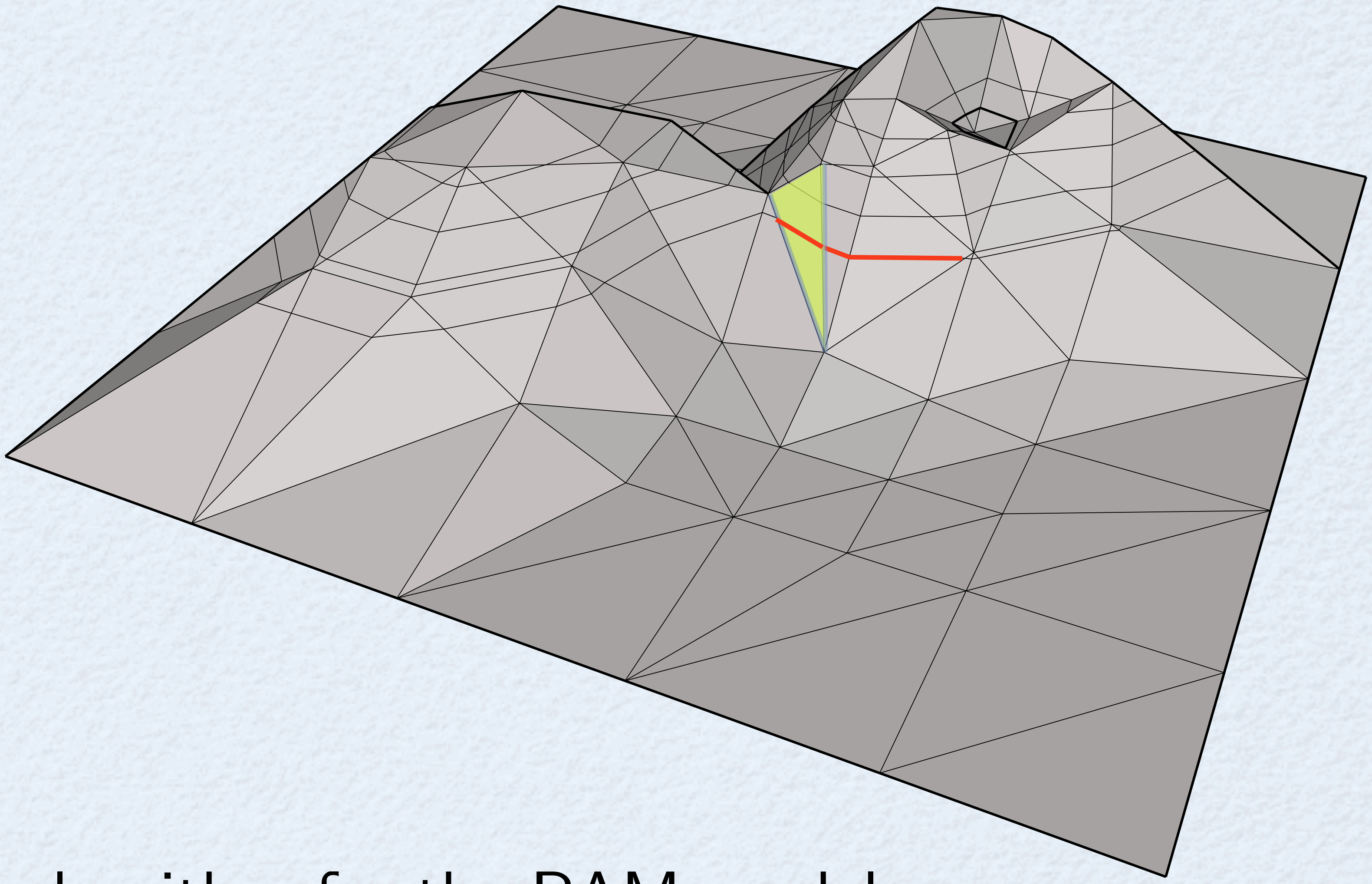

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I/O Complexity:

$$O(N/B \cdot \log_B |L| + T).$$

segments in the output

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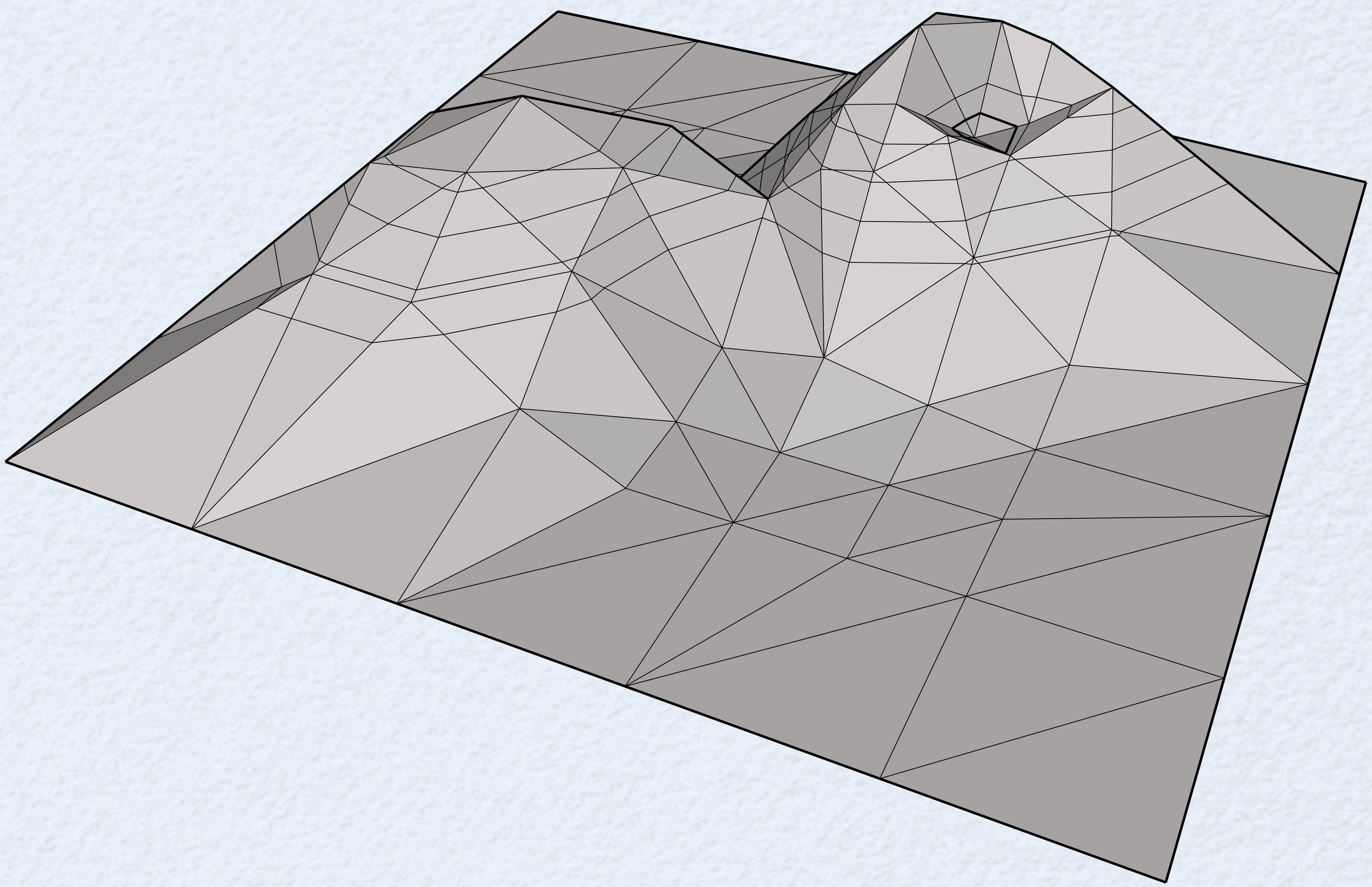
Scan the triangles (in the order laid out on the disk) and generate all segments. Then sort the output.



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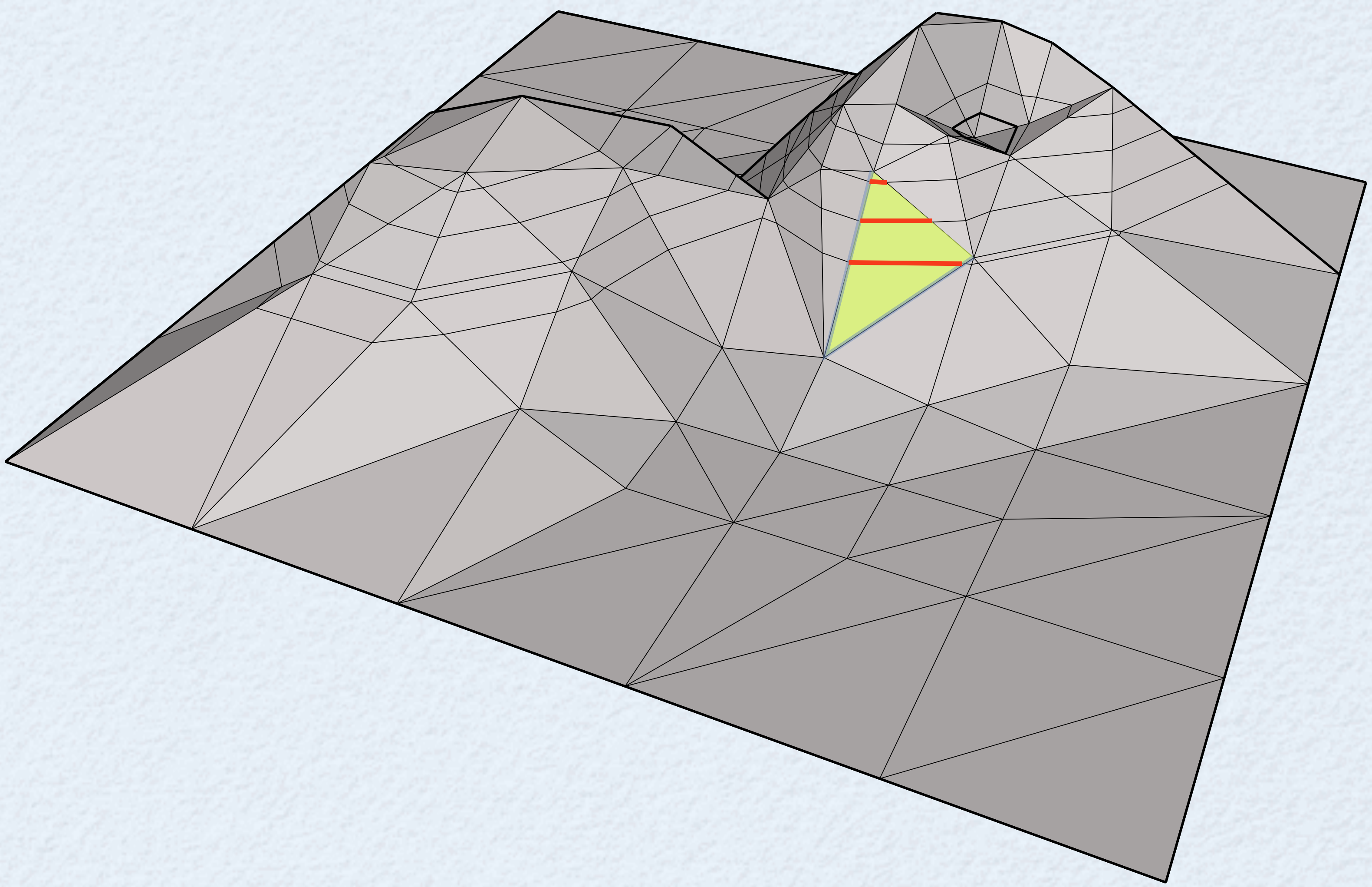

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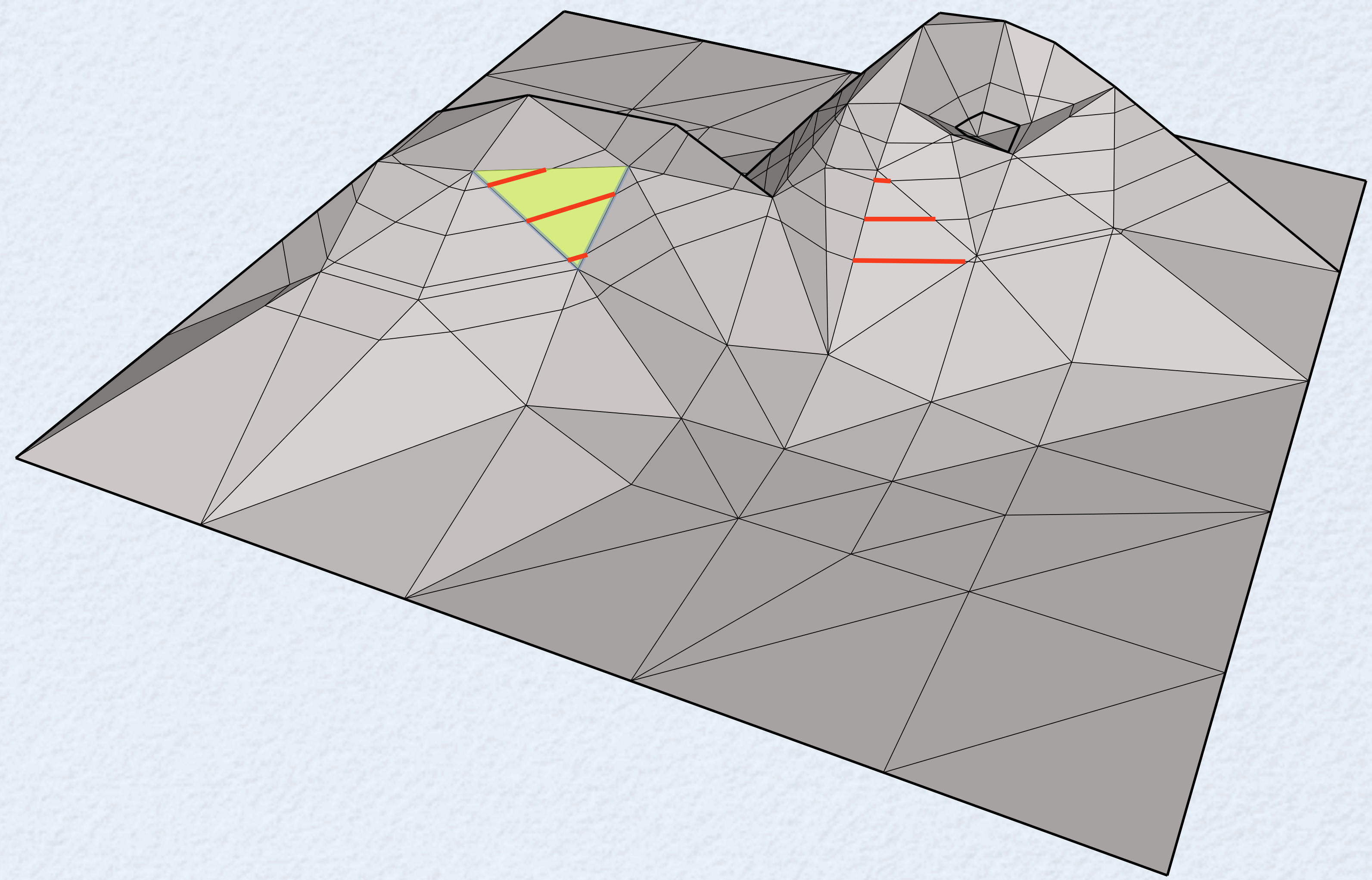

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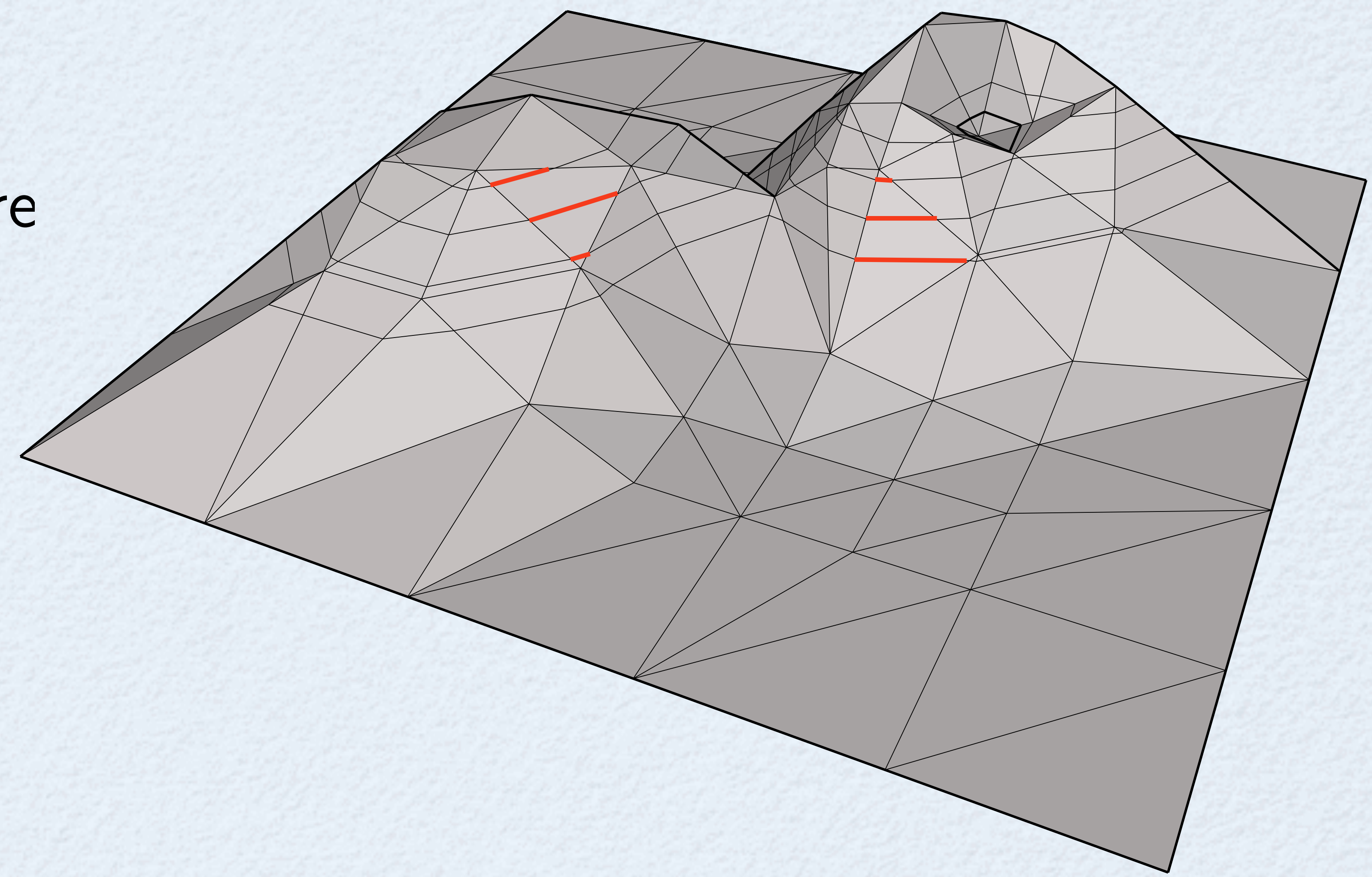
For segment s at level ℓ_i store pair (ℓ_i, s) plus the segments before and after s on contour containing s.



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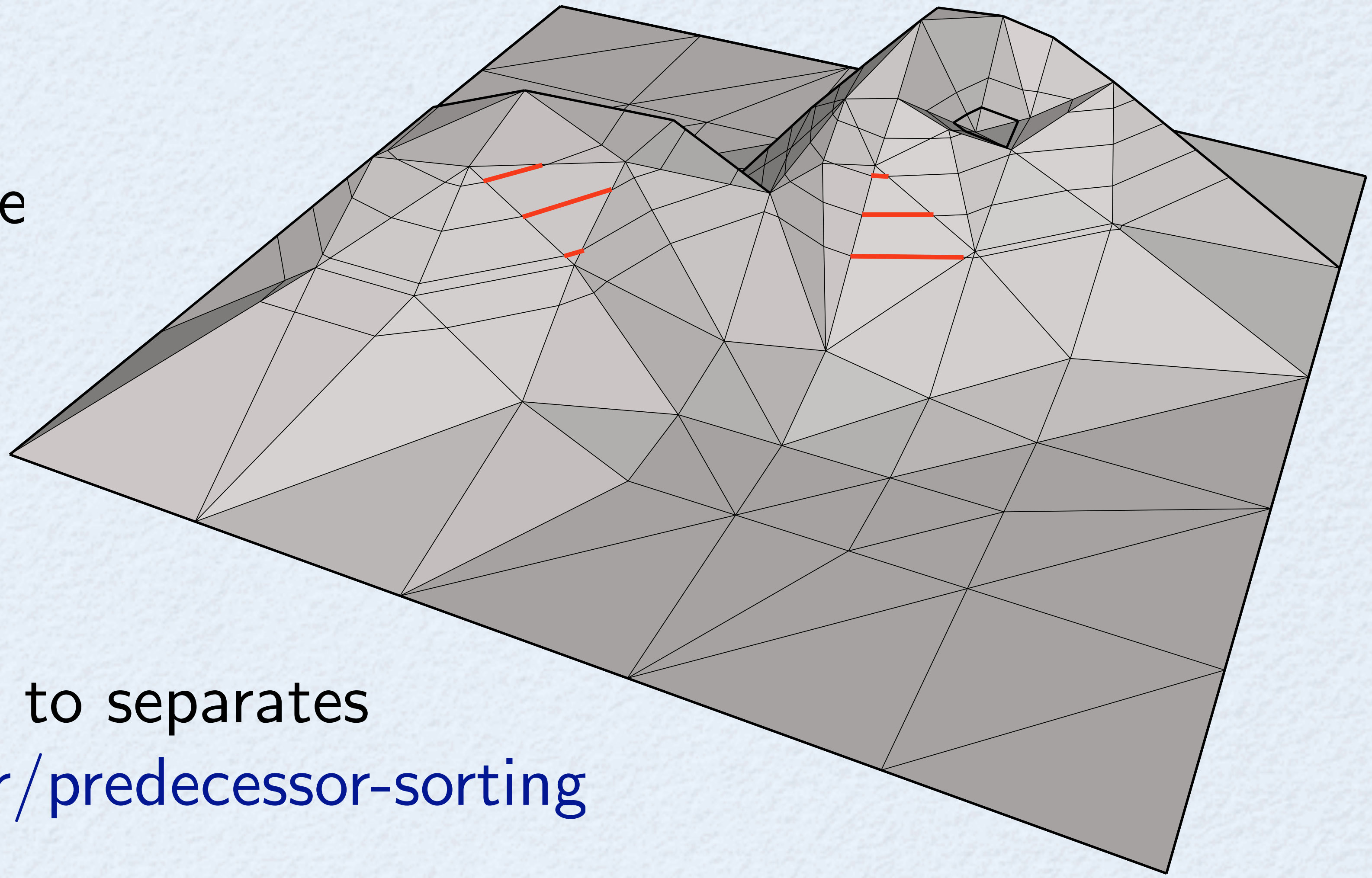

For segment s at level ℓ_i store pair (ℓ_i, s) plus the segments before and after s on contour containing s.

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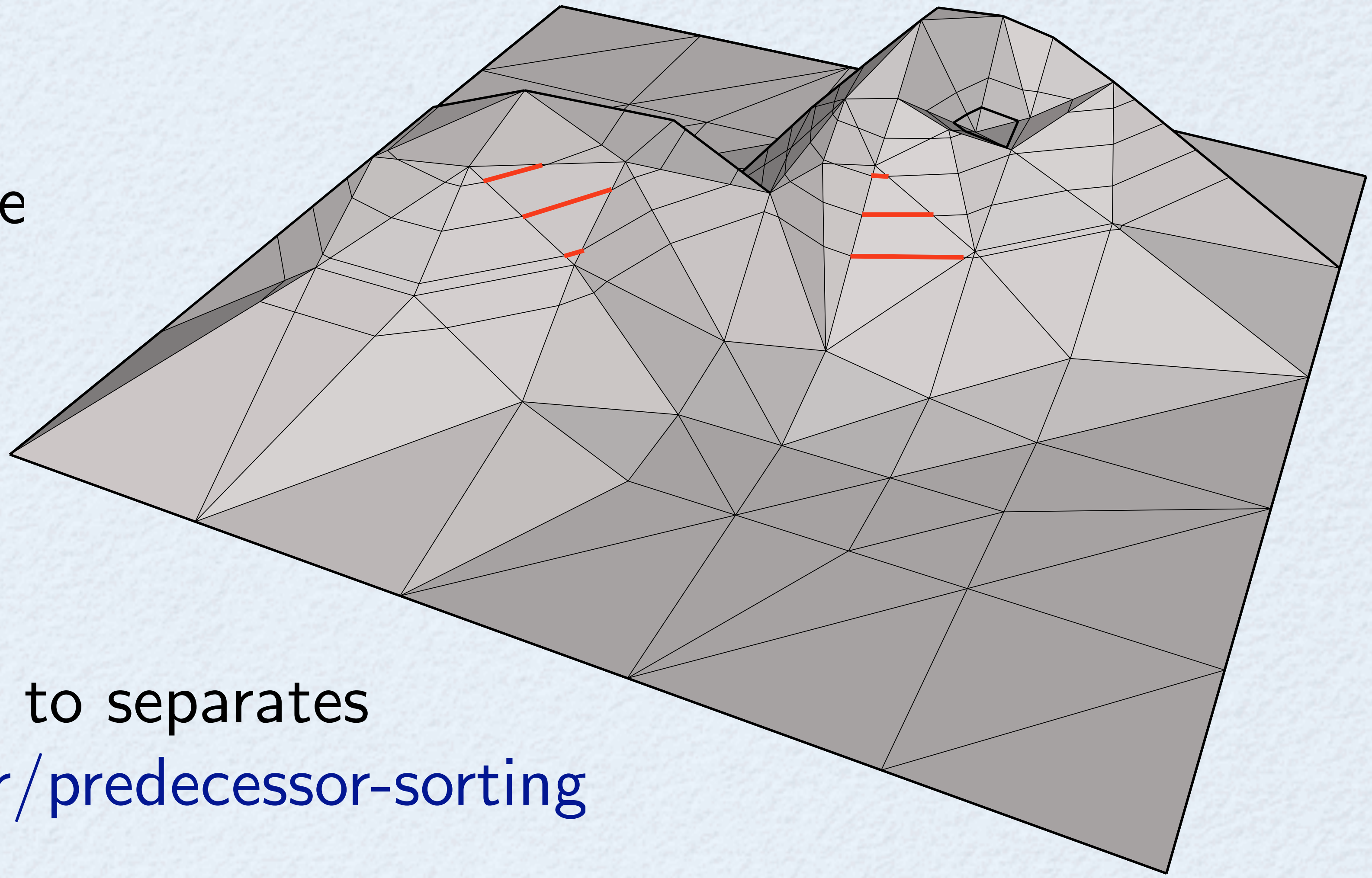

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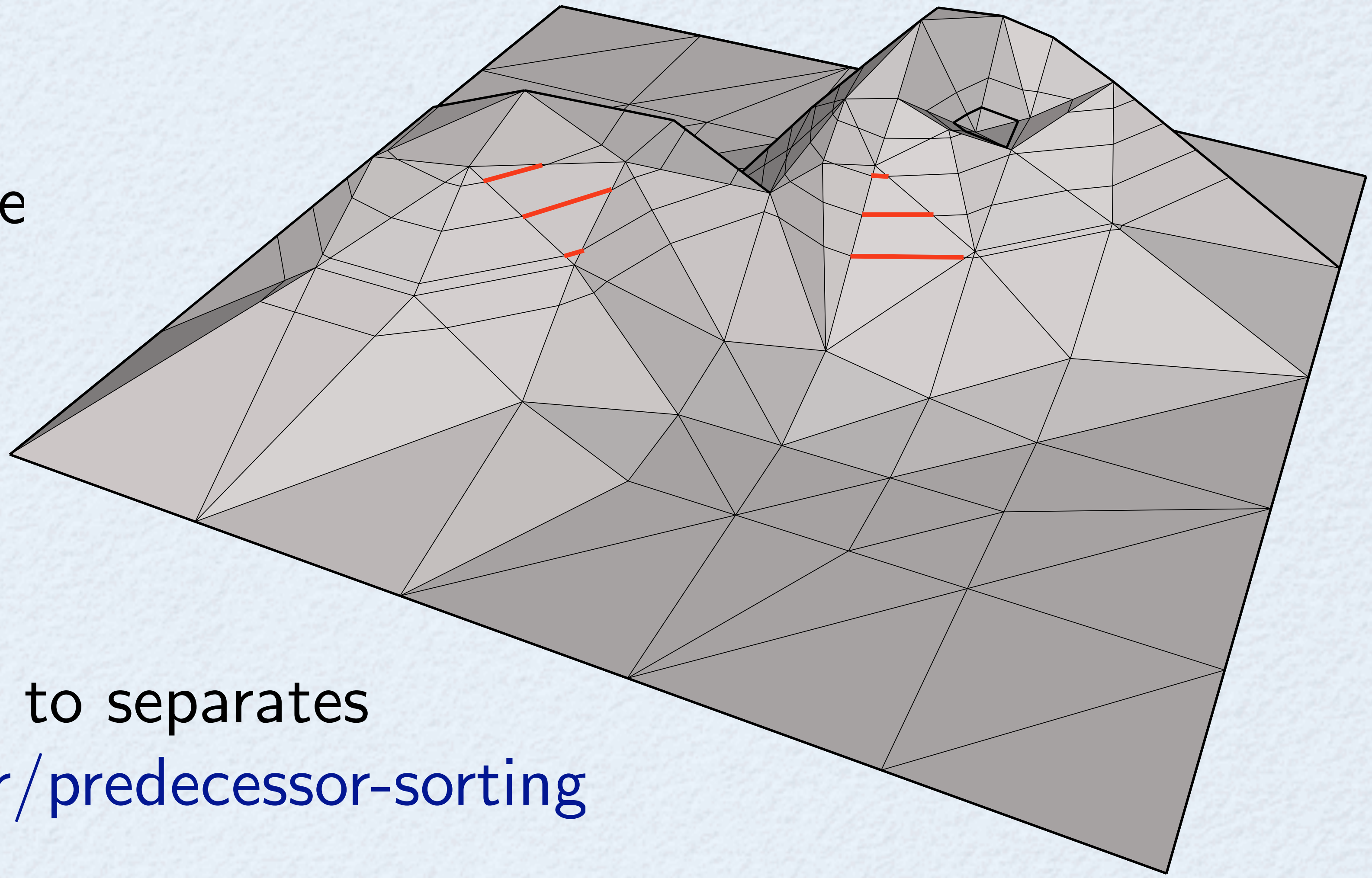

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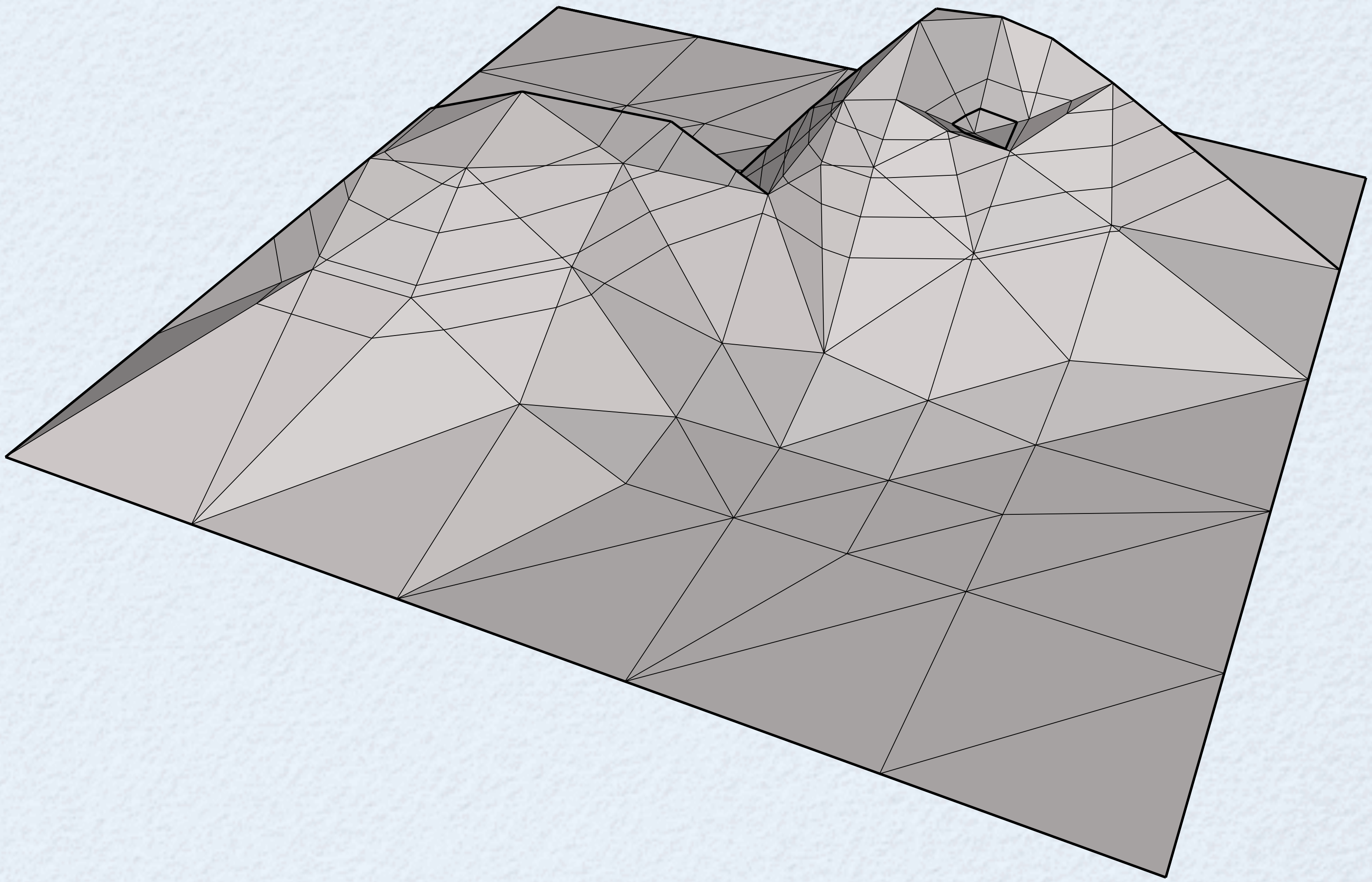

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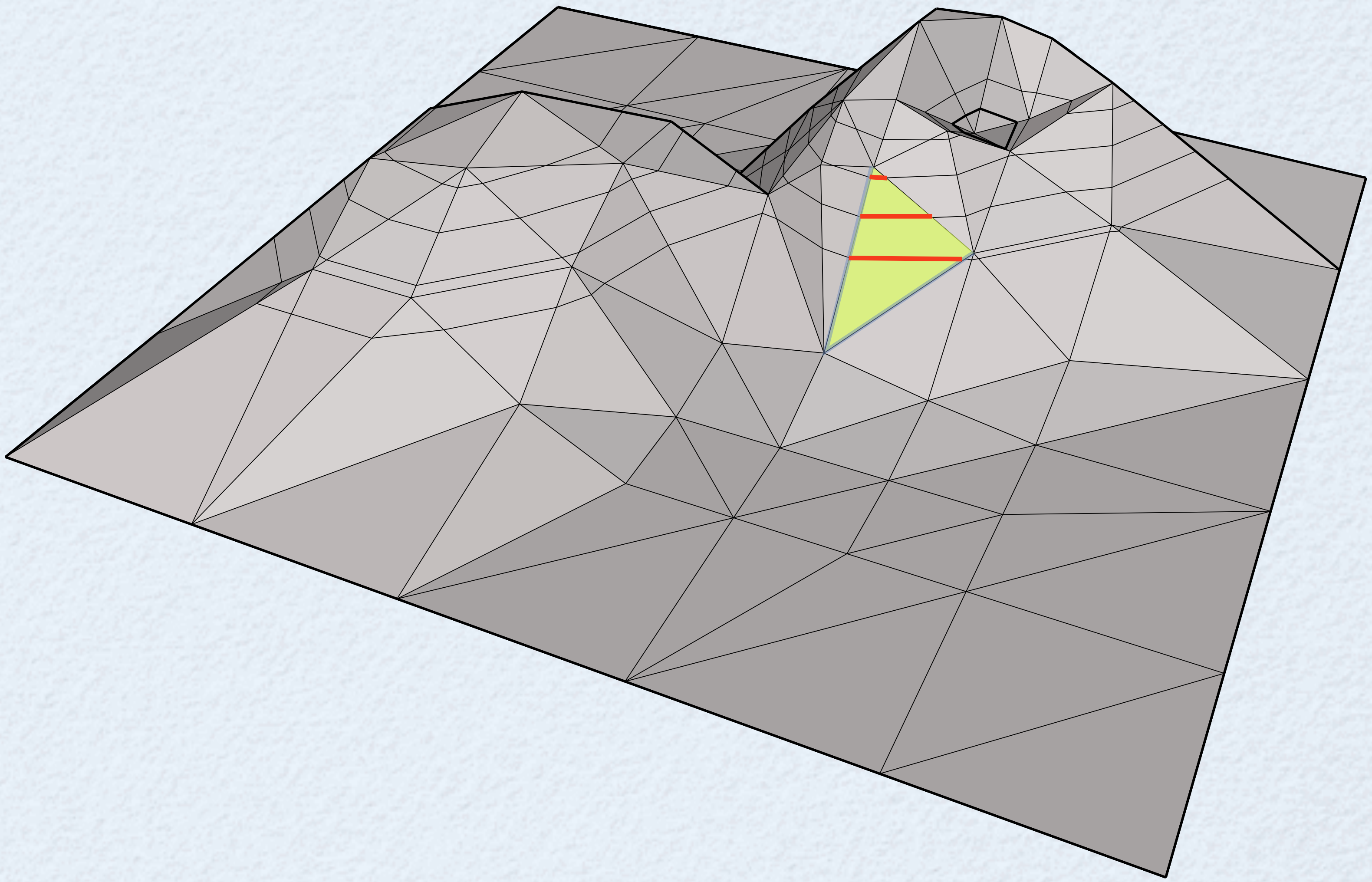

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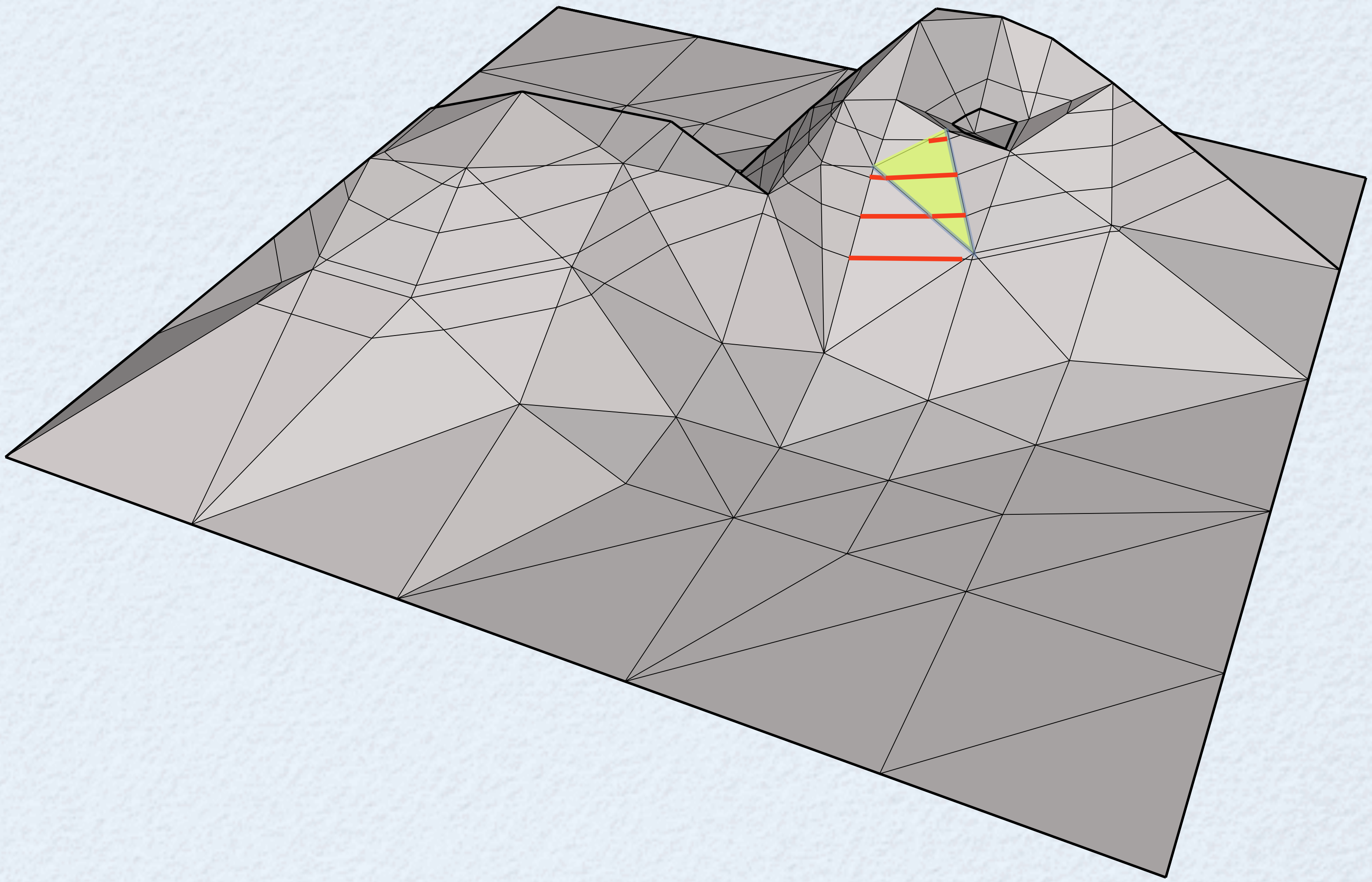

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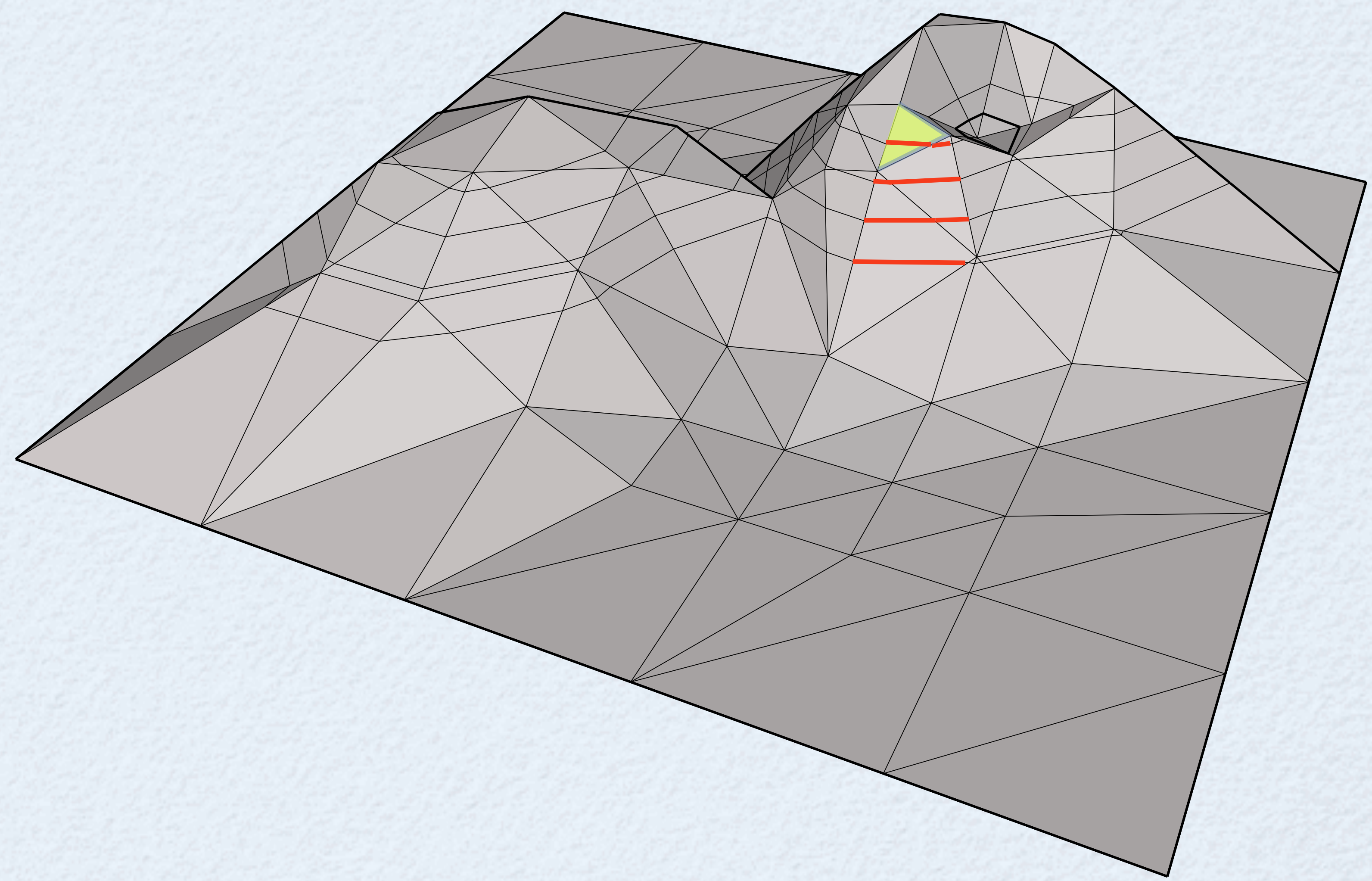

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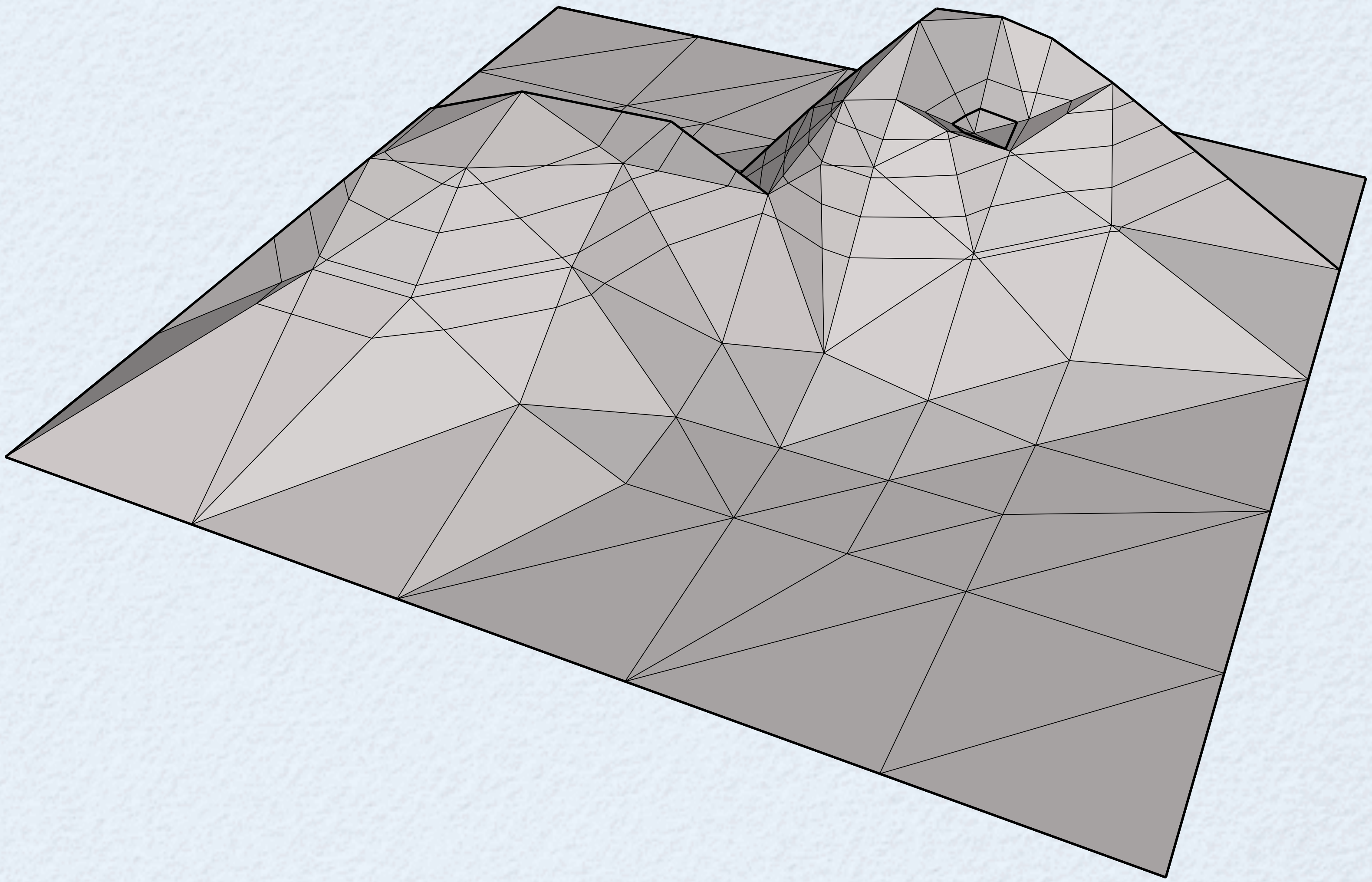

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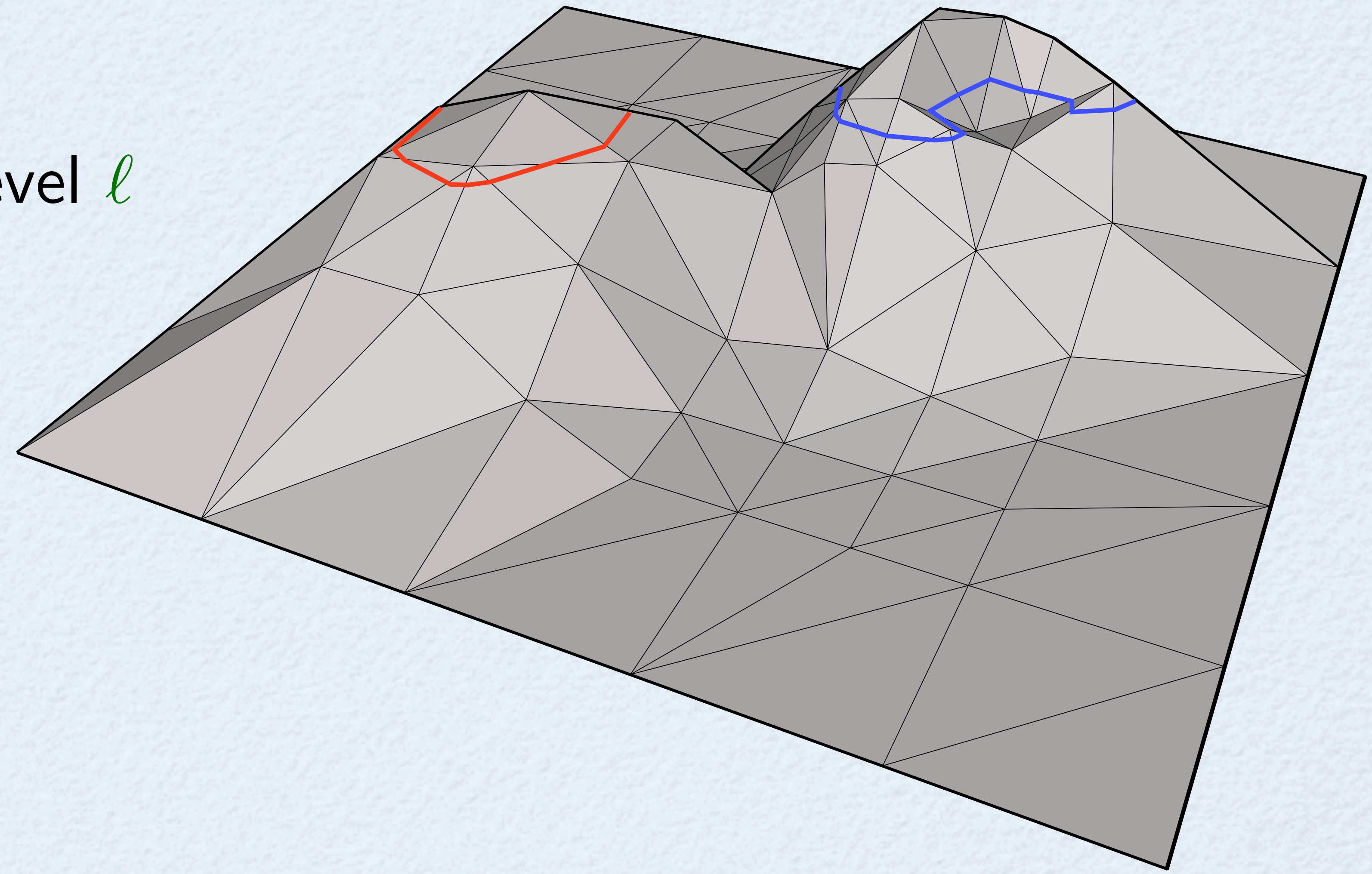

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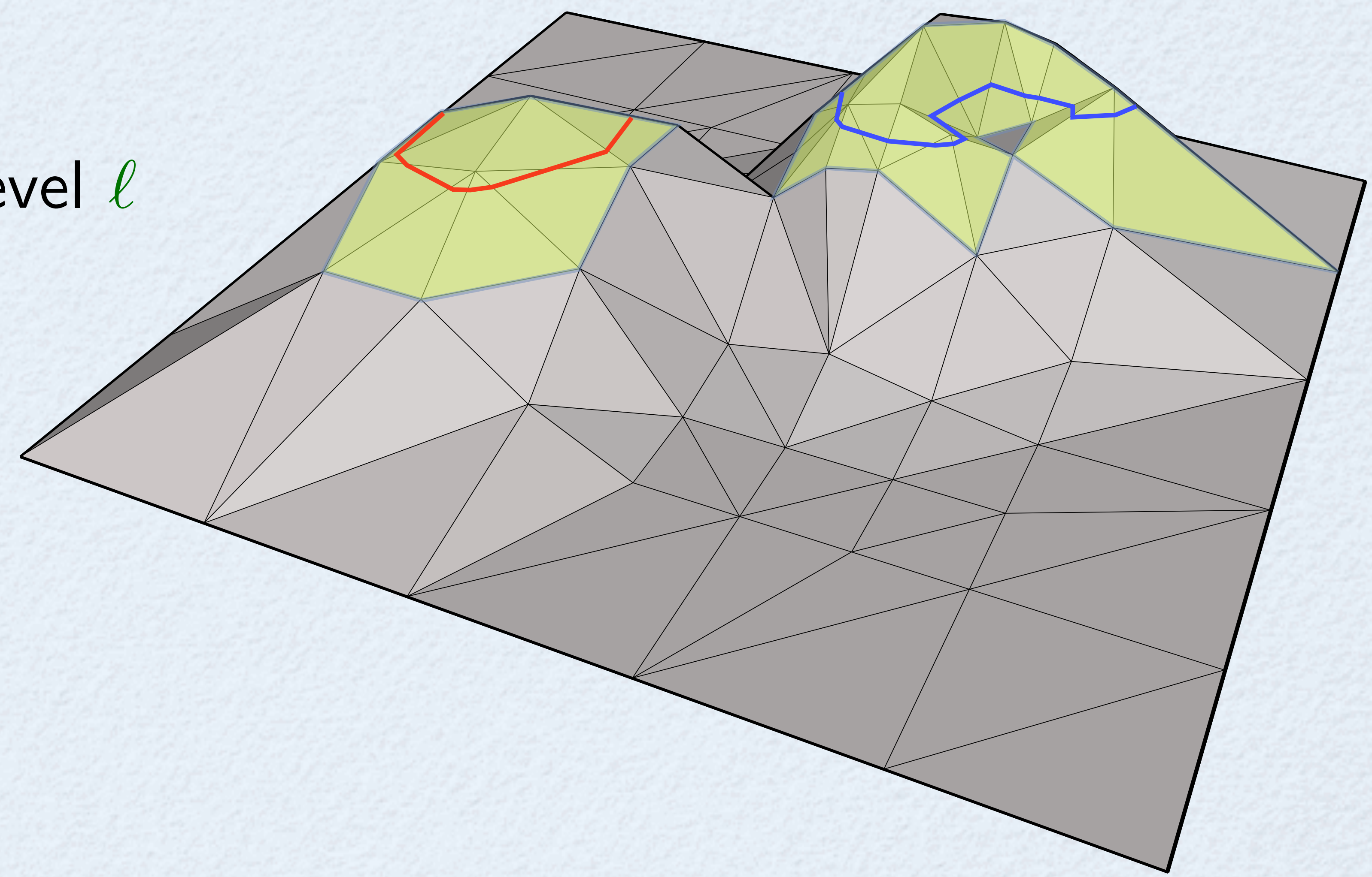

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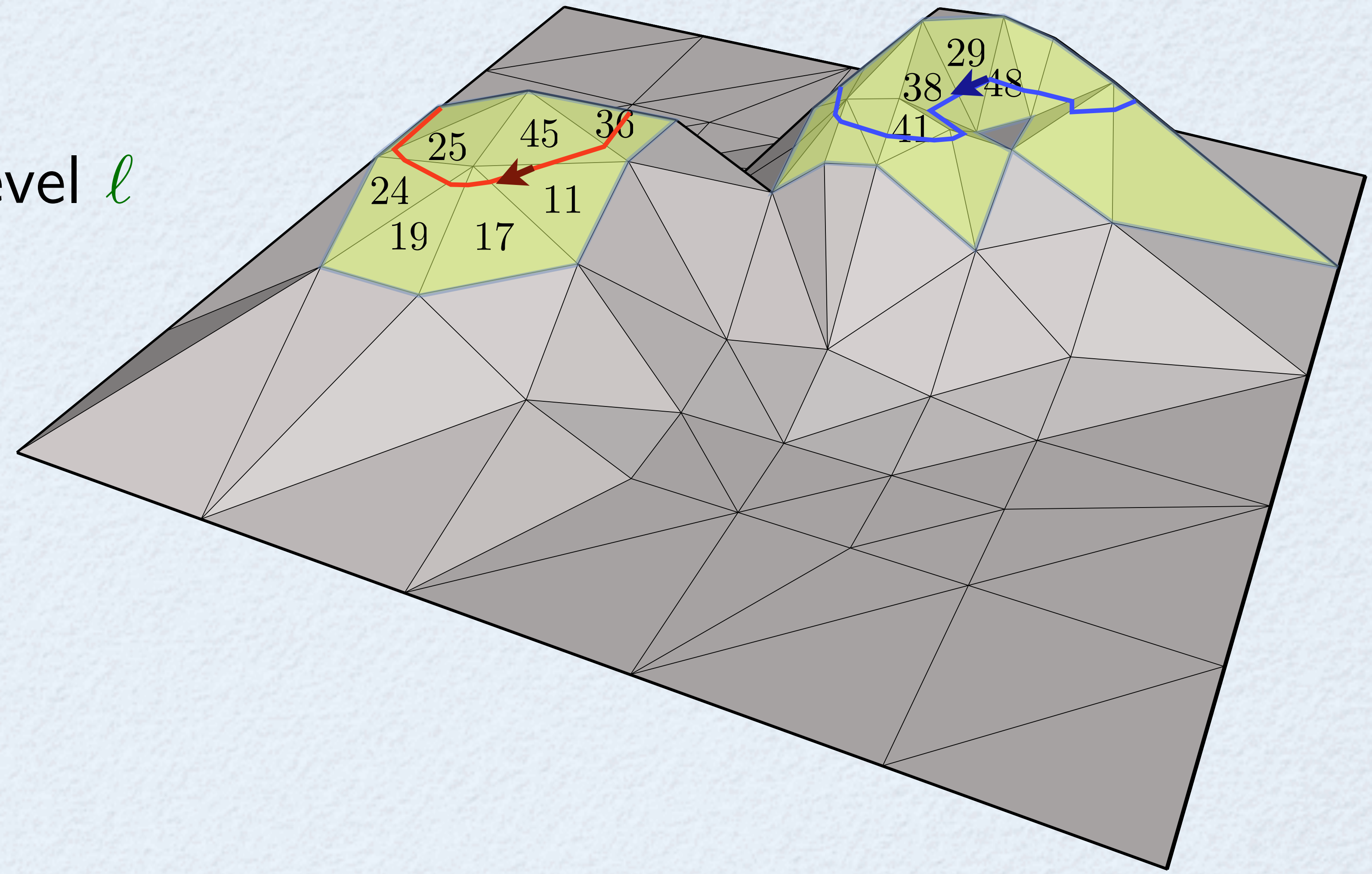

The restriction of $\prec$ to Δ_ℓ traverses each contour of $\mathbb{M}$ in circular order.

Level Ordering Theorem

Theorem. For any terrain $\mathbb{M}$, there is
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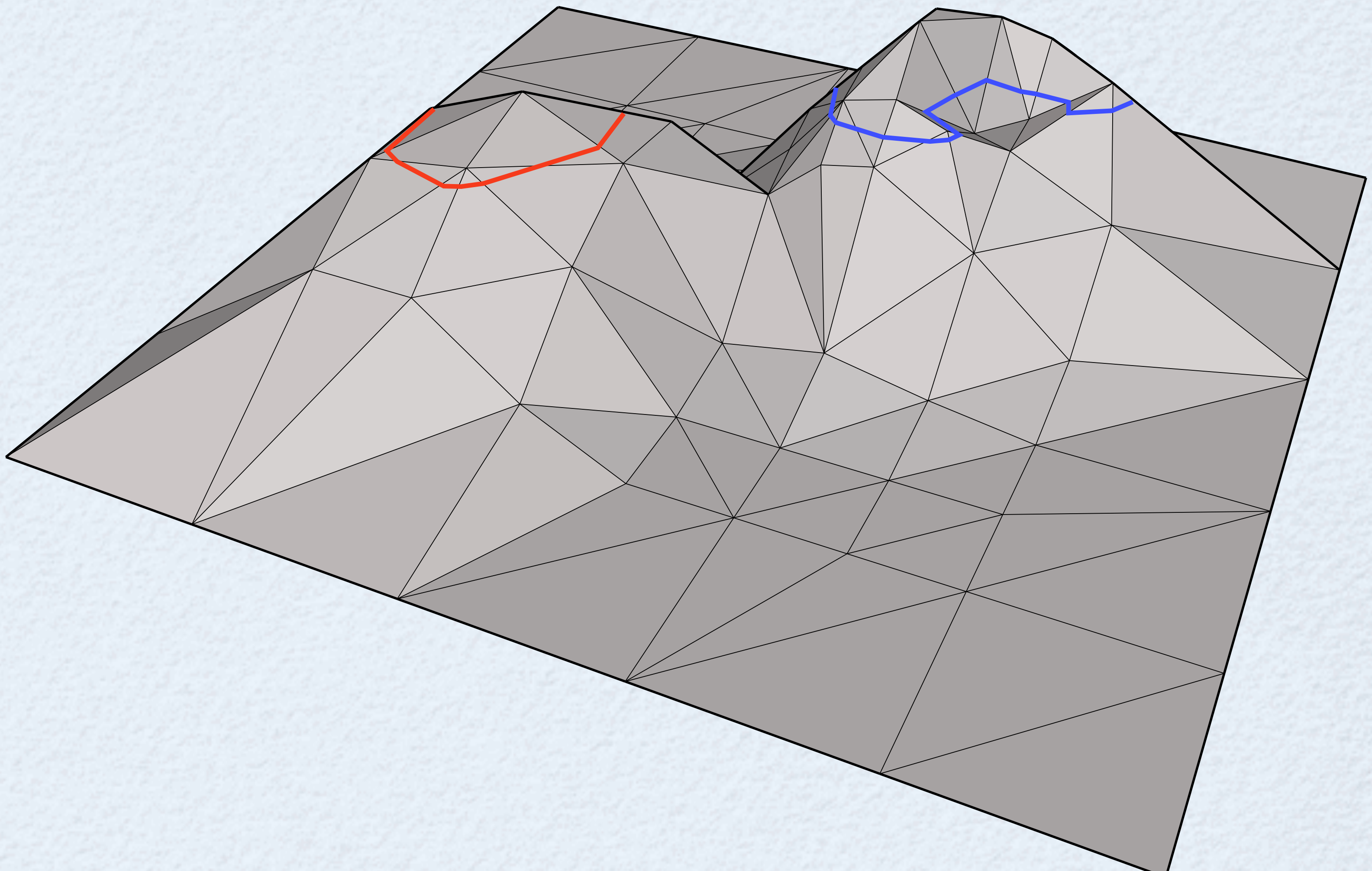

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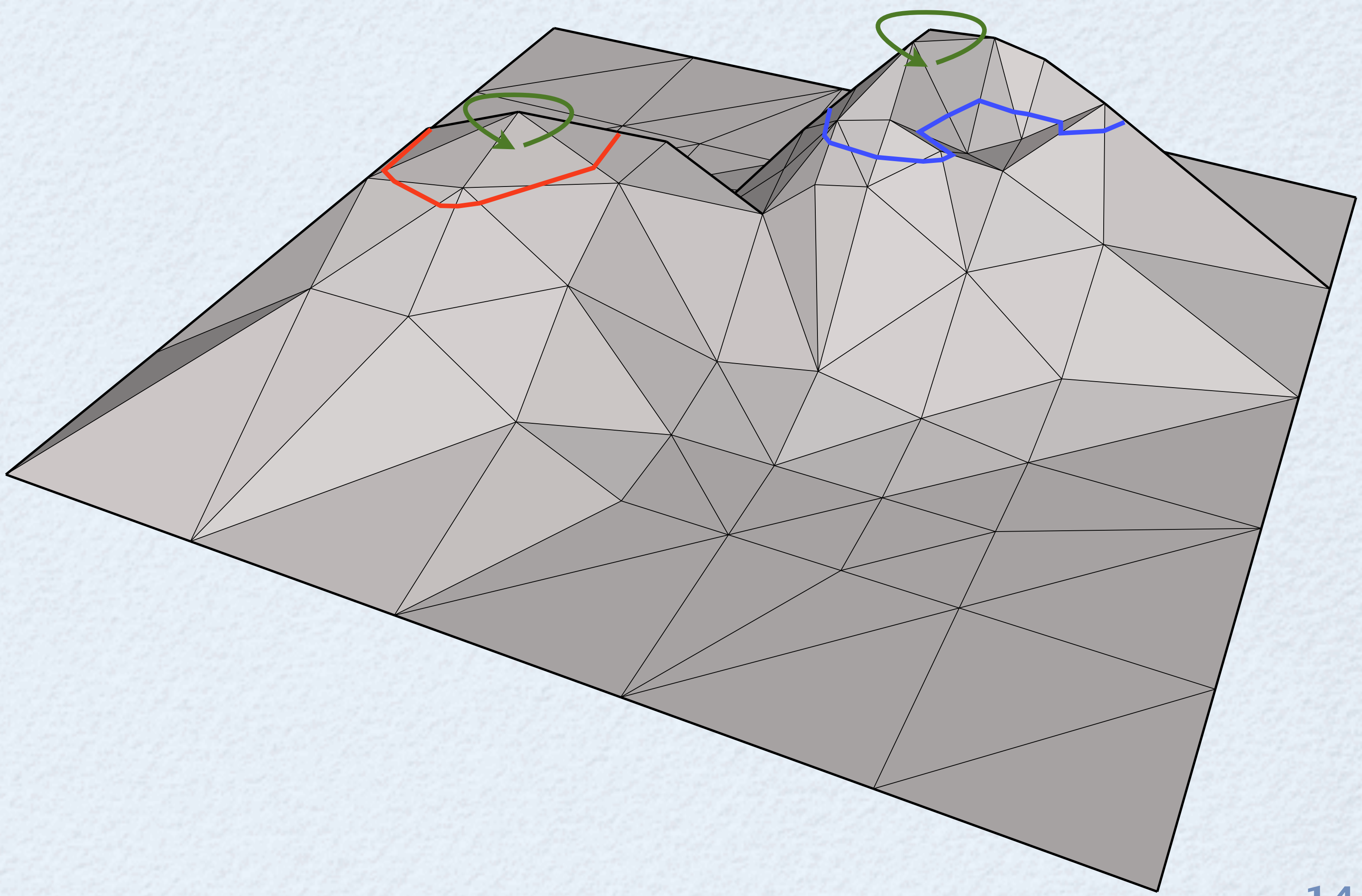

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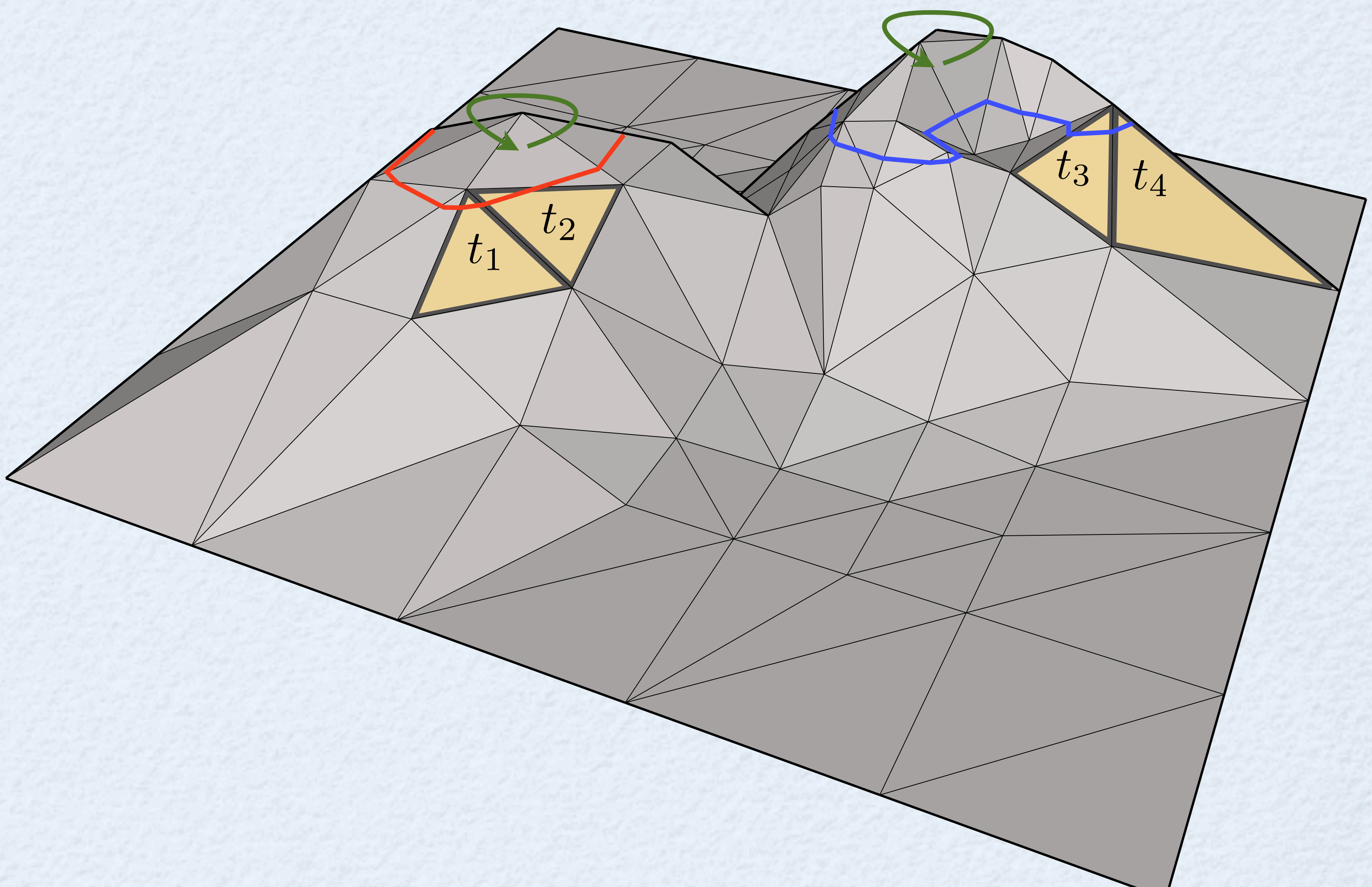

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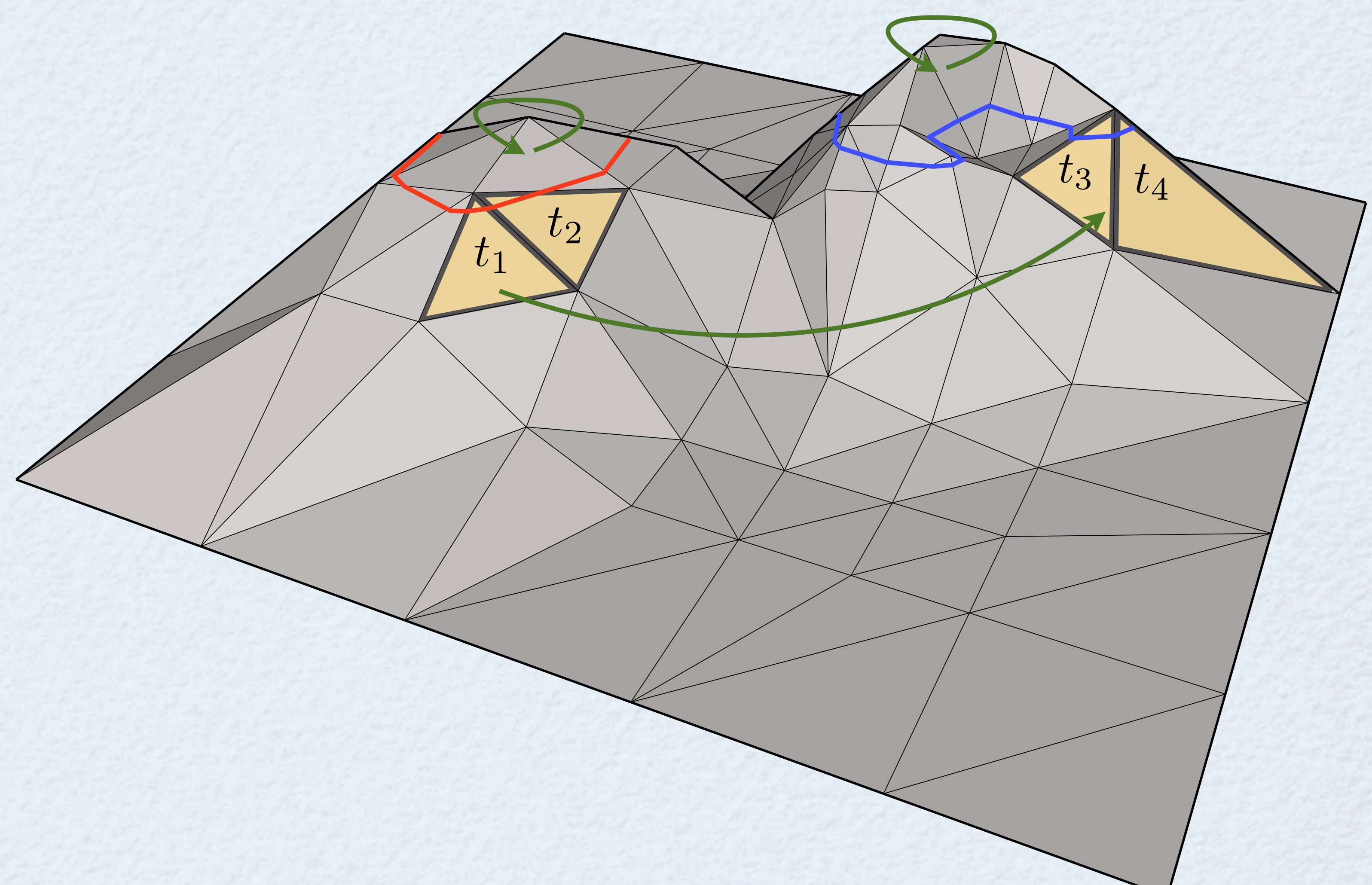

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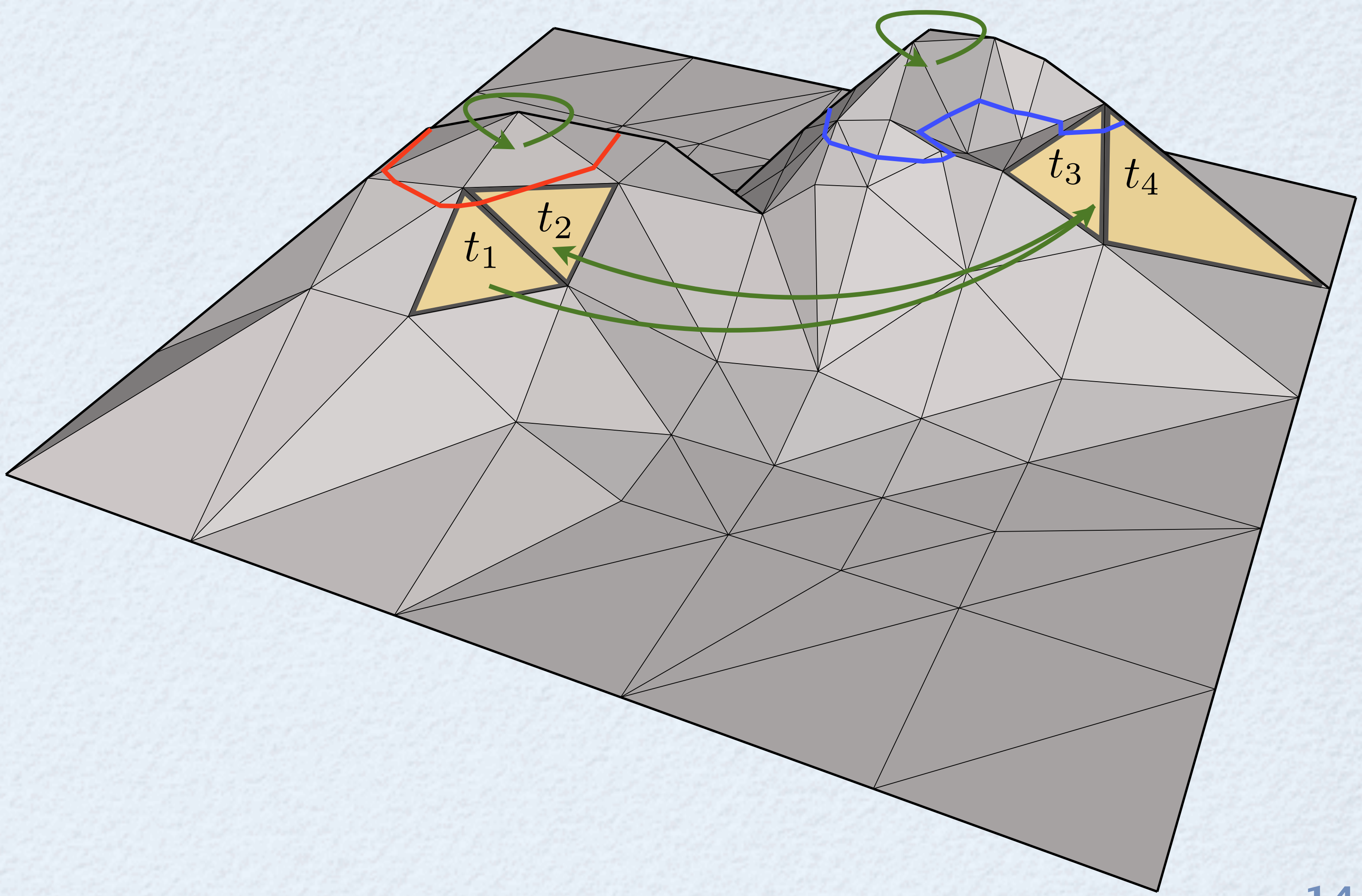

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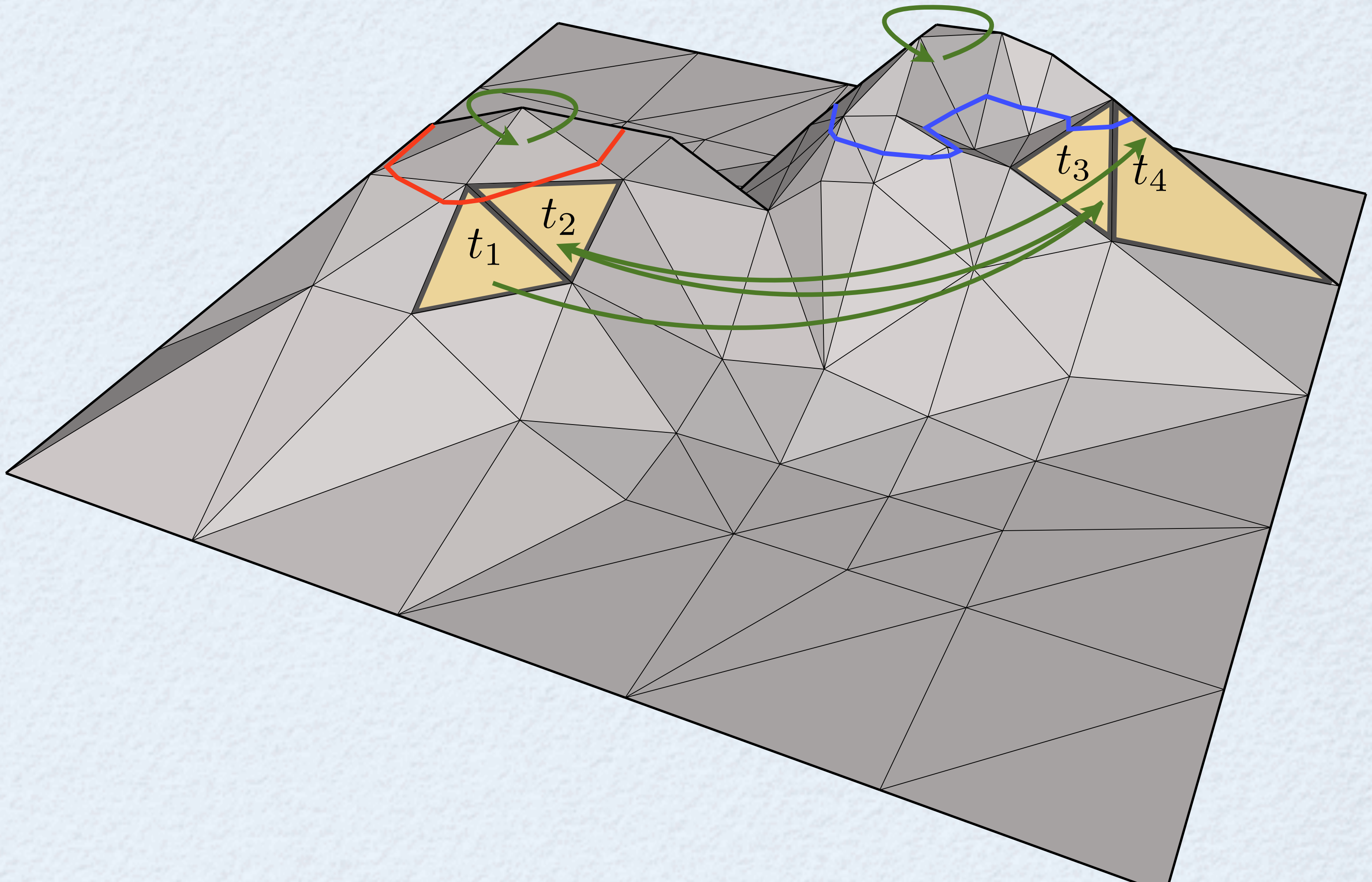

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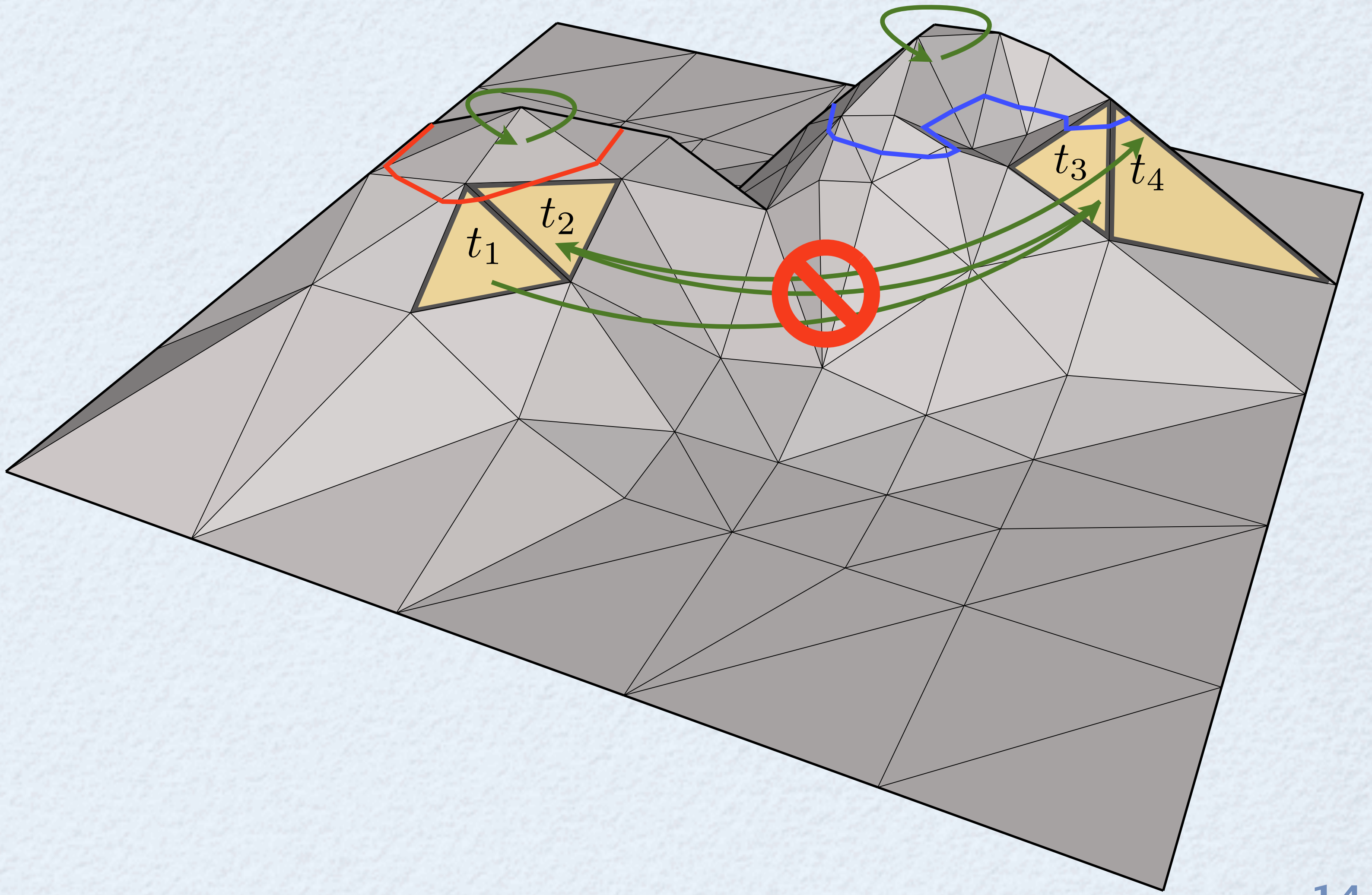

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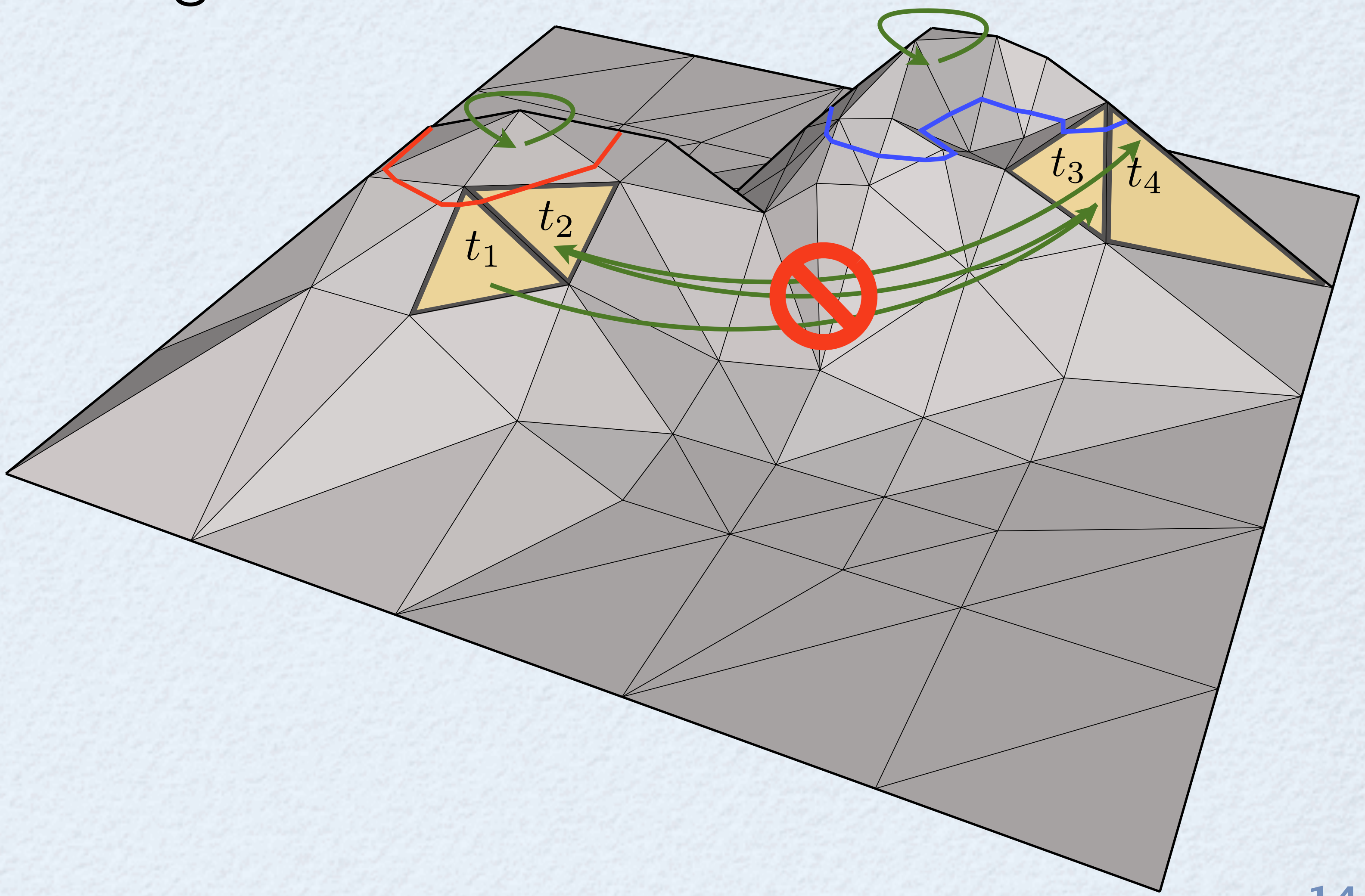

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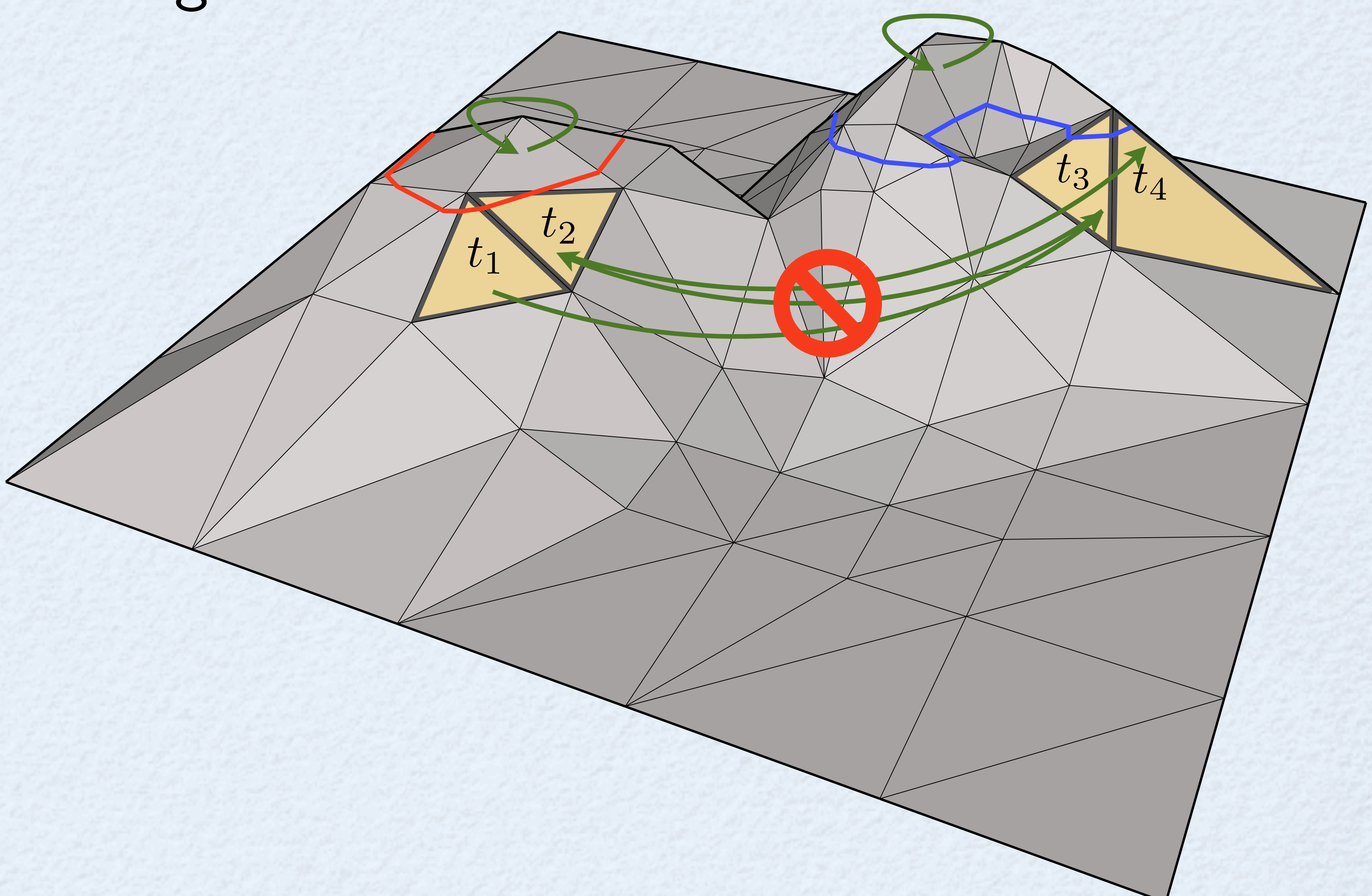

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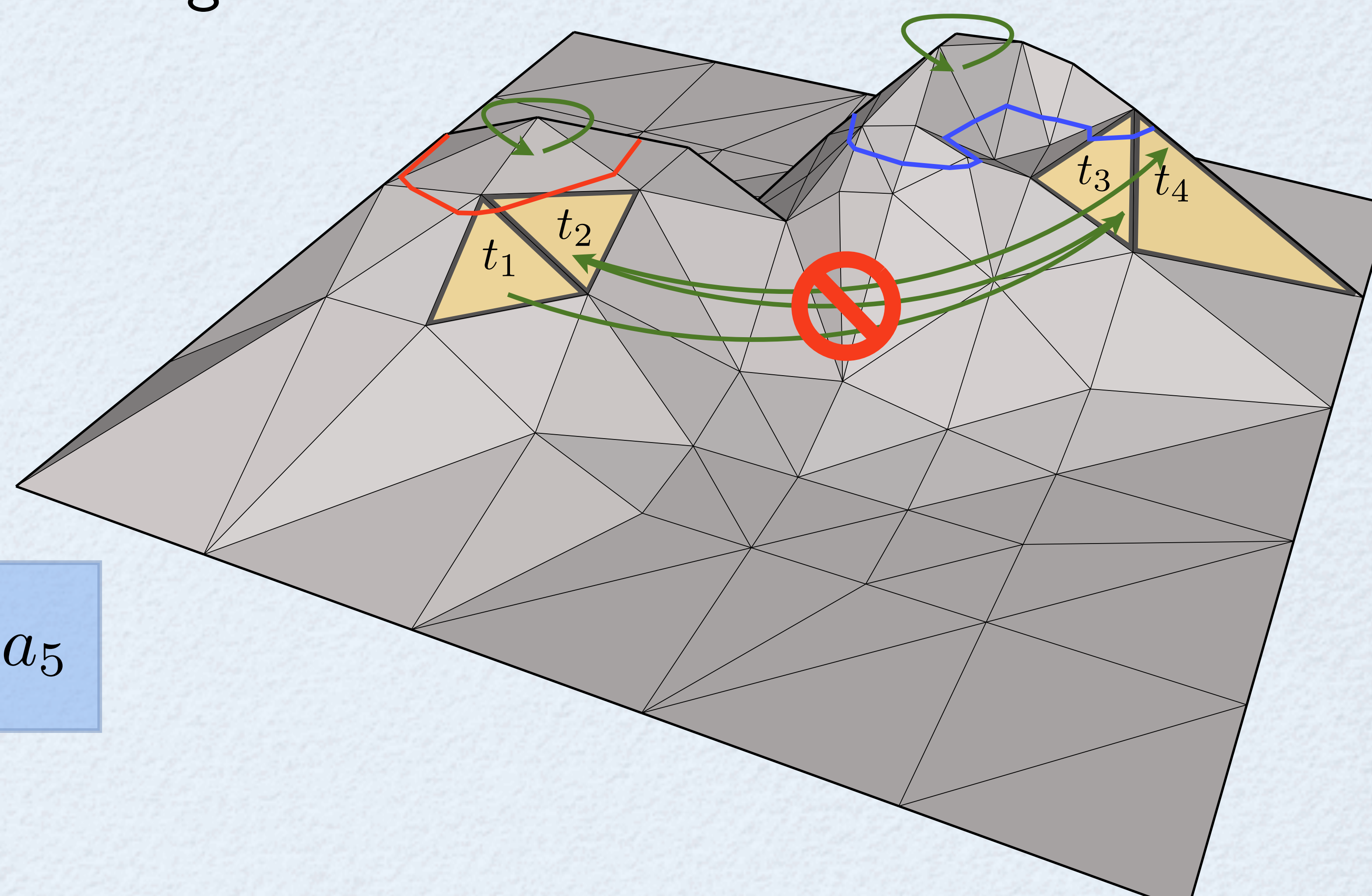

$a_1 a_2 b_1 c_1 c_2 c_3 b_2 b_3 d_1 d_2 b_4 a_3 a_4 a_5$

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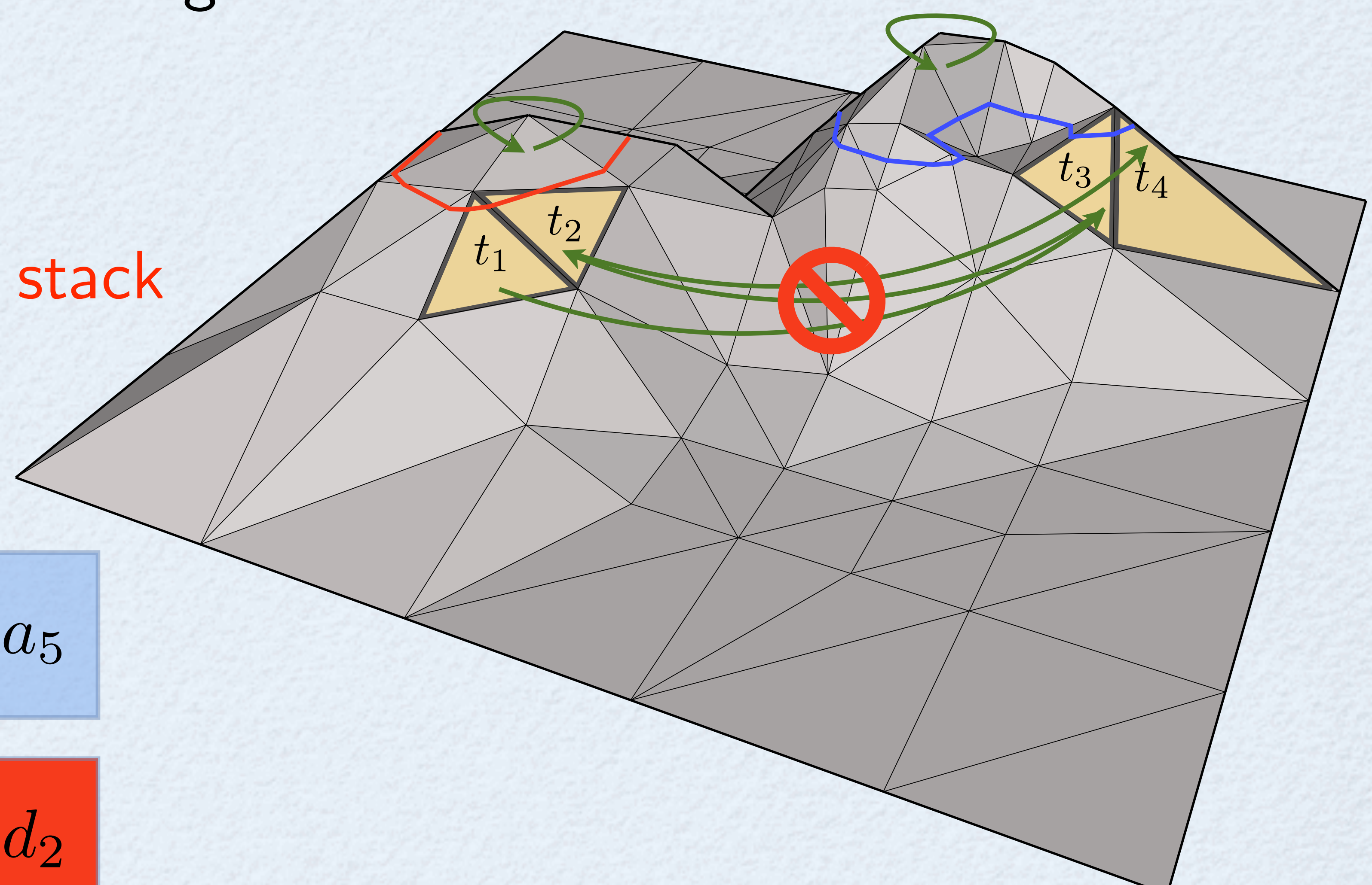


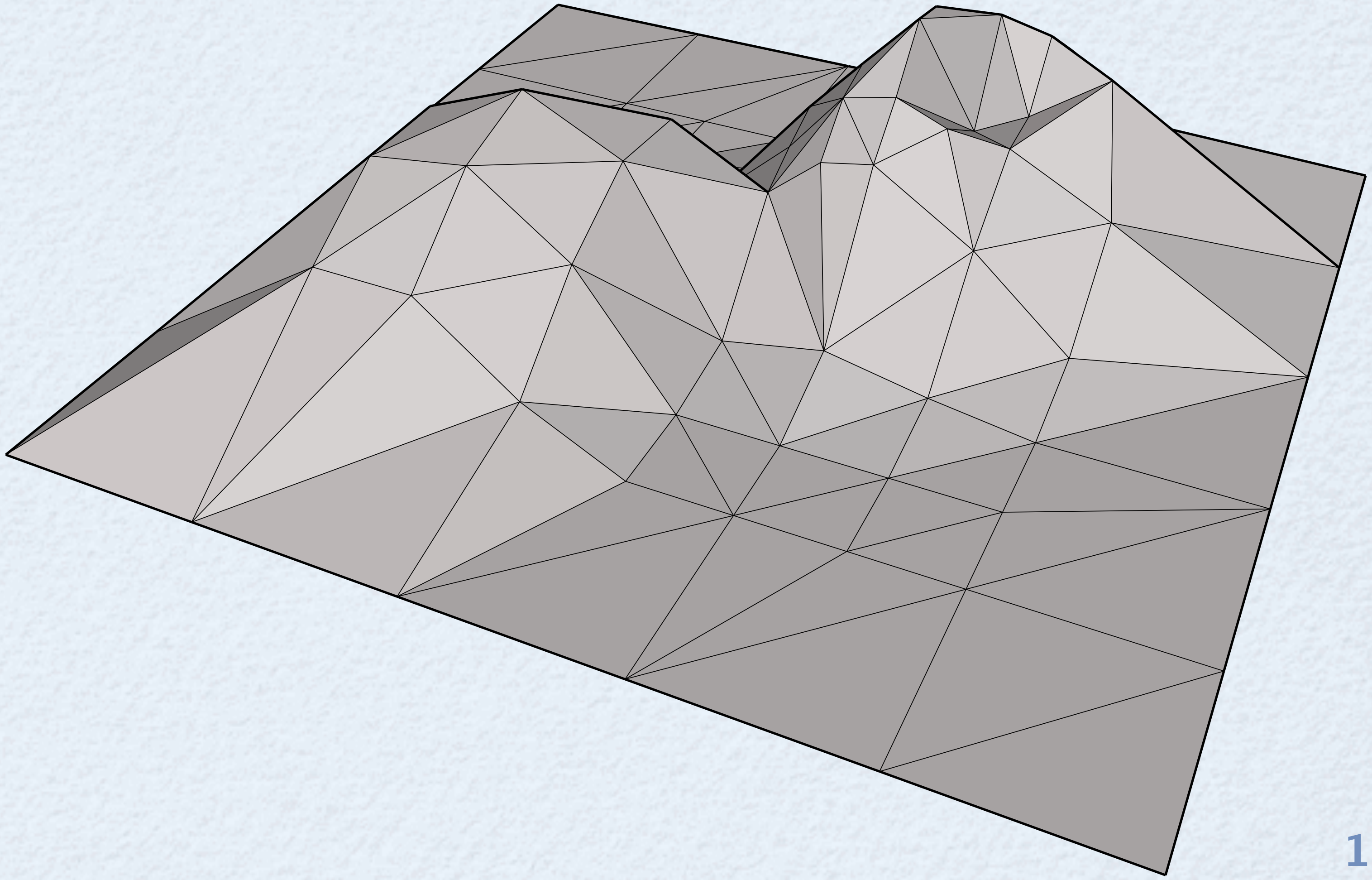
Can separate contours using a stack in $O(T/B)$ I/Os.

$a_1 a_2 b_1 c_1 c_2 c_3 b_2 b_3 d_1 d_2 b_4 a_3 a_4 a_5$

$a_1 a_2 a_3 a_4 a_5 b_1 b_2 b_3 b_4 c_1 c_2 c_3 d_1 d_2$

The Algorithm

1. Sweep the terrain by a horizontal plane in the $+z$ direction.



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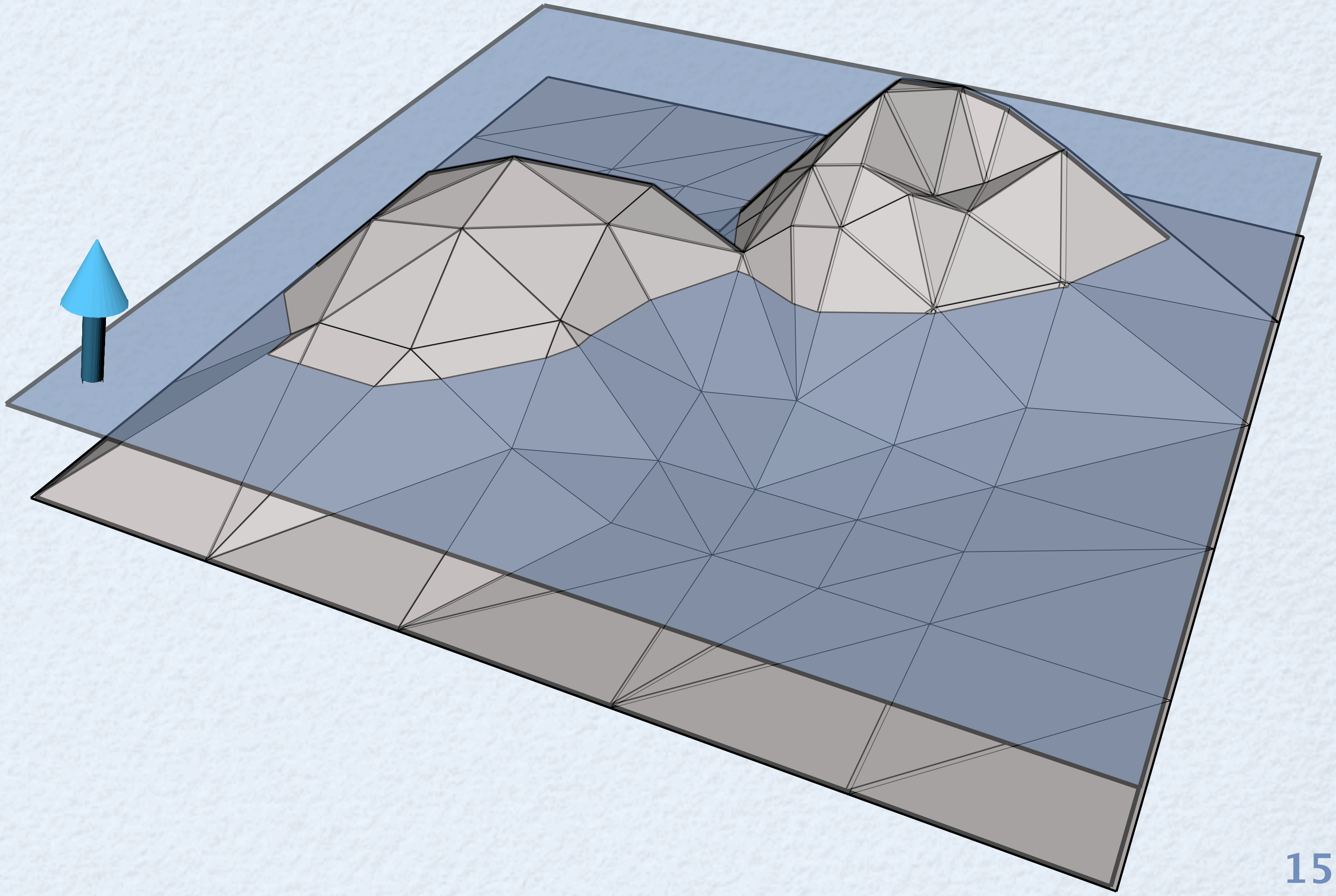

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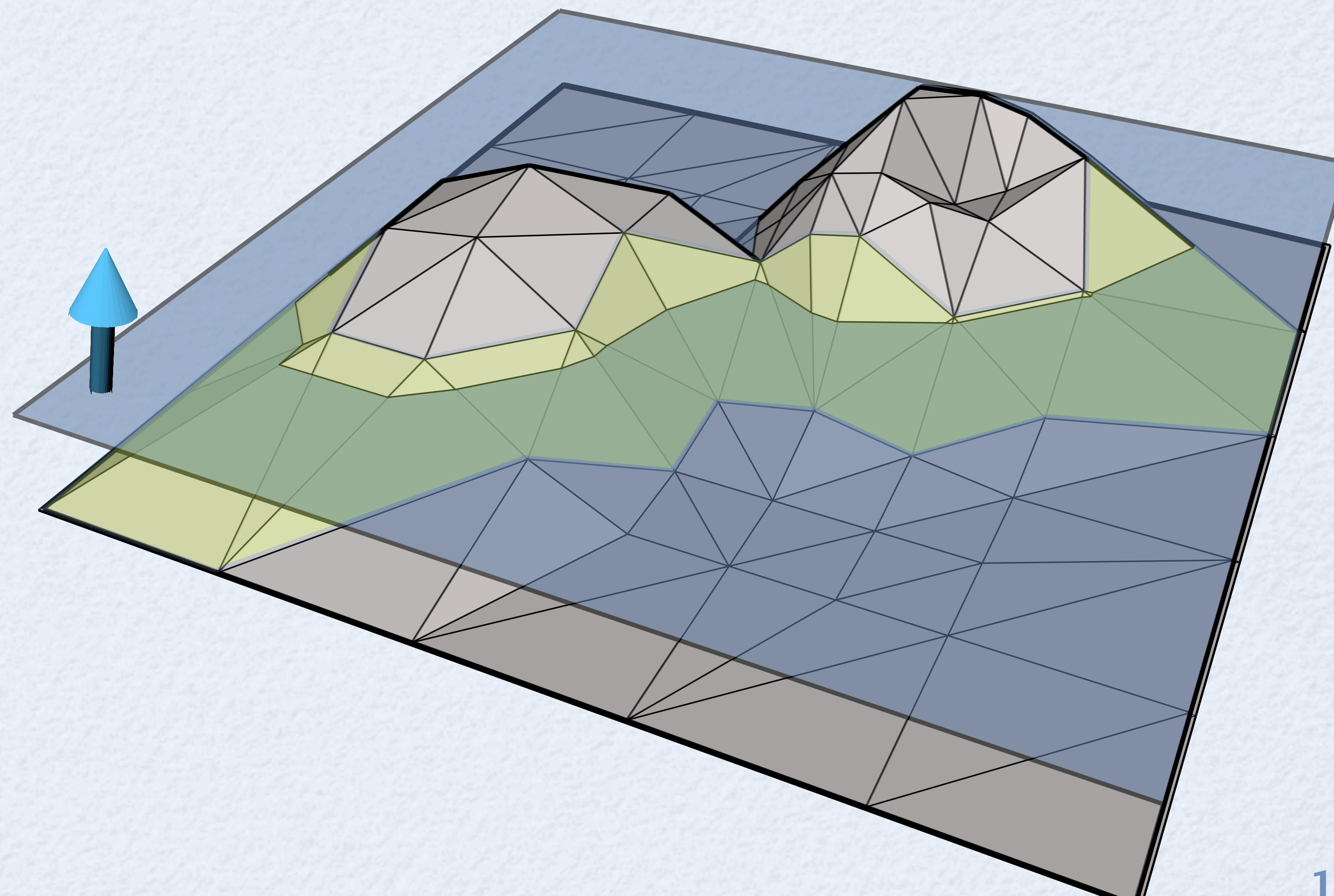

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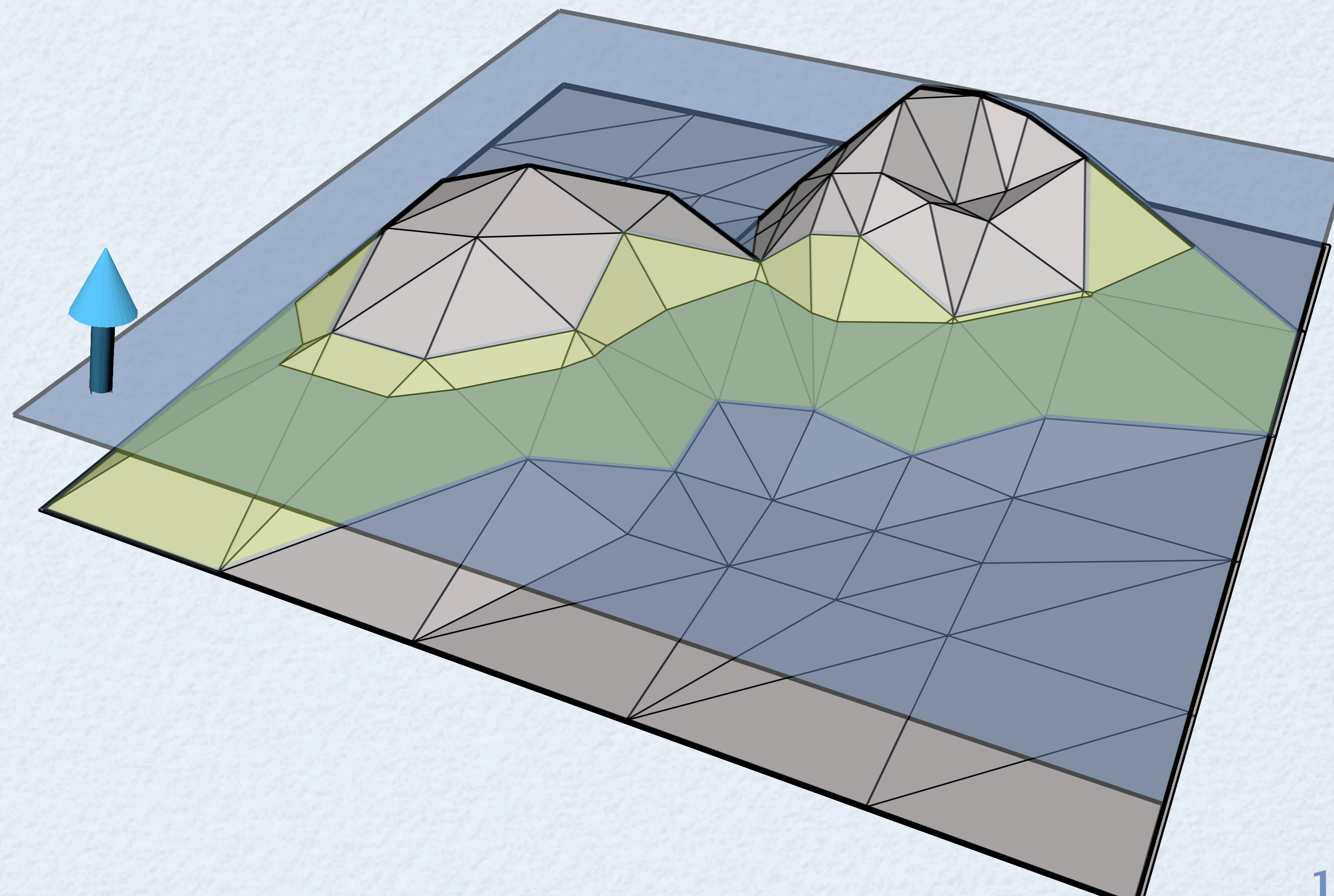
3. When passing target level $\ell_i \in L$, dump contents of tree to disk.



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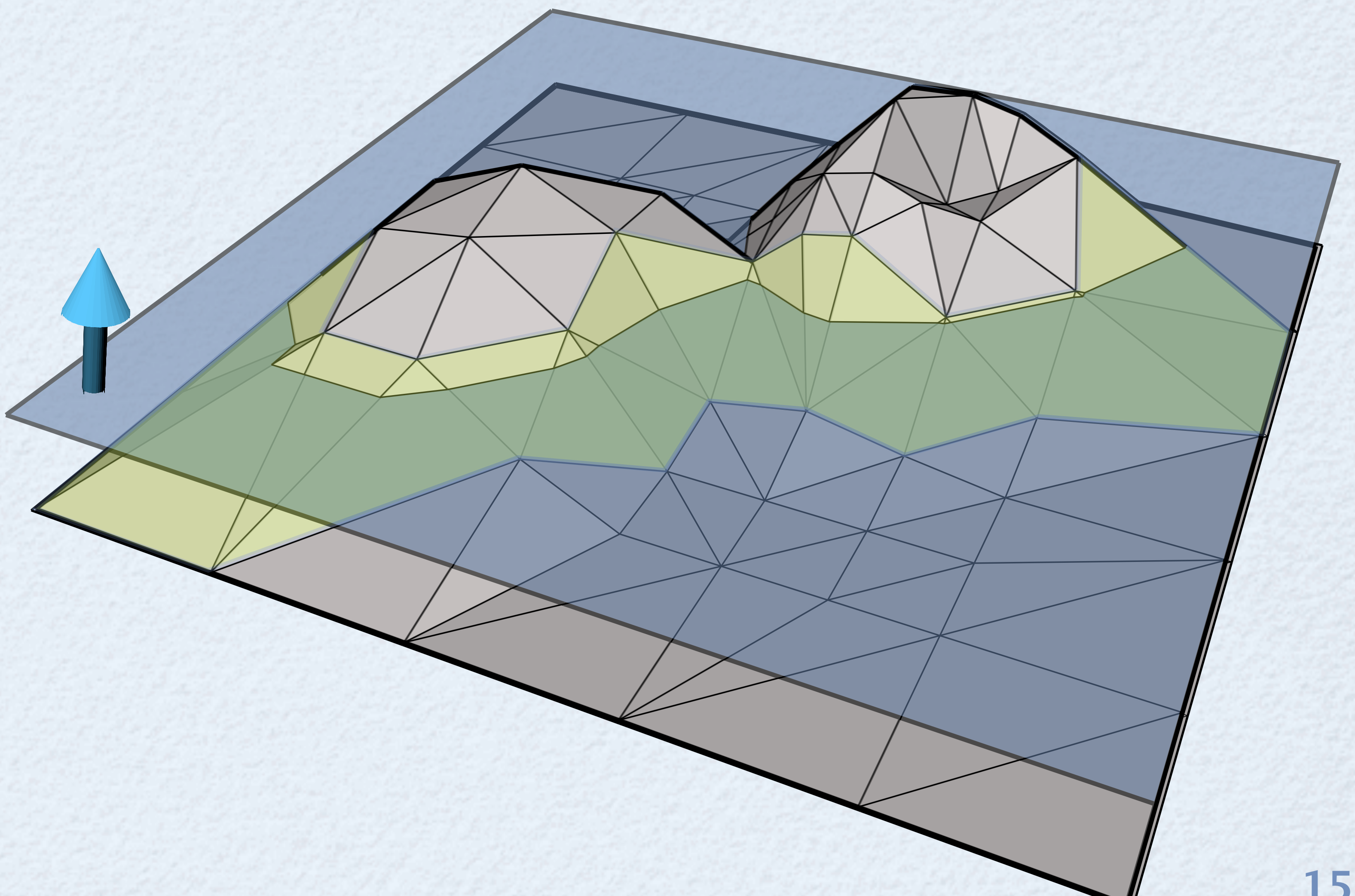

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Buffer Tree [Arge'95]

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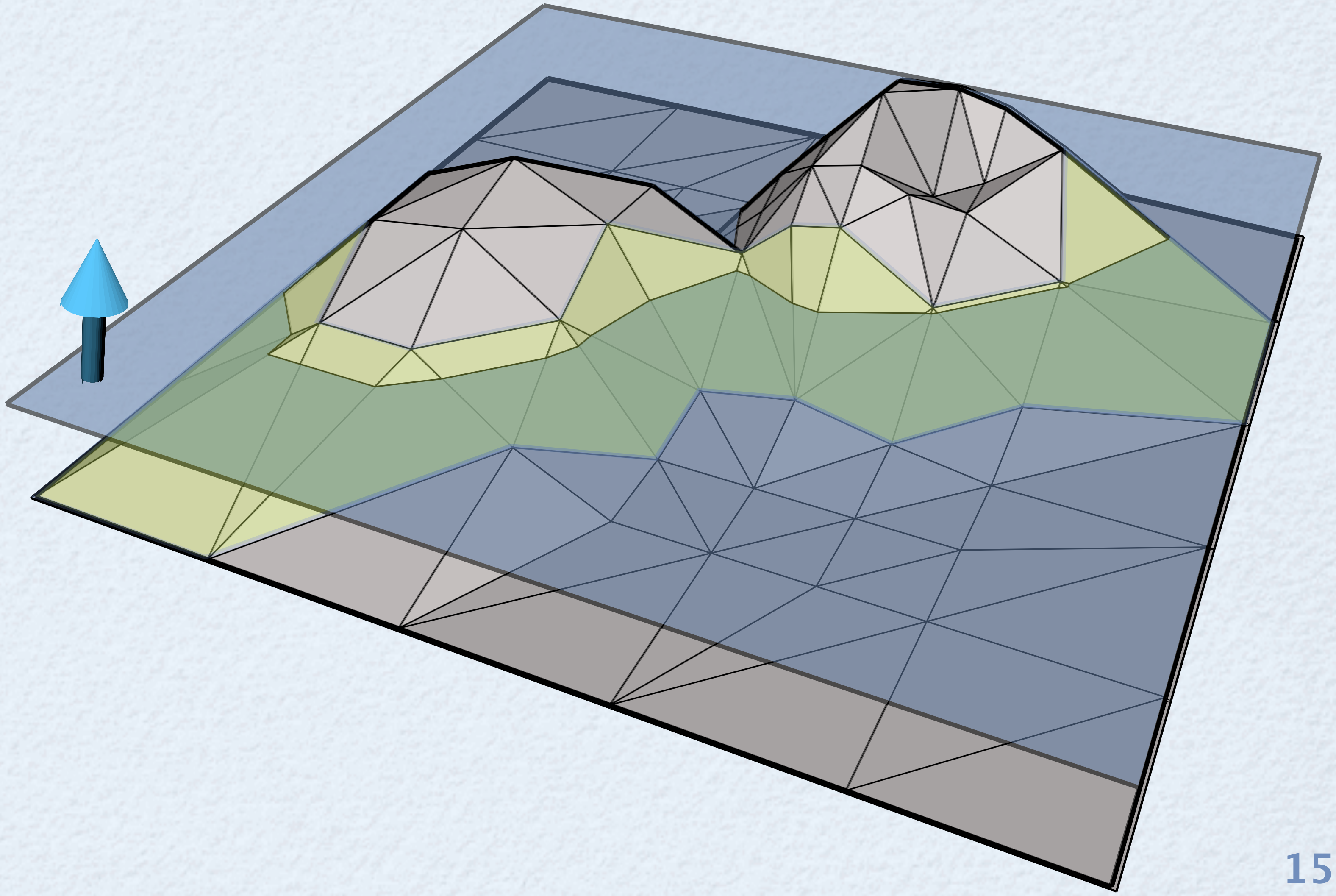

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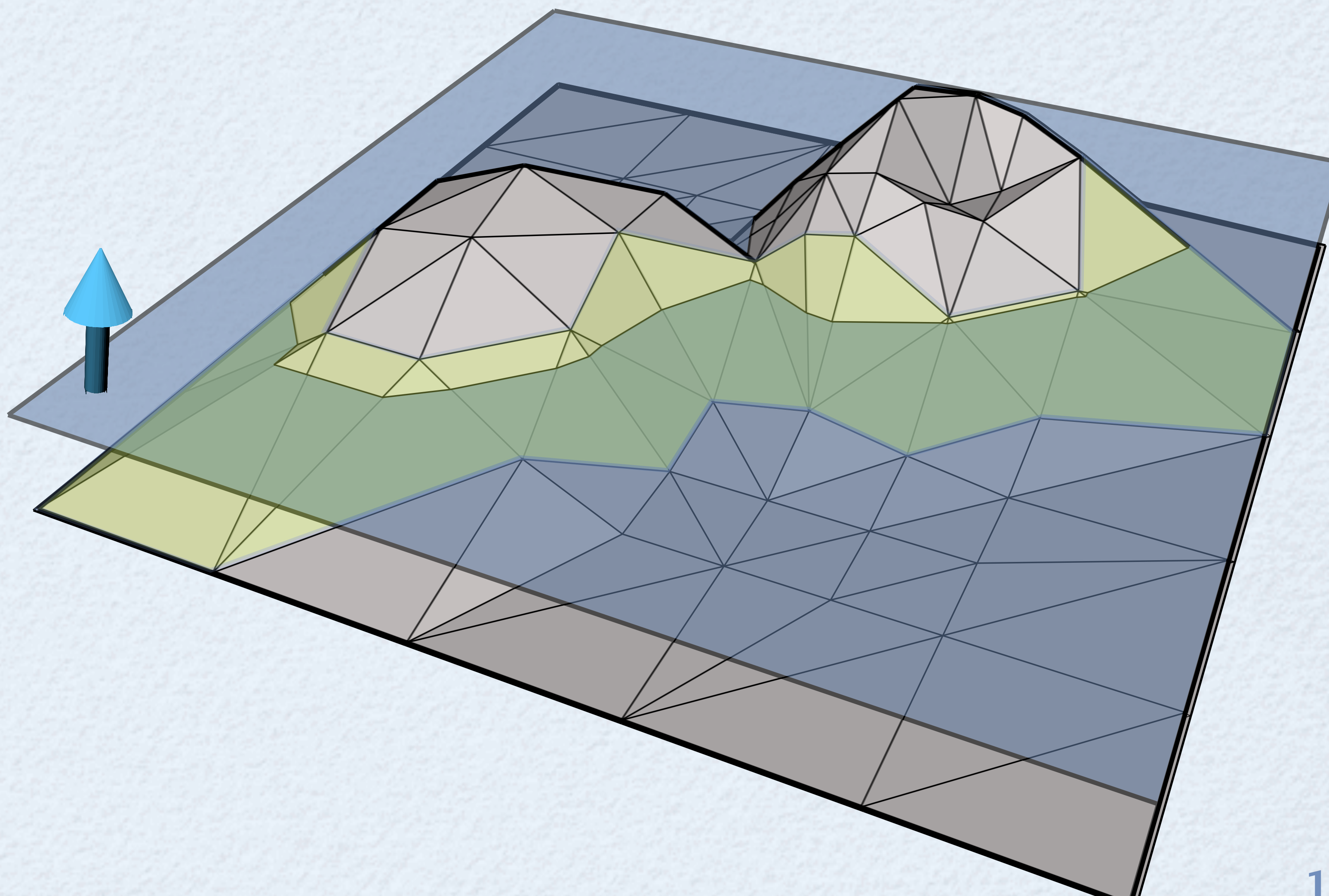

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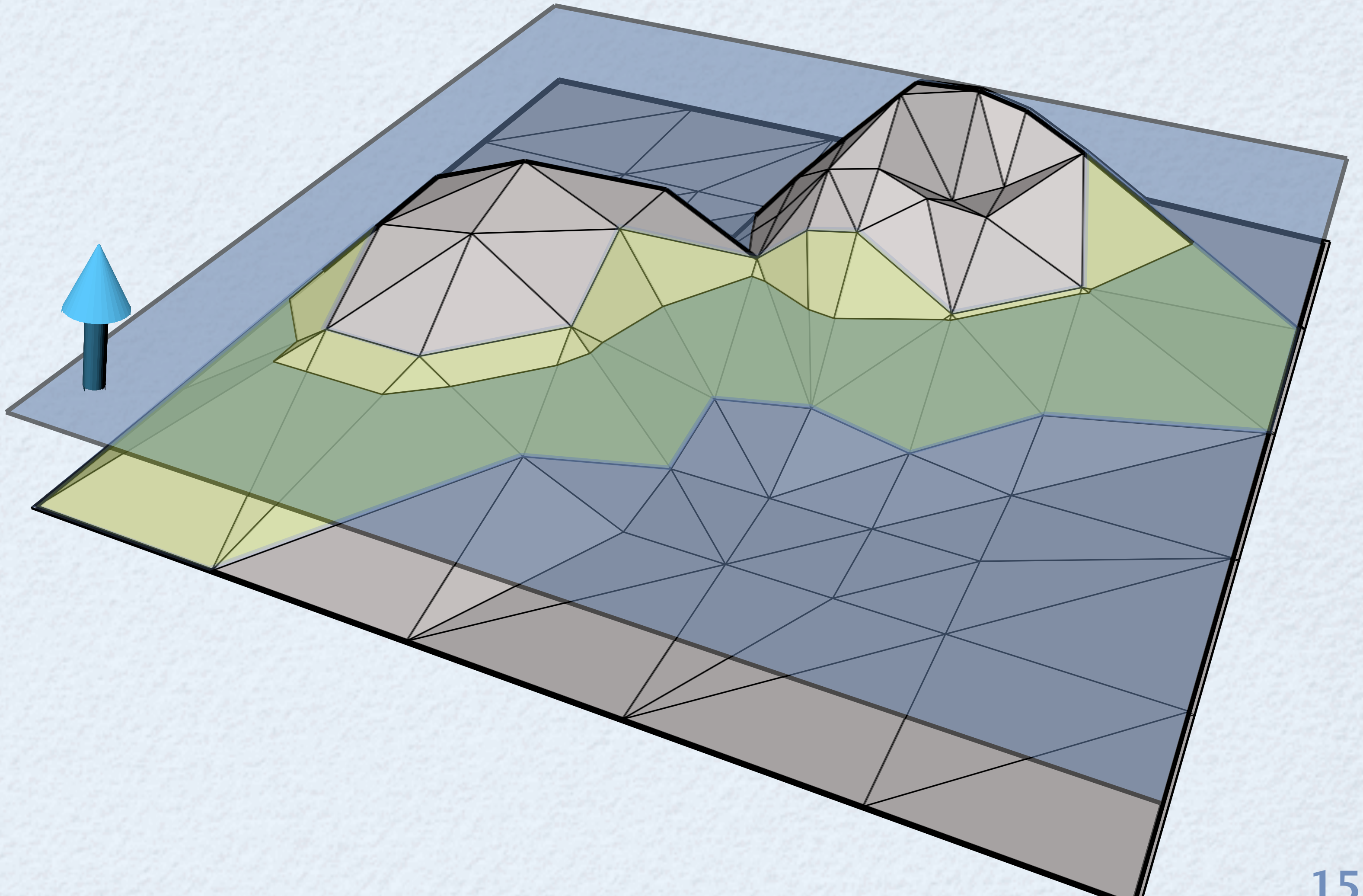

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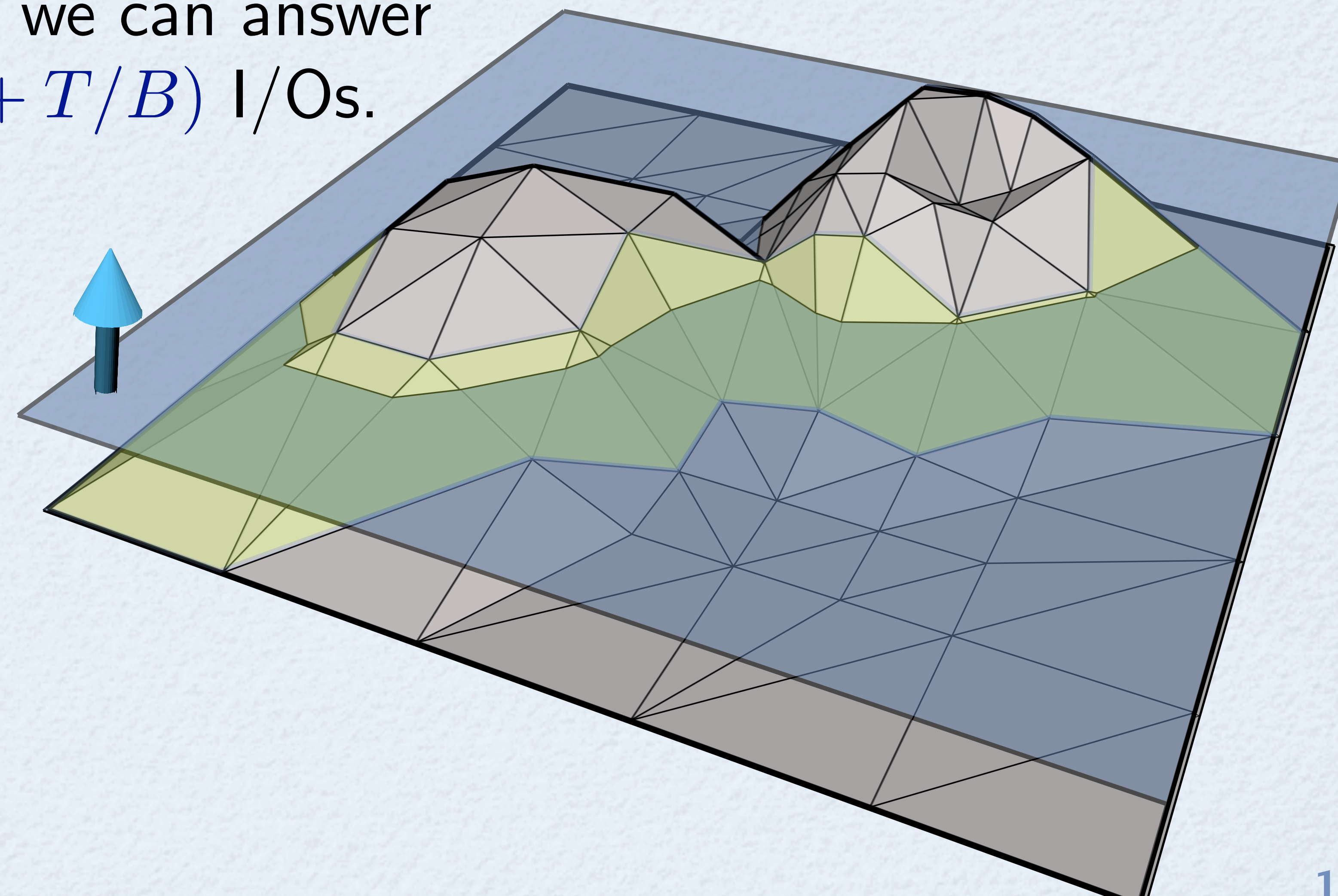
Using a persistent search tree, we can answer contour queries in $O(\log_B N + T/B)$ I/Os.



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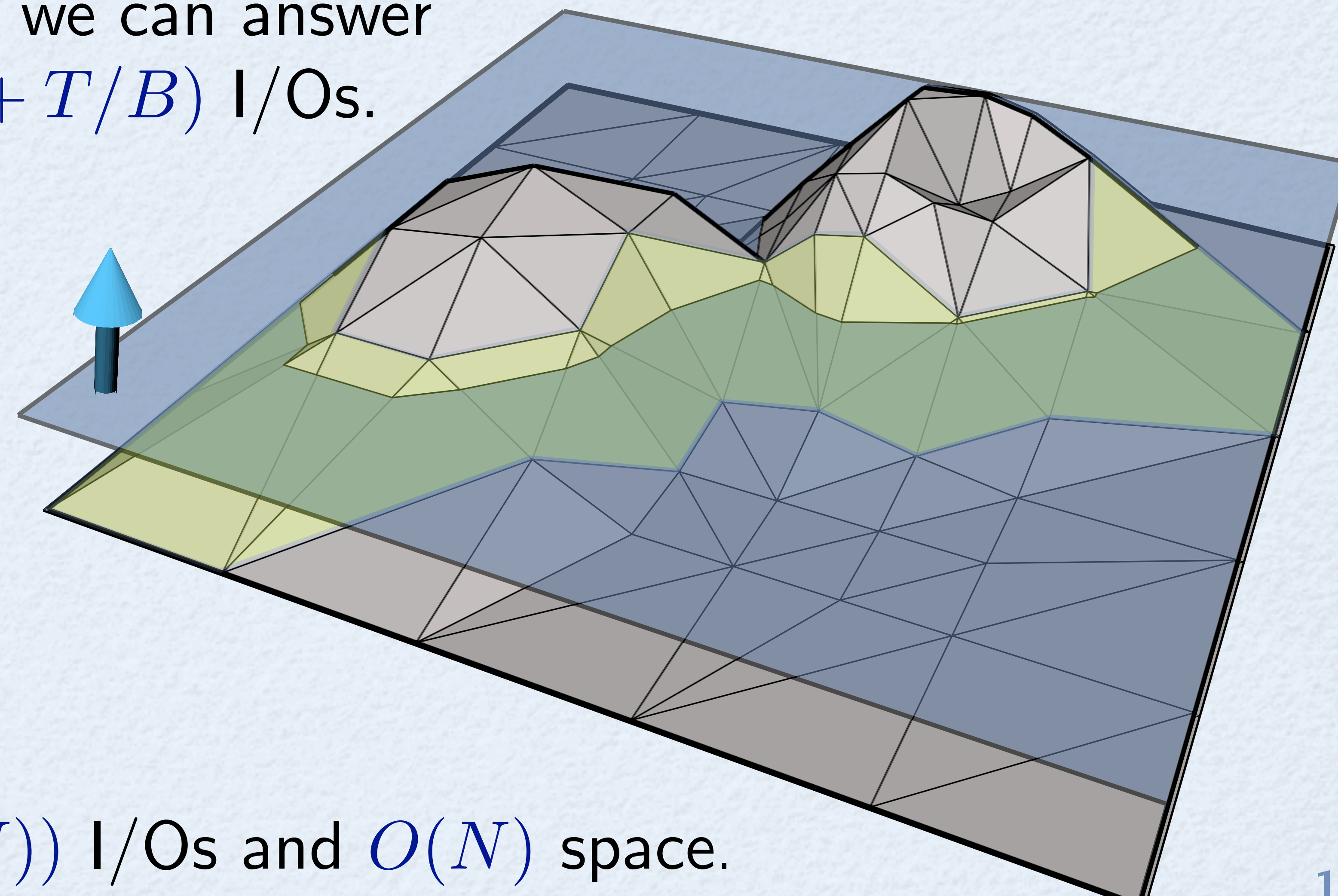

Preprocessing needs $O(\text{Sort}(N))$ I/Os and $O(N)$ space.

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The amortized cost of N intermixed insert and delete operations on an initially empty buffer tree is $O\left(\frac{N}{B}\log_{M/B}\frac{N}{B}\right) = O(\mathsf{Sort}(N))$ I/Os.

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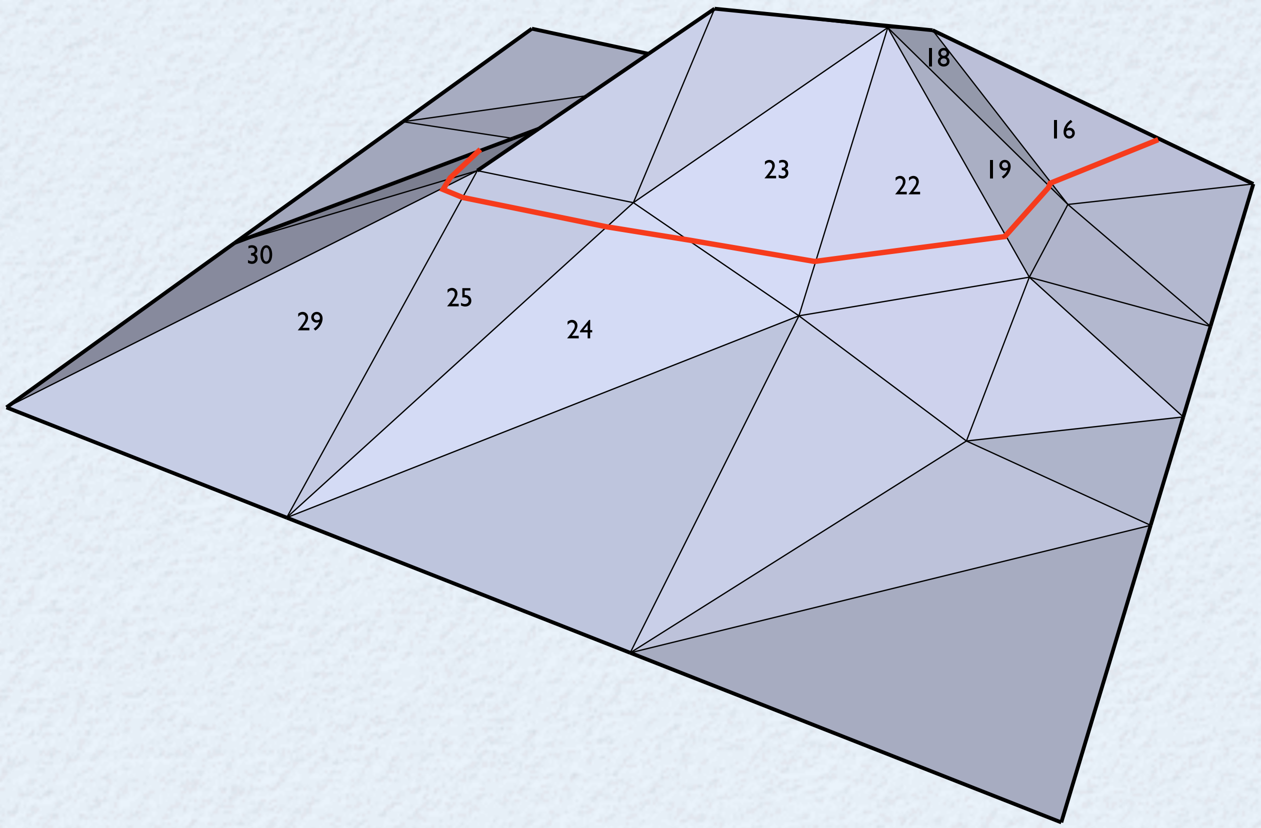
Flushing the tree writes the entire content of the tree in sorted order on disk, and this takes $O(N/B)$ I/Os.



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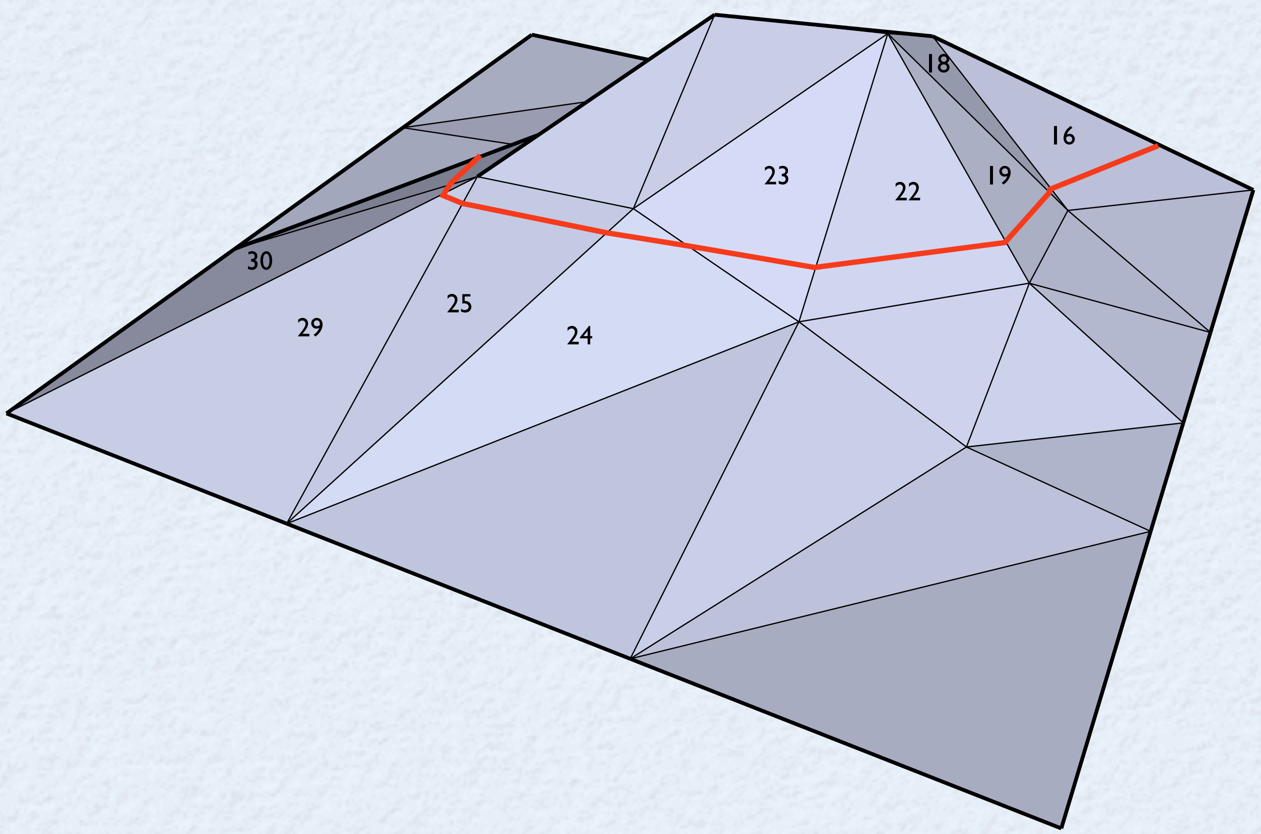
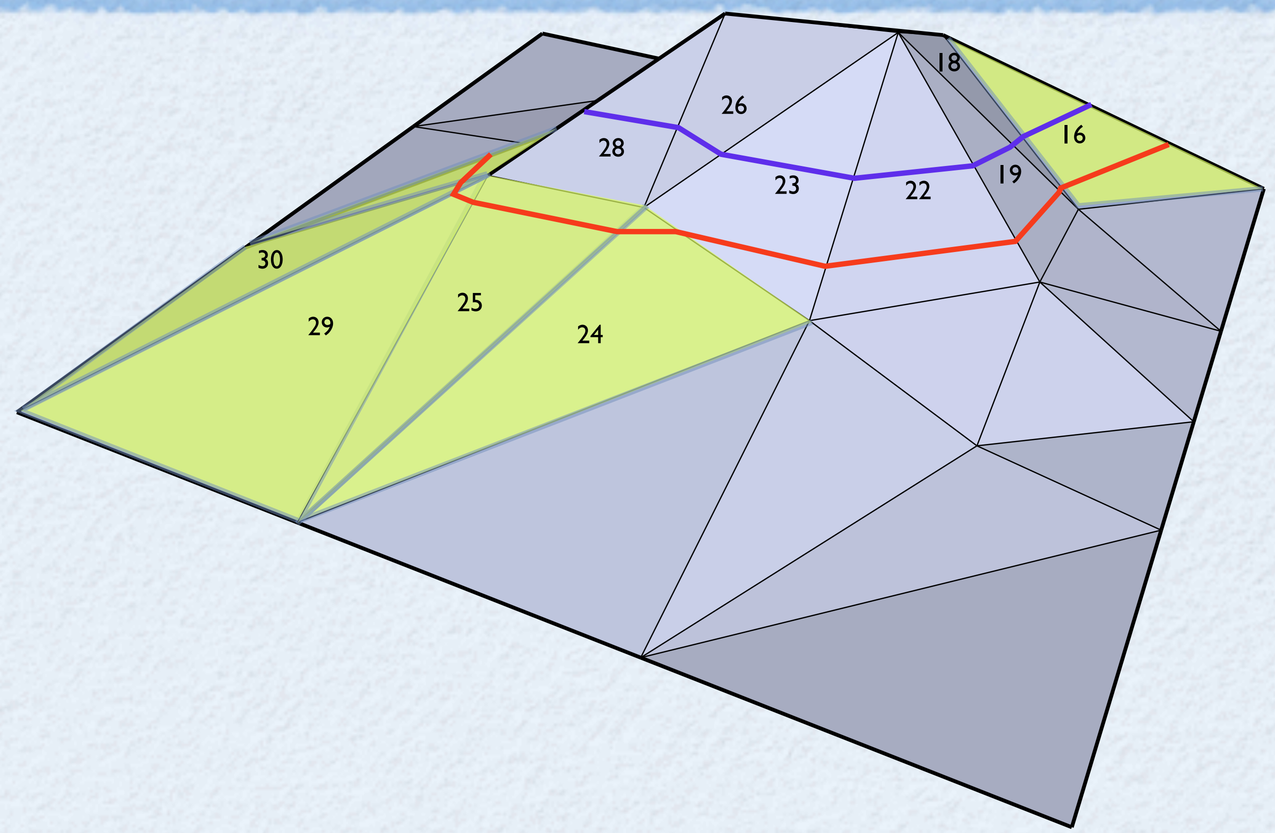


It is possible to make a persistent tree such that each version can be retrieved in $O(\log_B N)$ I/Os.

In More Detail



0. Compute a level ordering of $\mathbb{M}$ and the rank of each triangle.
1. Sort the vertices in the ascending order of their heights.
2. Let $\ell = \ell_1$ be the first level of interest in $L = \{\ell_1, \ldots, \ell_k\}$.
3. Scan the sorted list of vertices: at vertex v
4. If $h(v) > \ell$:
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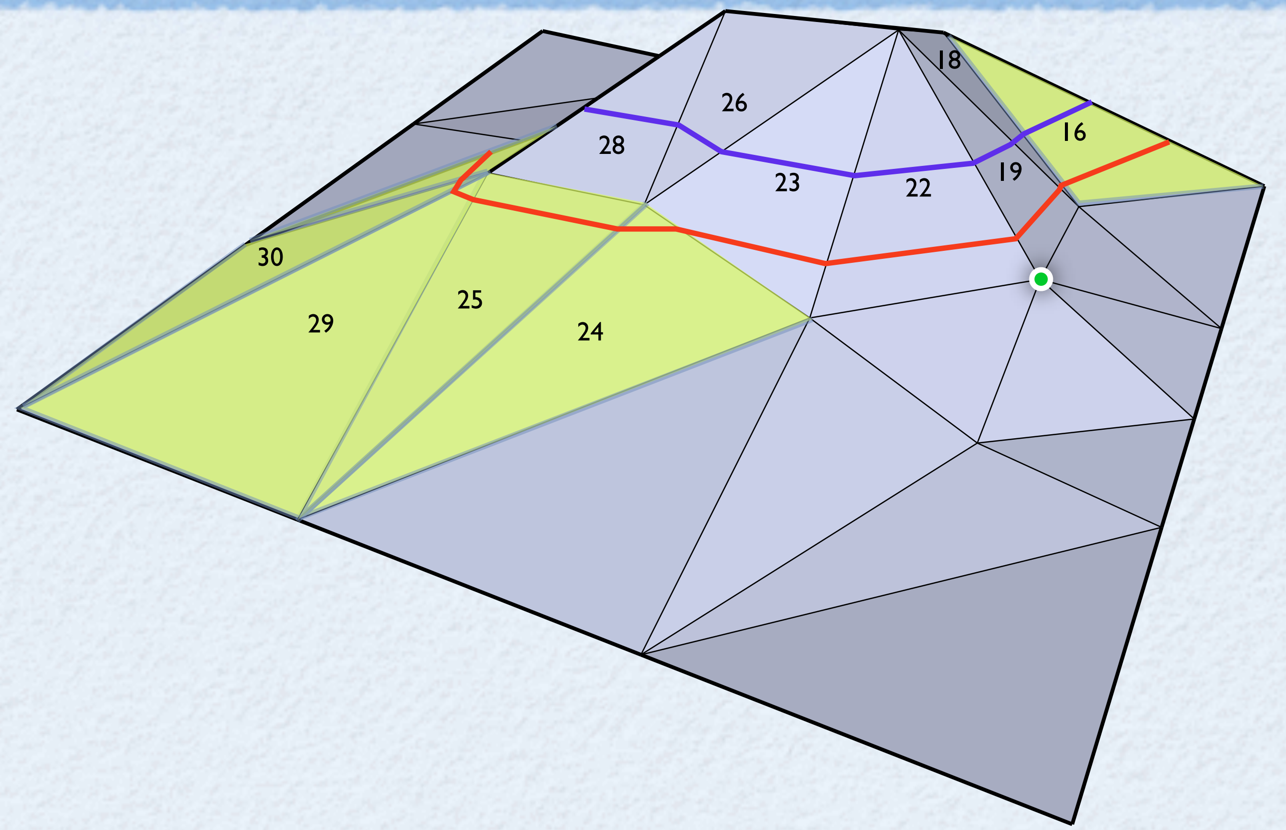

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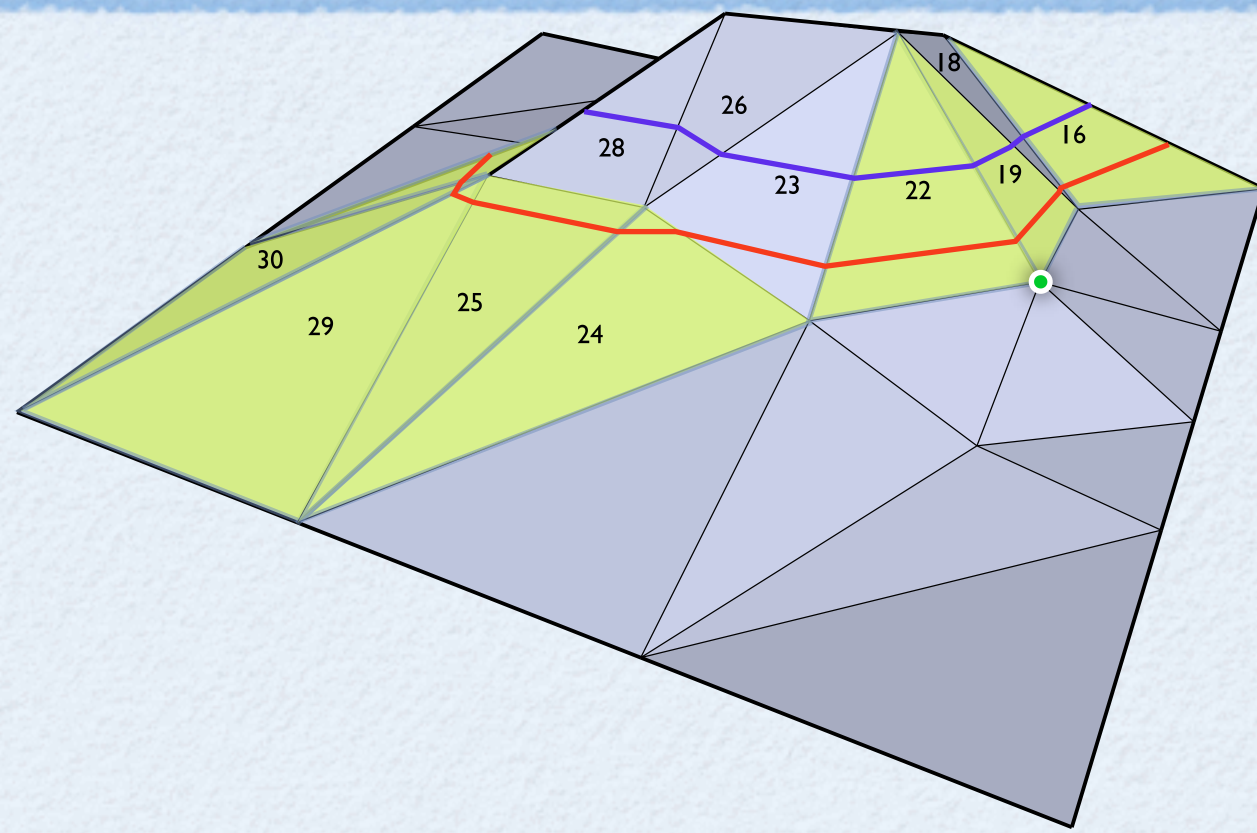

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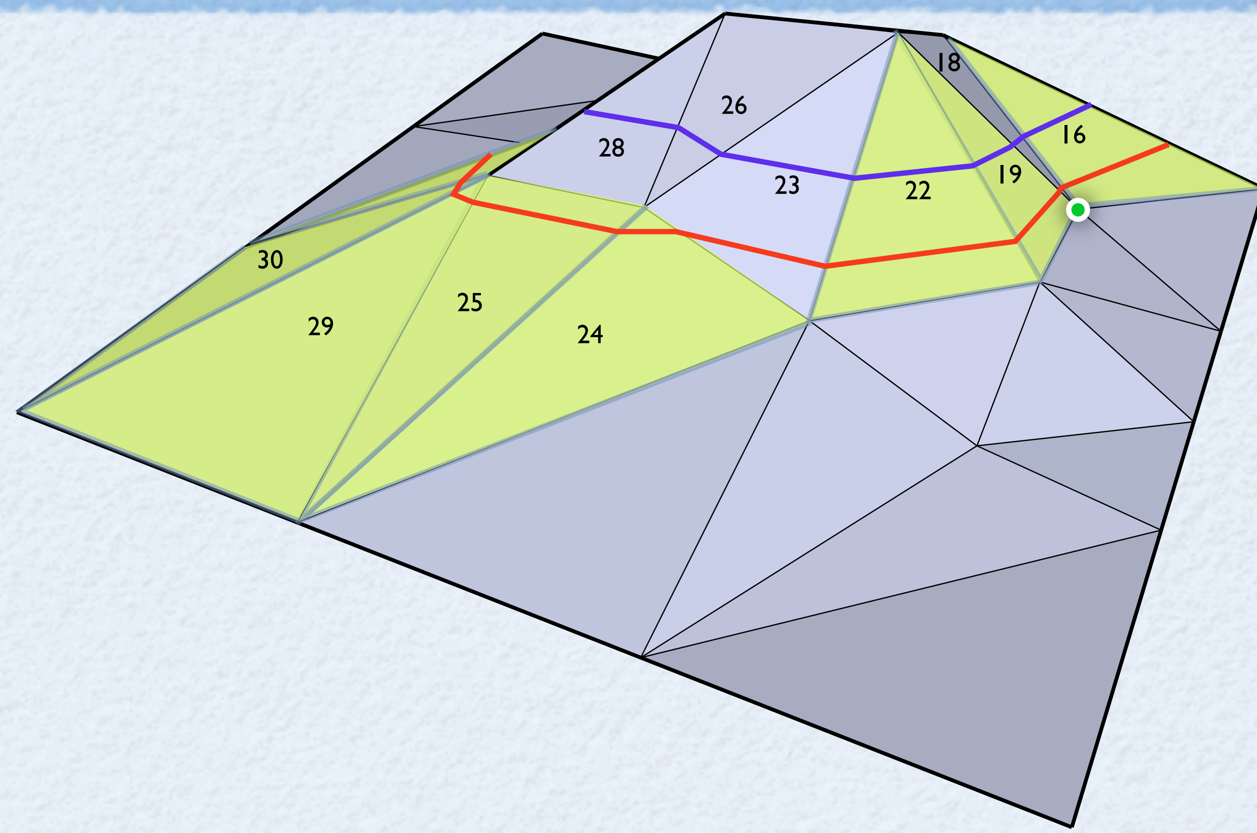

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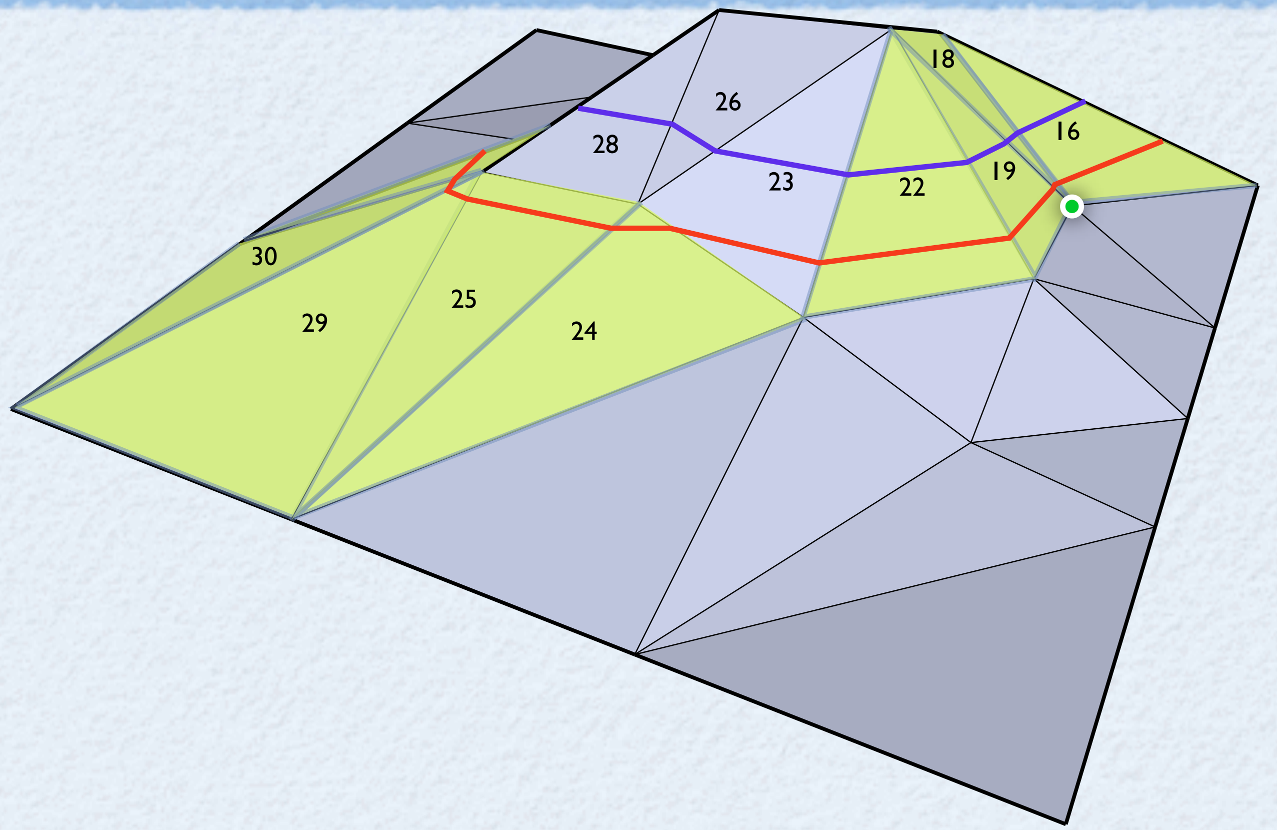

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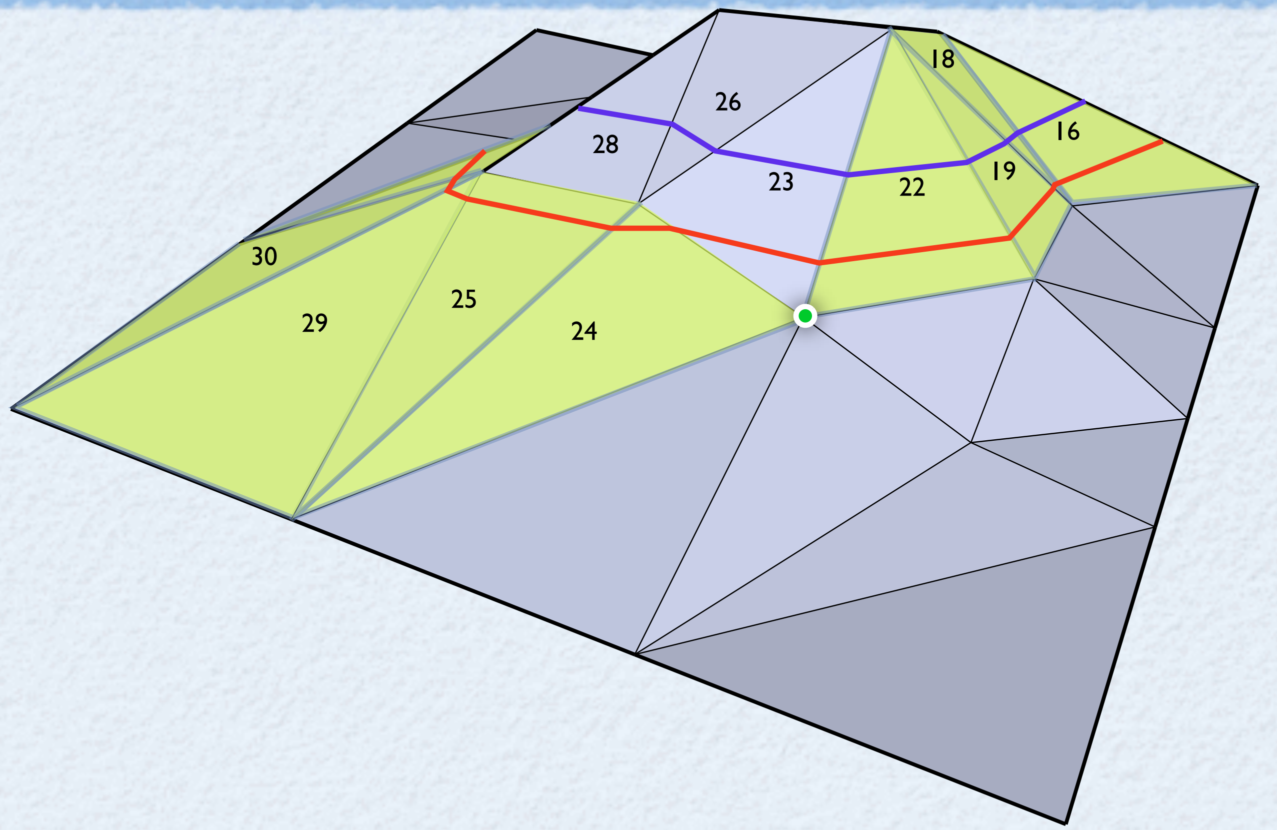

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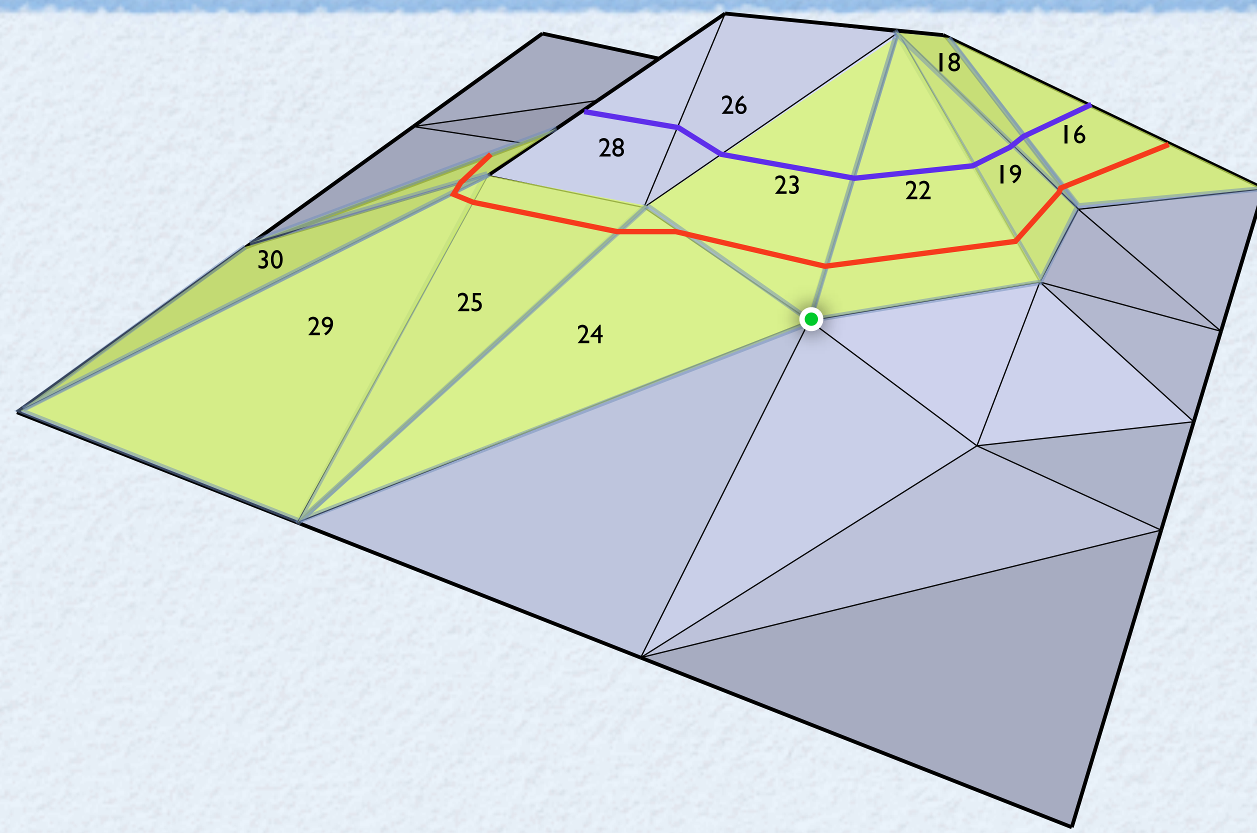

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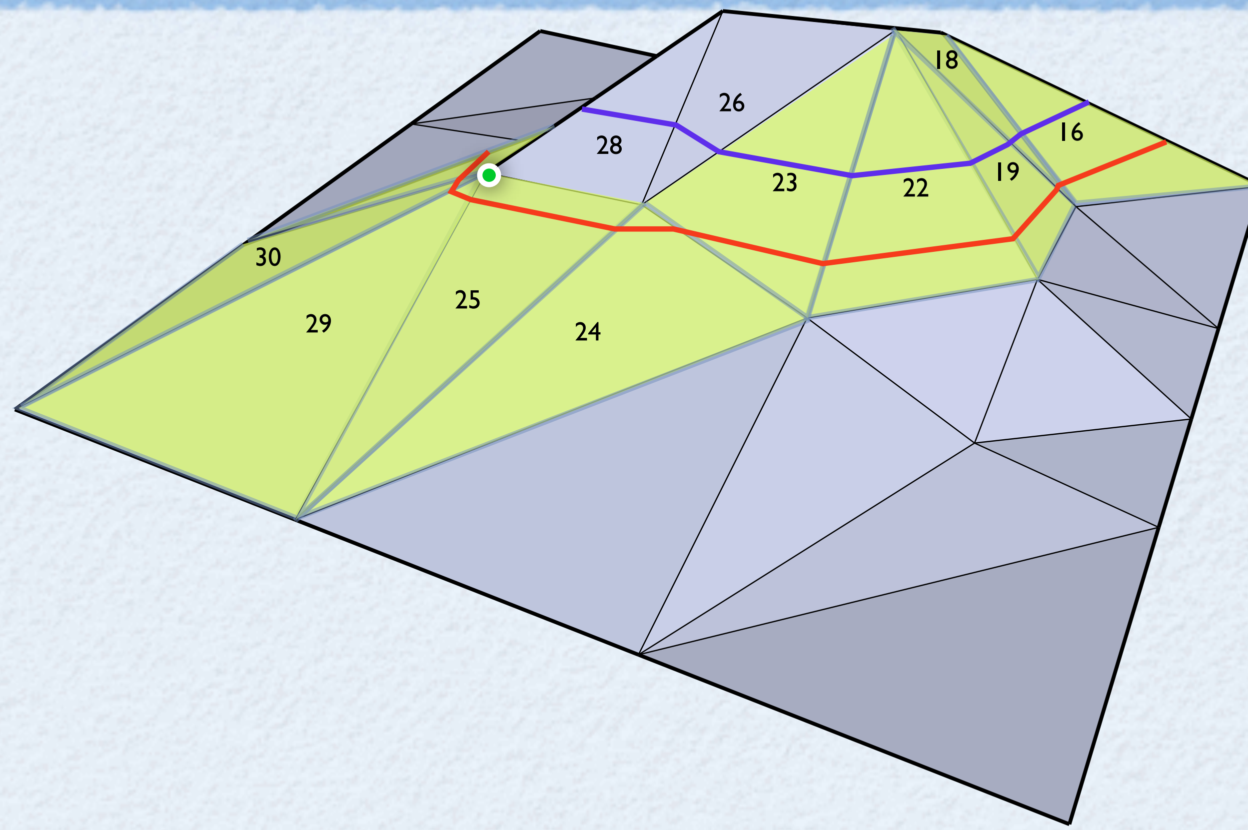

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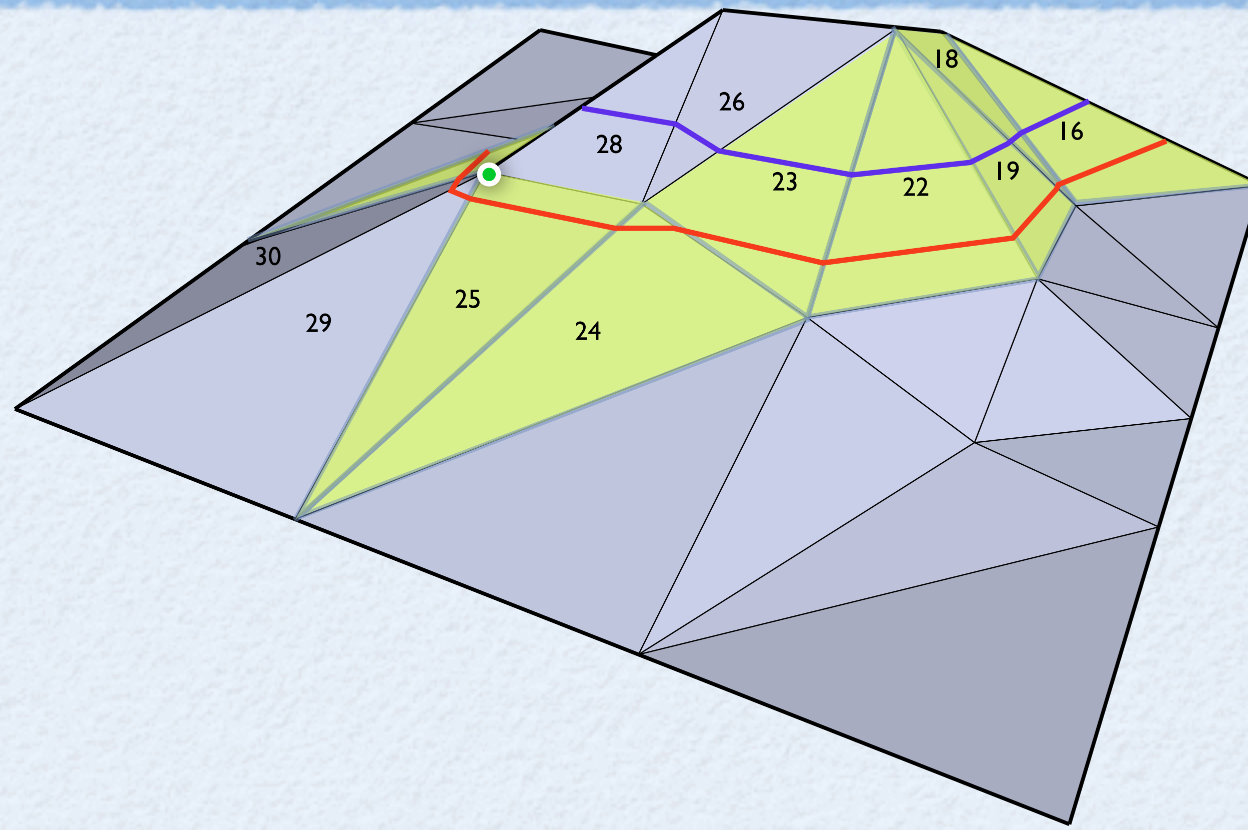

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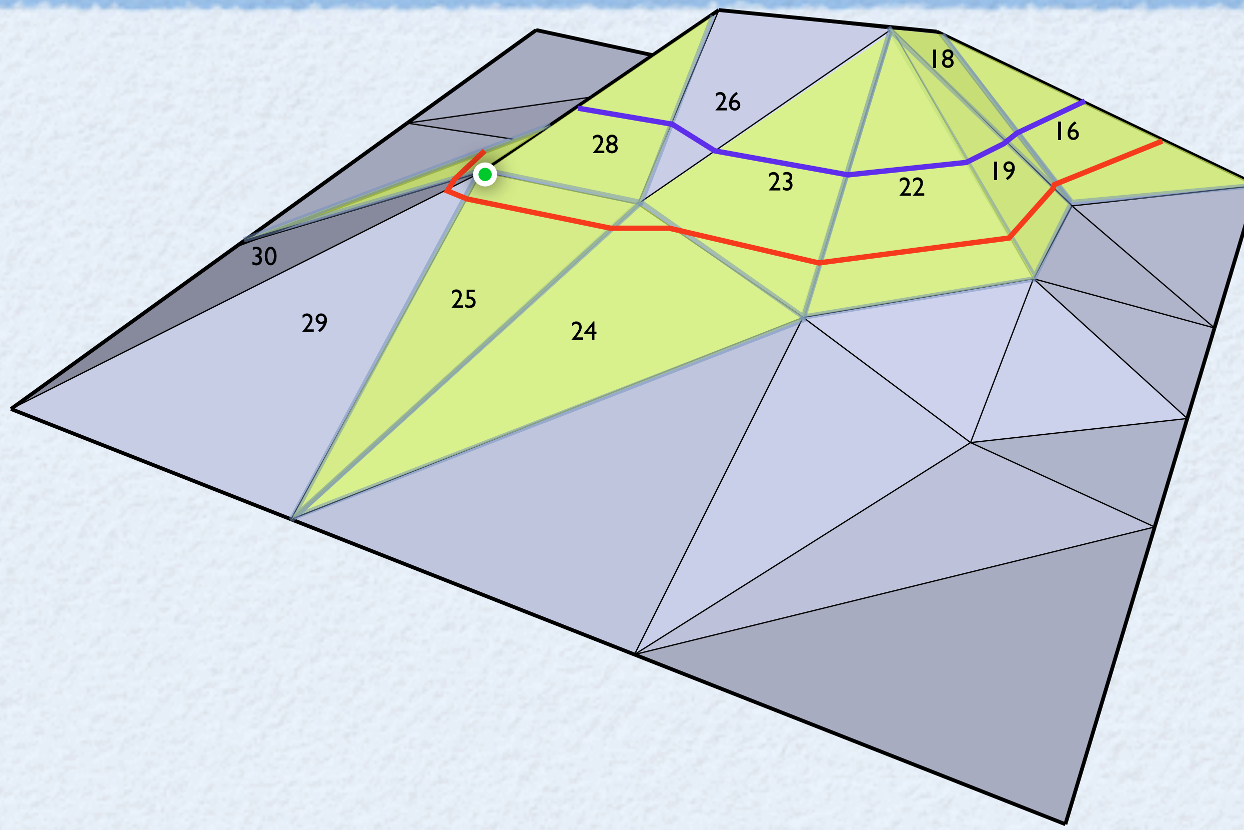

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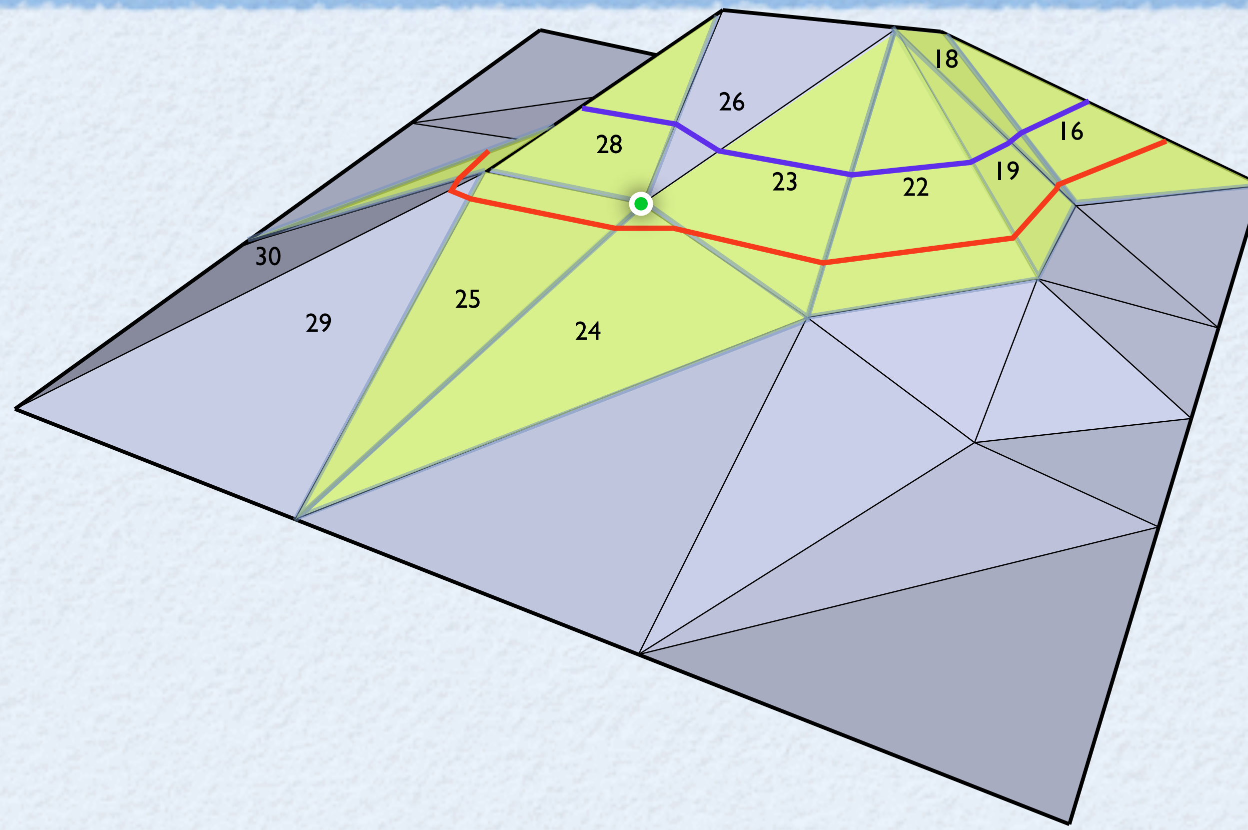

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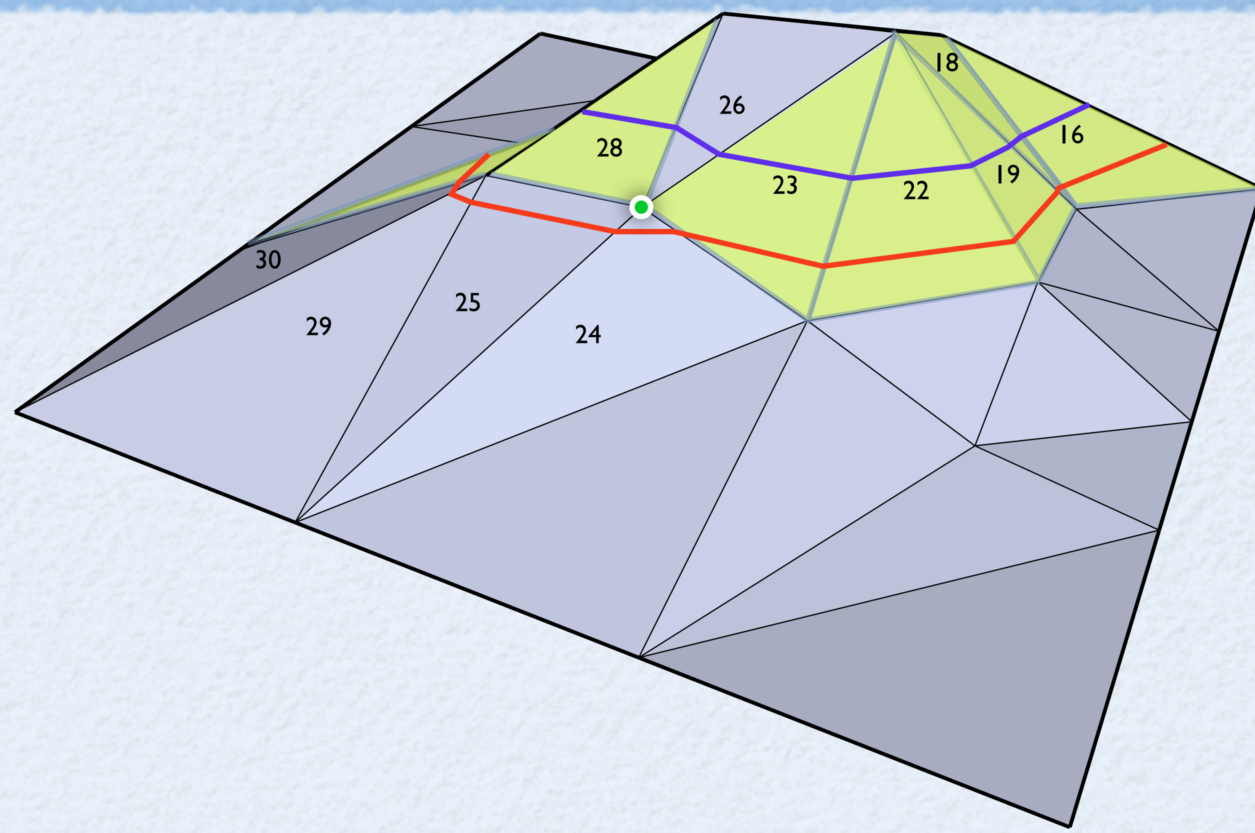

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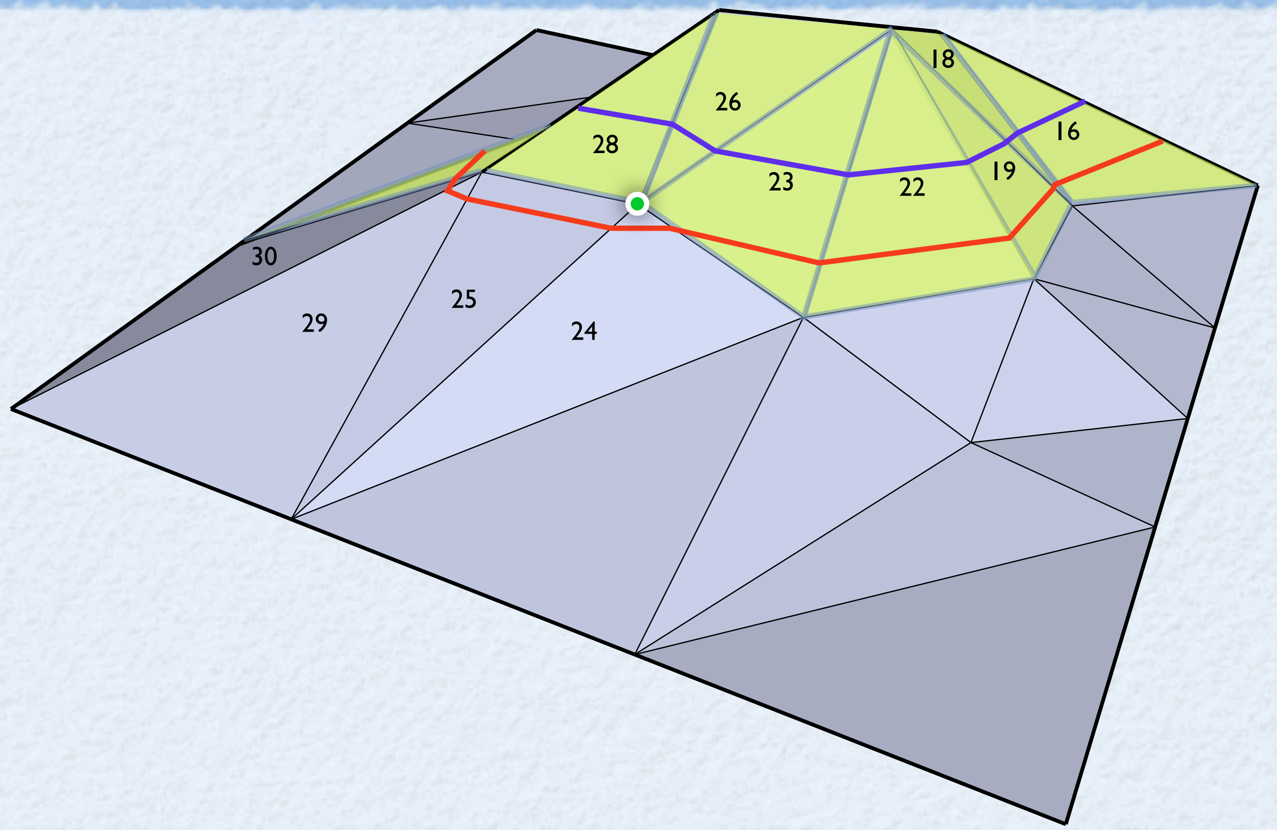

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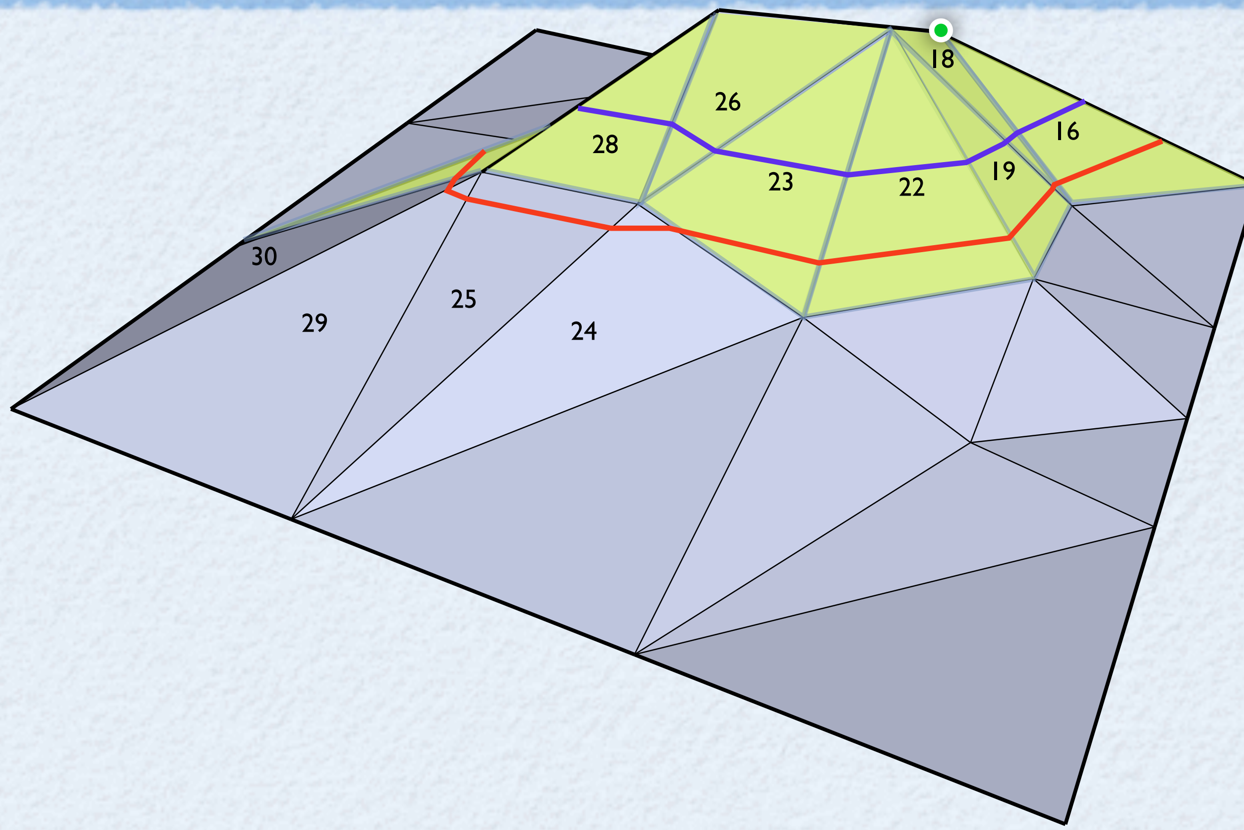

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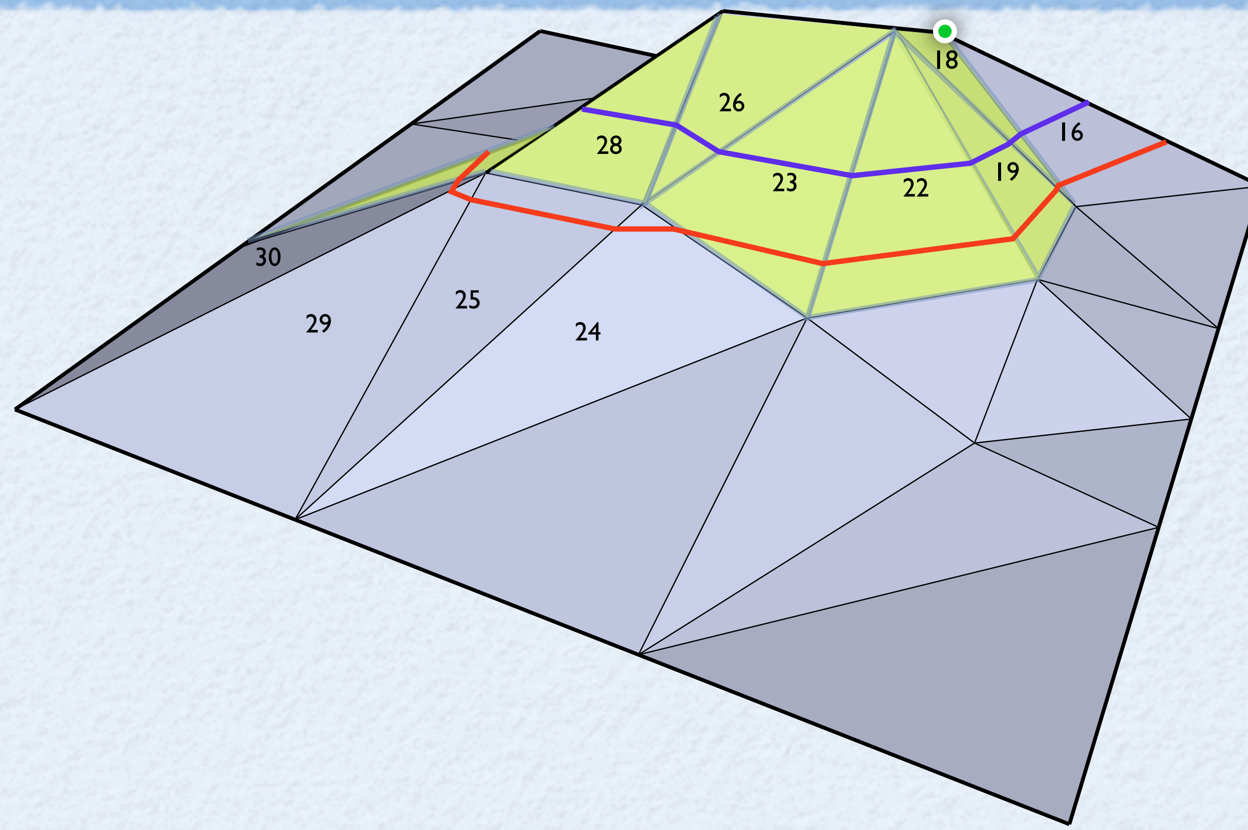

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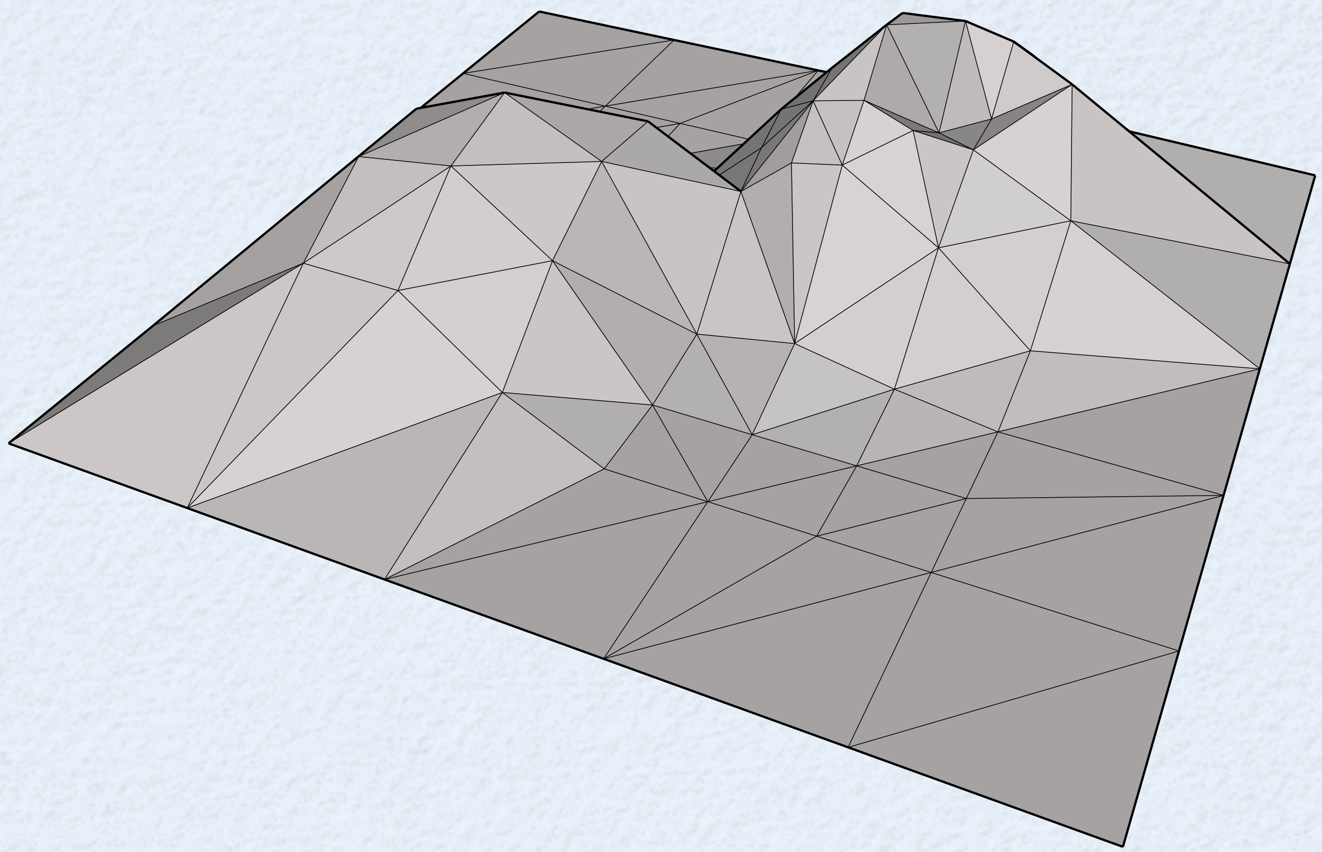
Critical Points of a Terrain



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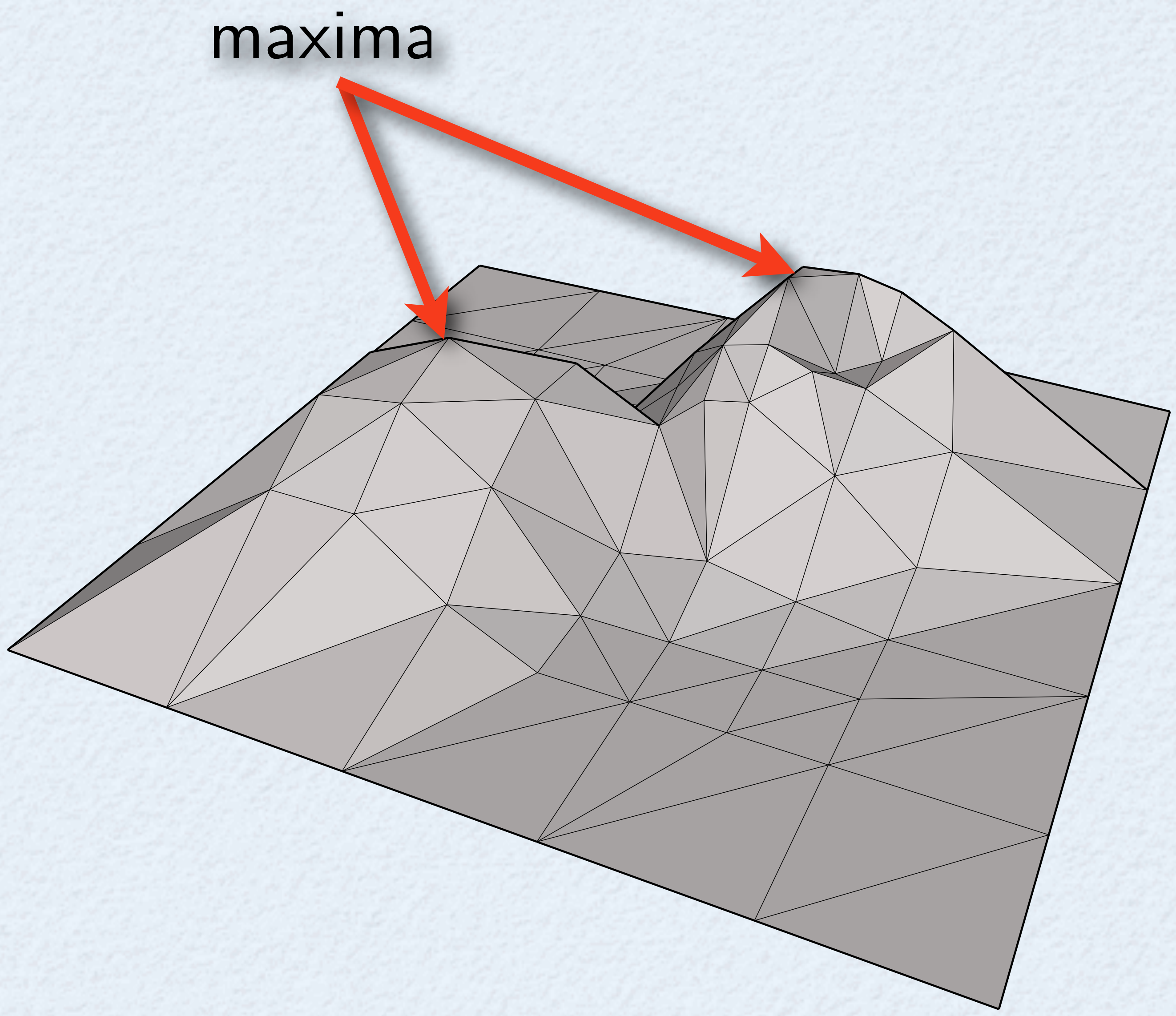

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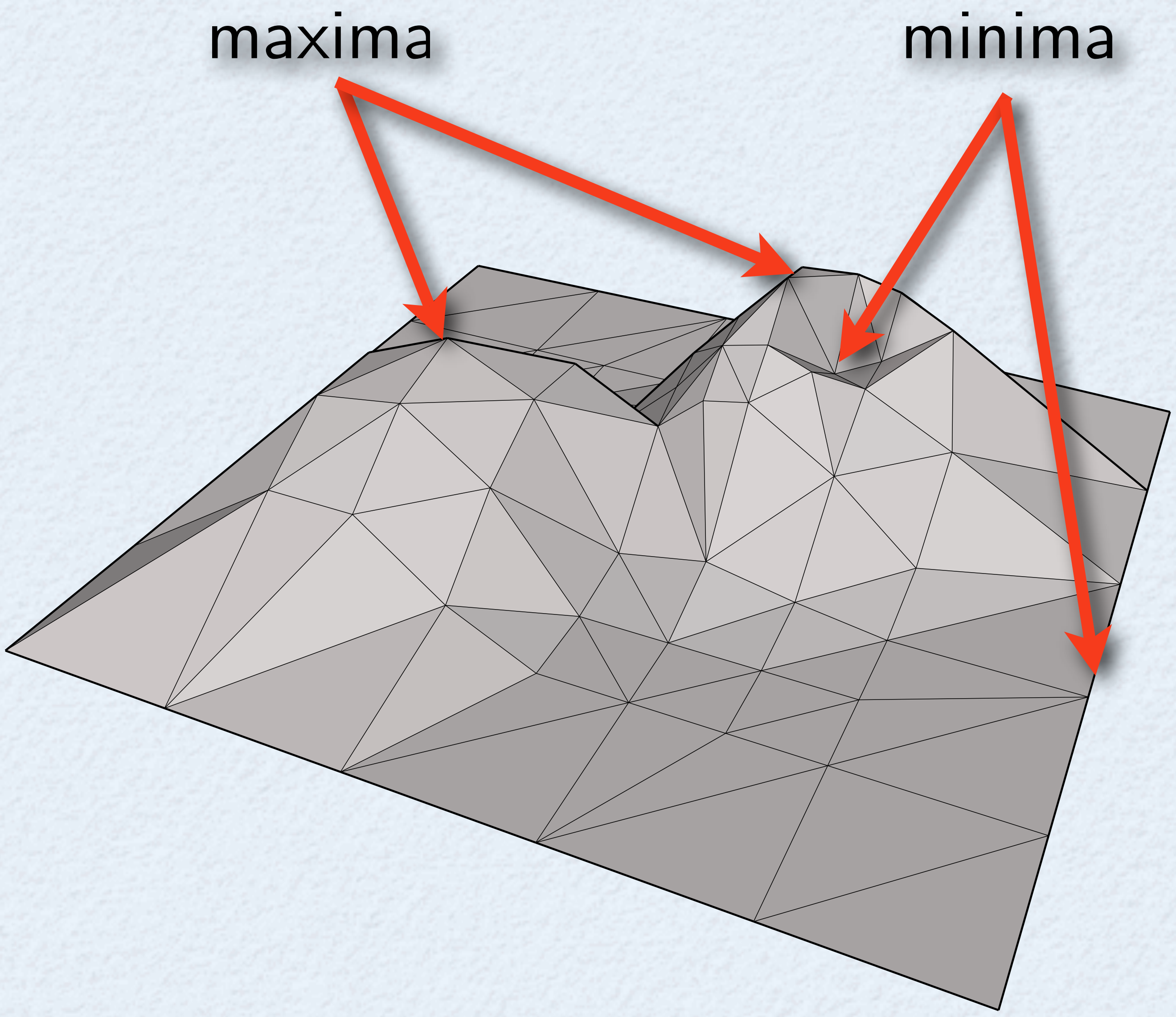

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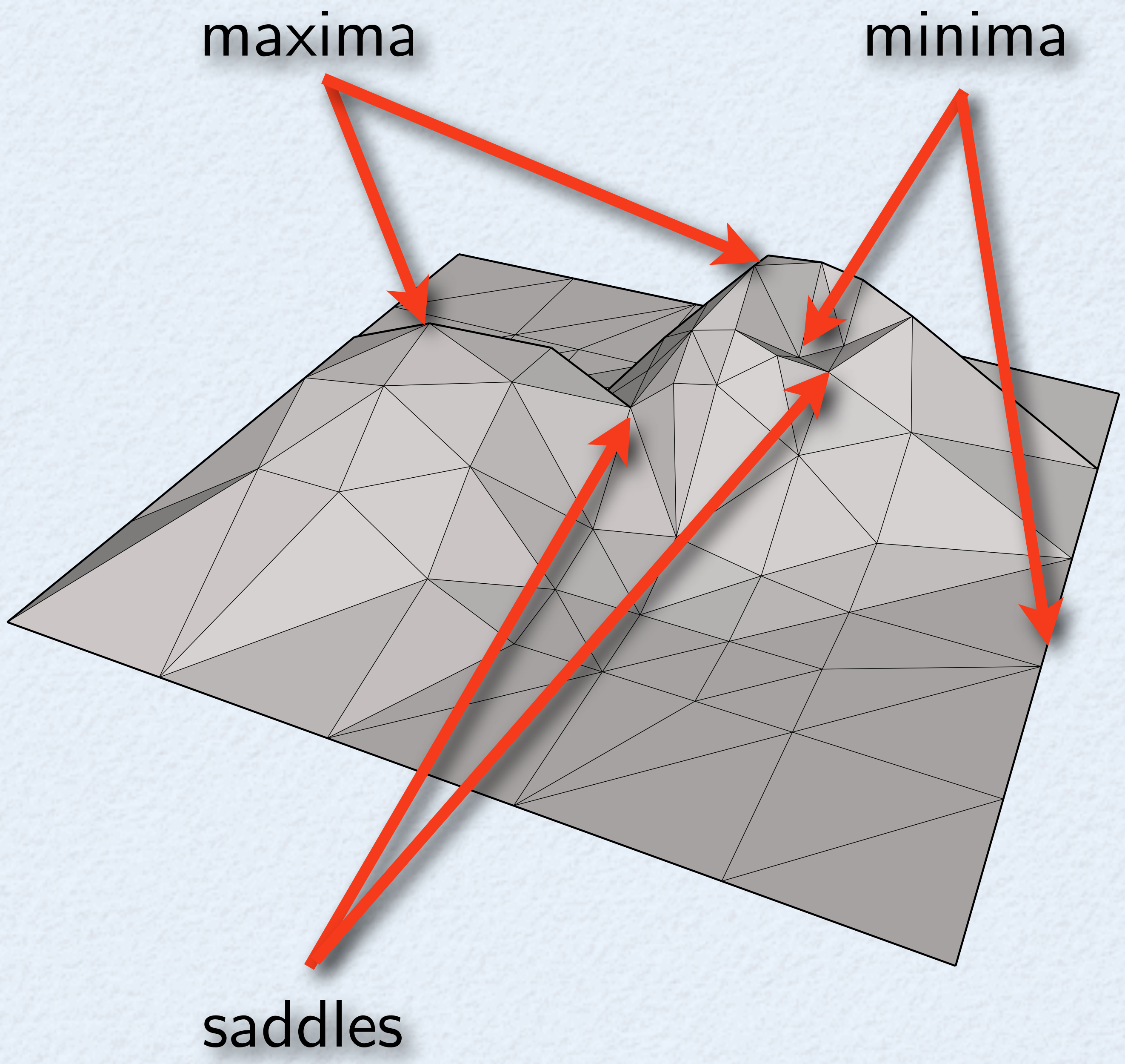

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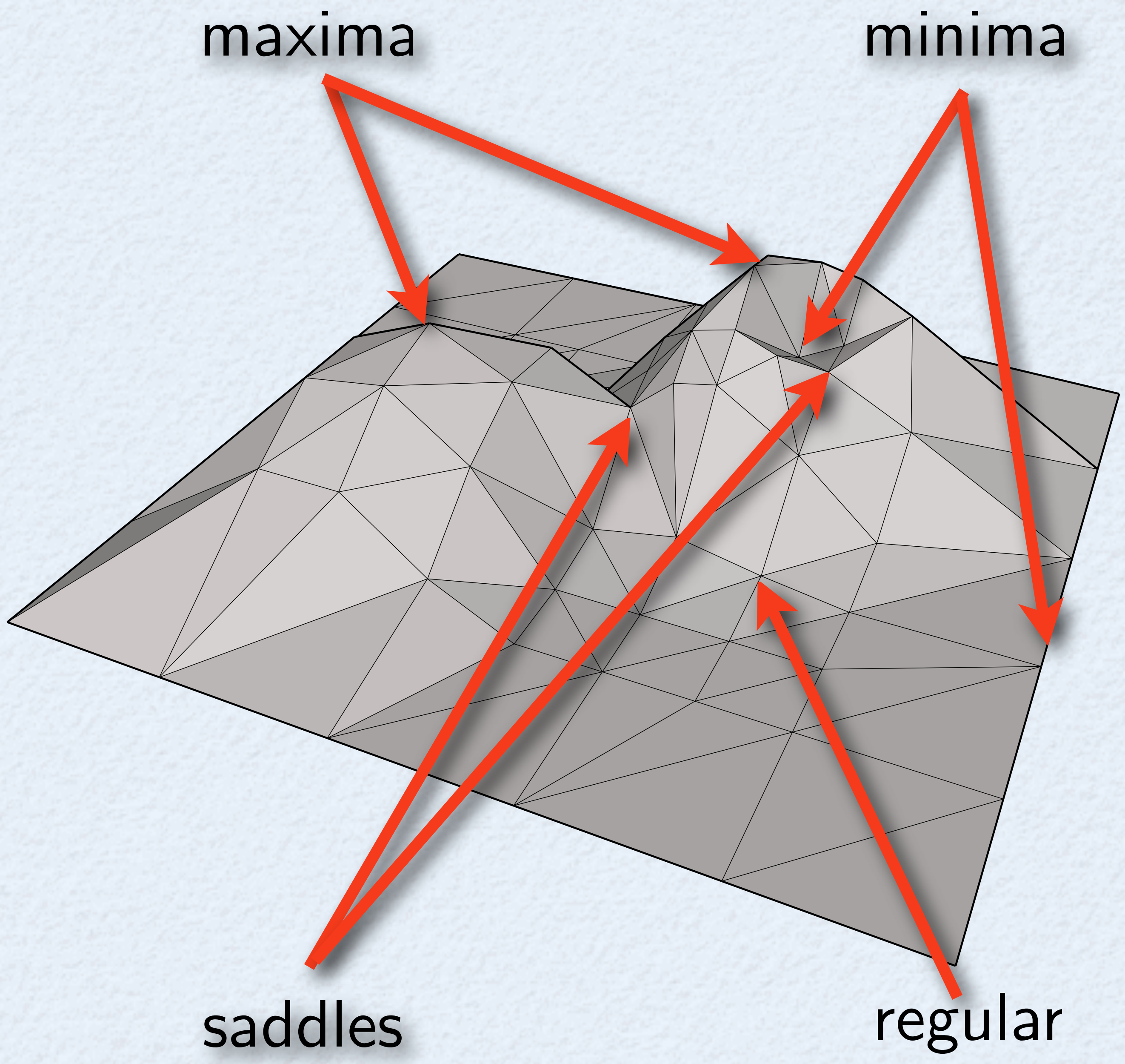

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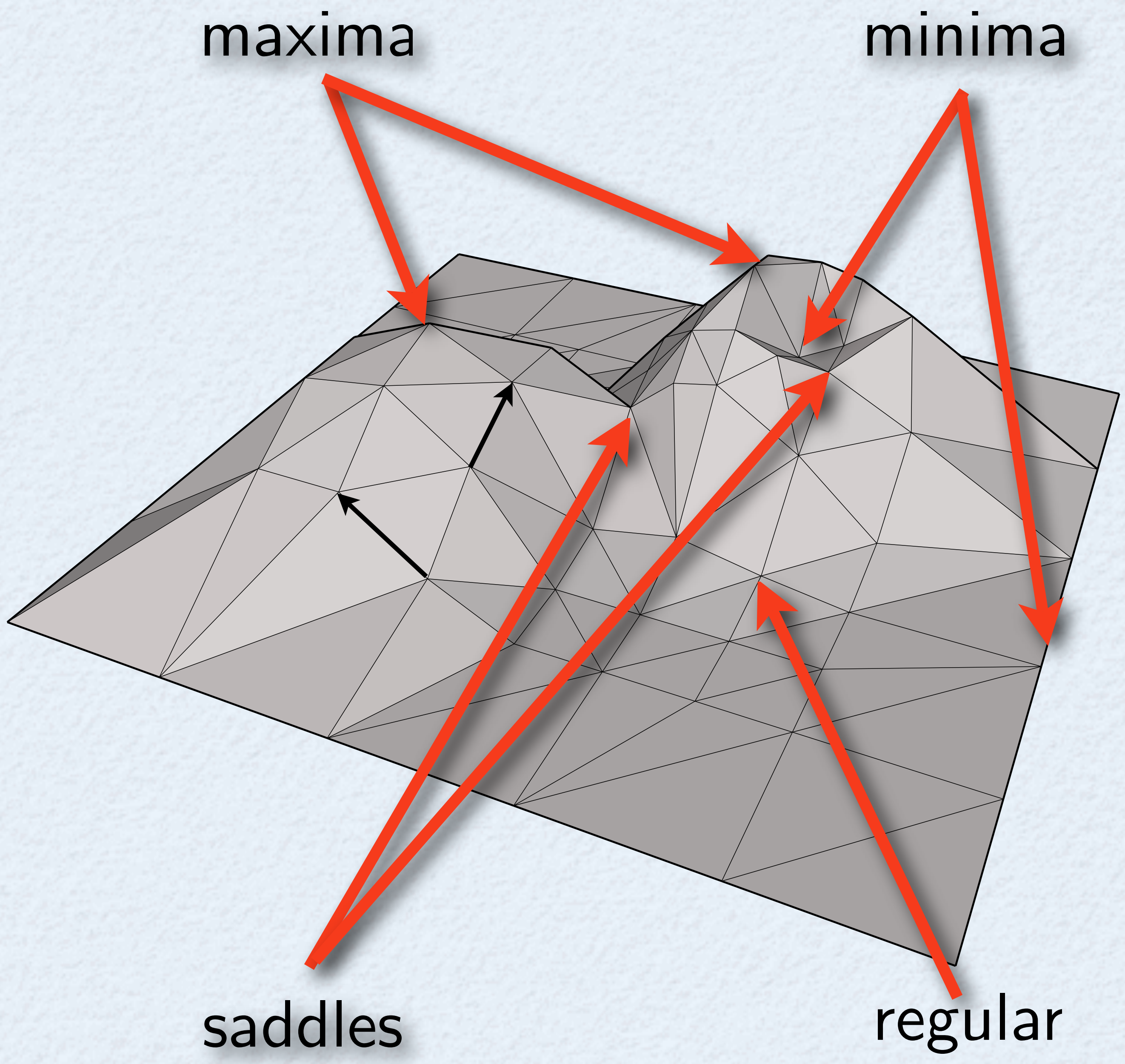

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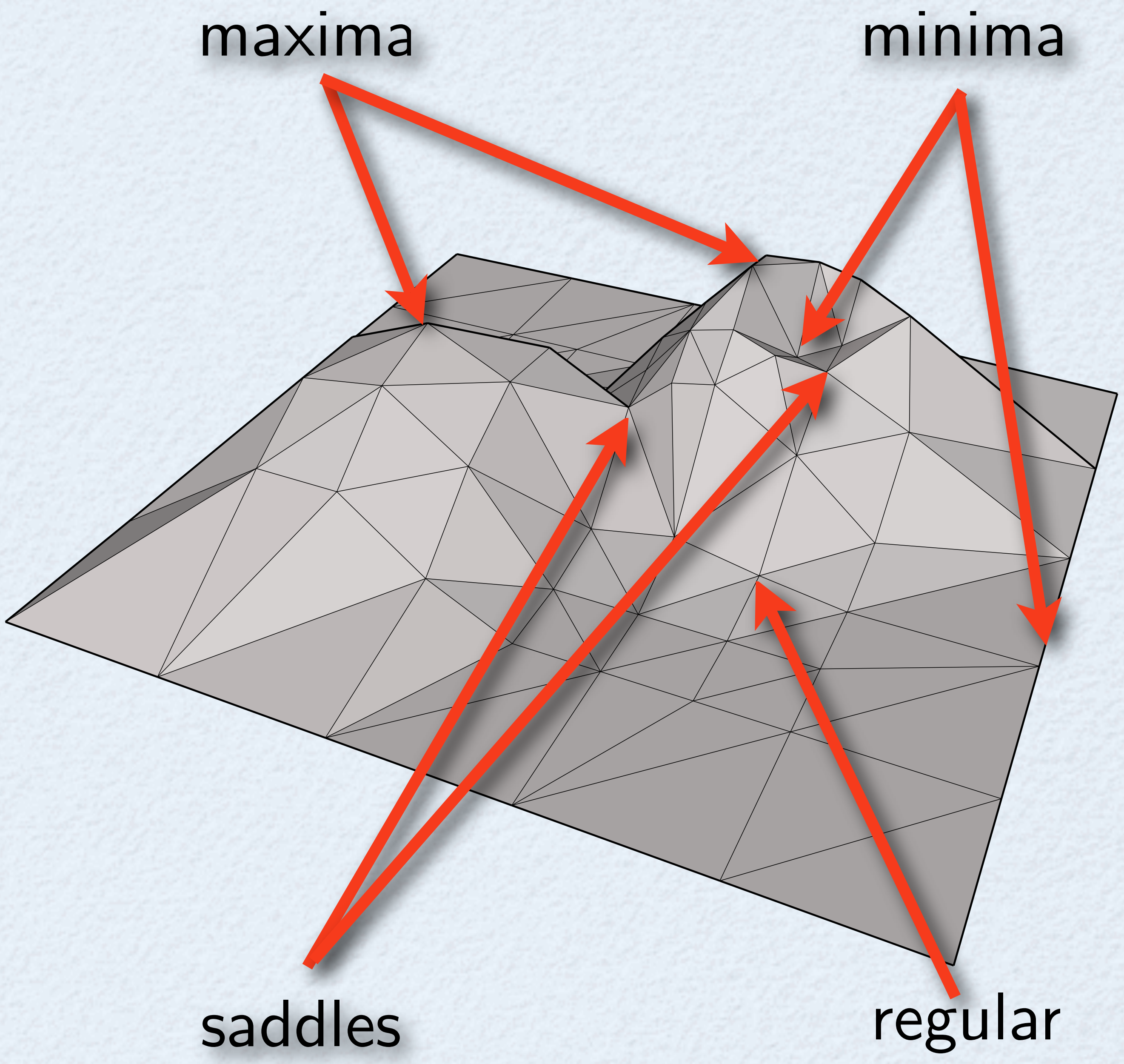

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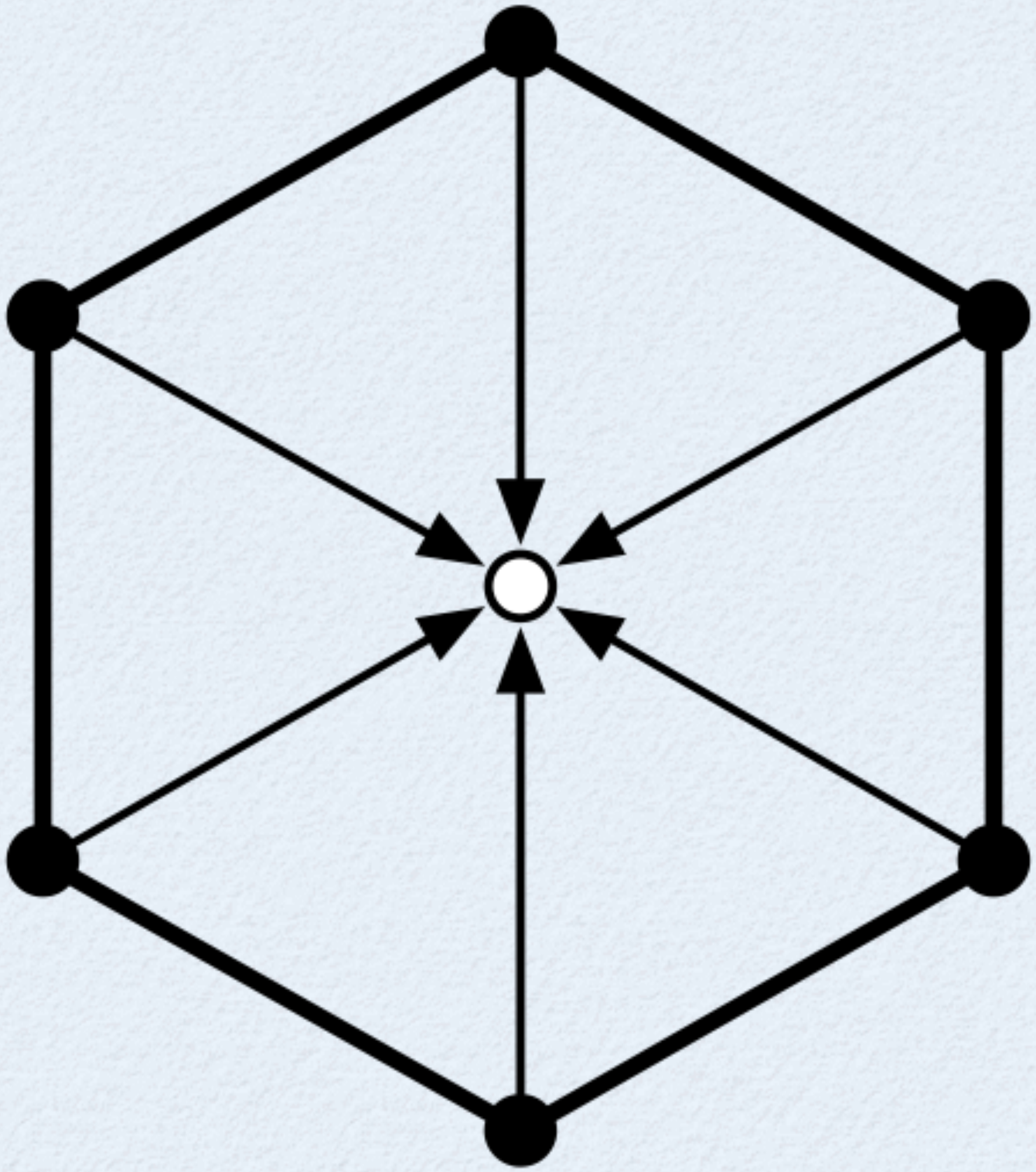

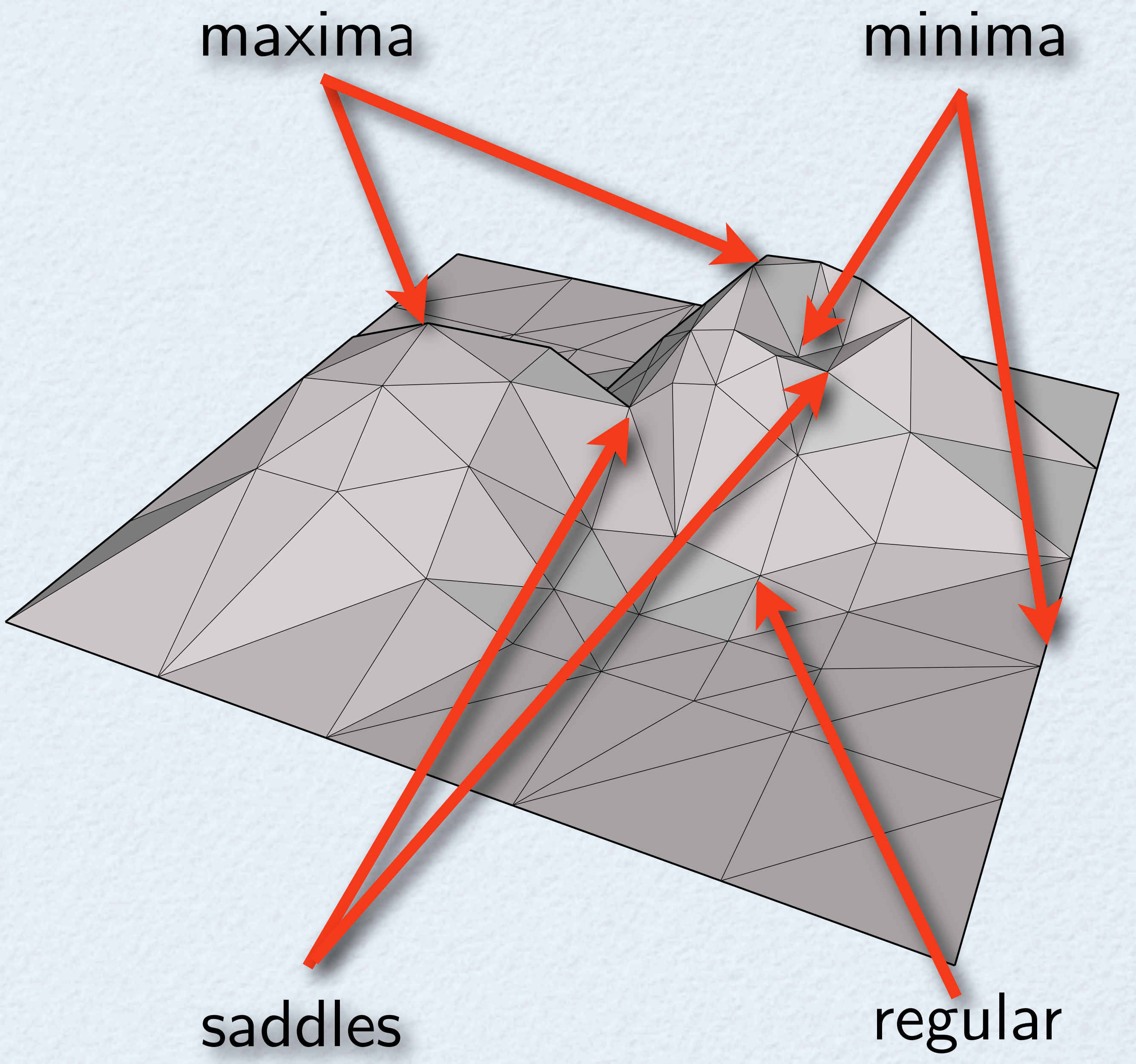

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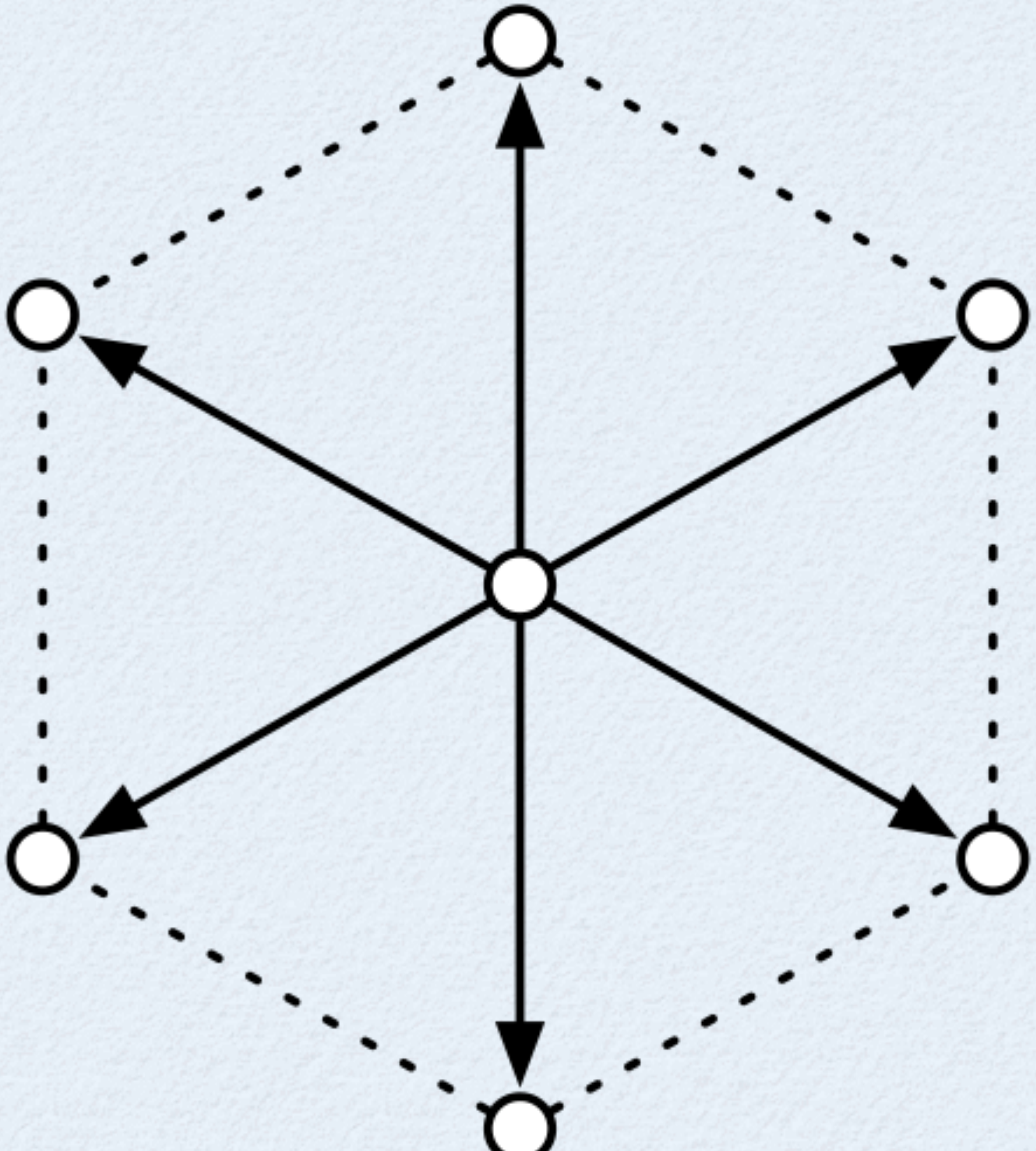

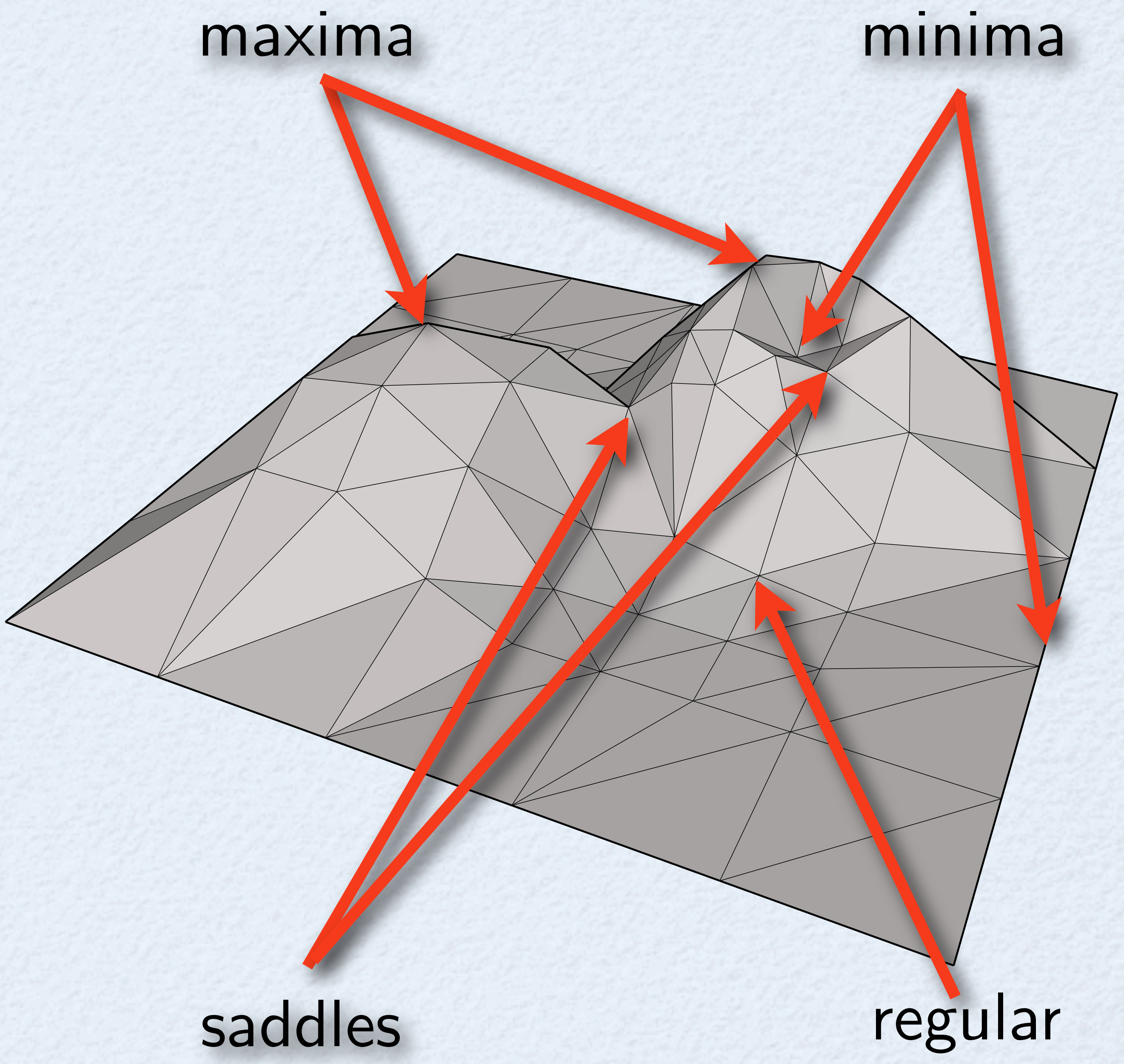

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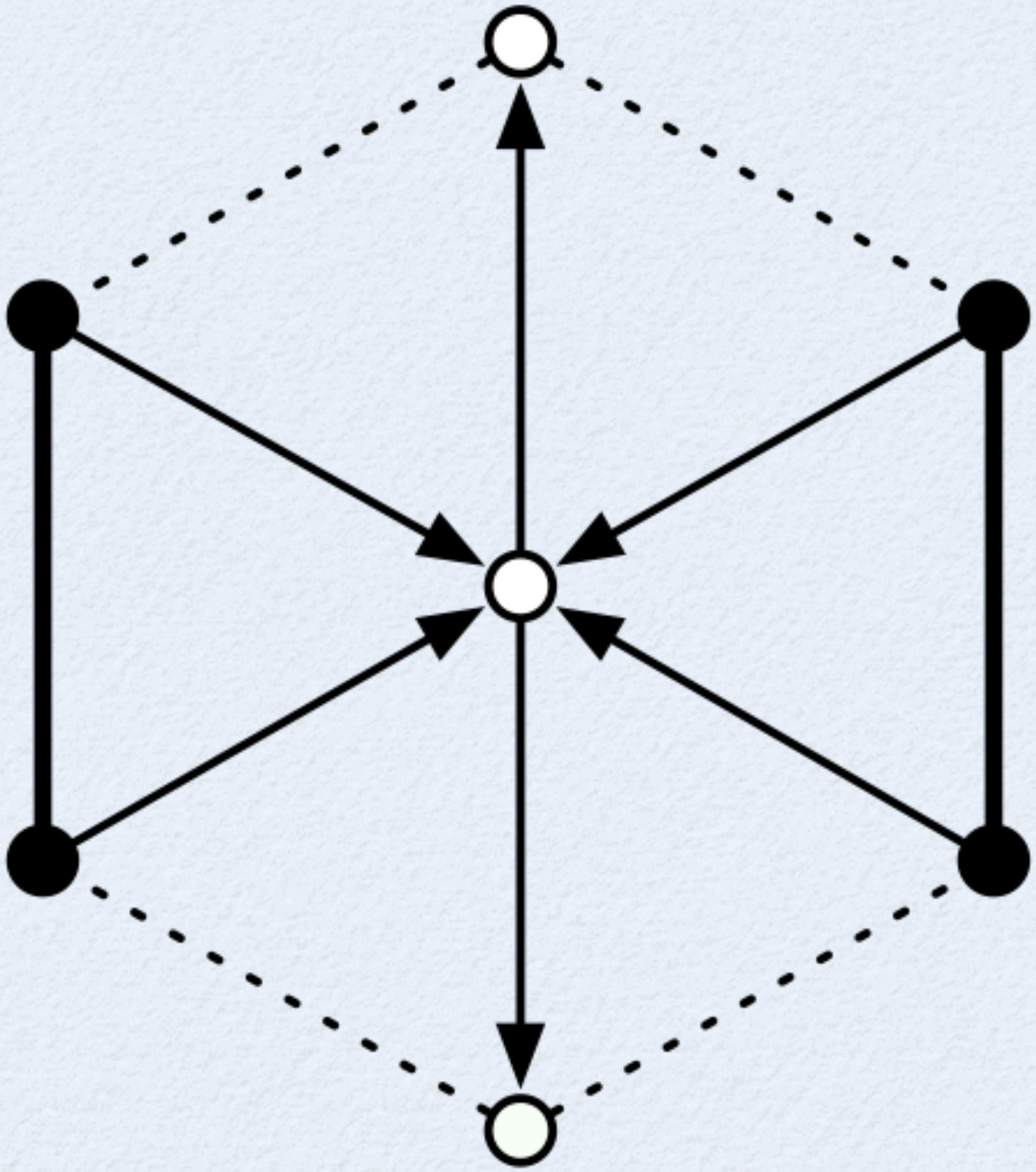

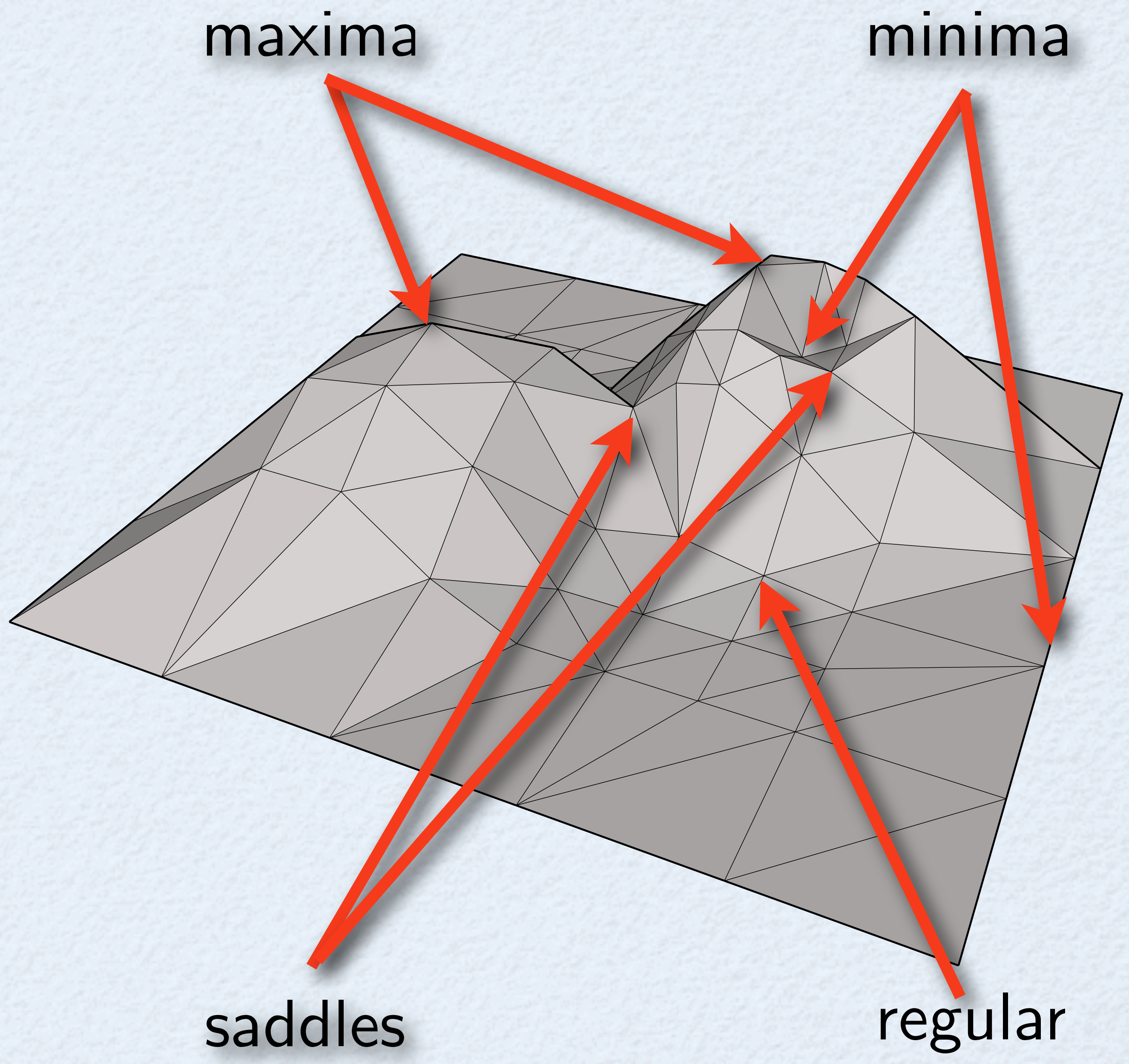

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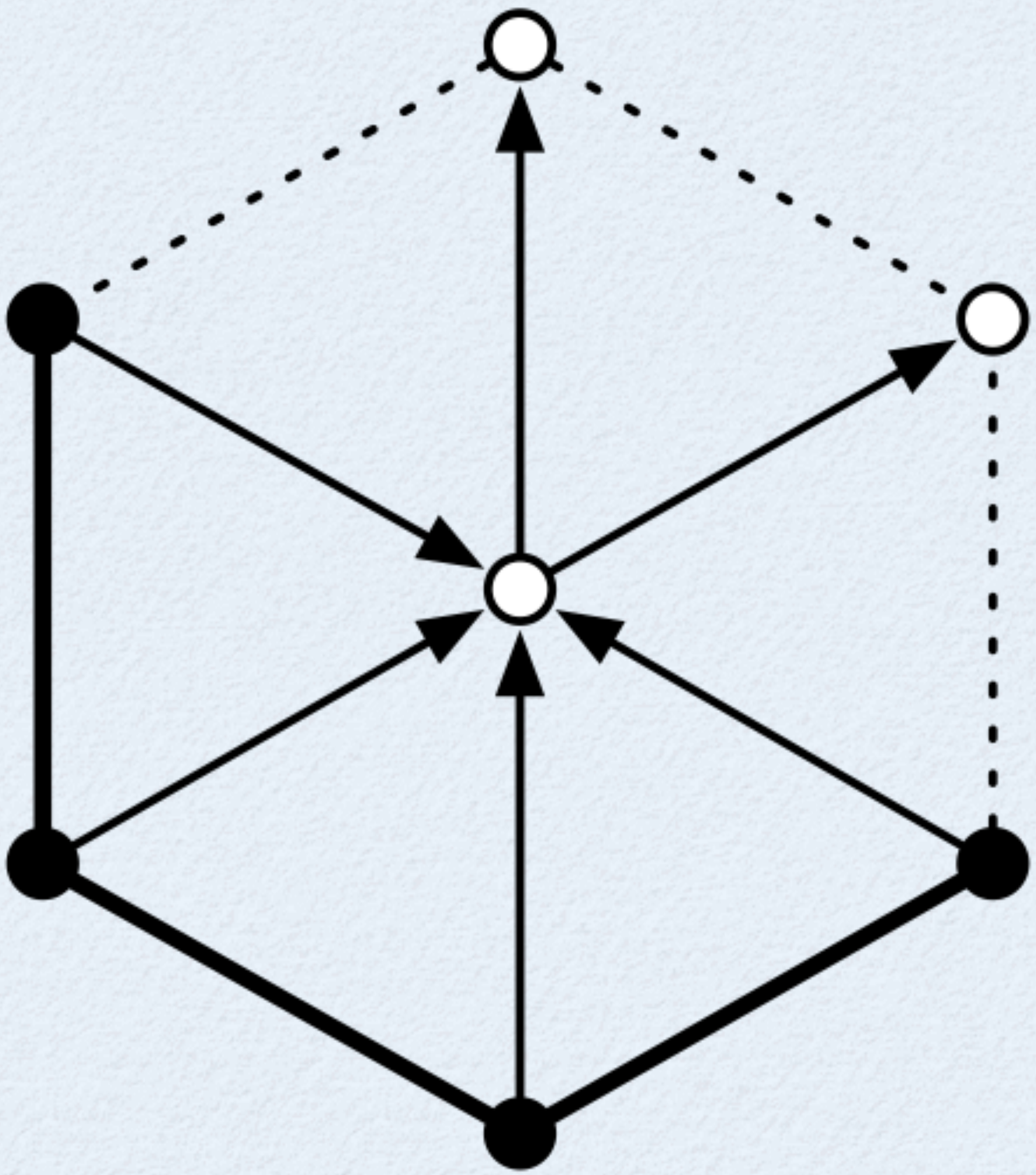

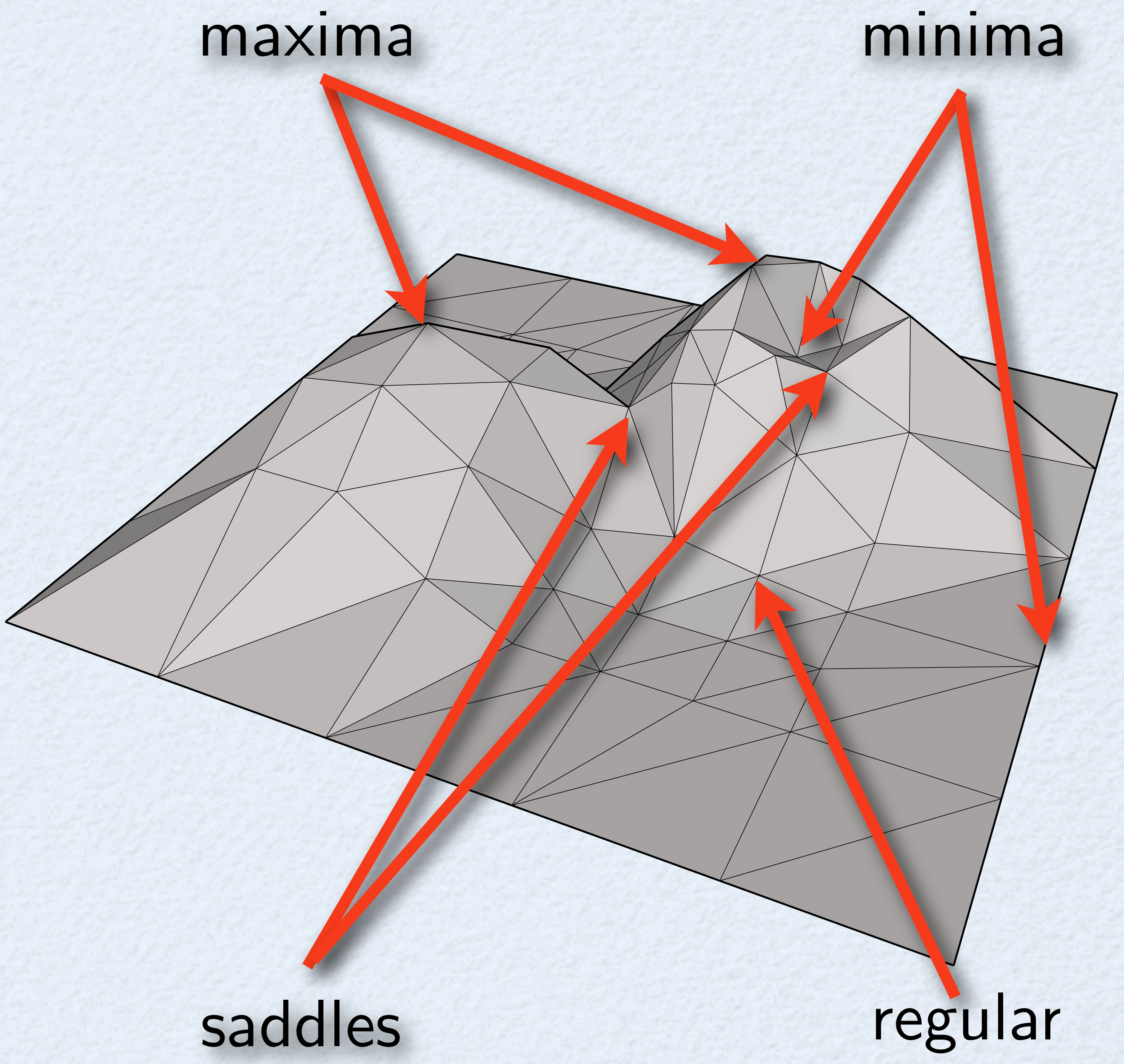

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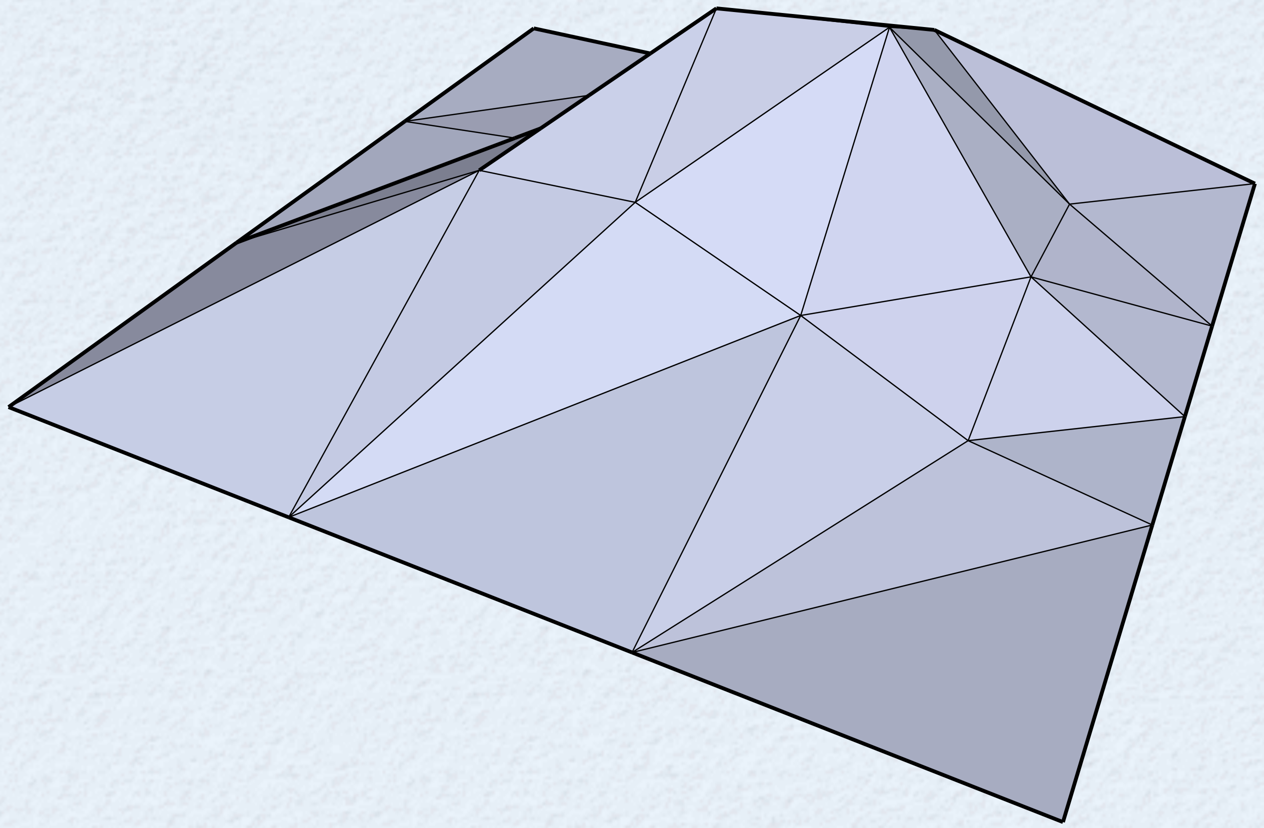

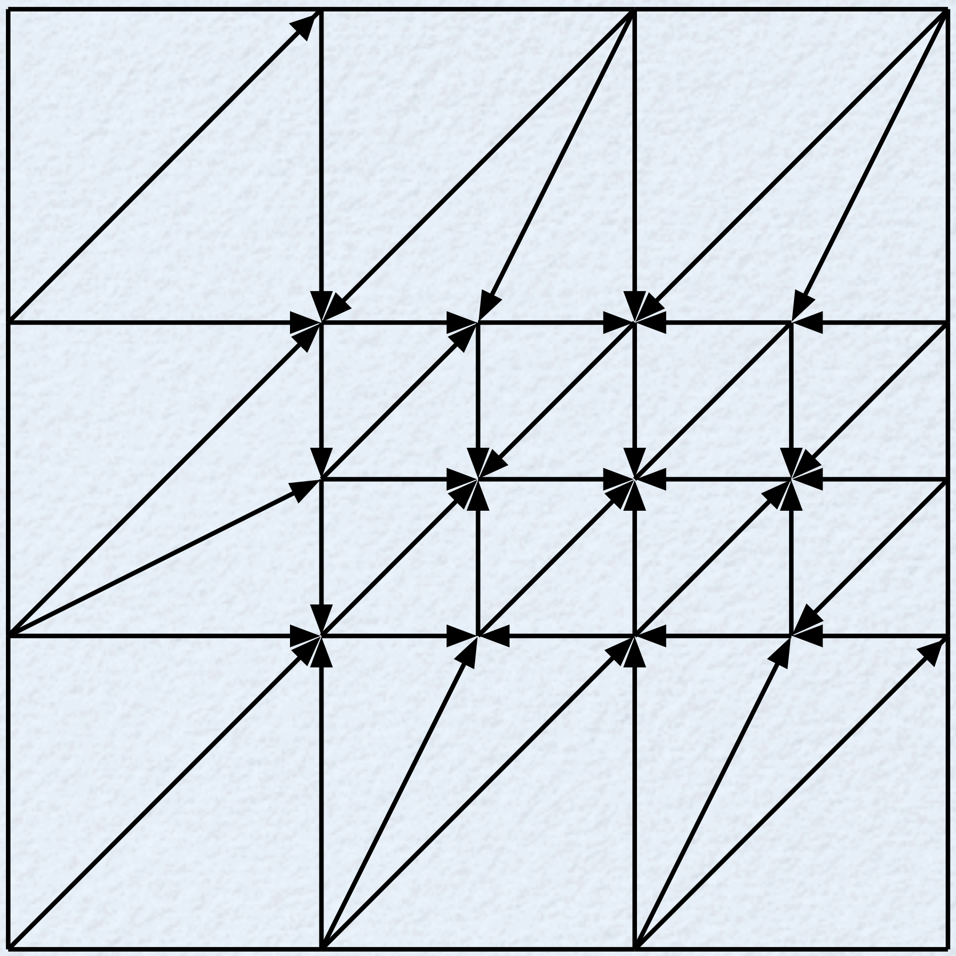

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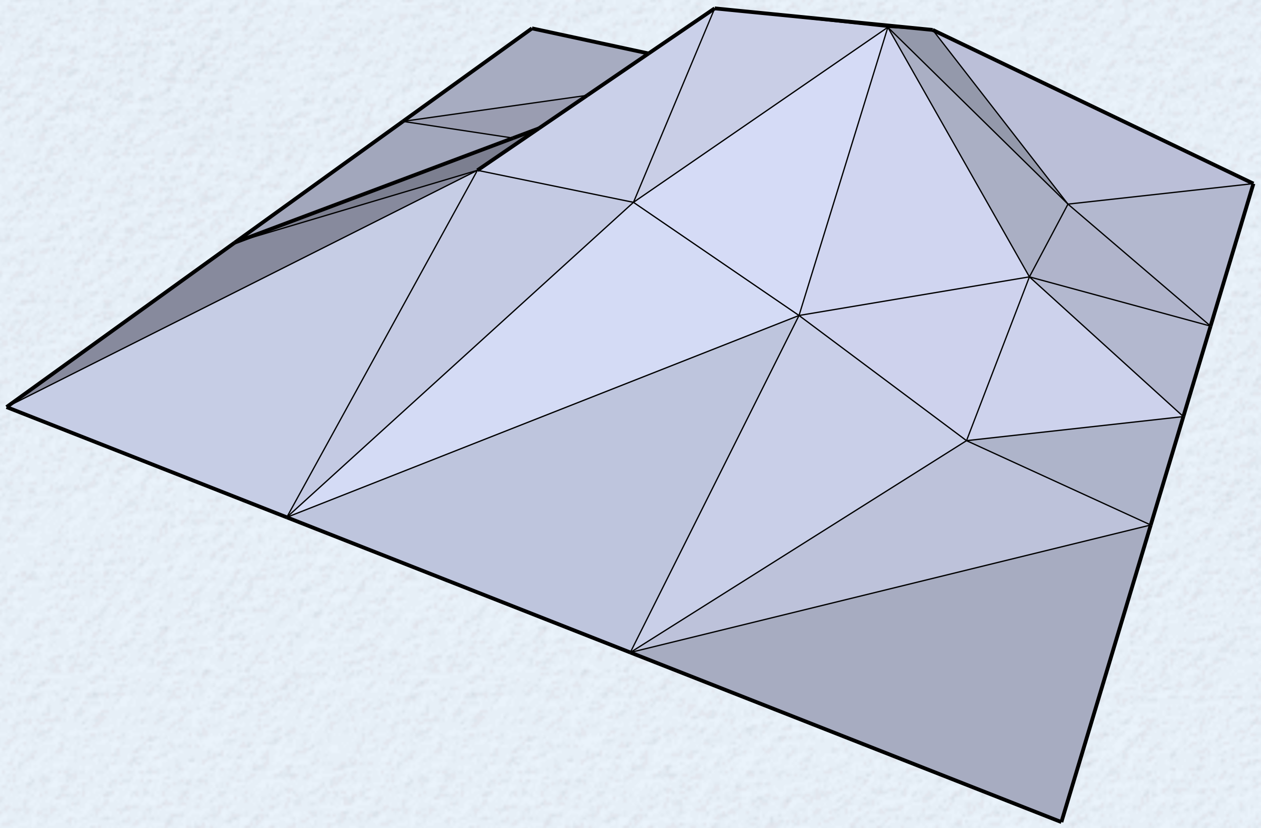

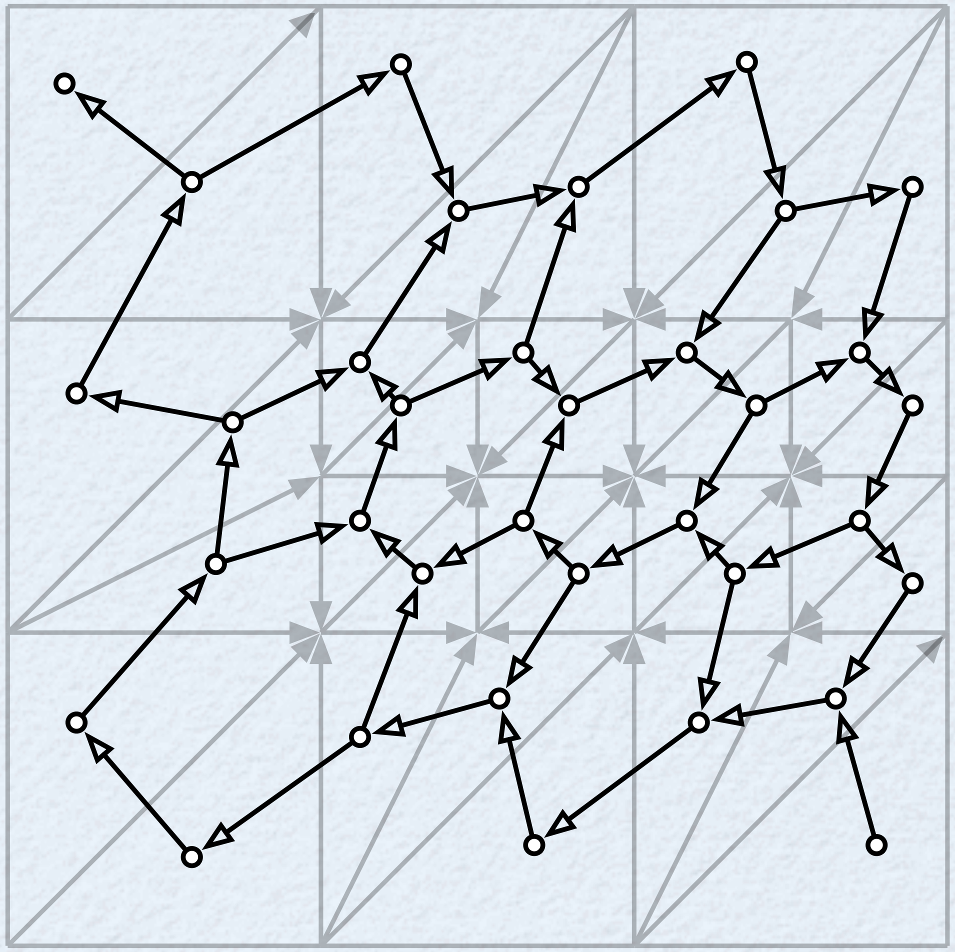

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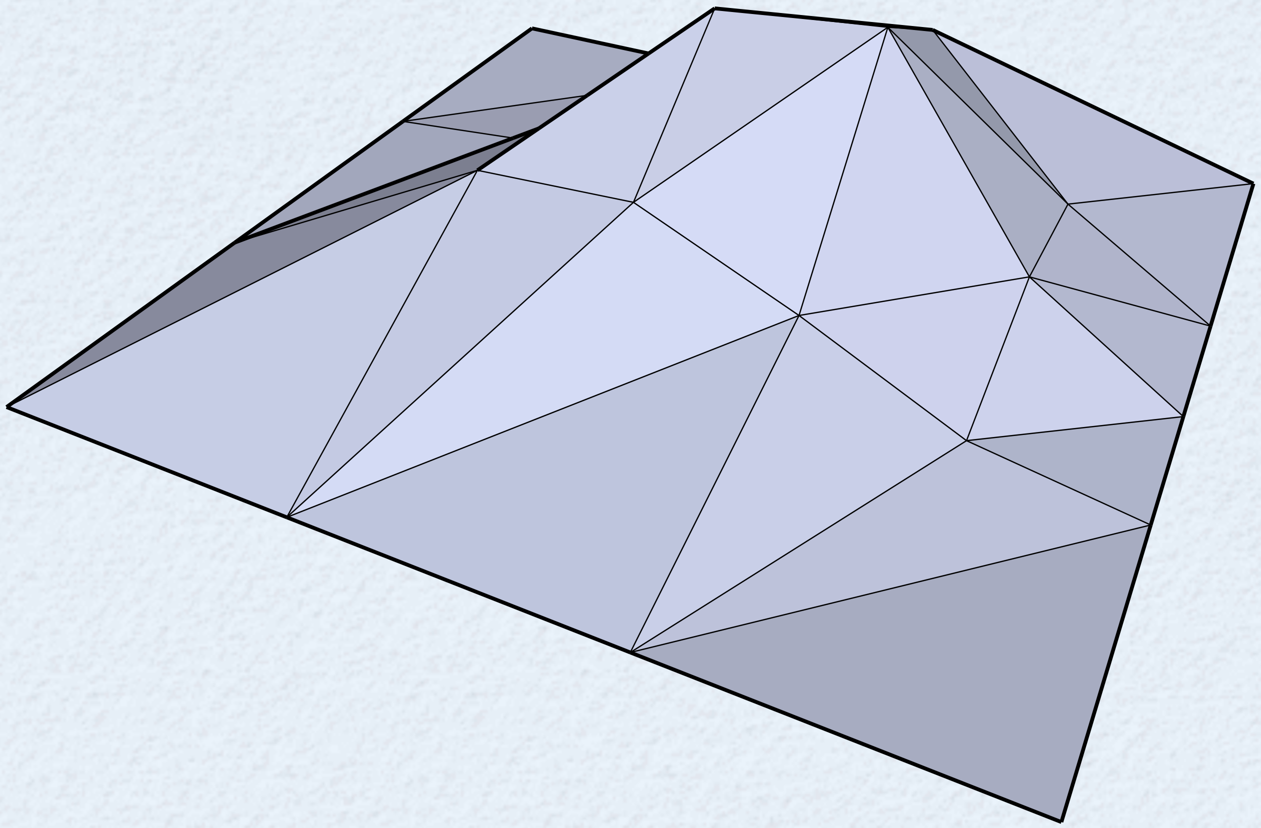

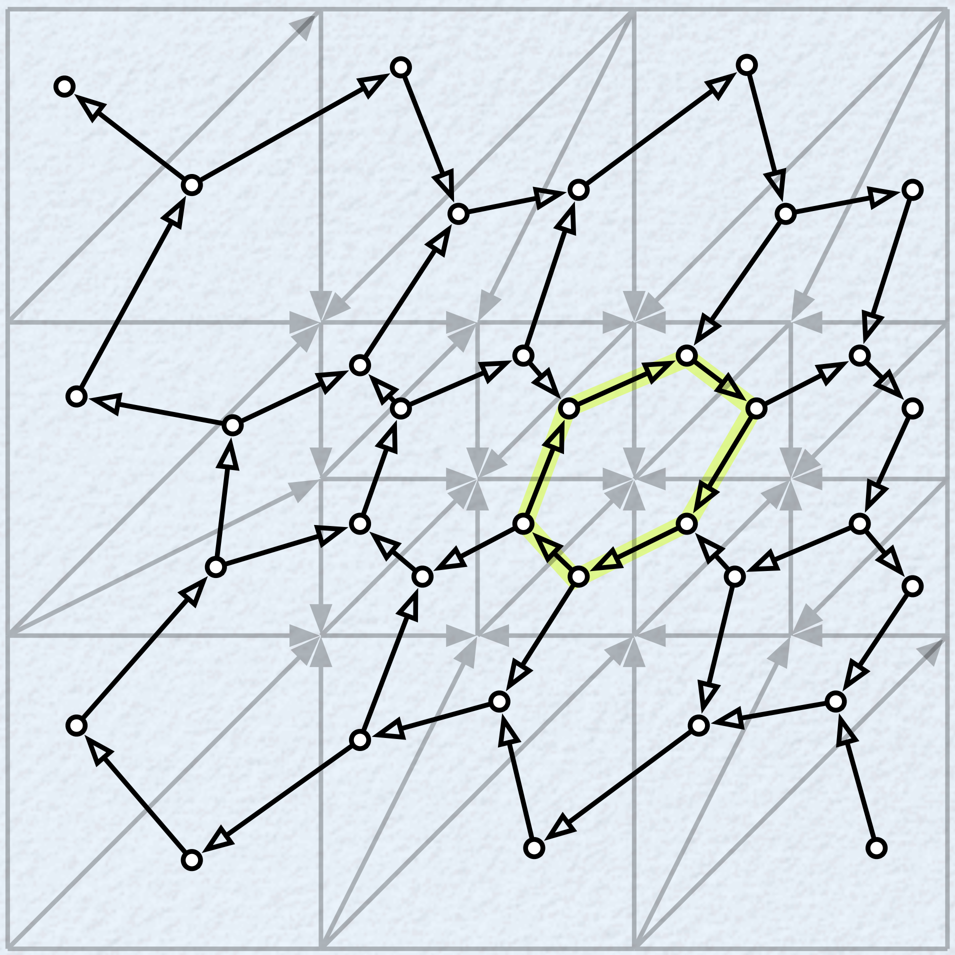

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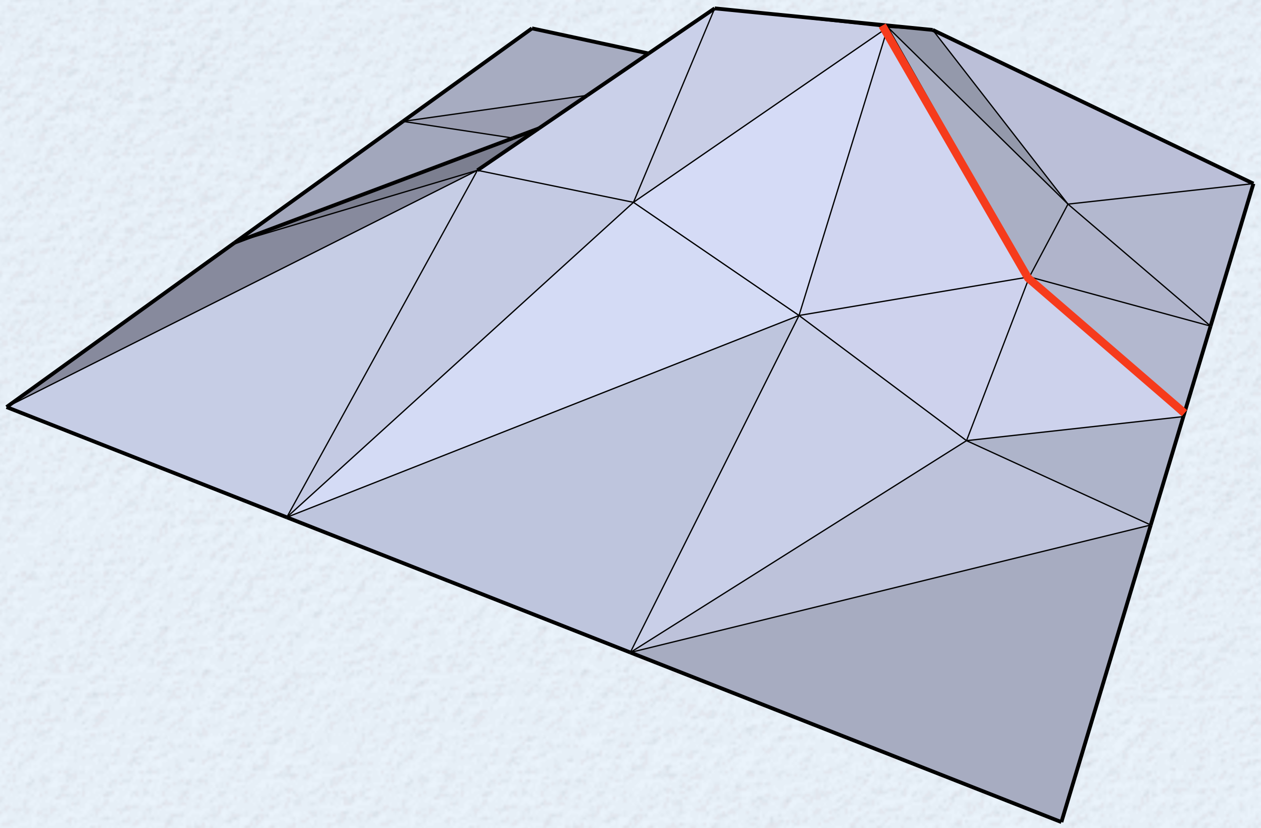

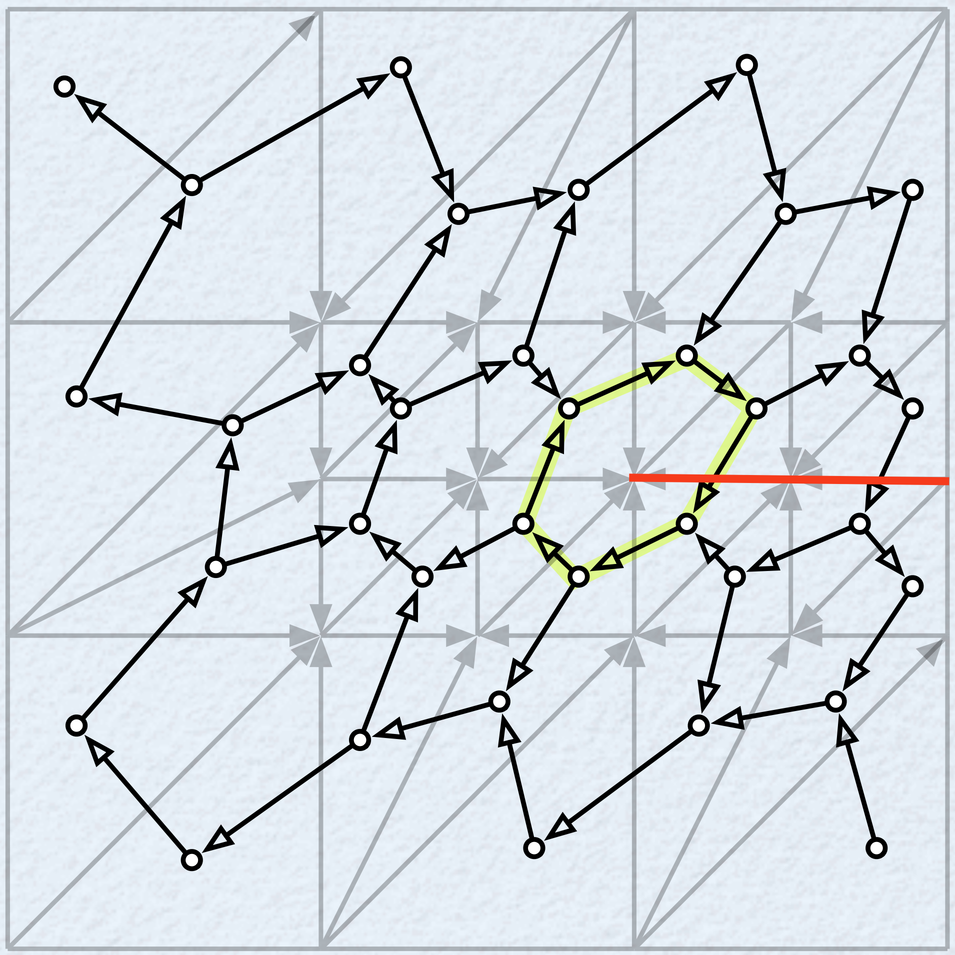

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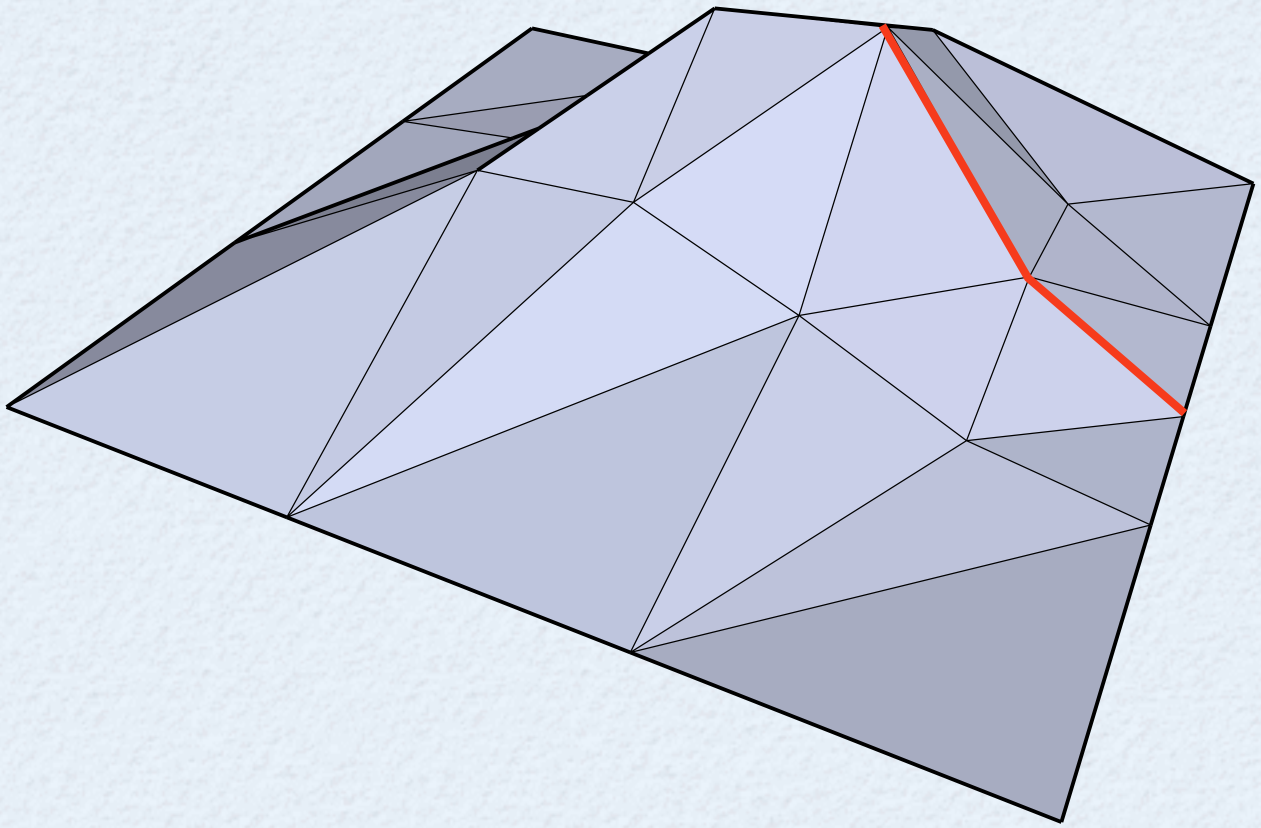

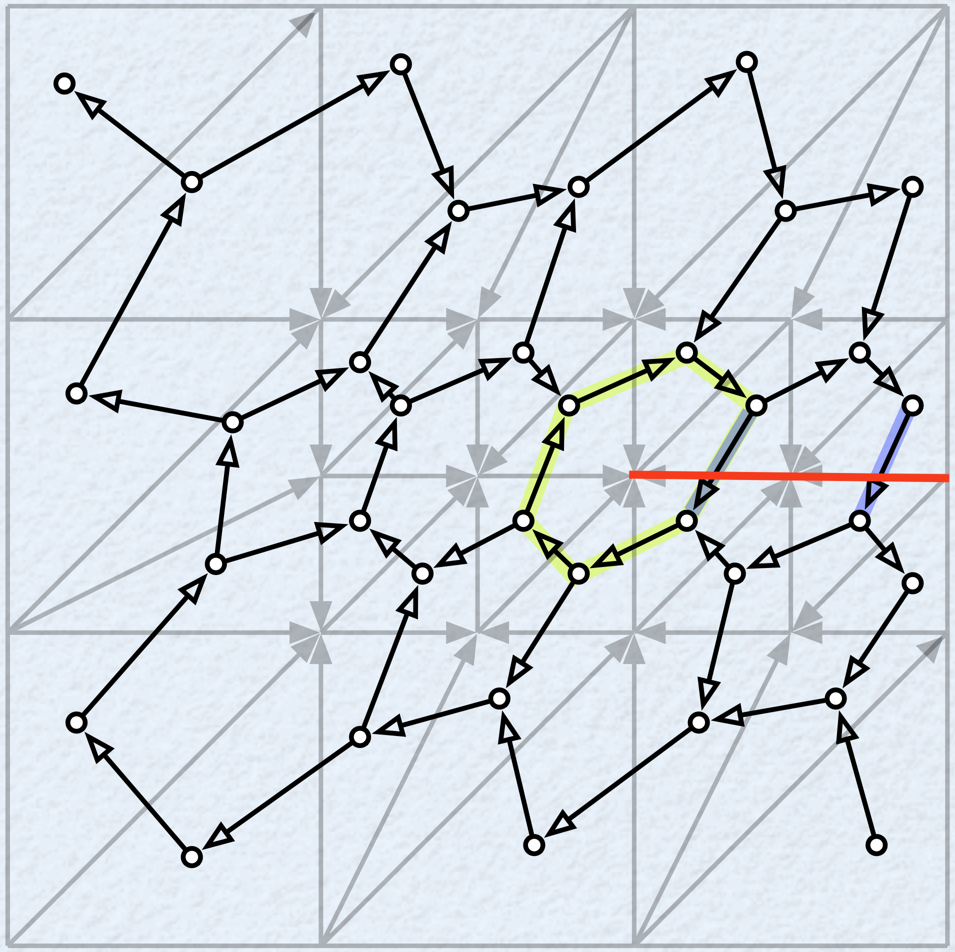

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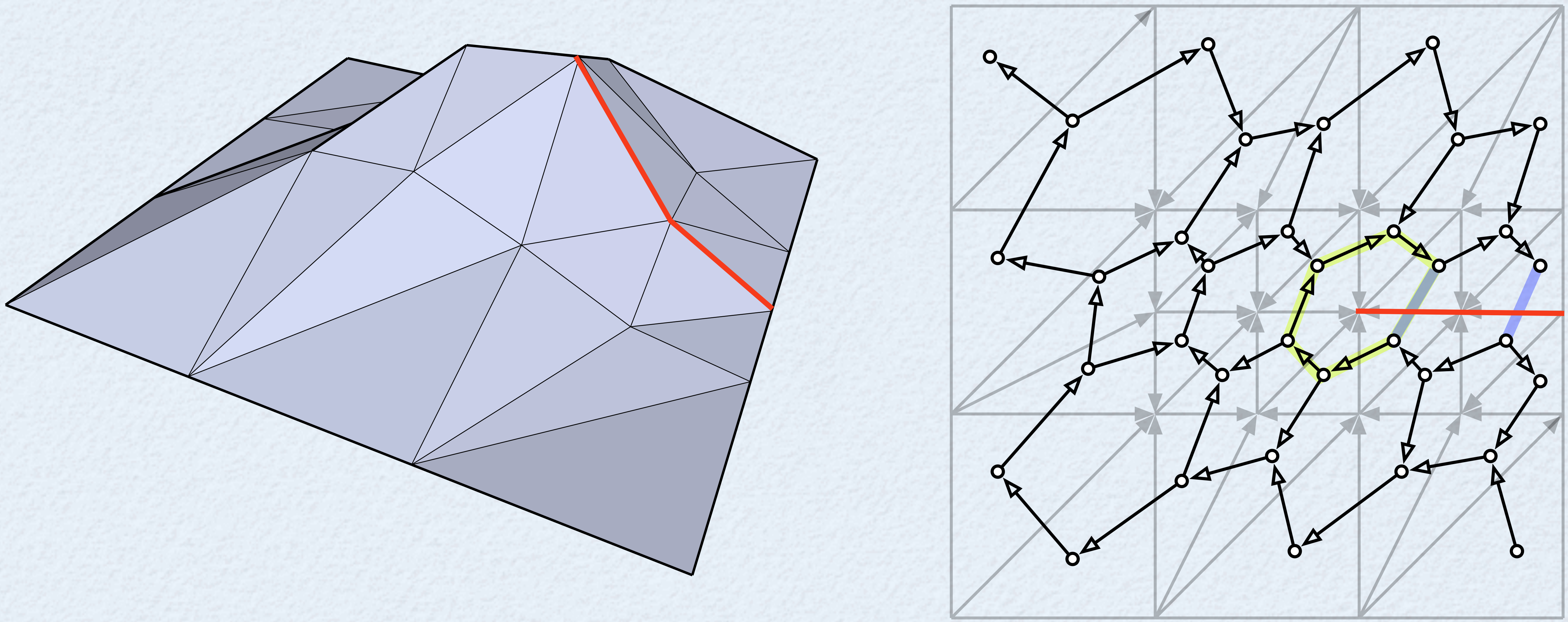

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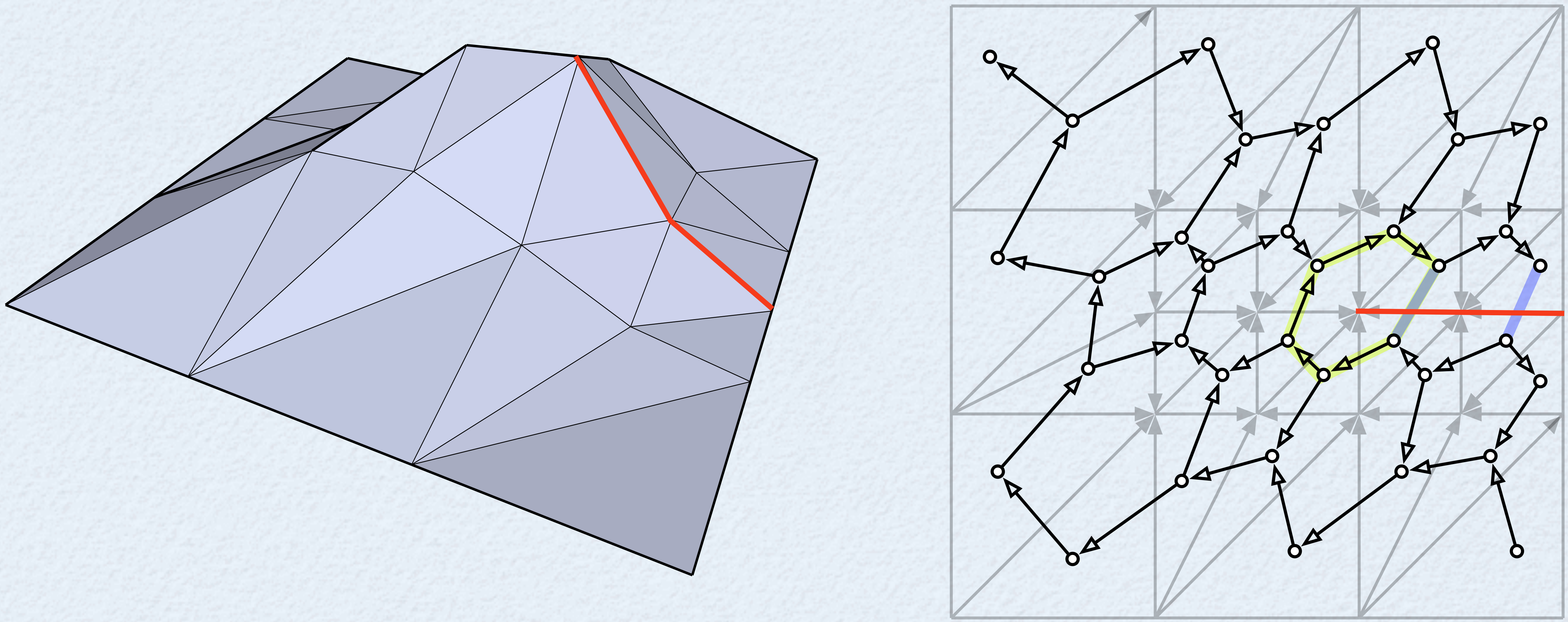
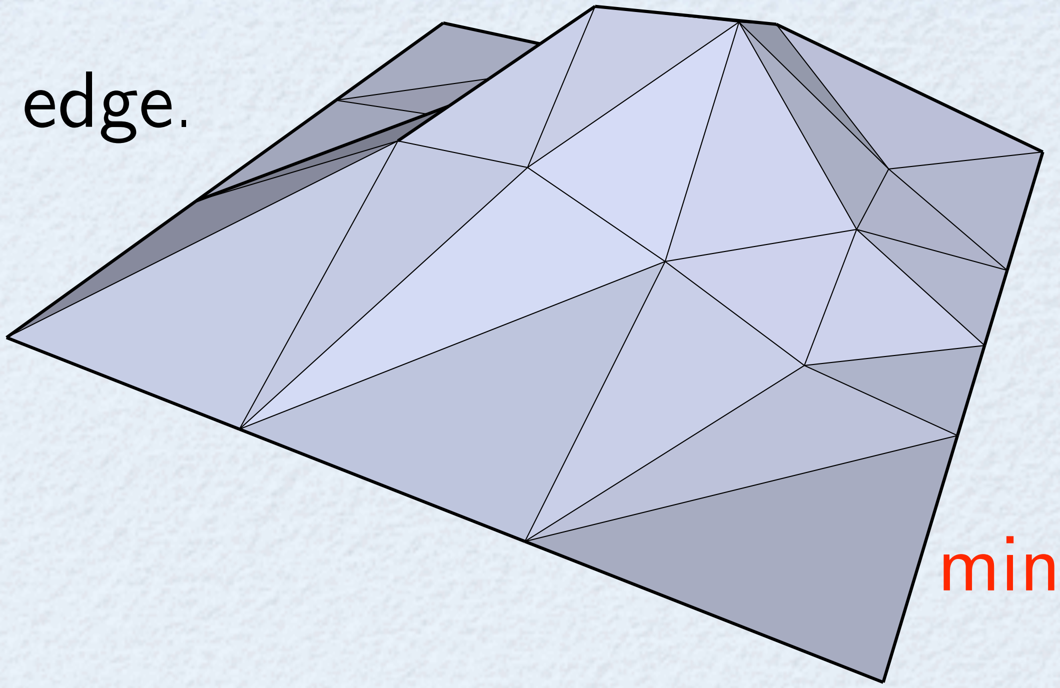


Lemma. Every cycle of $\mathbb{M}^*$ loses precisely one edge.

Partial Order on Triangles



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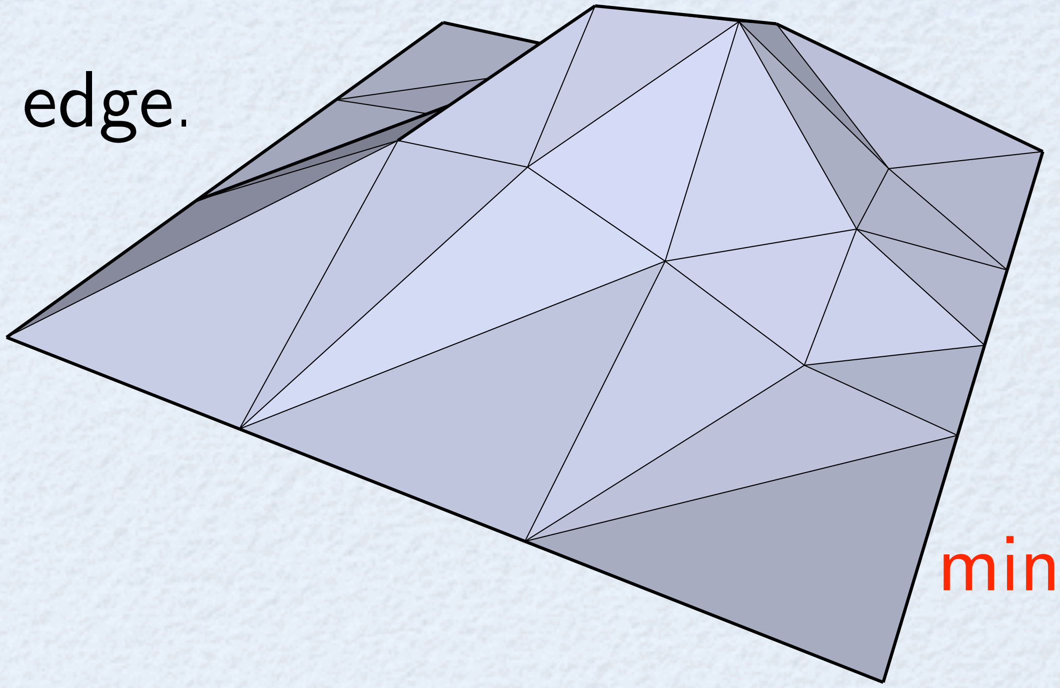

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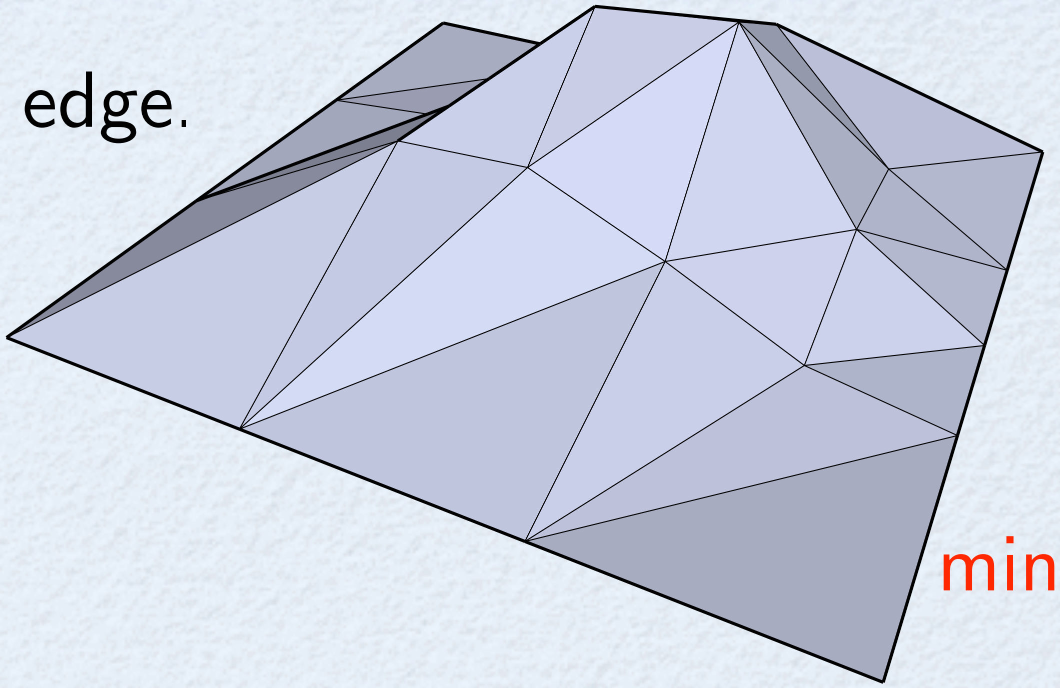

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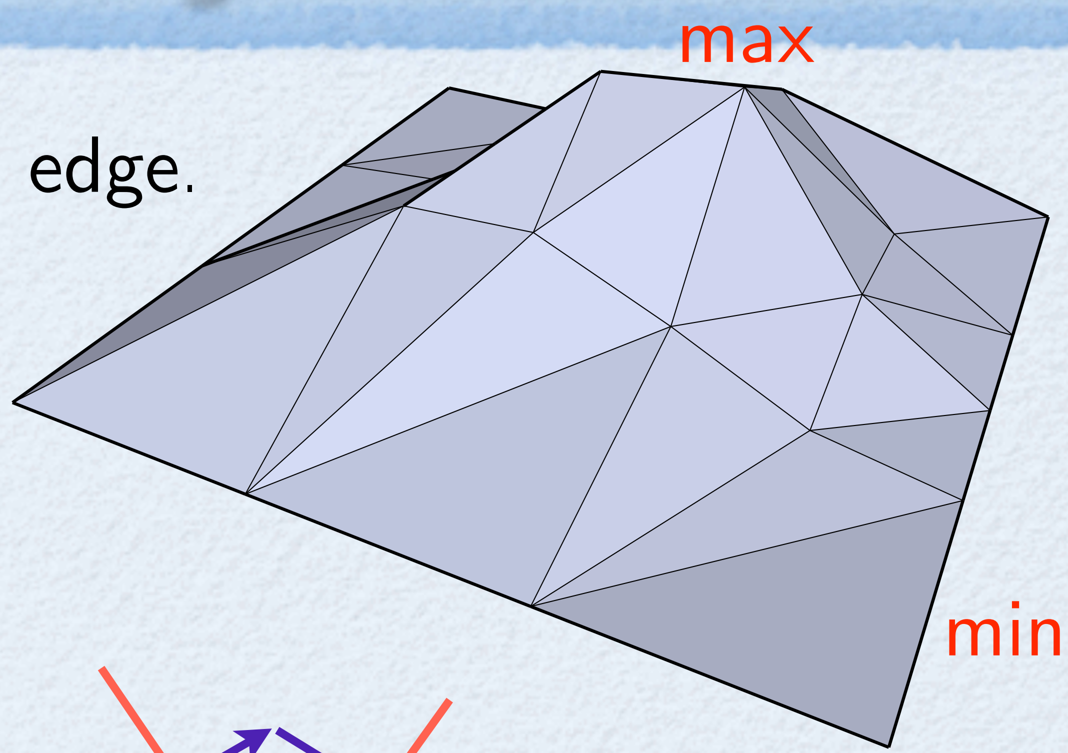

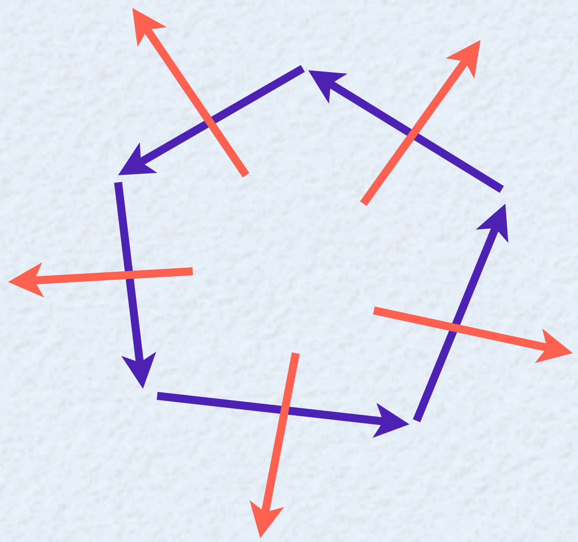

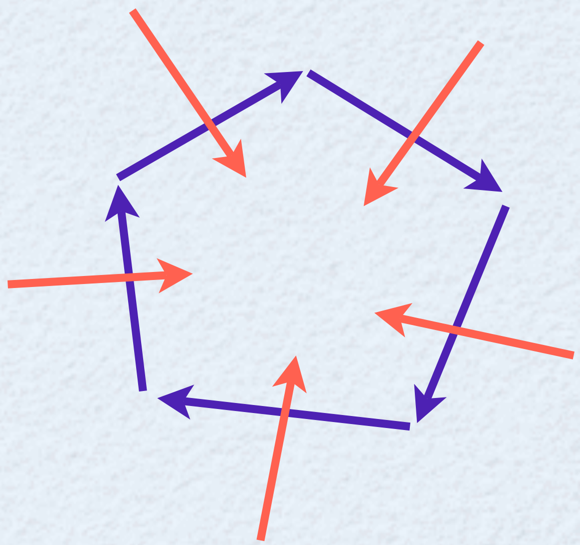

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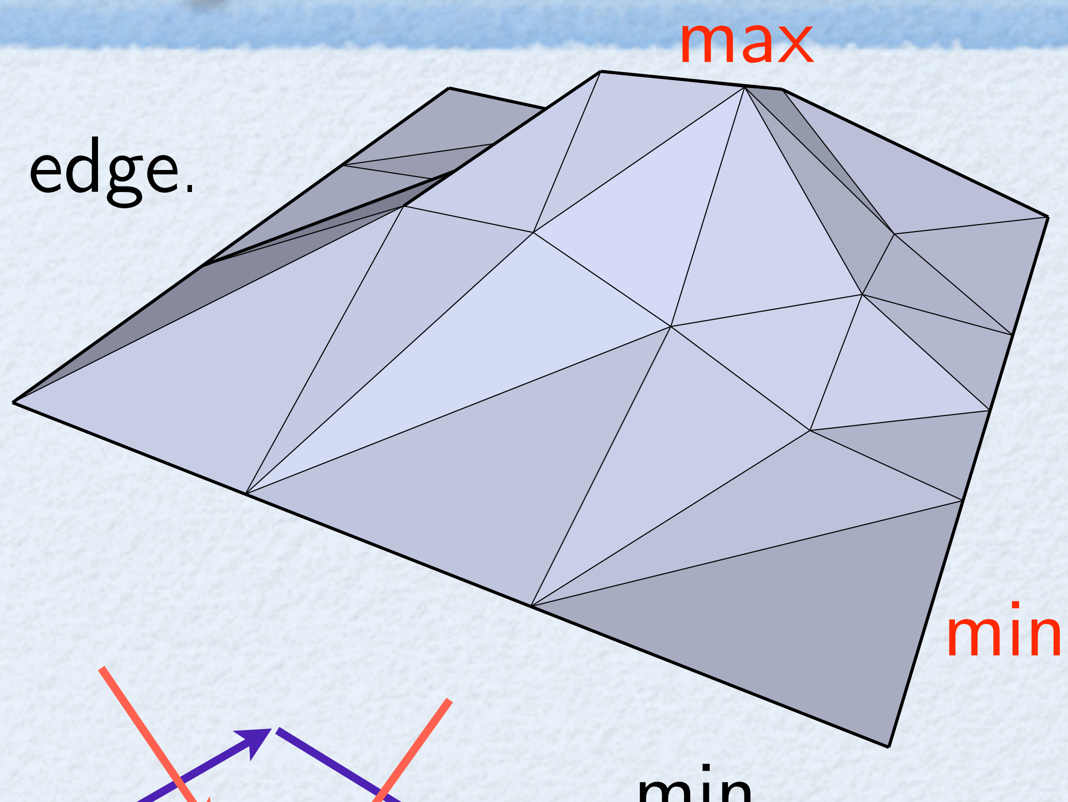

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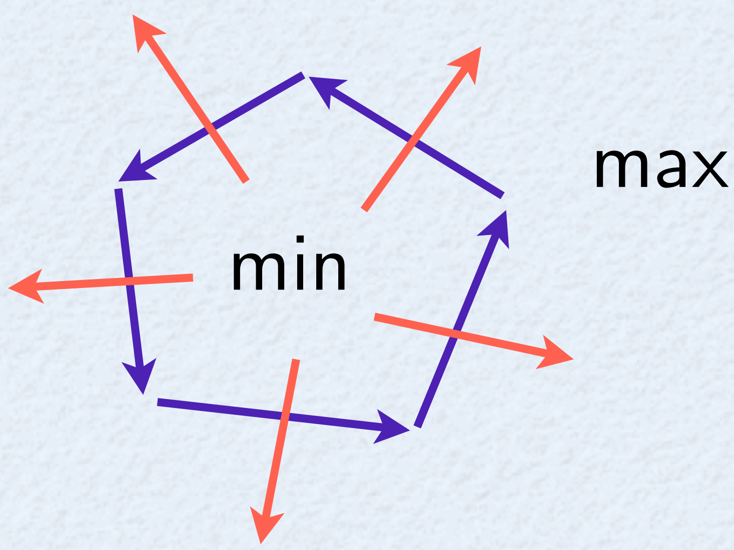

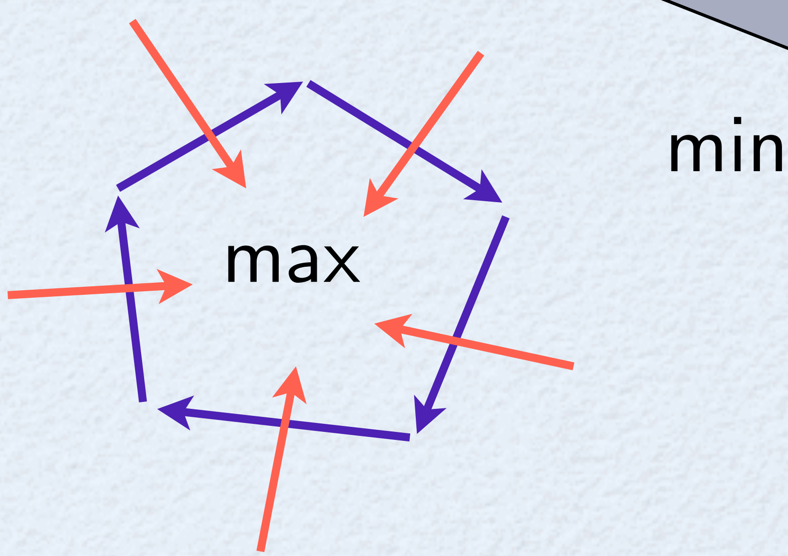

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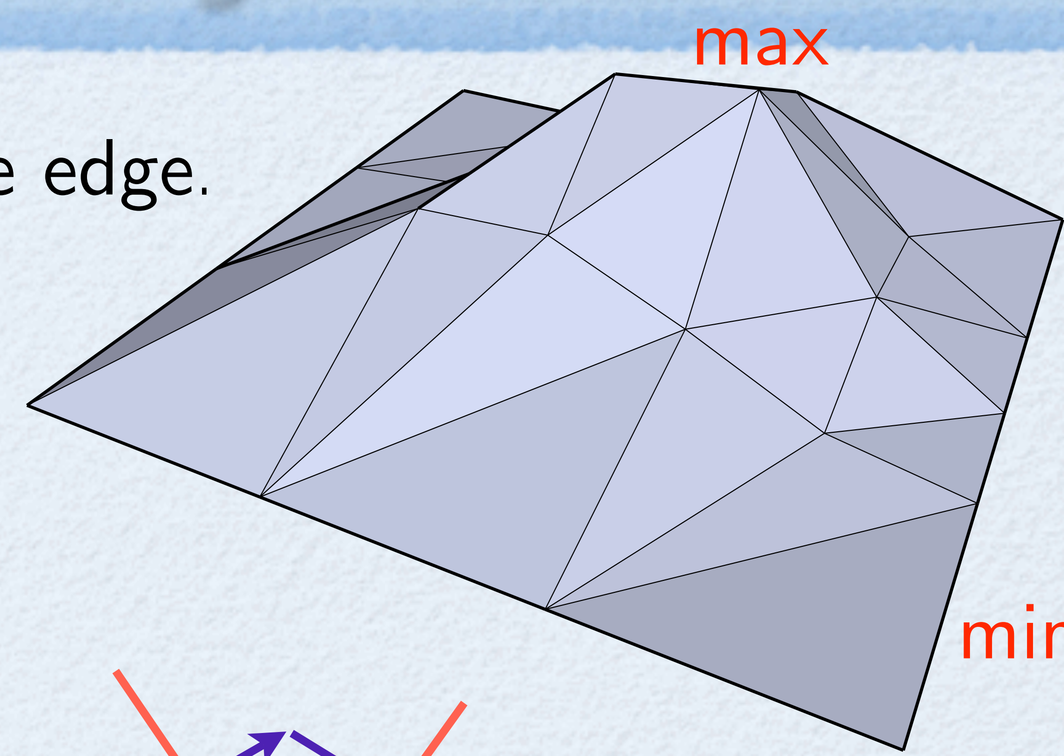

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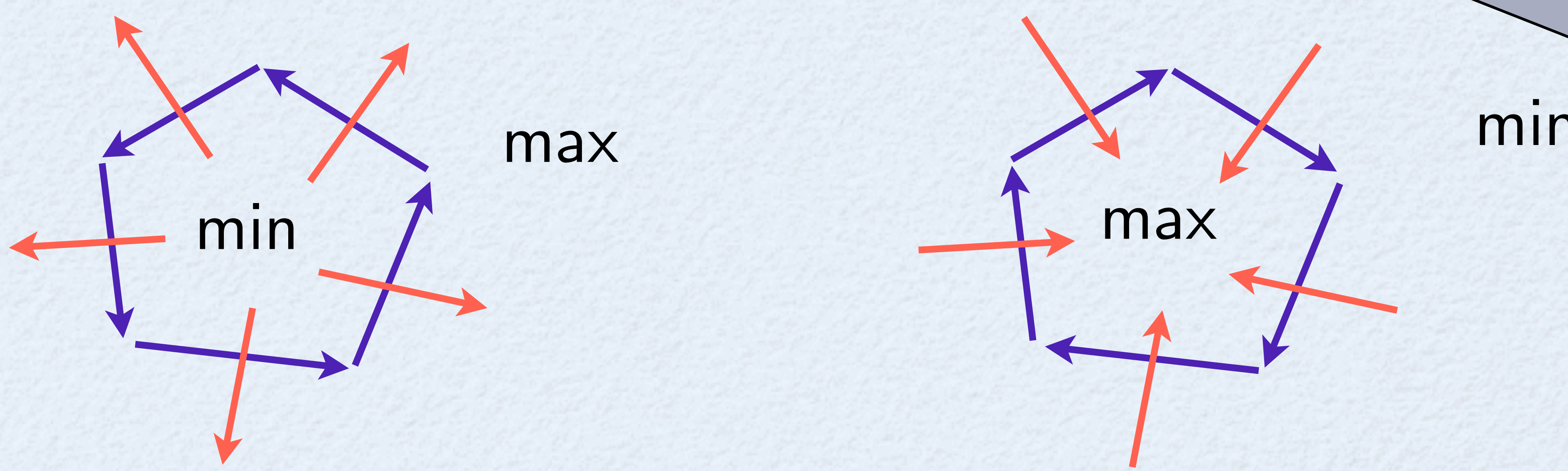
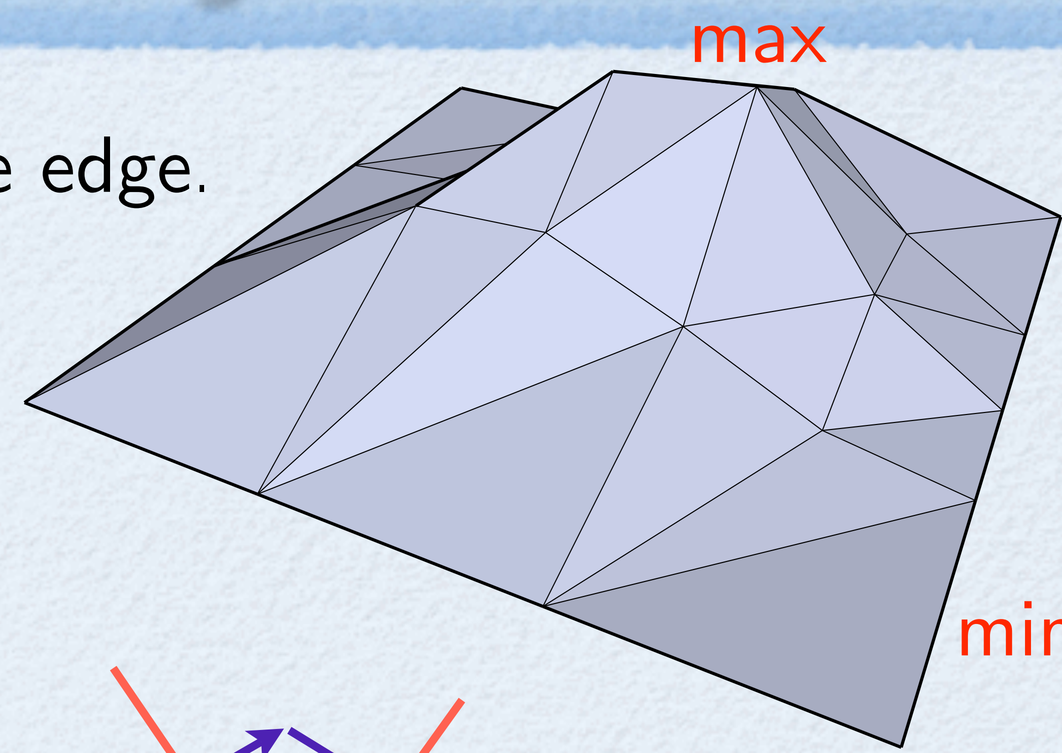


3. The min-max path can only cross the cycle once.

Partial Order on Triangles

Lemma. Every cycle of $\mathbb{M}^*$ loses precisely one edge.



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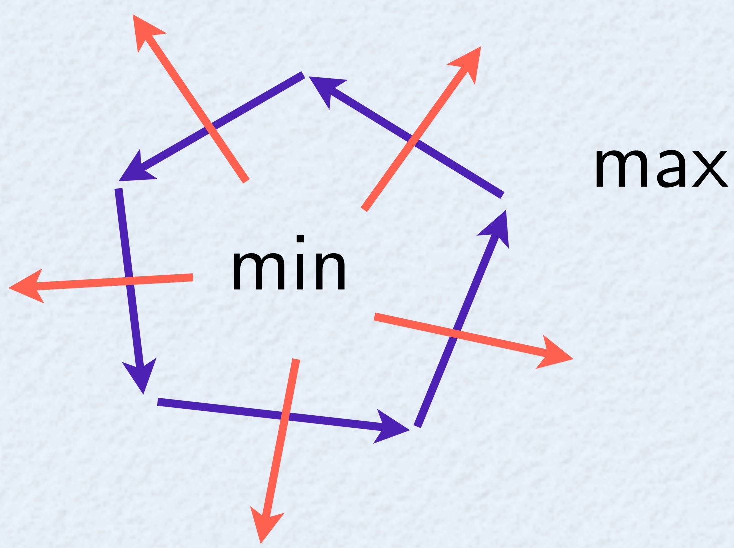

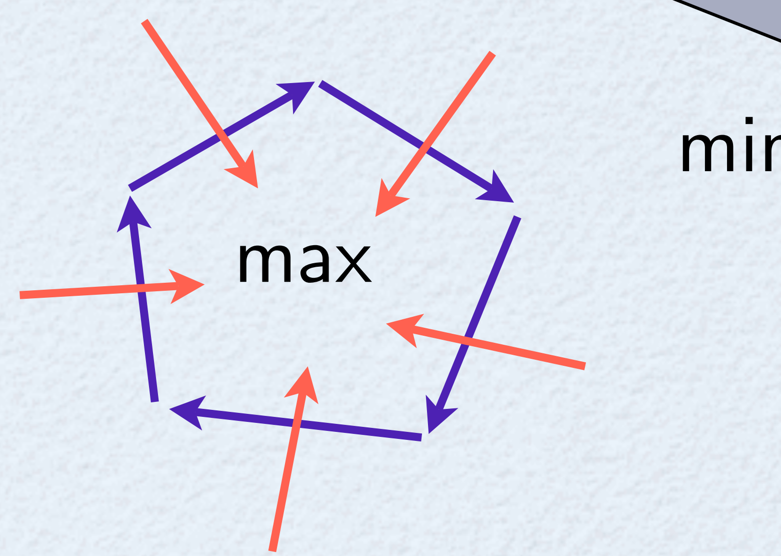

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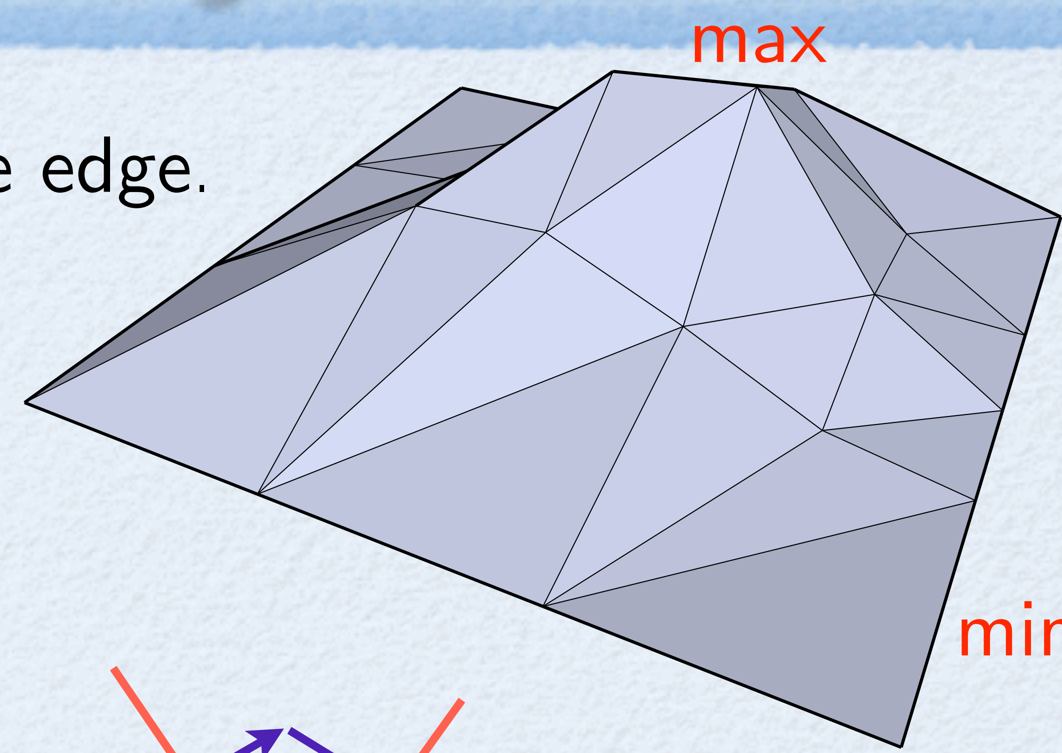

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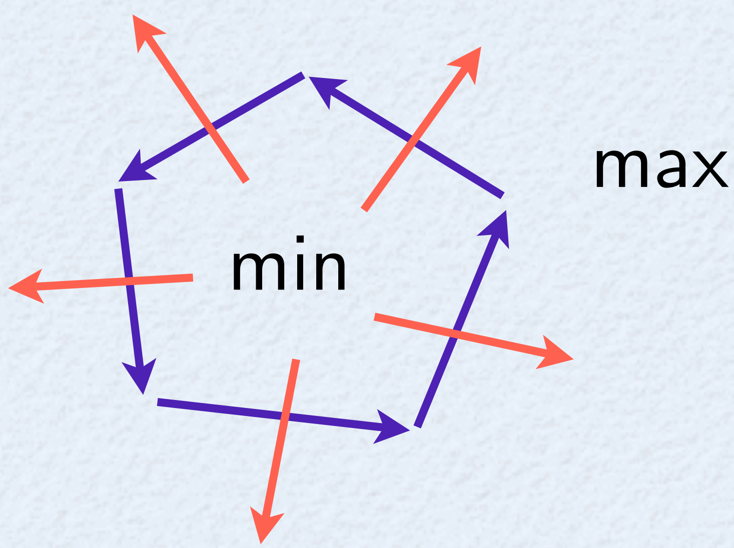

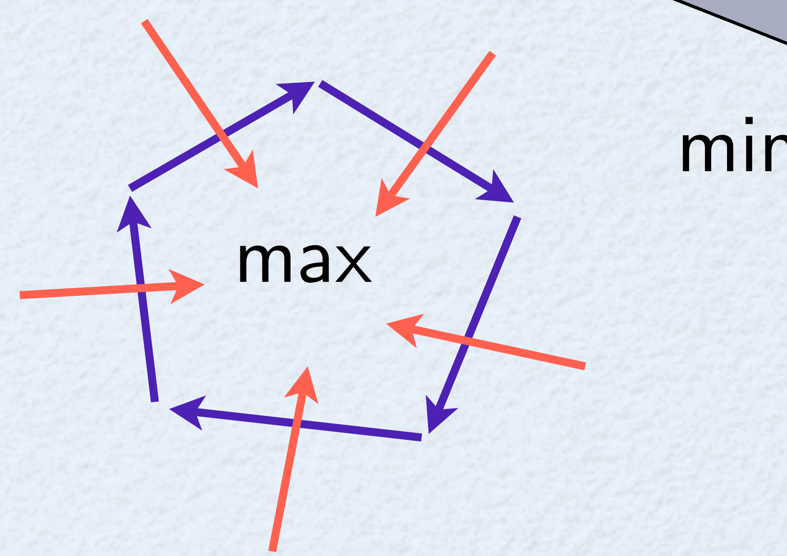

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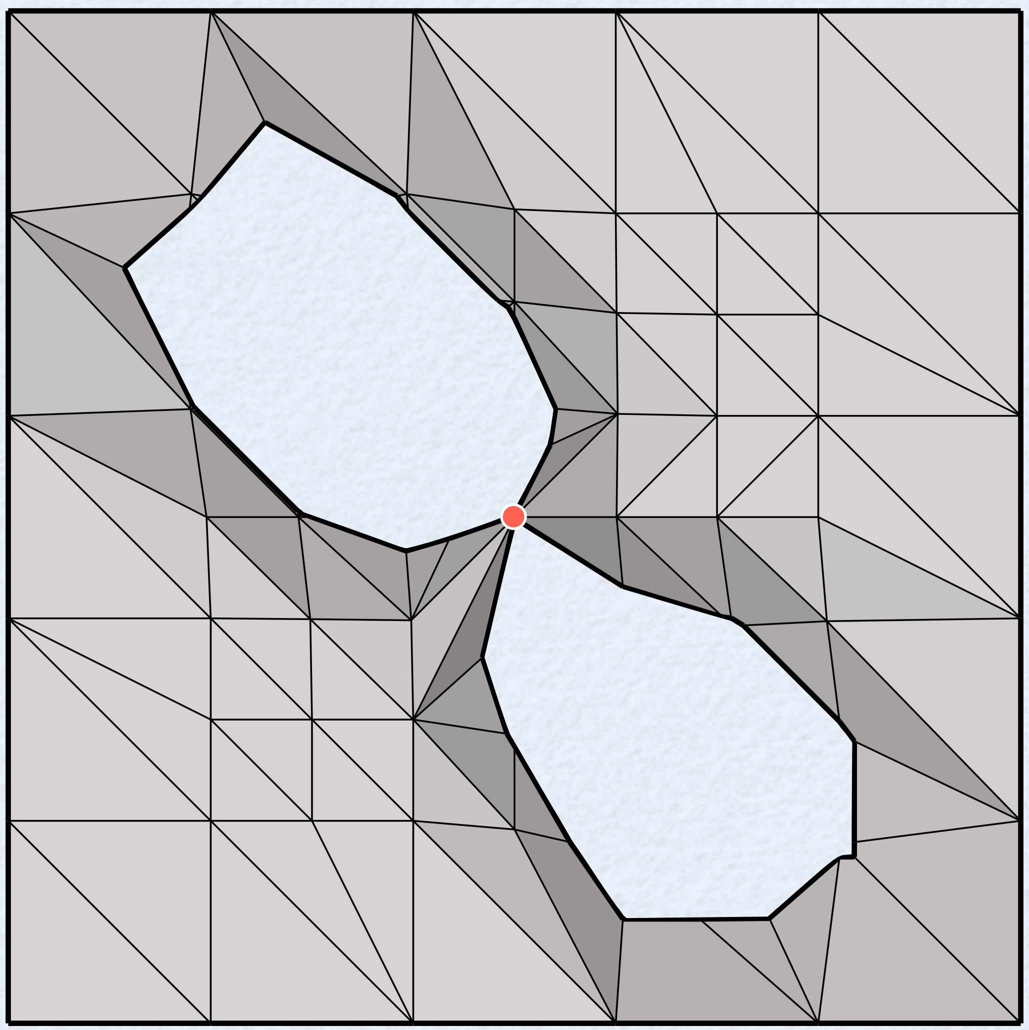
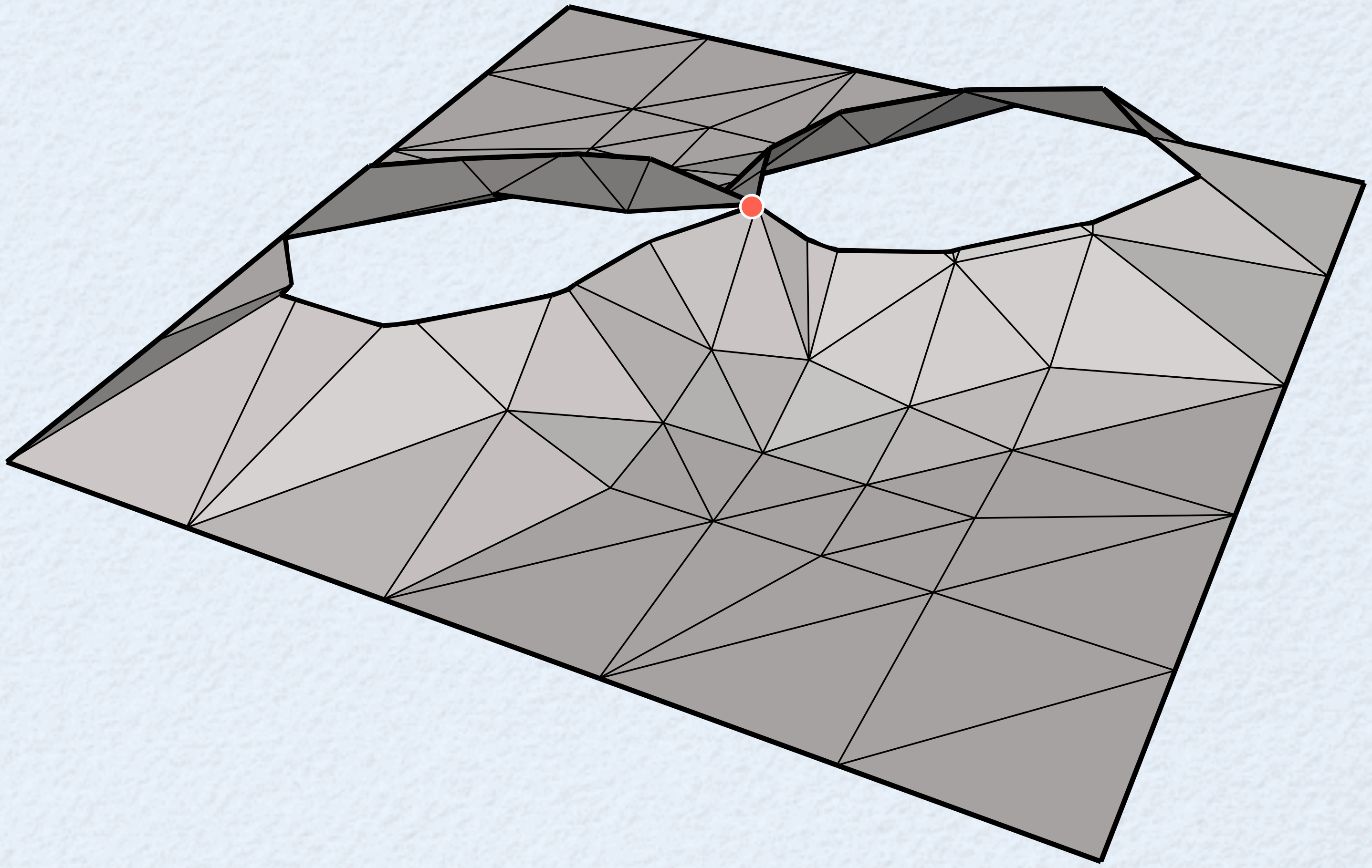
[Arge, Toma, Zeh'03]

What about terrains with saddles?

A saddle is negative if it joins two disjoint connected components of its sublevel-set and positive otherwise.





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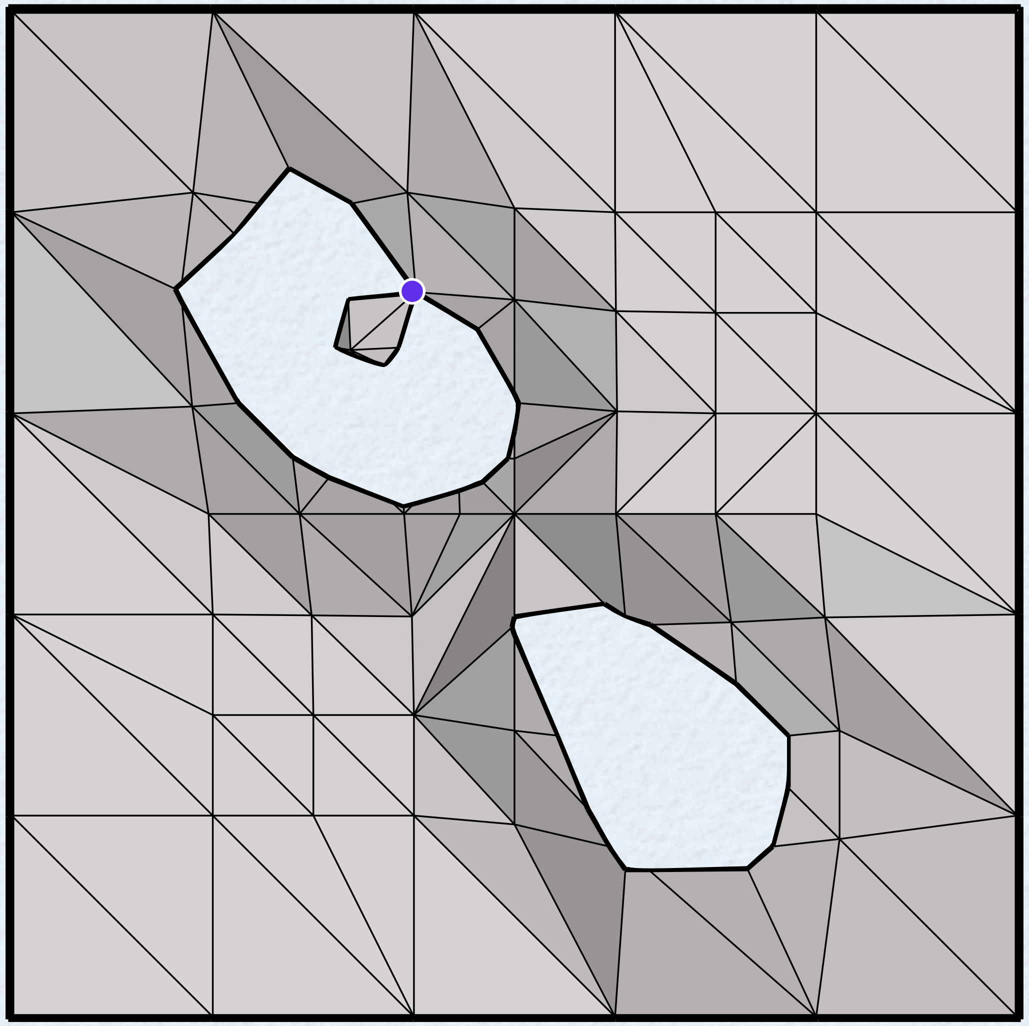

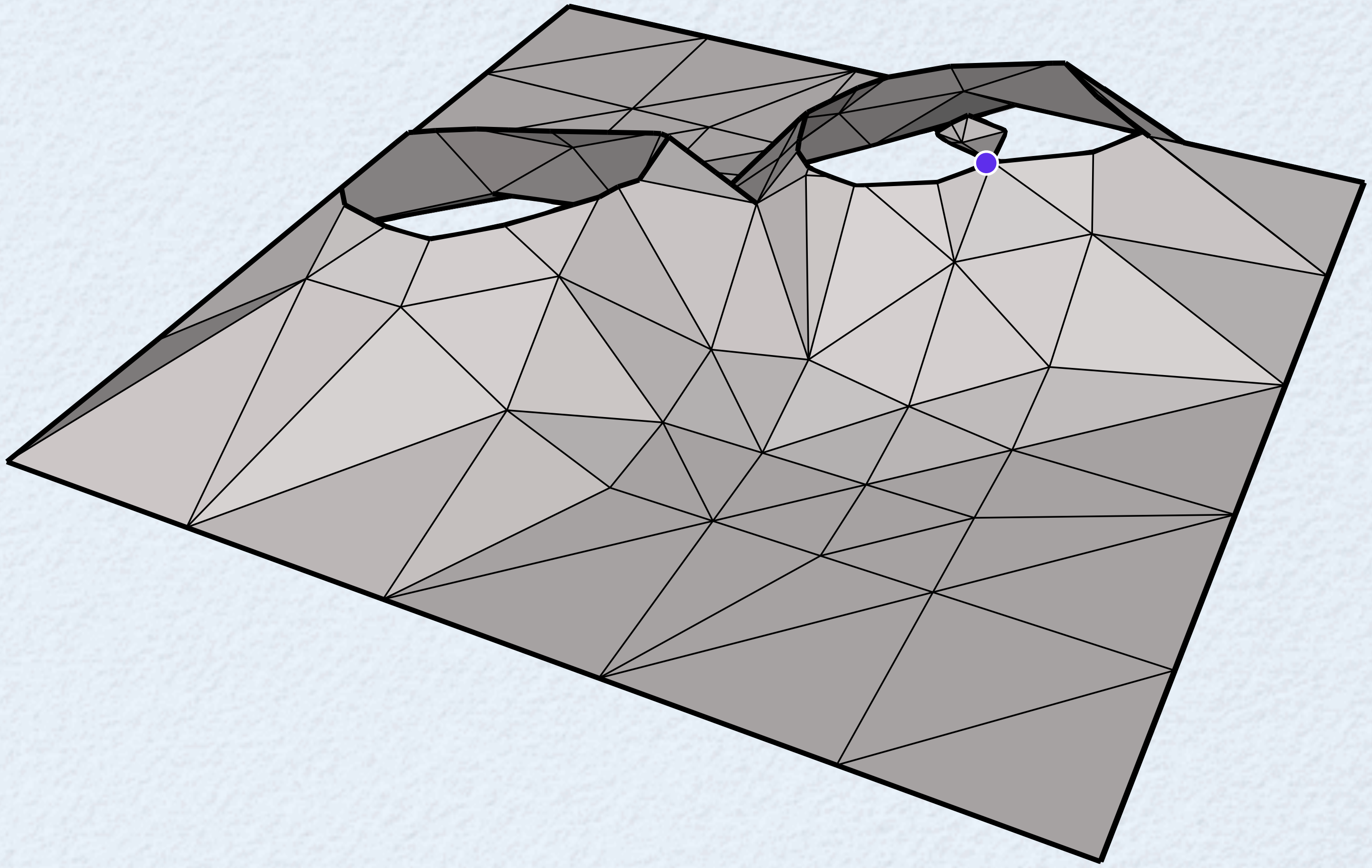

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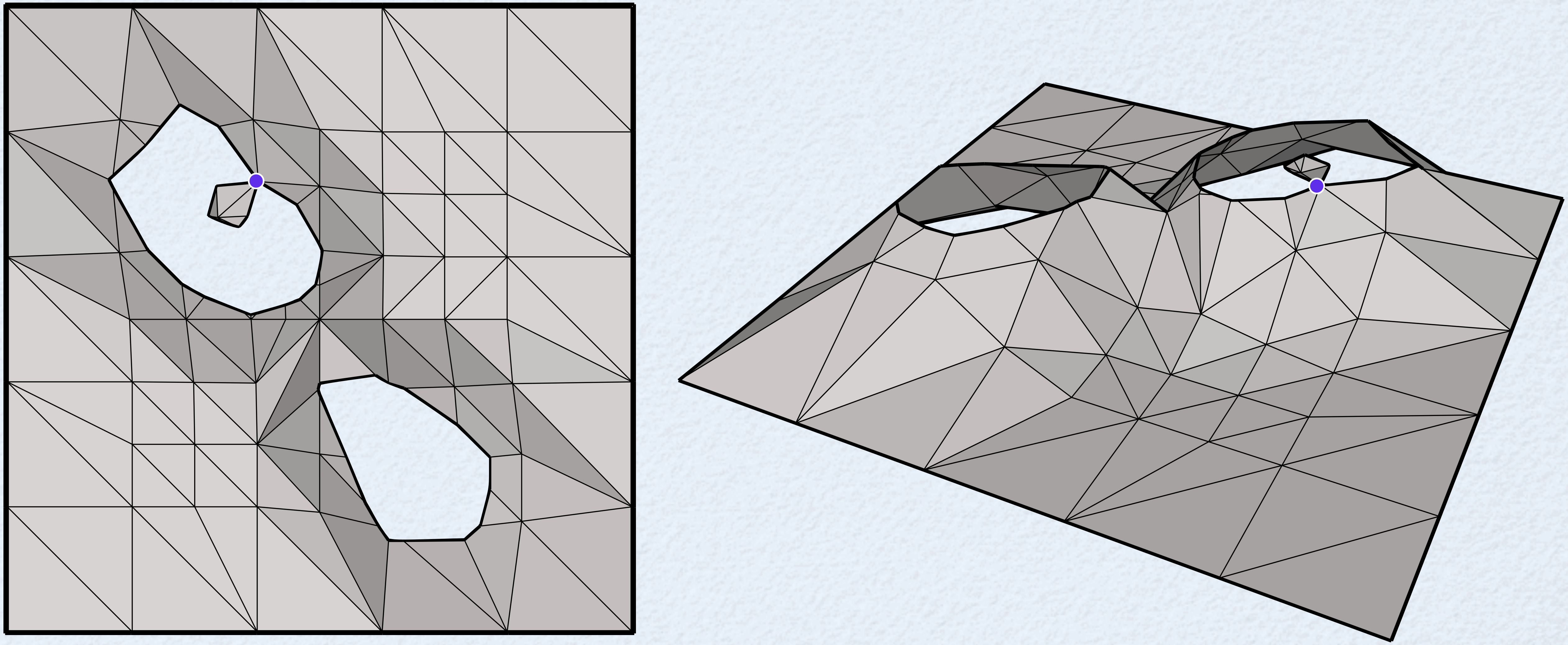

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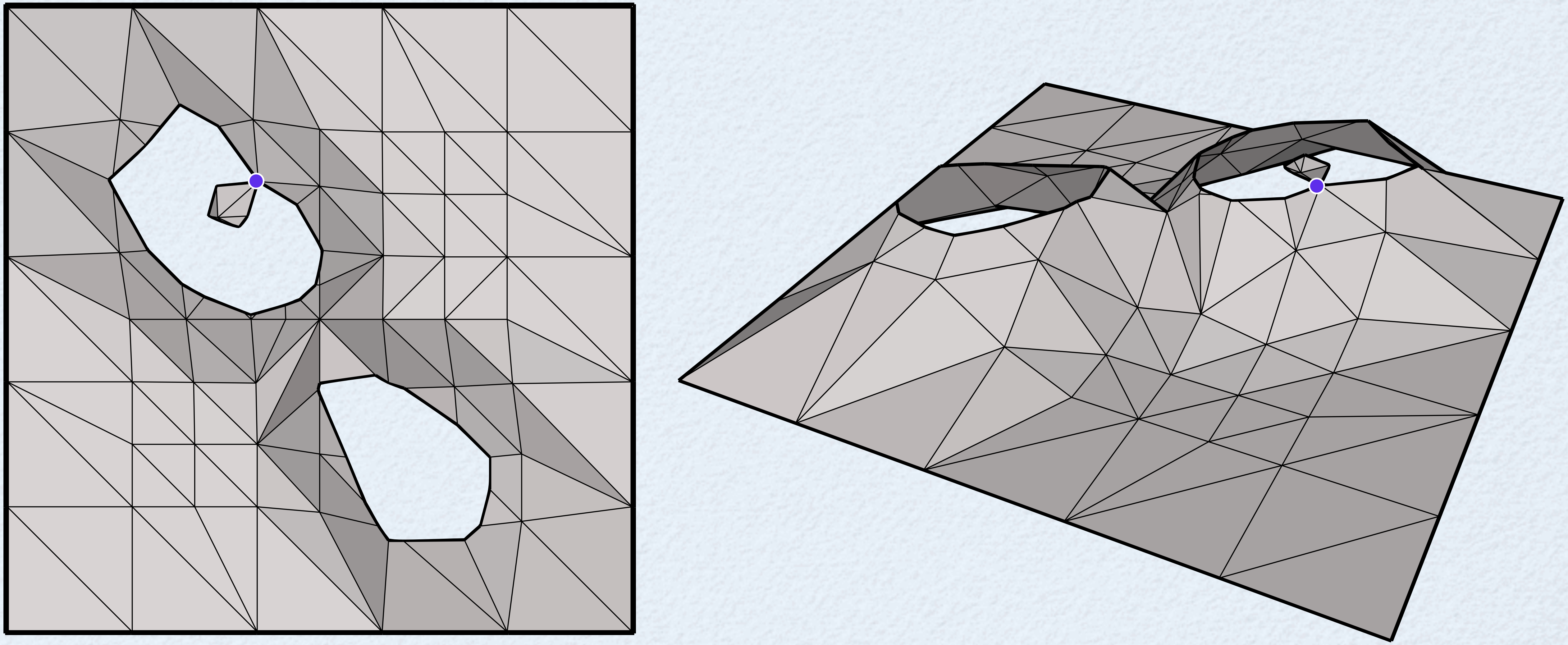

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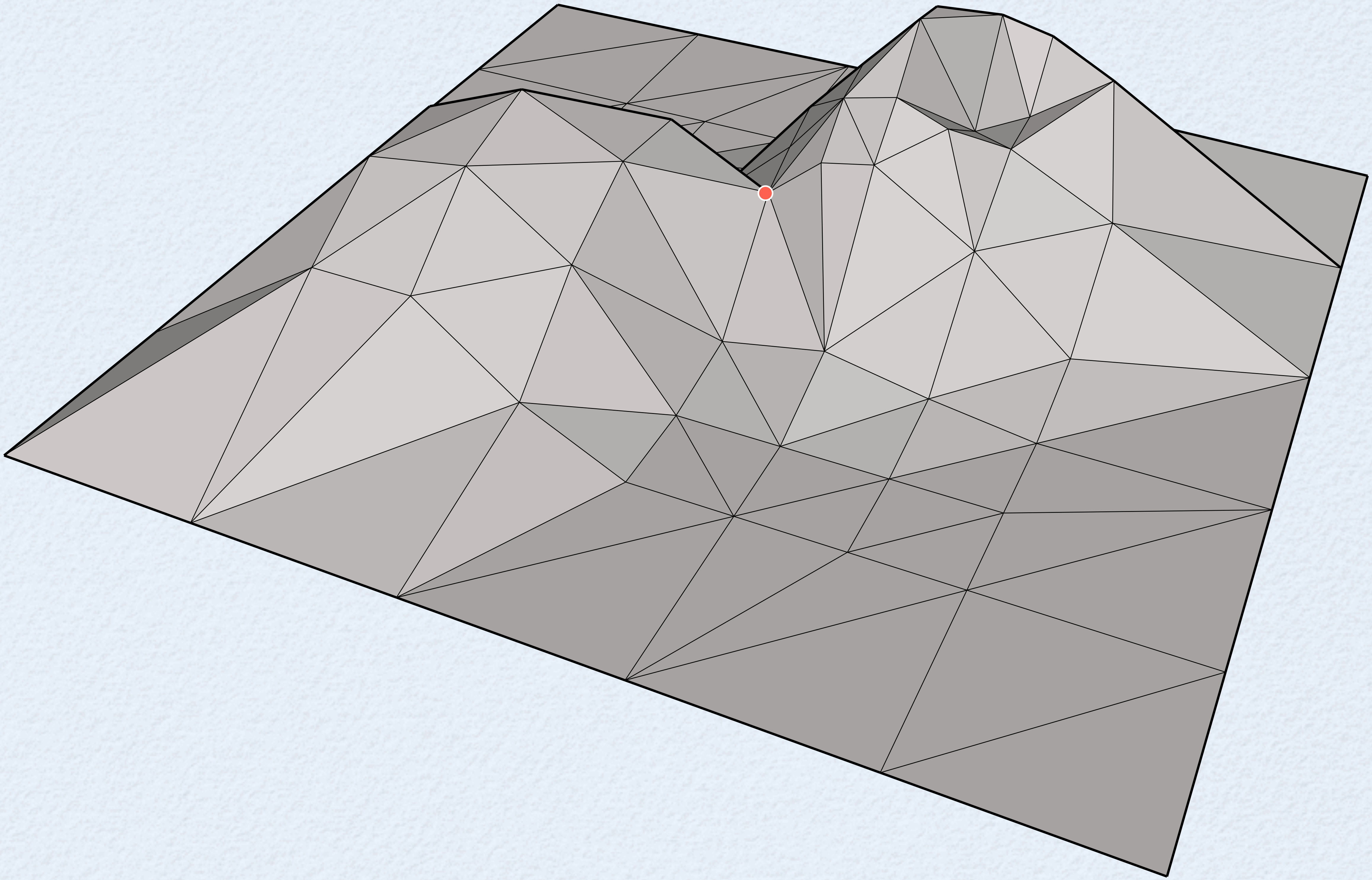
[Agarwal, Arge, Yi '06] Positive and negative saddle points can be found in $O(\text{Sort}(N))$ I/Os.

Positive and Negative Cut-Trees

Positive Cut-Tree: follow an ascending path in every connected component of the upper link of every positive saddle, joining paths when they collide.



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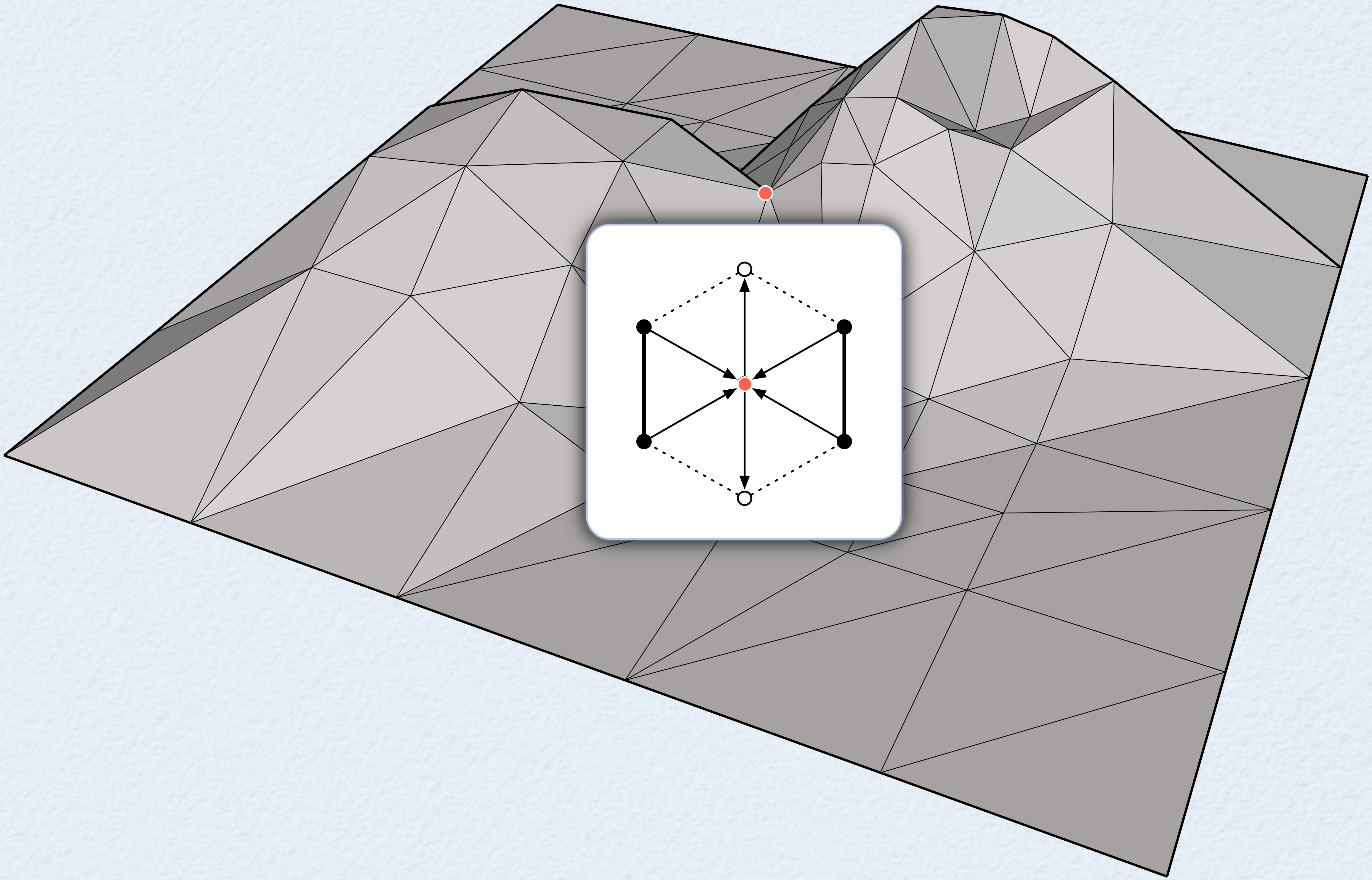

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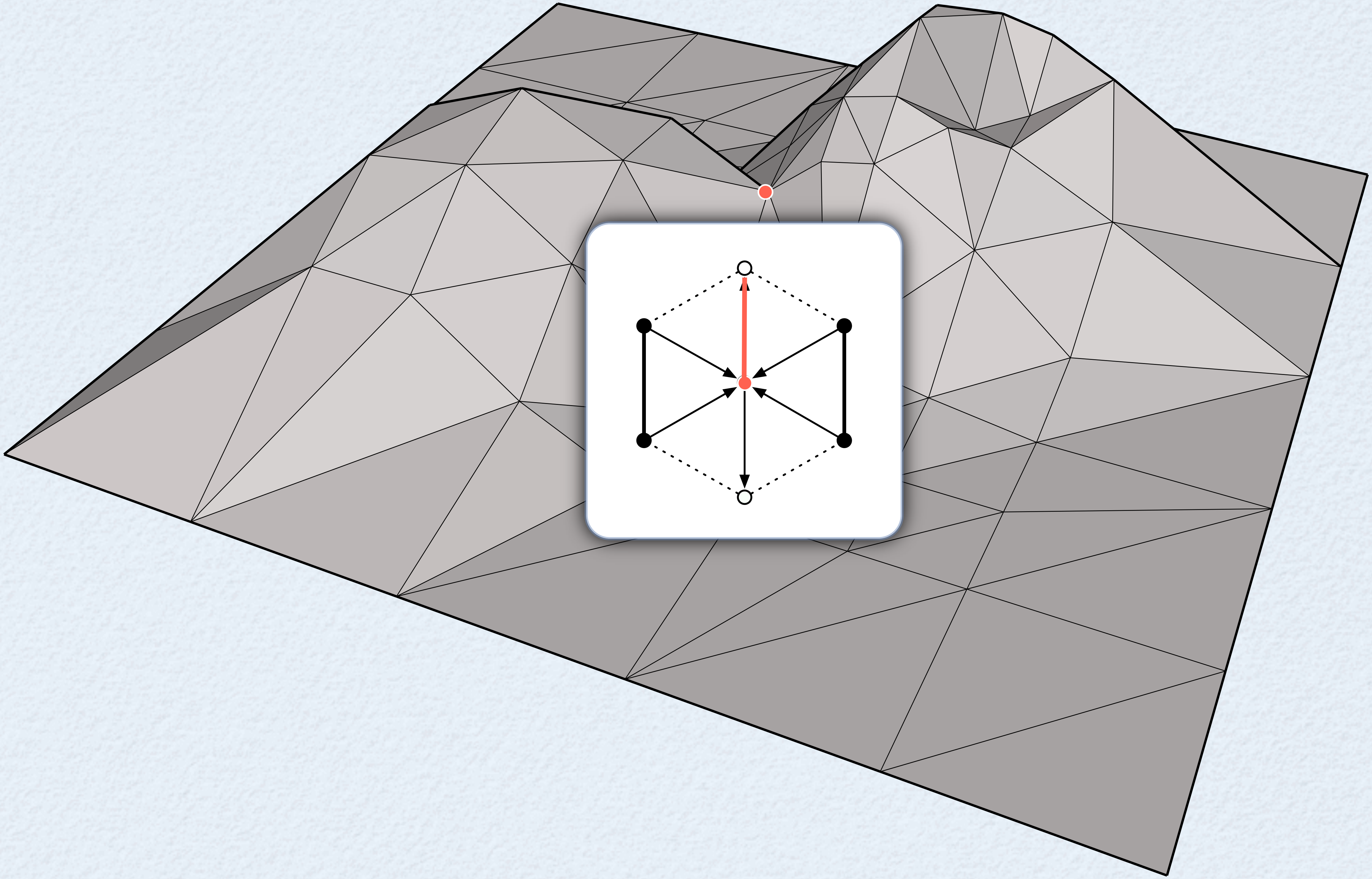

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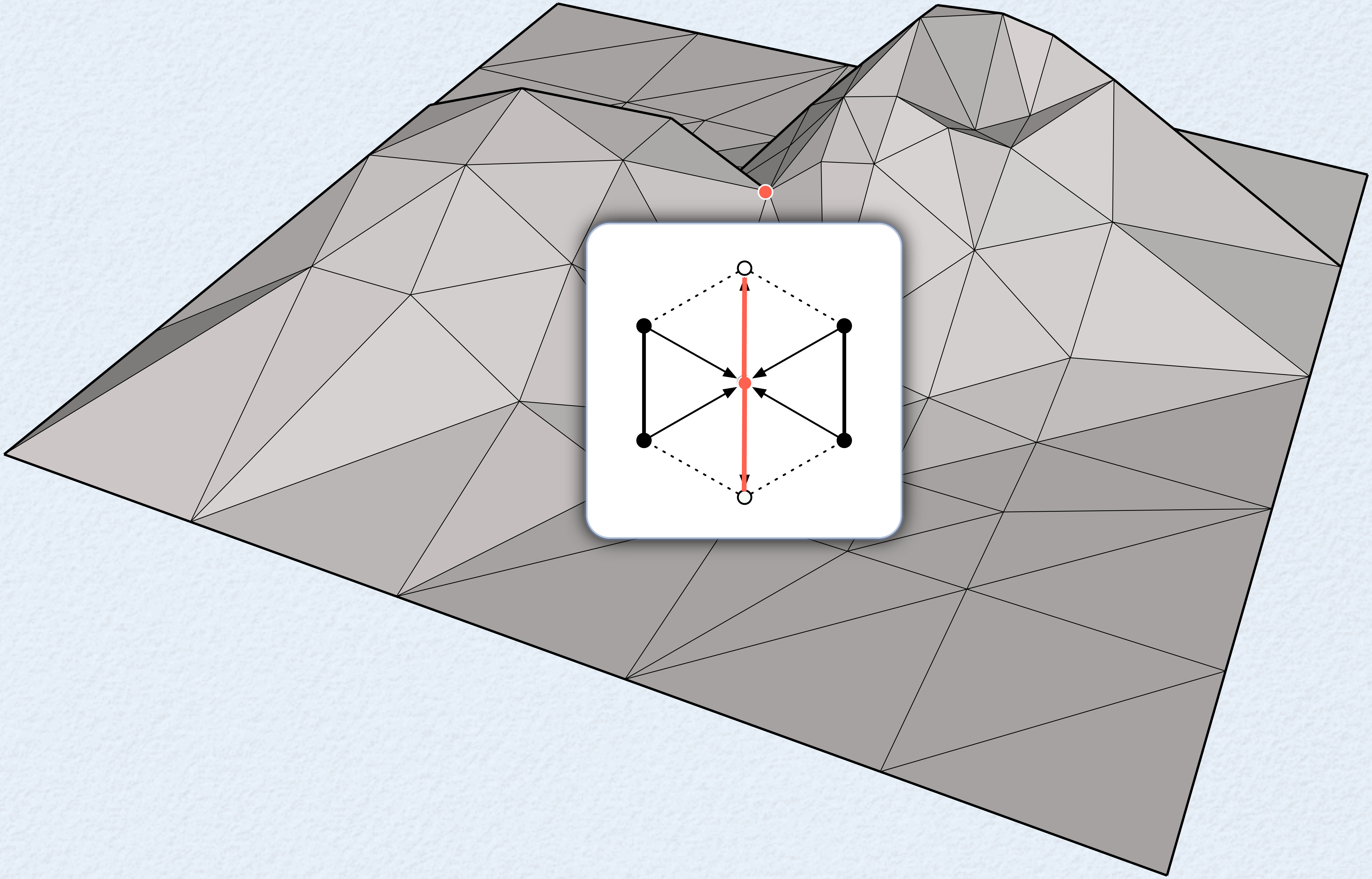

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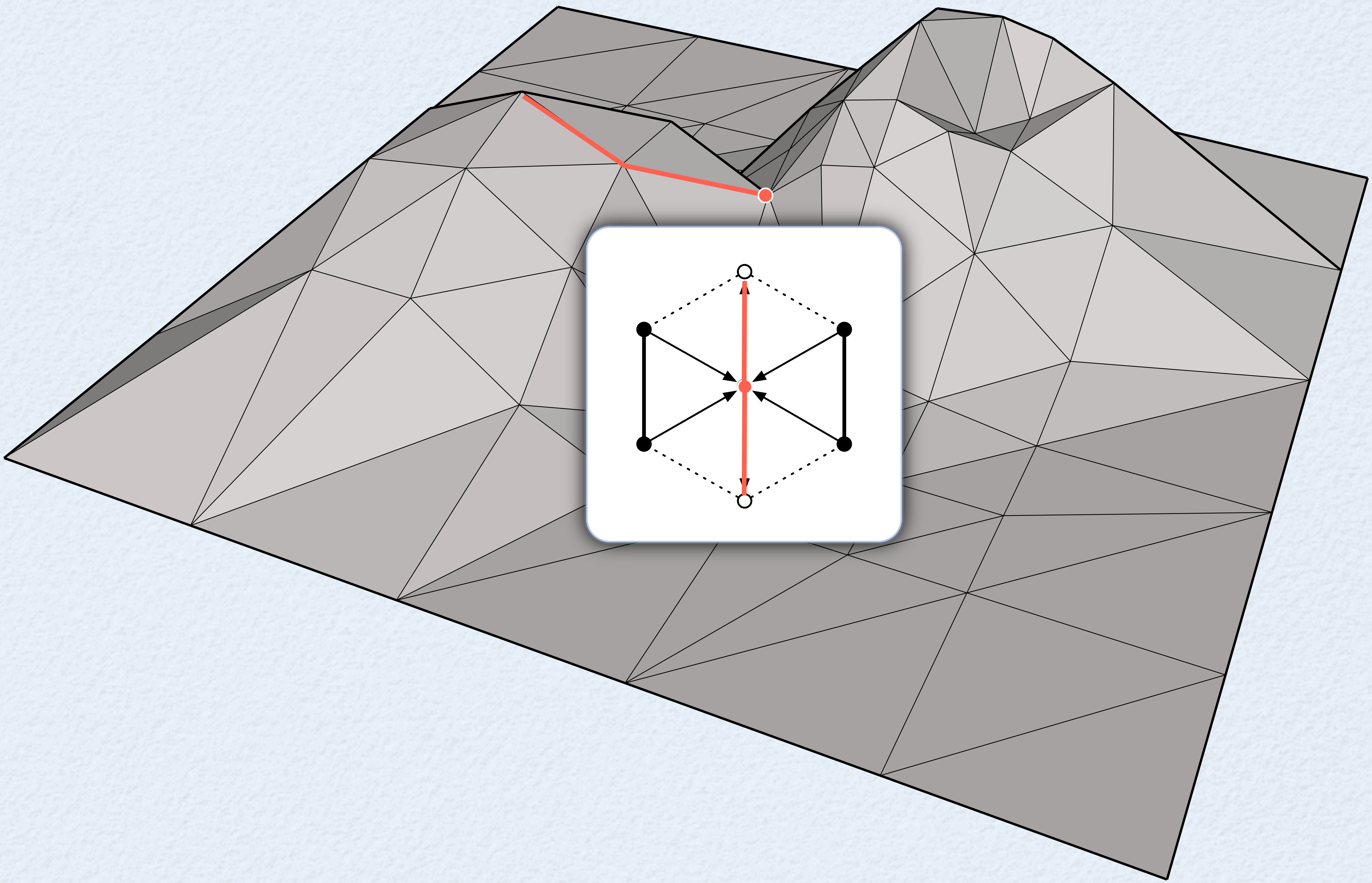

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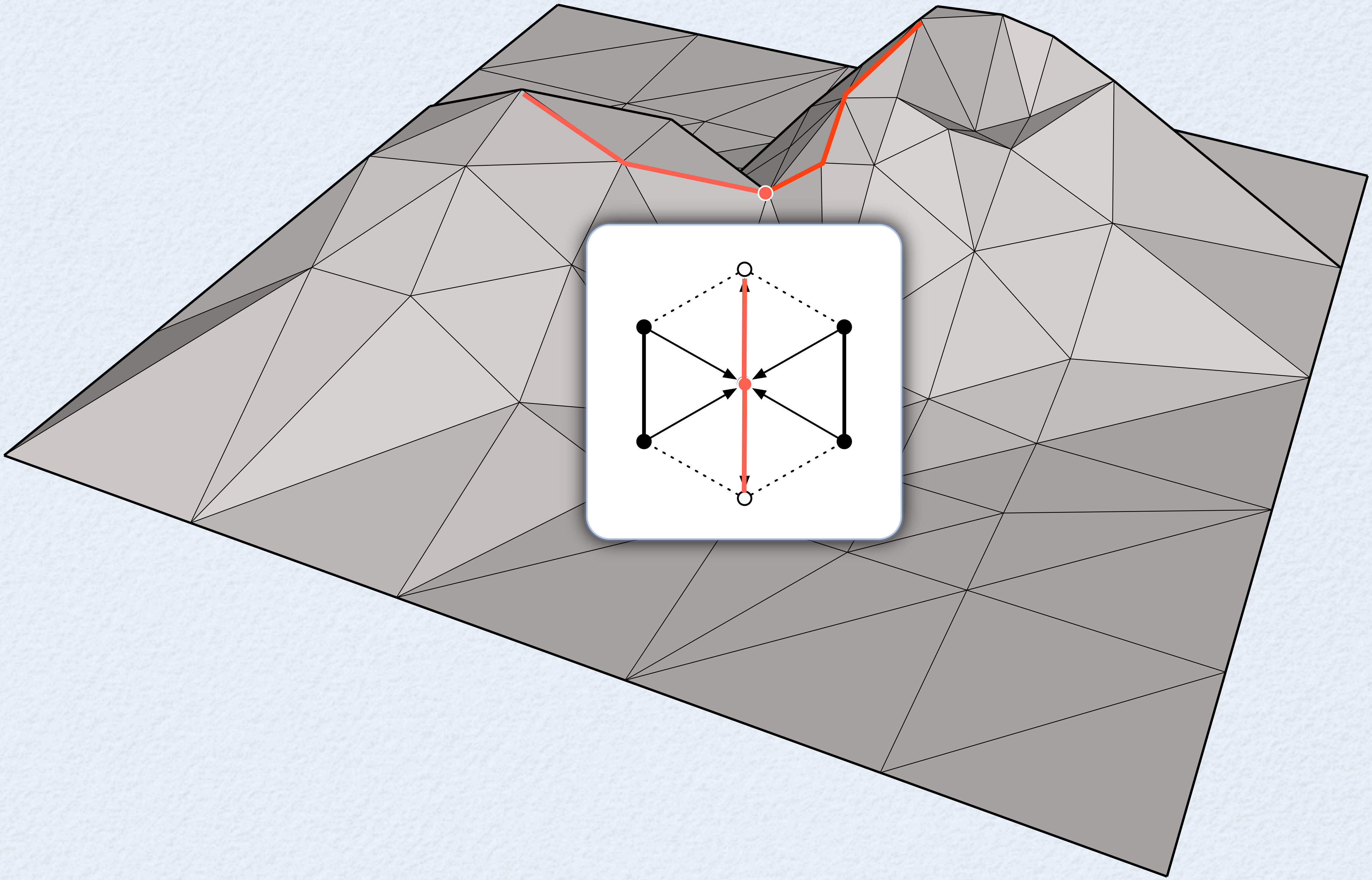

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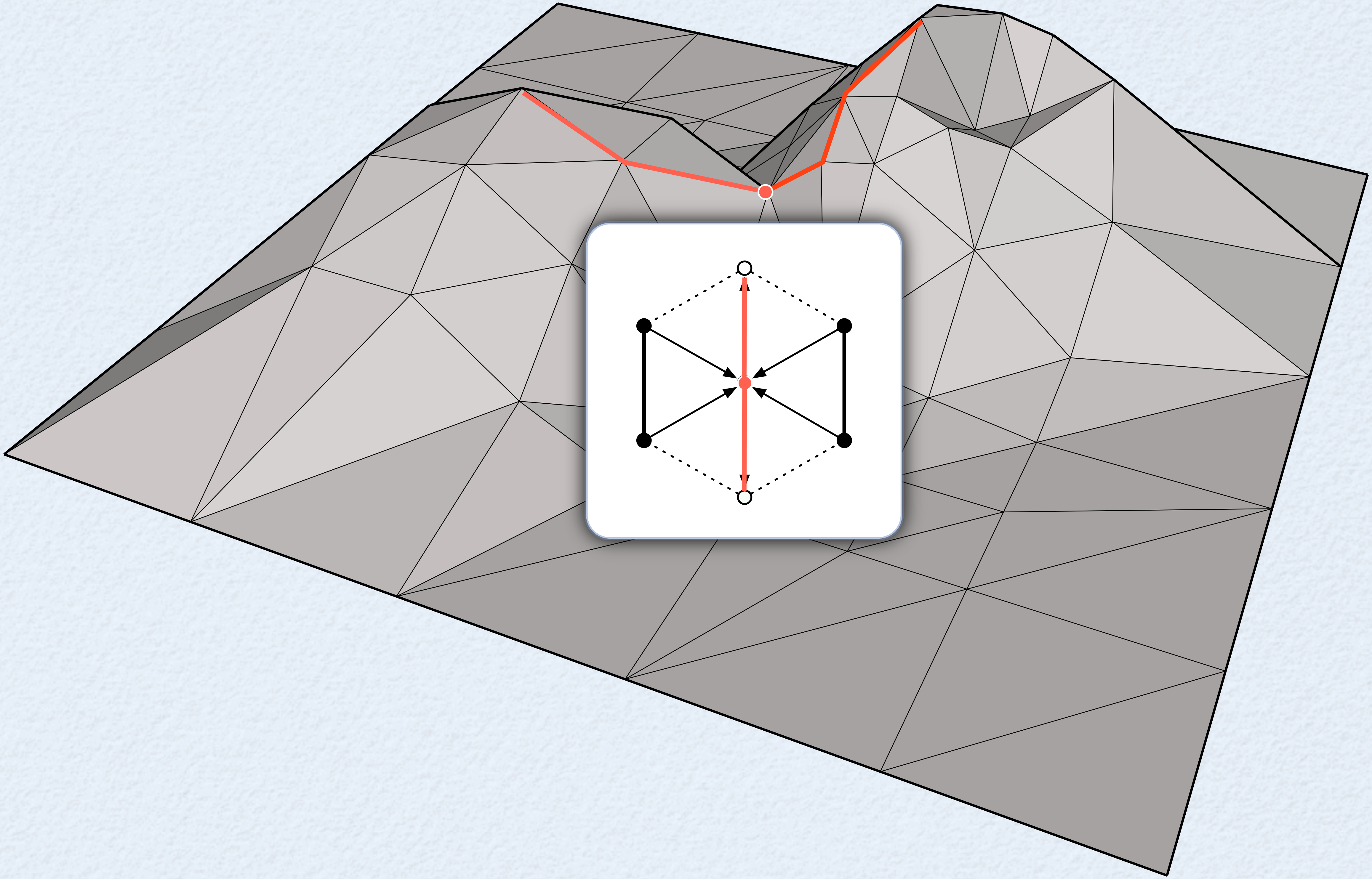

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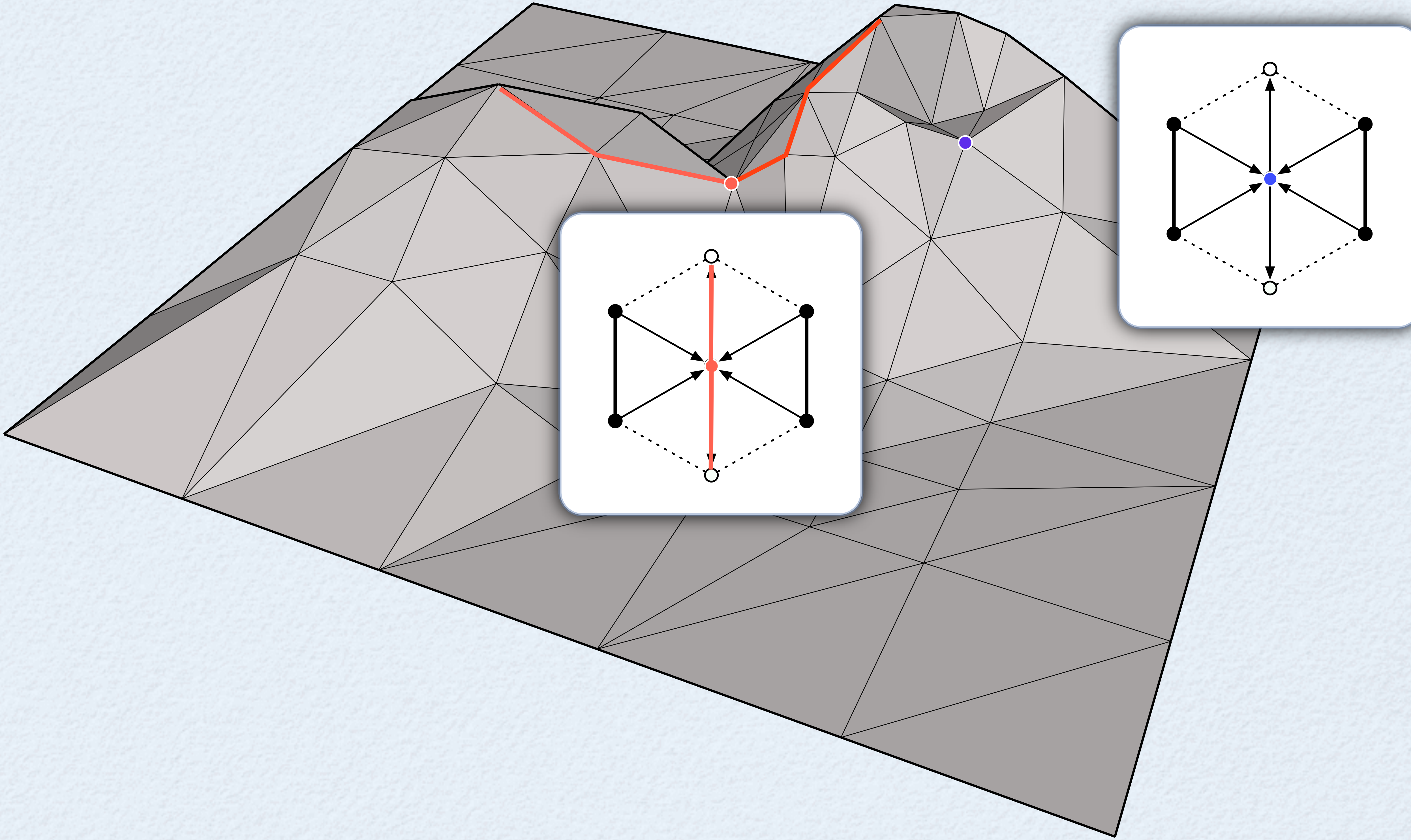

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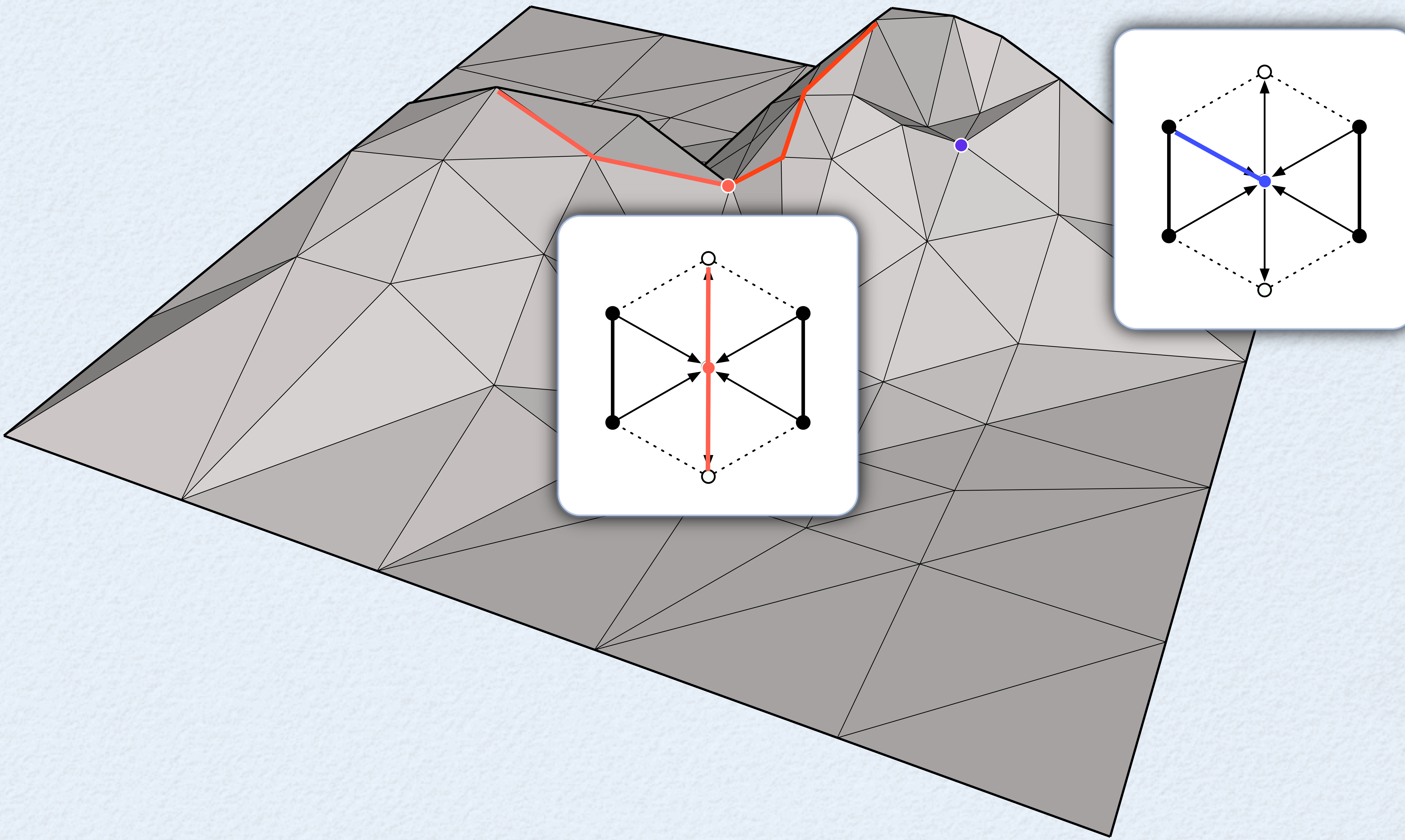

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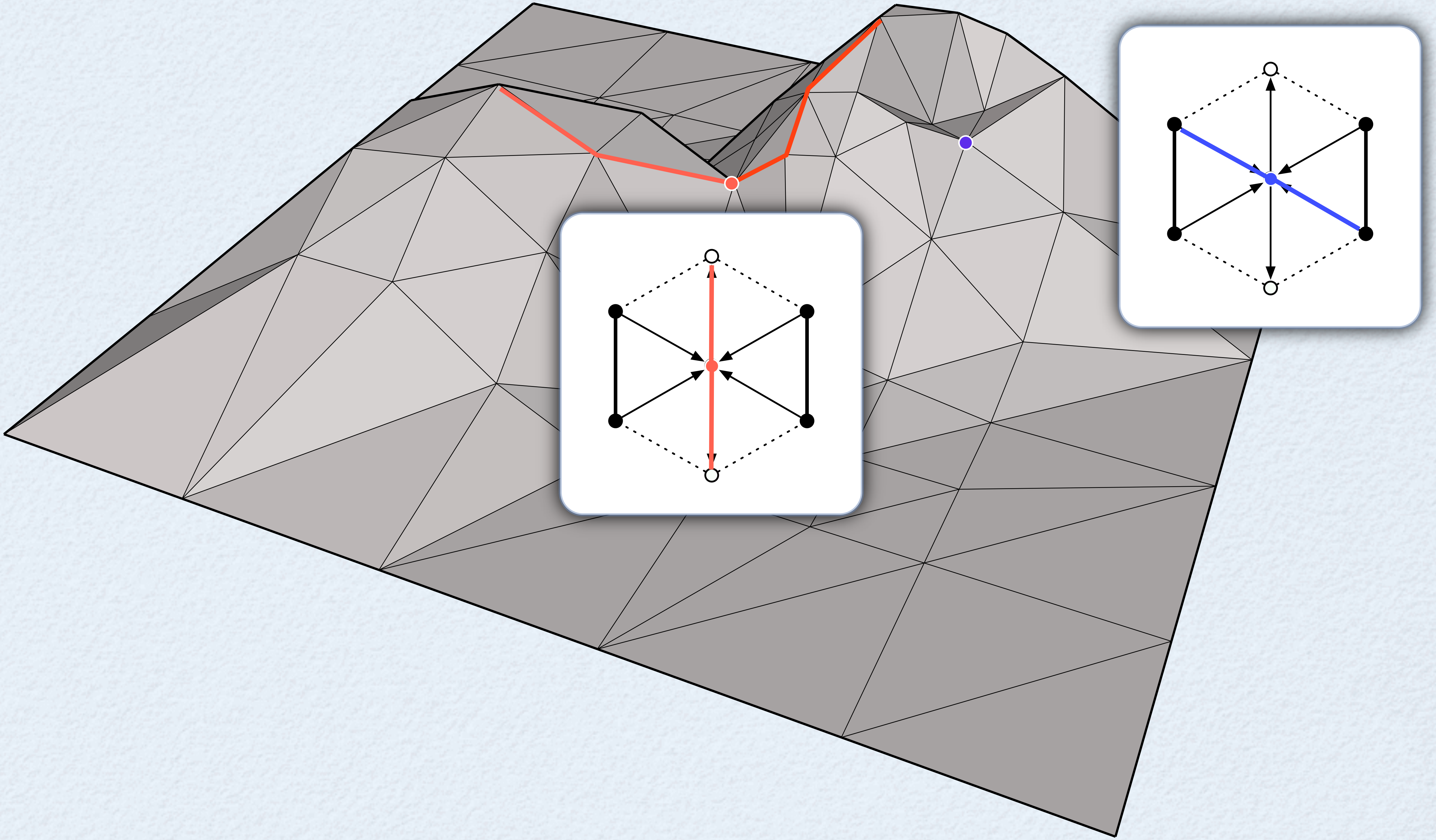

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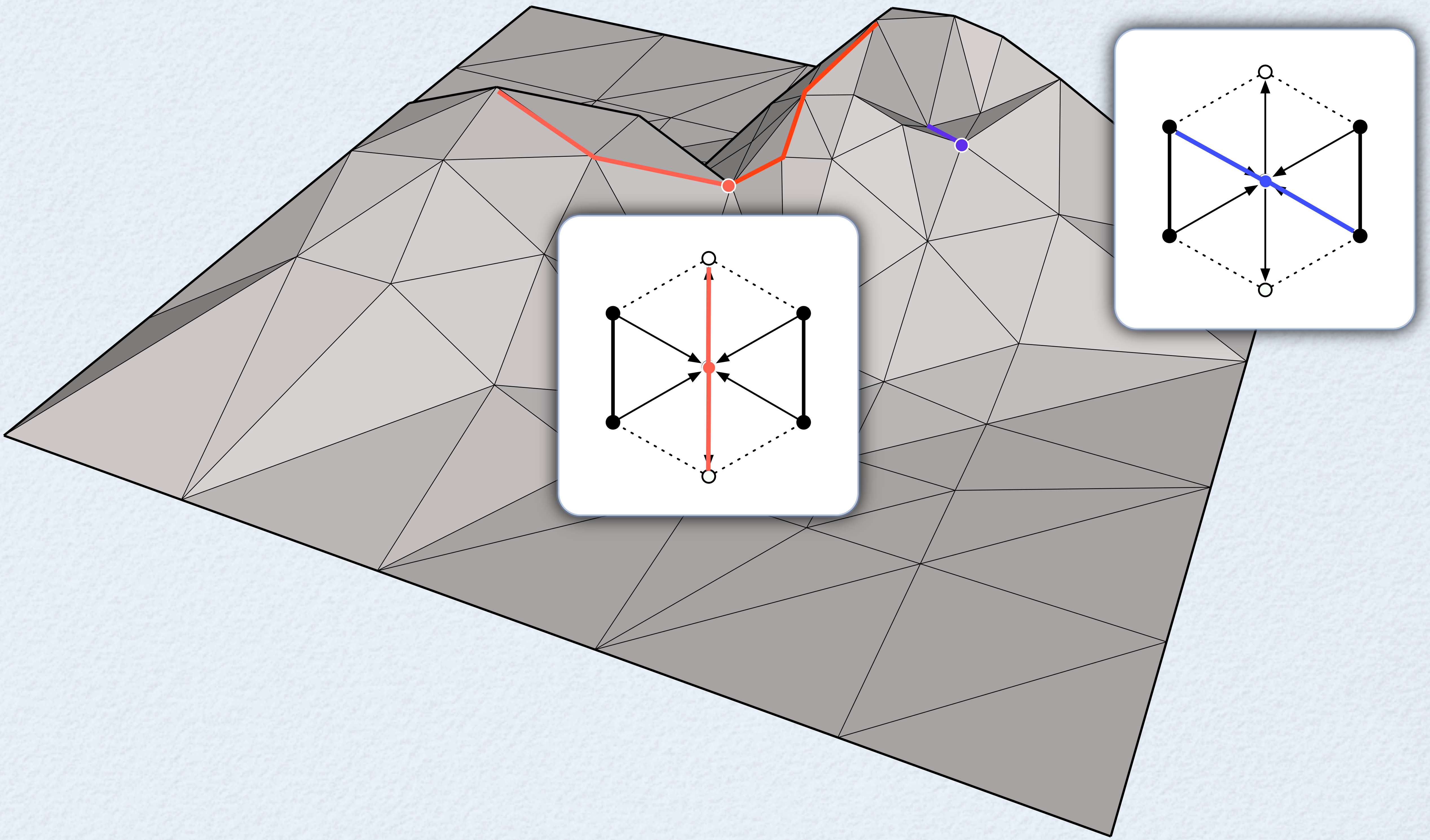

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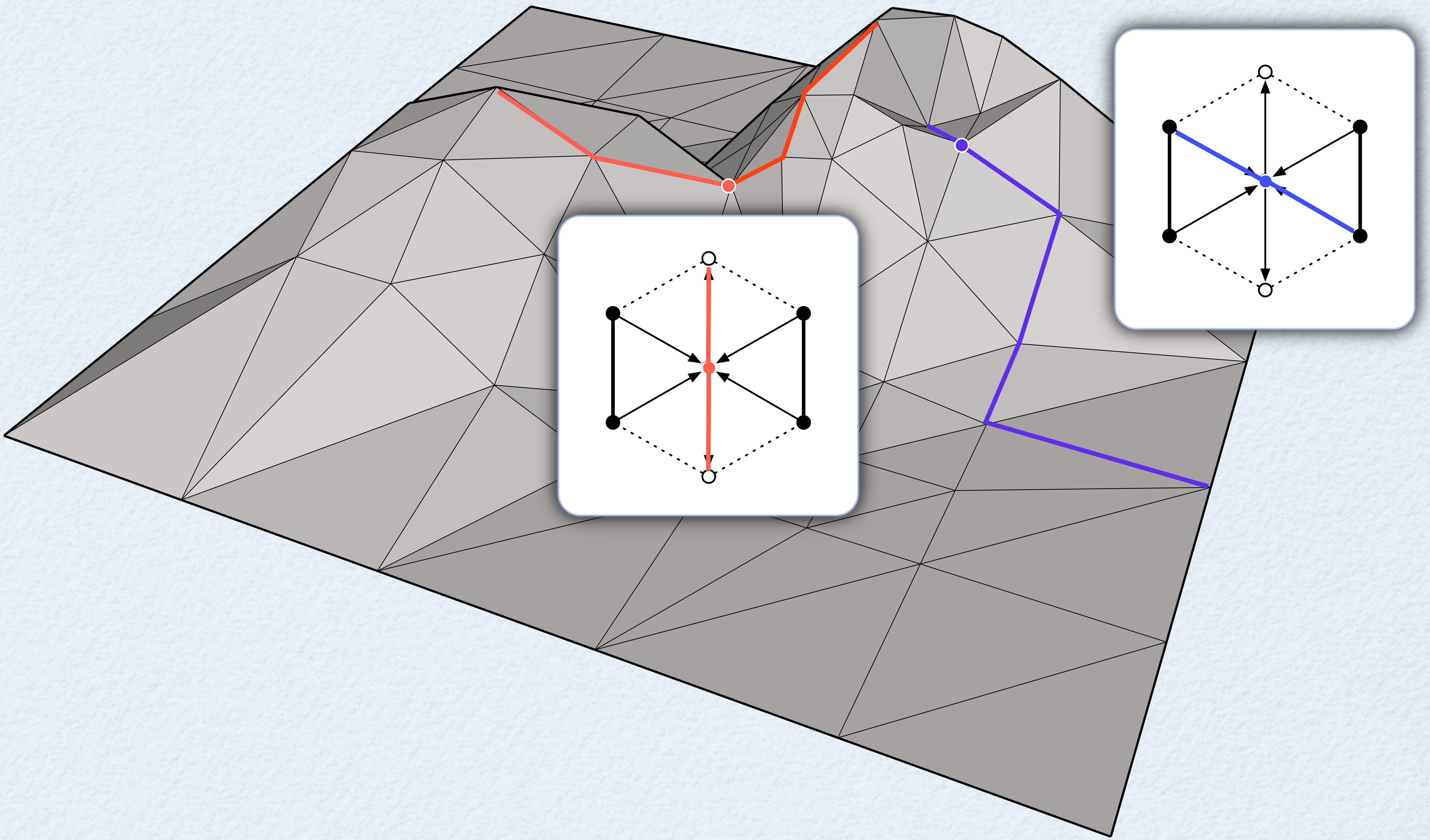

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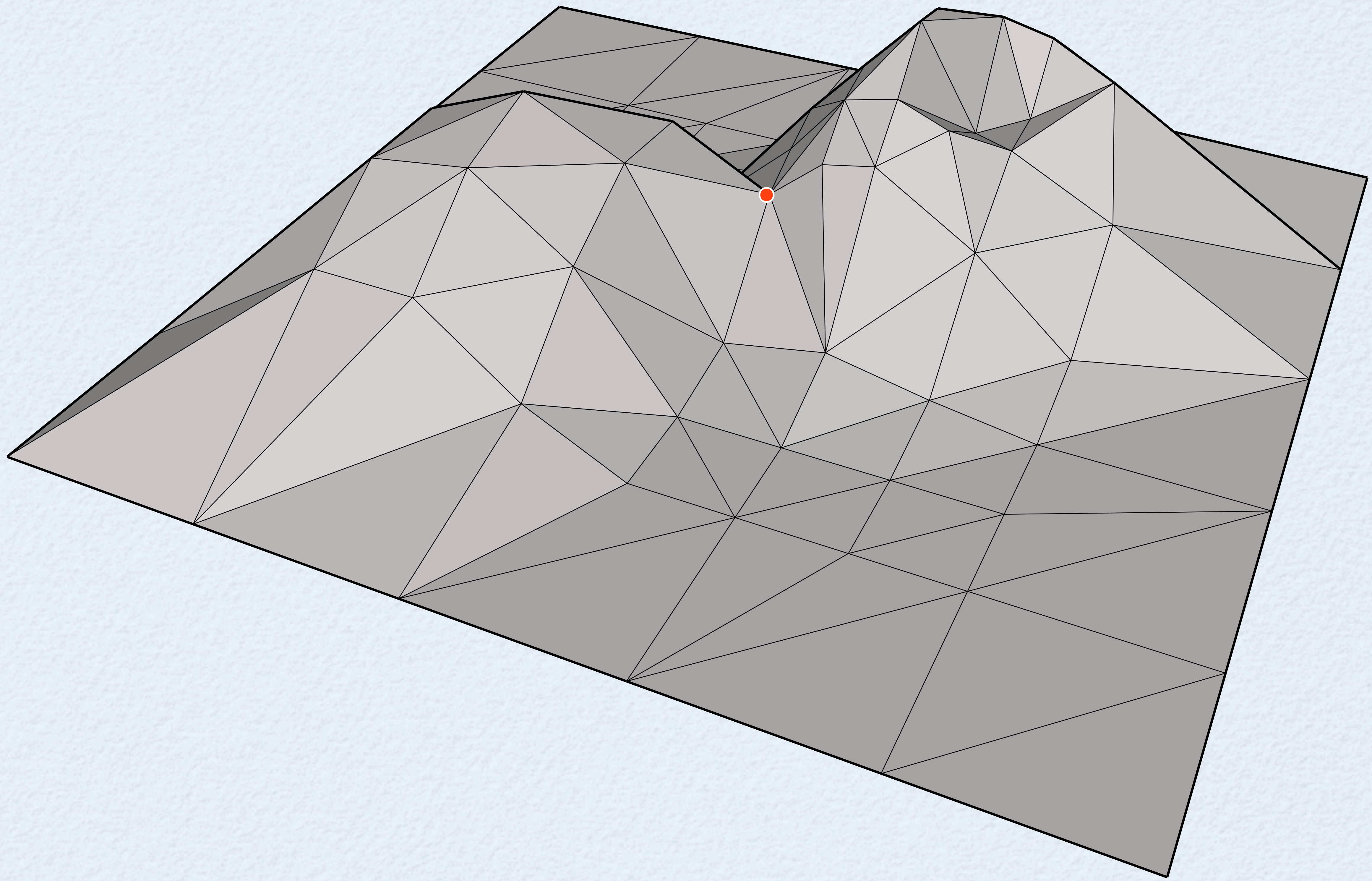
Turning a terrain into an elementary one by surgery



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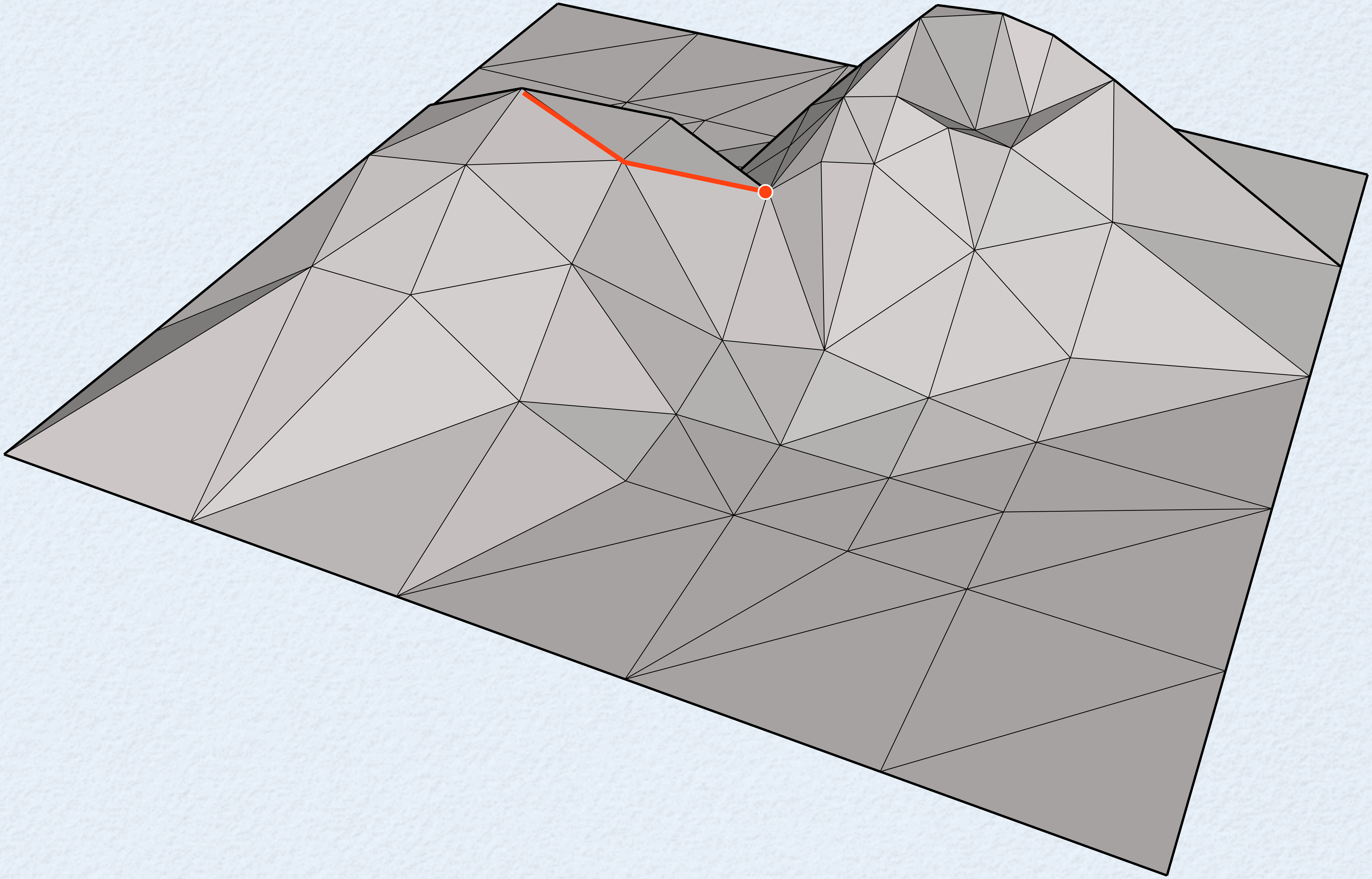

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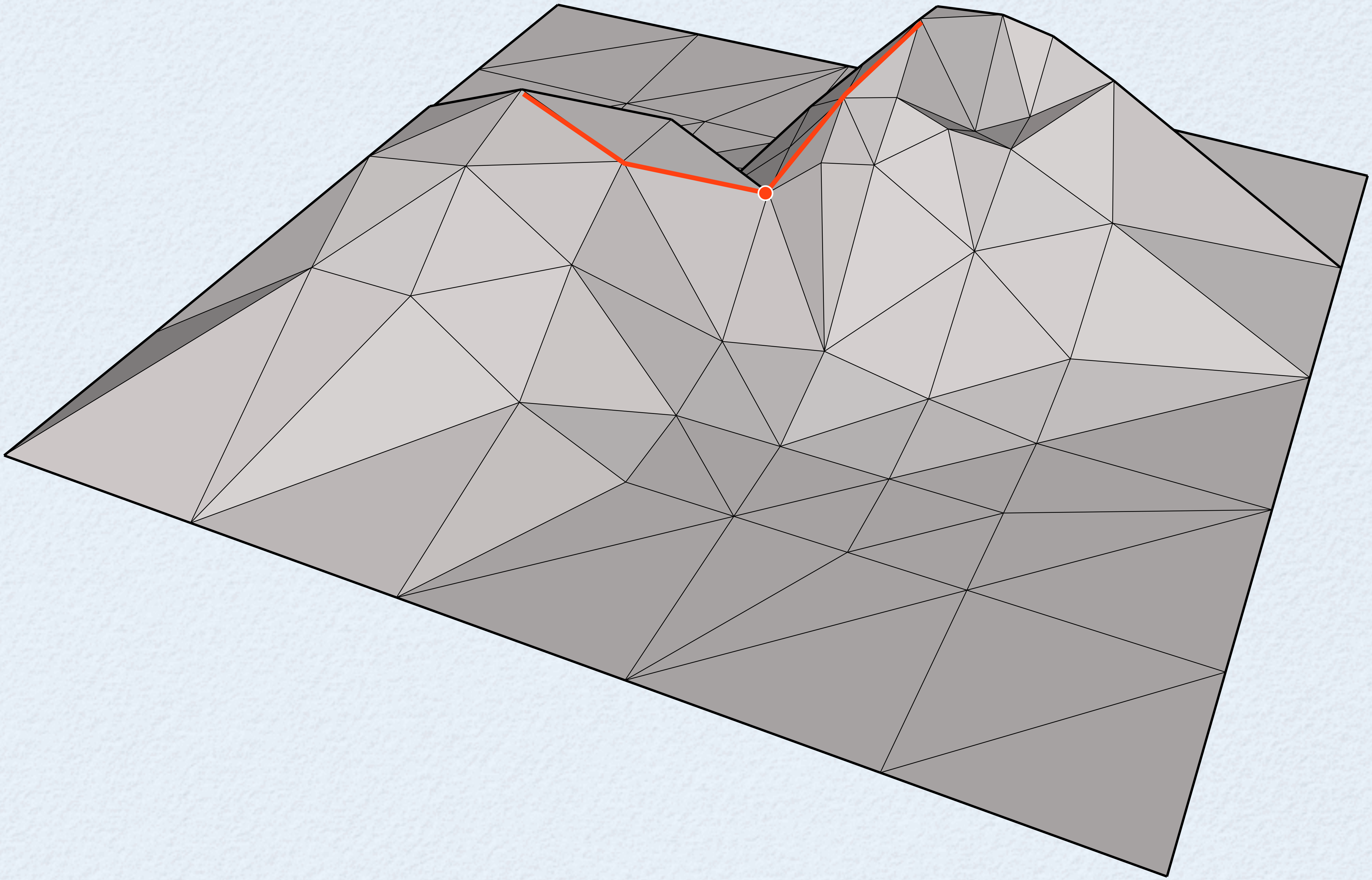

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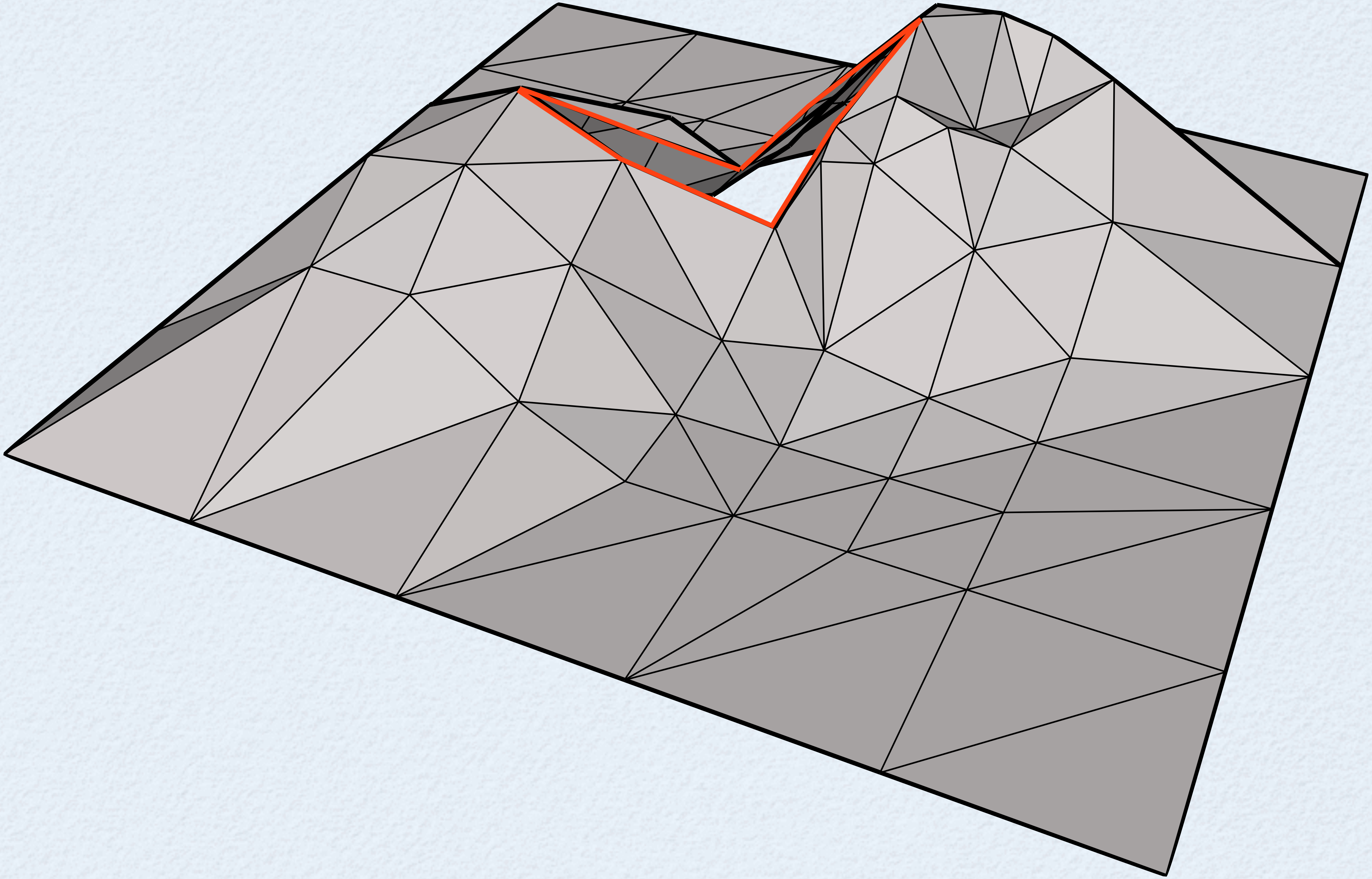

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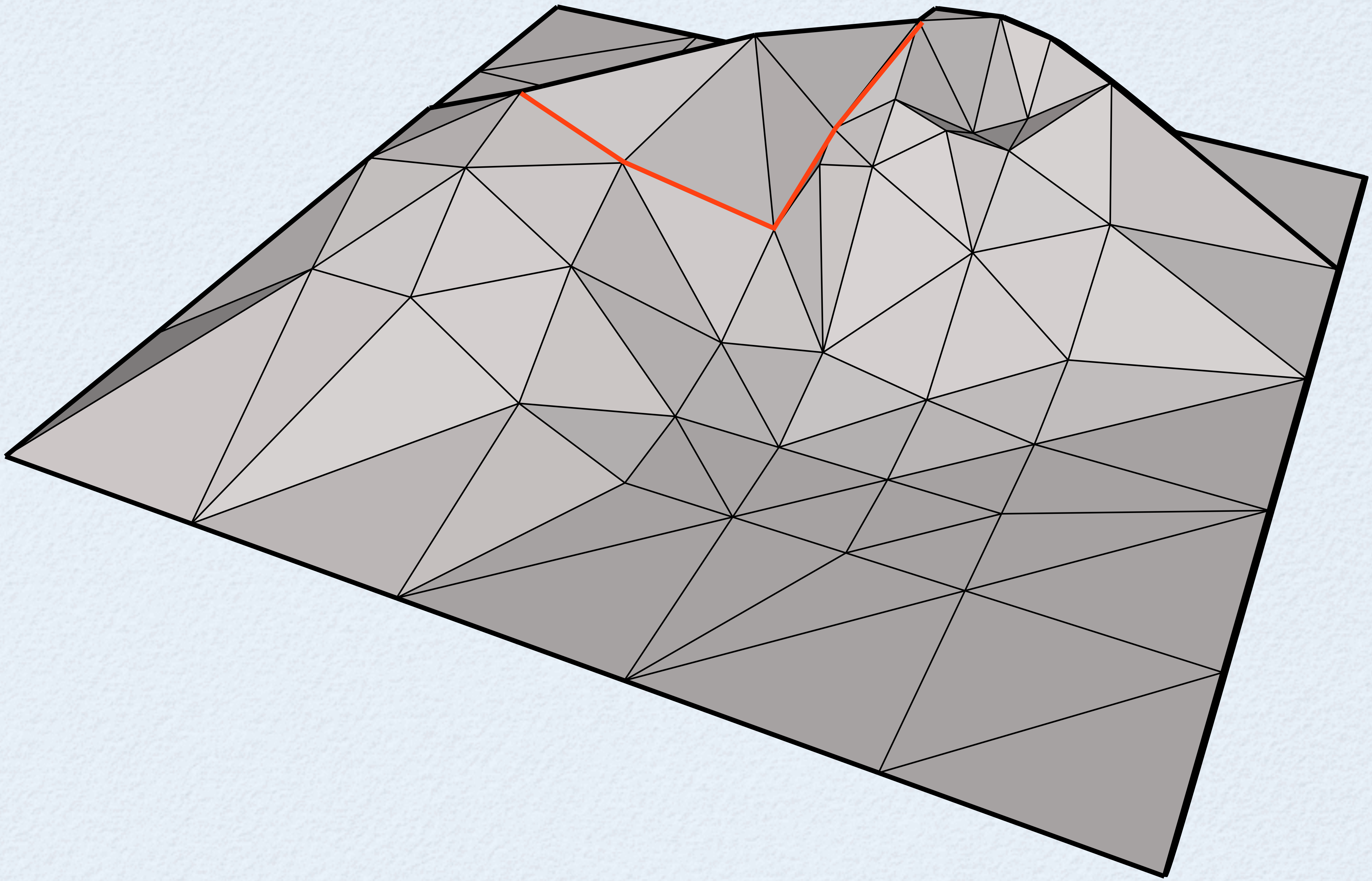

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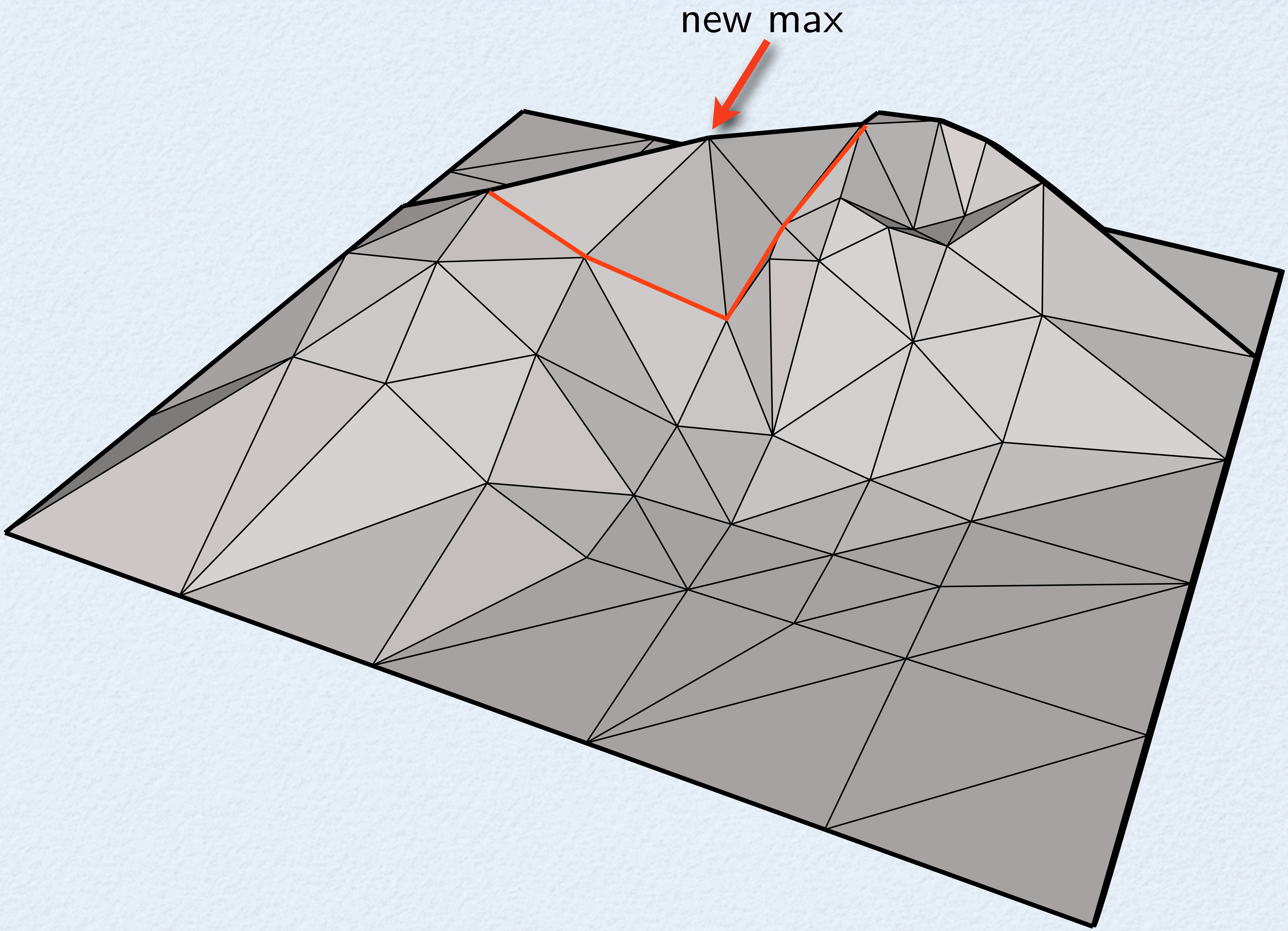

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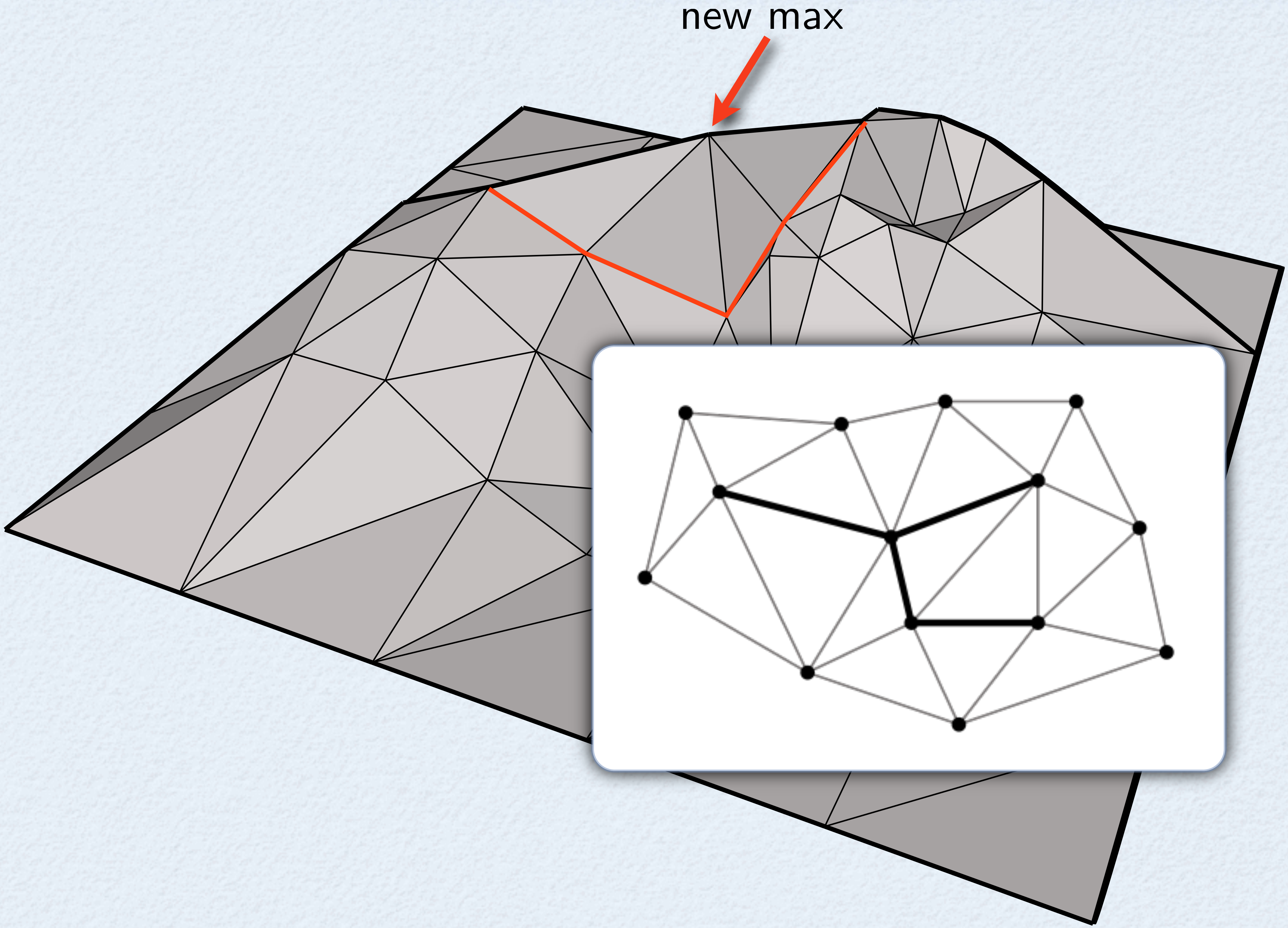

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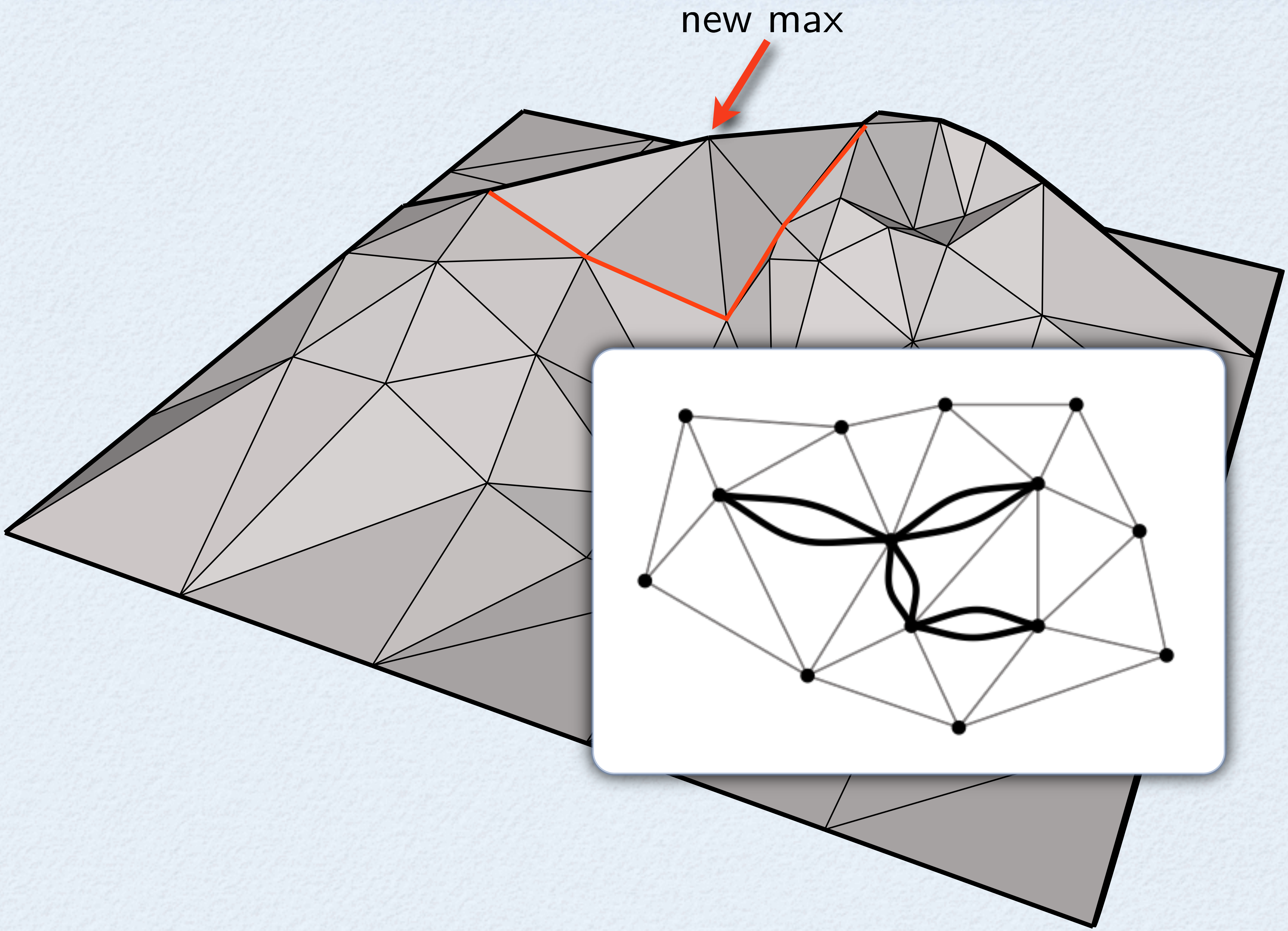

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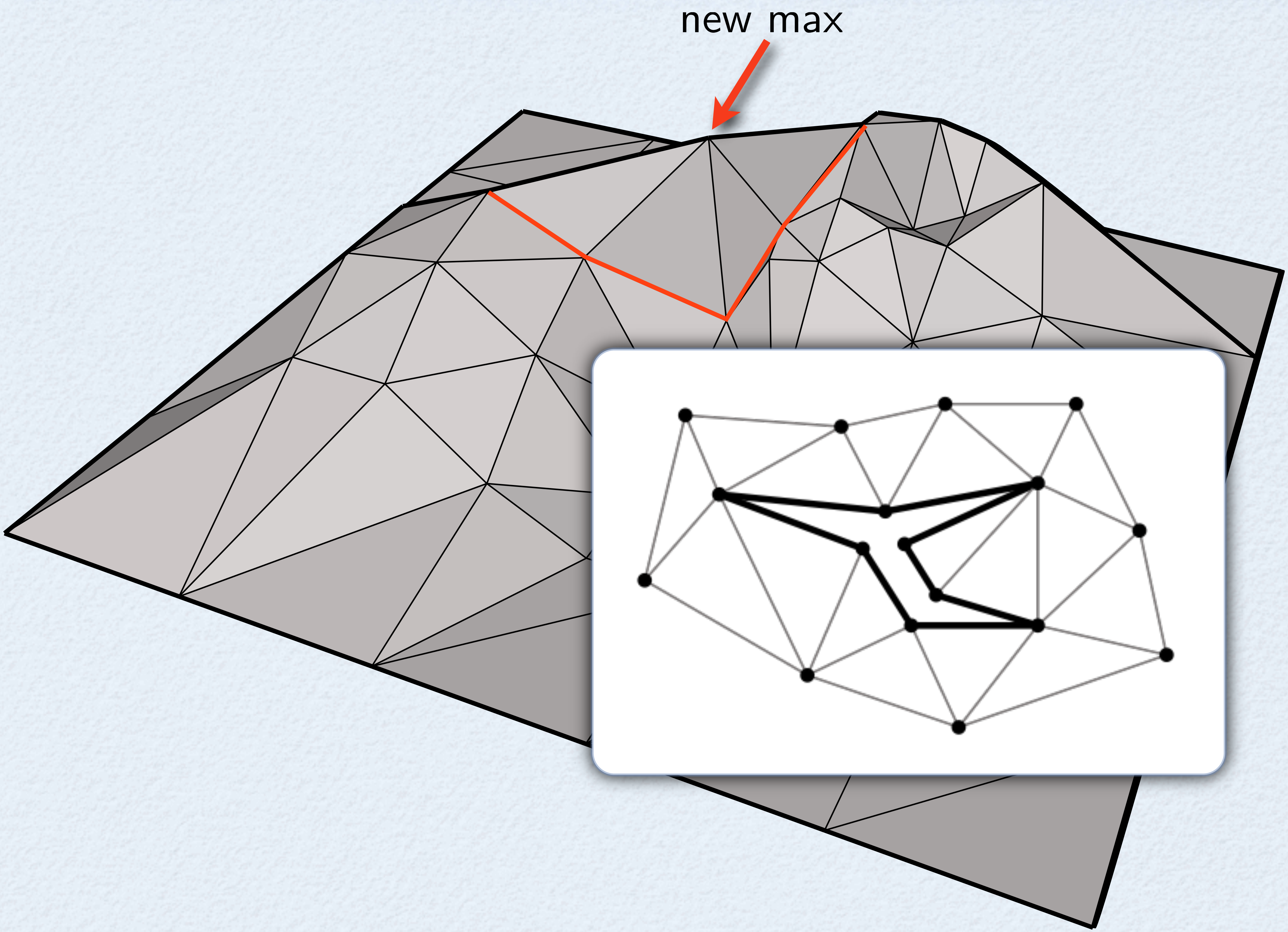

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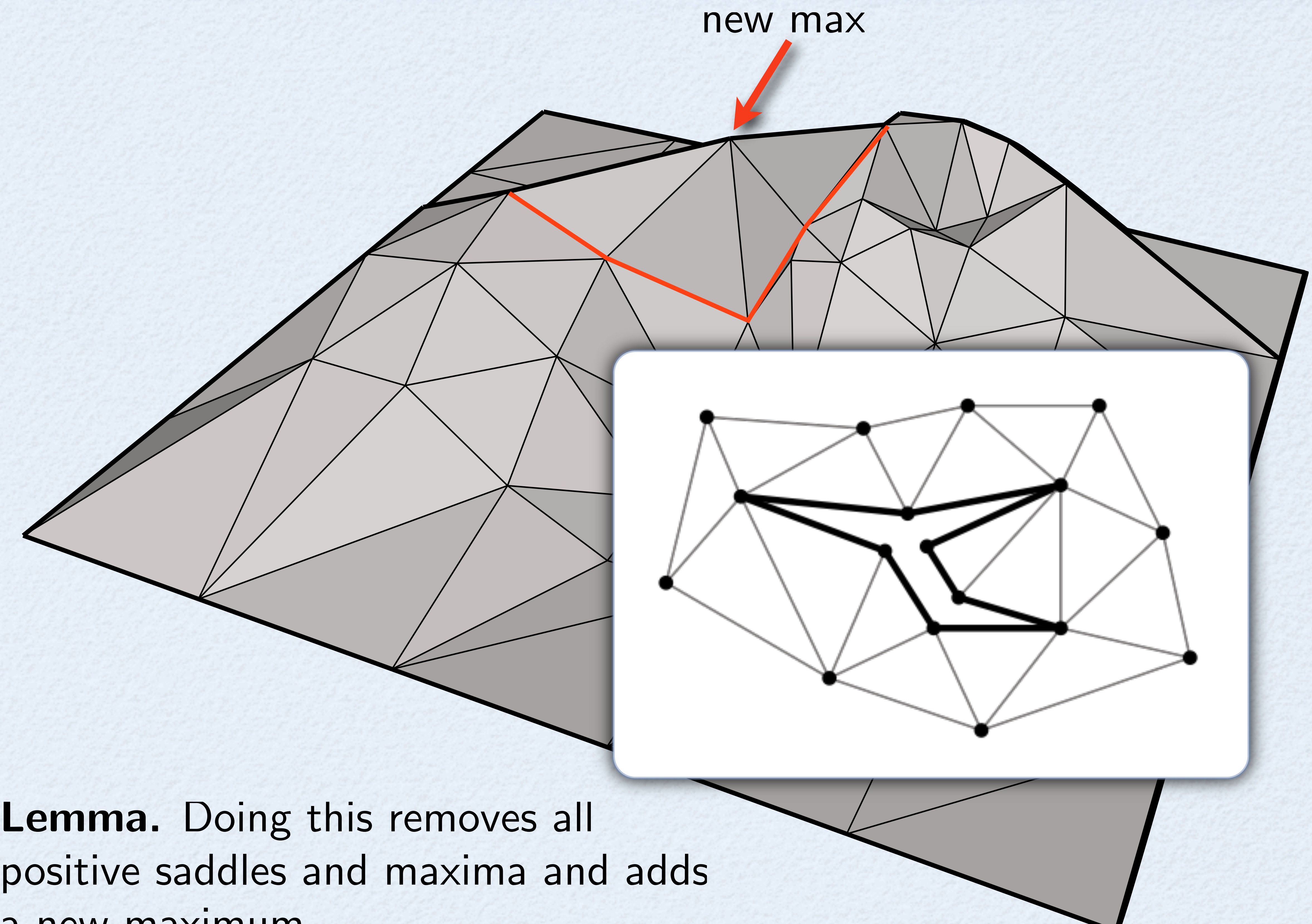
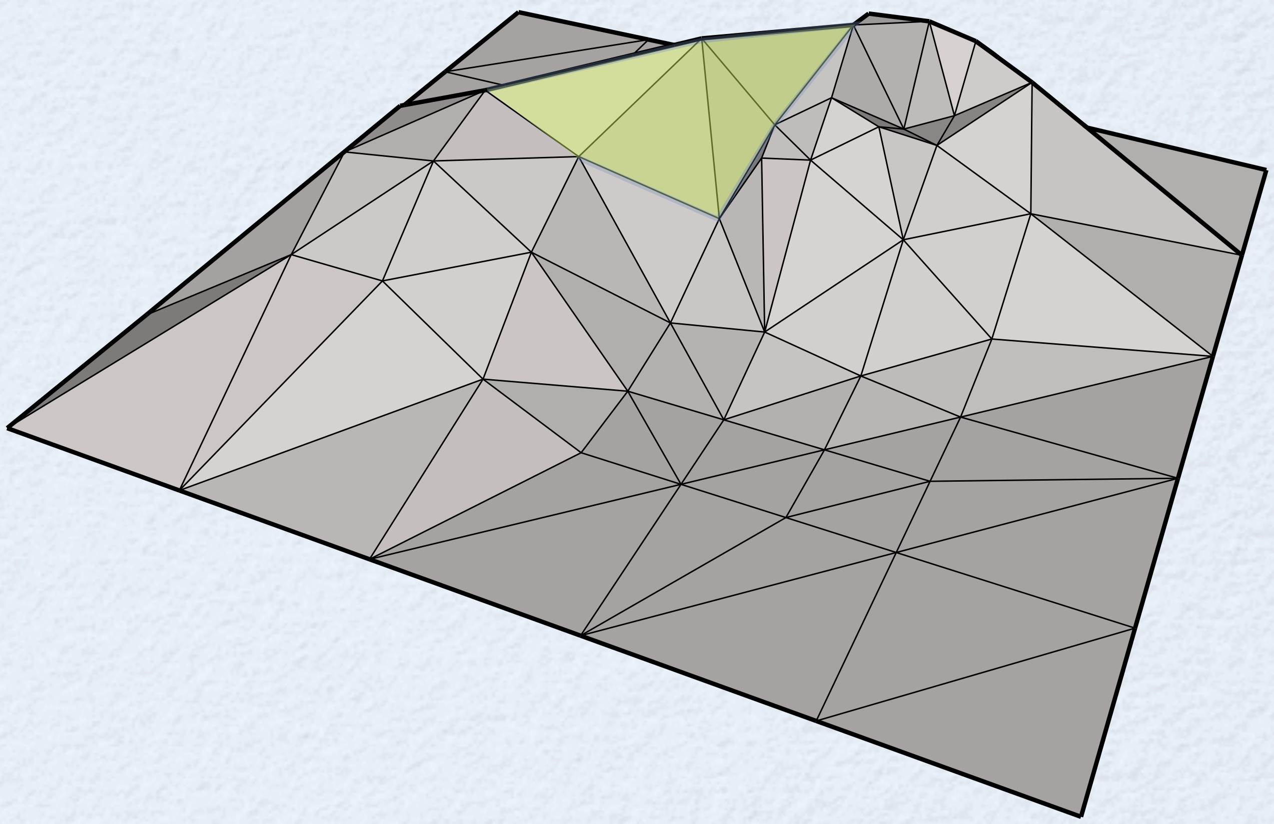


Lemma. Doing this removes all positive saddles and maxima and adds a new maximum.

What surgery does to contours?

The elementary terrain $\mathbb{M}'$ has all the triangles of $\mathbb{M}$ plus some of "new" triangles.



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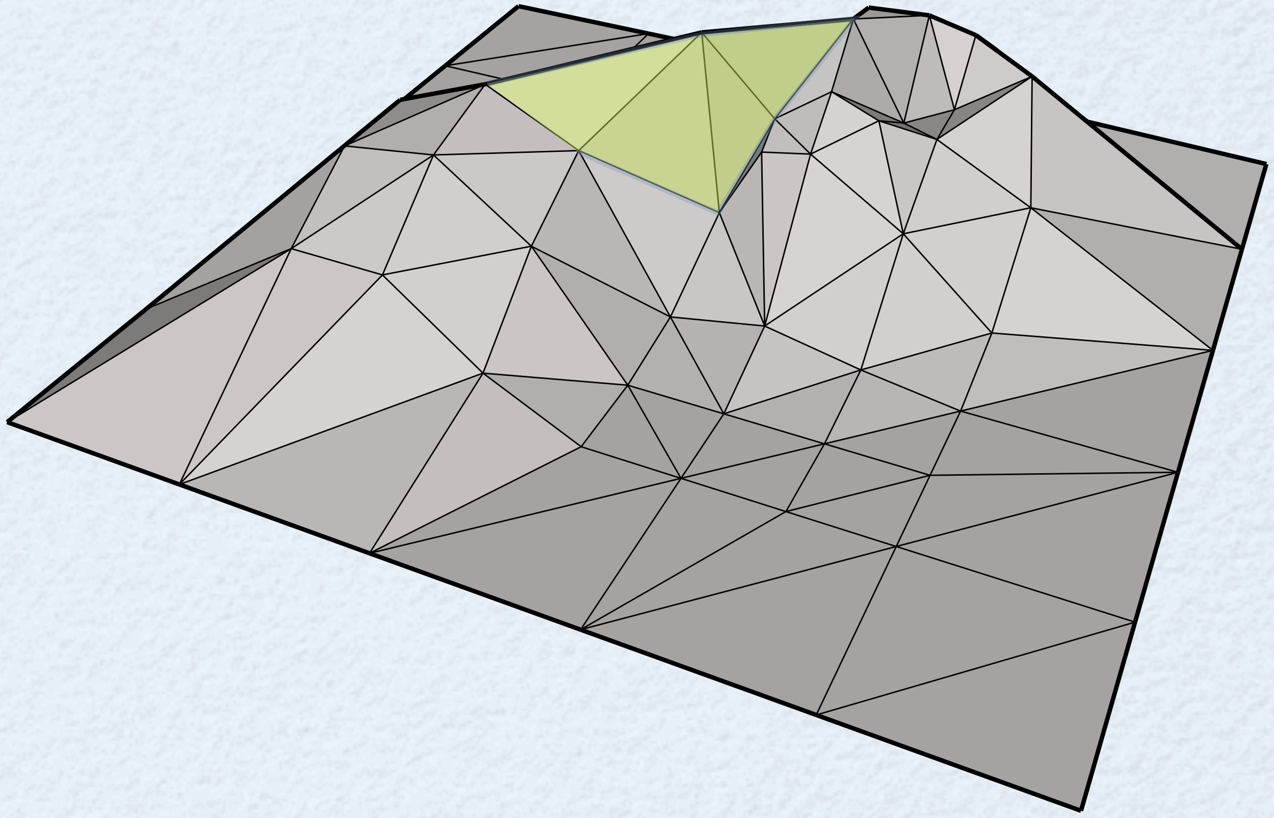

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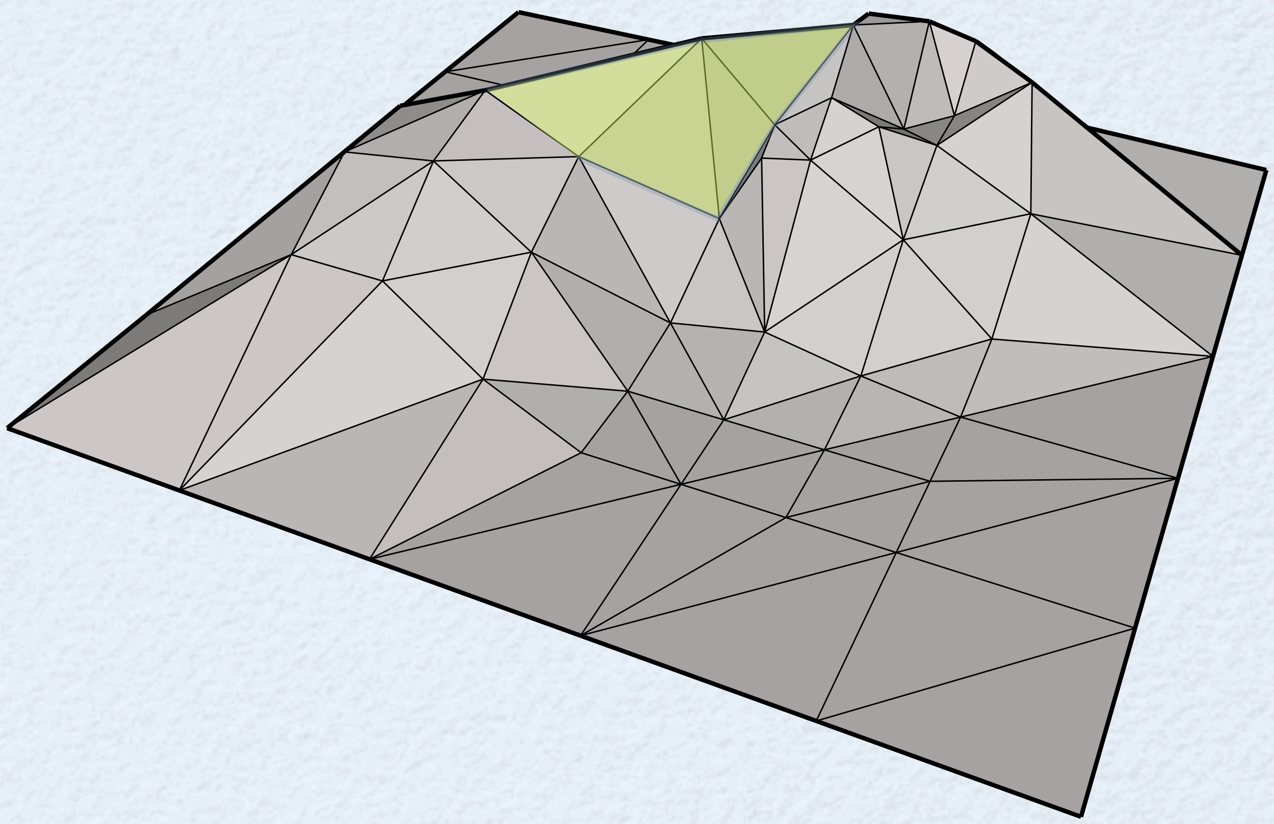

Theorem. In a contour of $\mathbb{M}'$, corresponding contours of $\mathbb{M}$ are broken (by segments from new triangles) in a nested (parenthesized) manner.

$$a_1a_2 * * b_1 * * * c_1c_2c_3 * * * * b_2b_3 * d_1d_2 * * b_4 * * a_3a_4a_5$$

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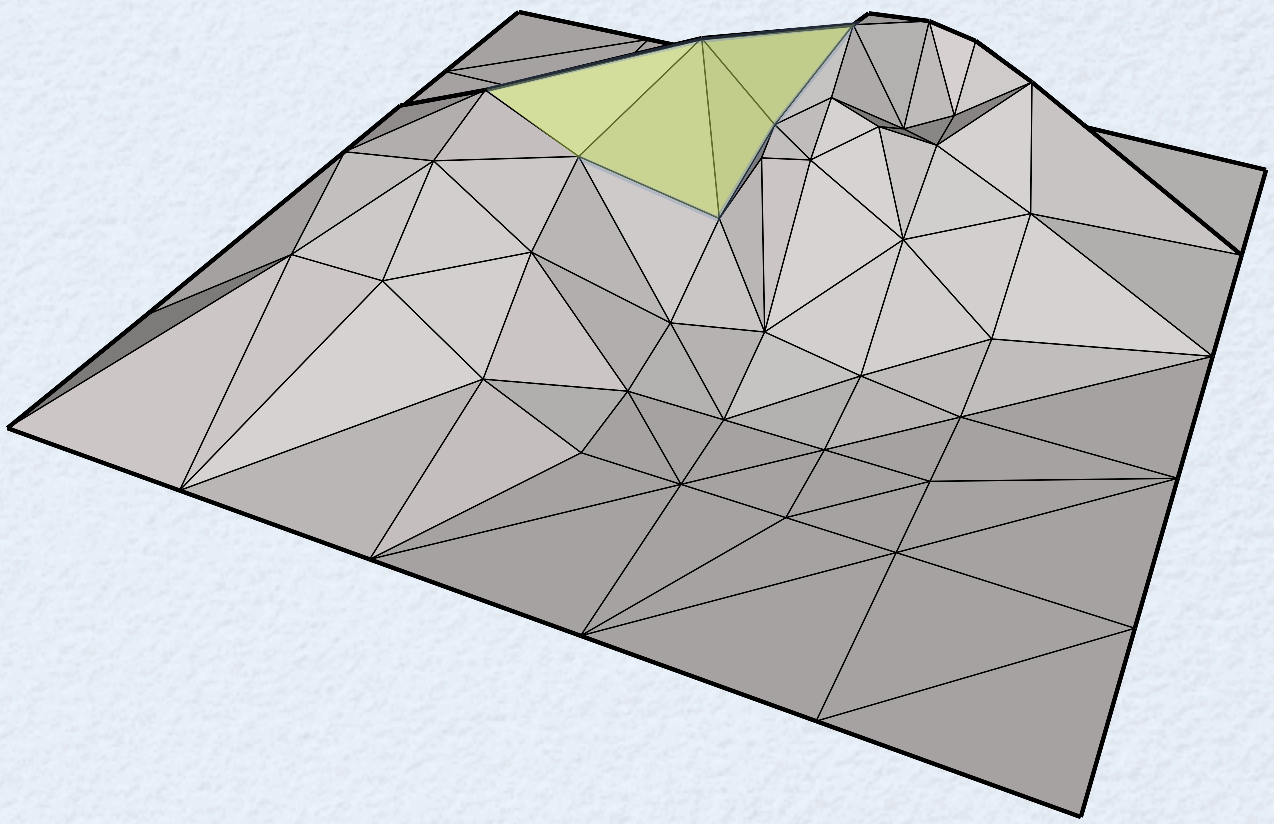

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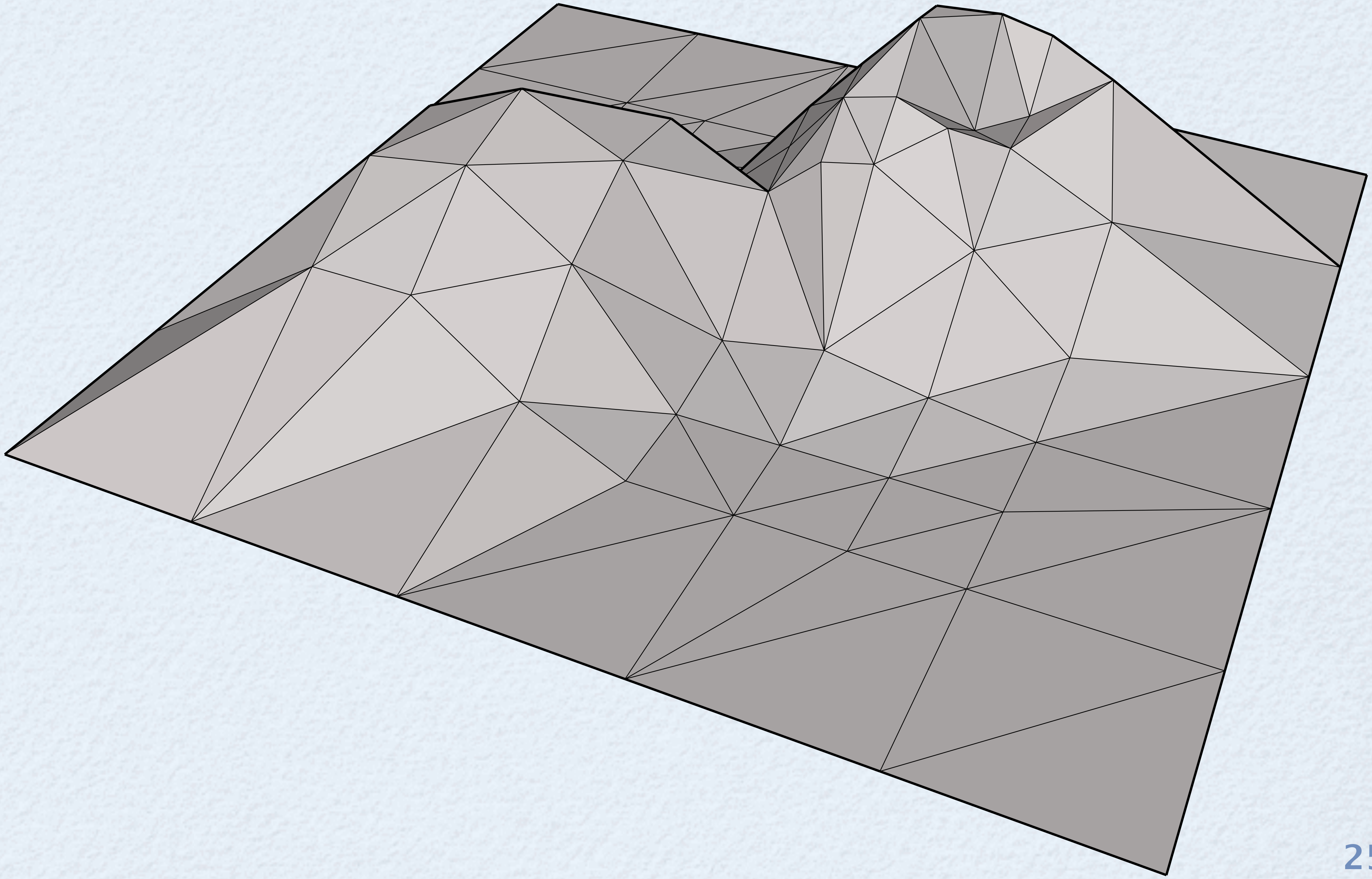

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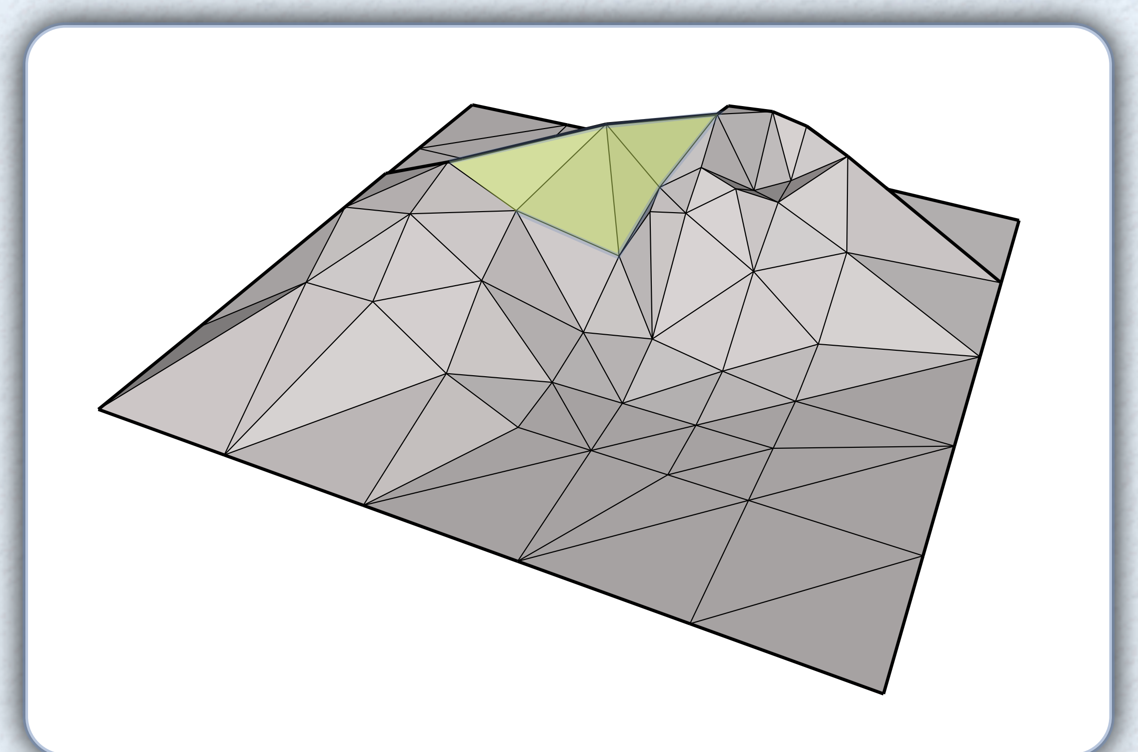

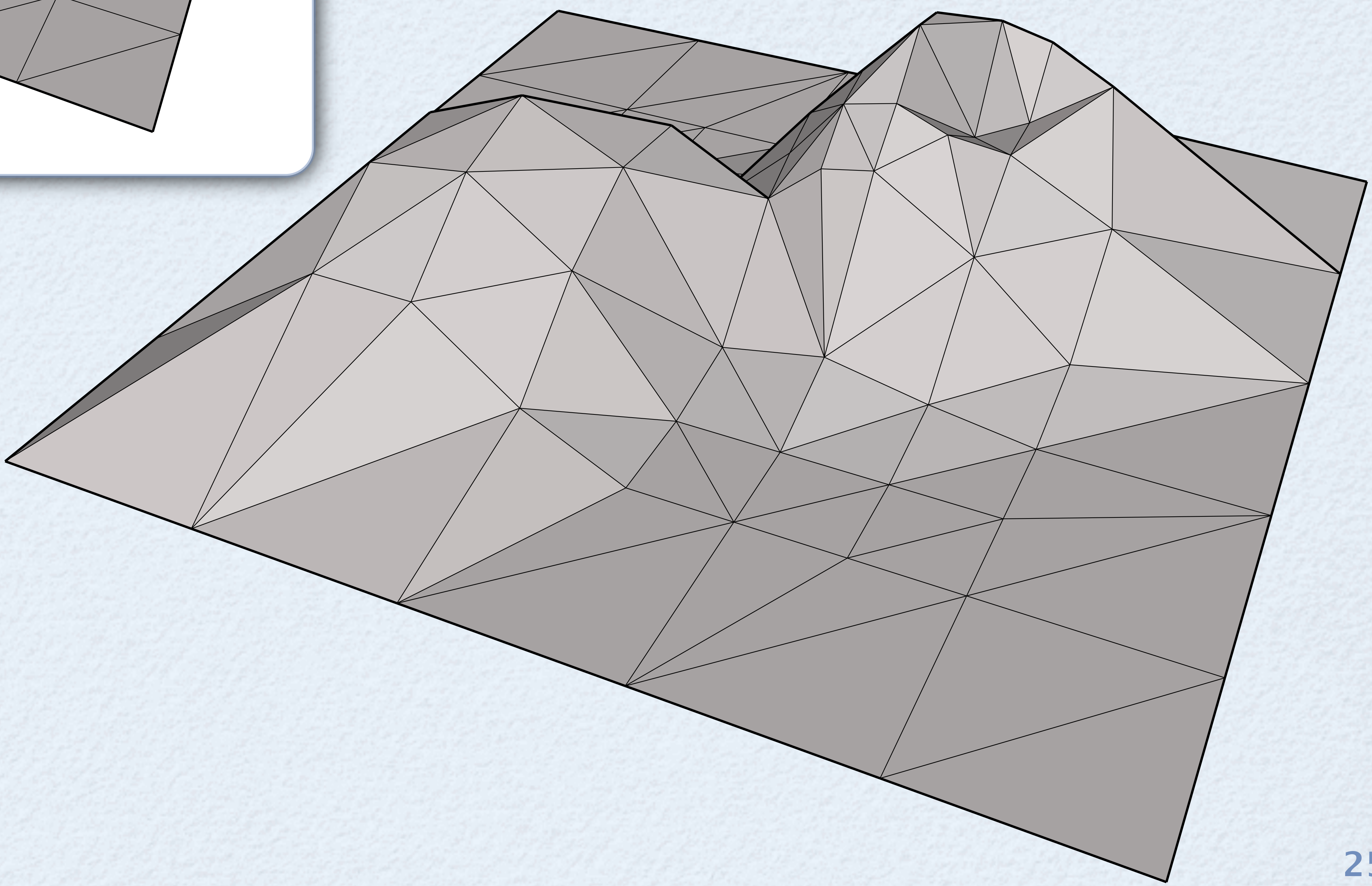

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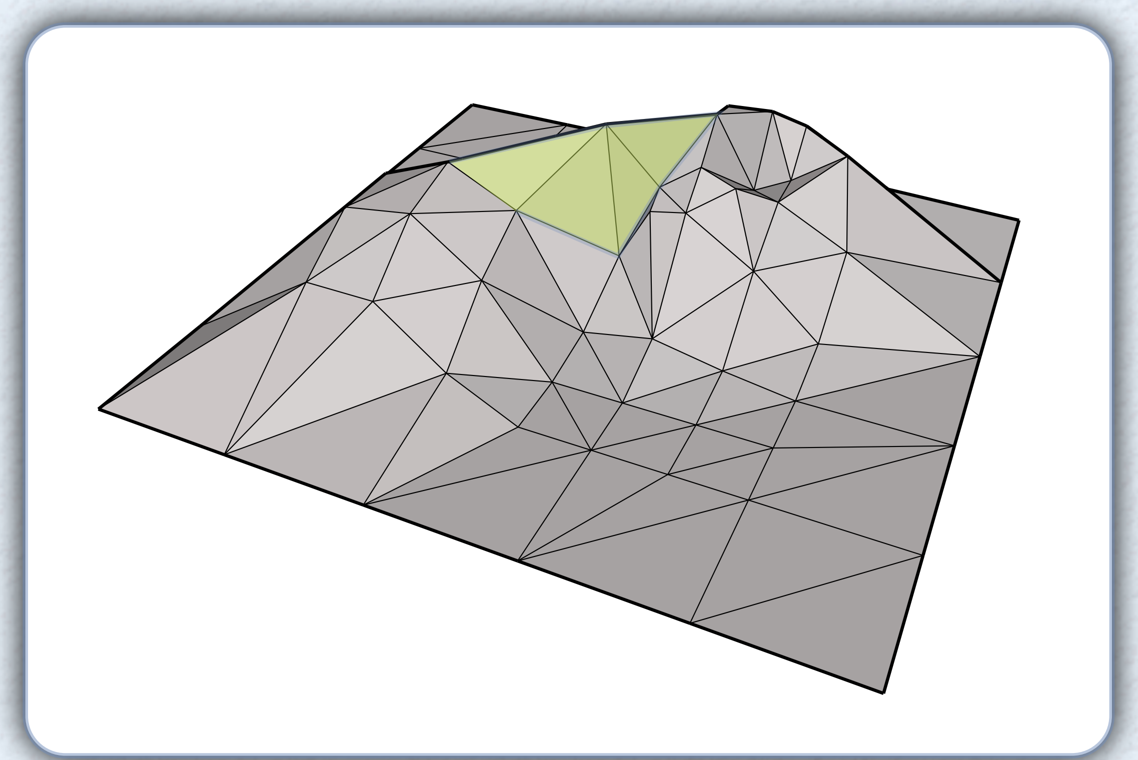

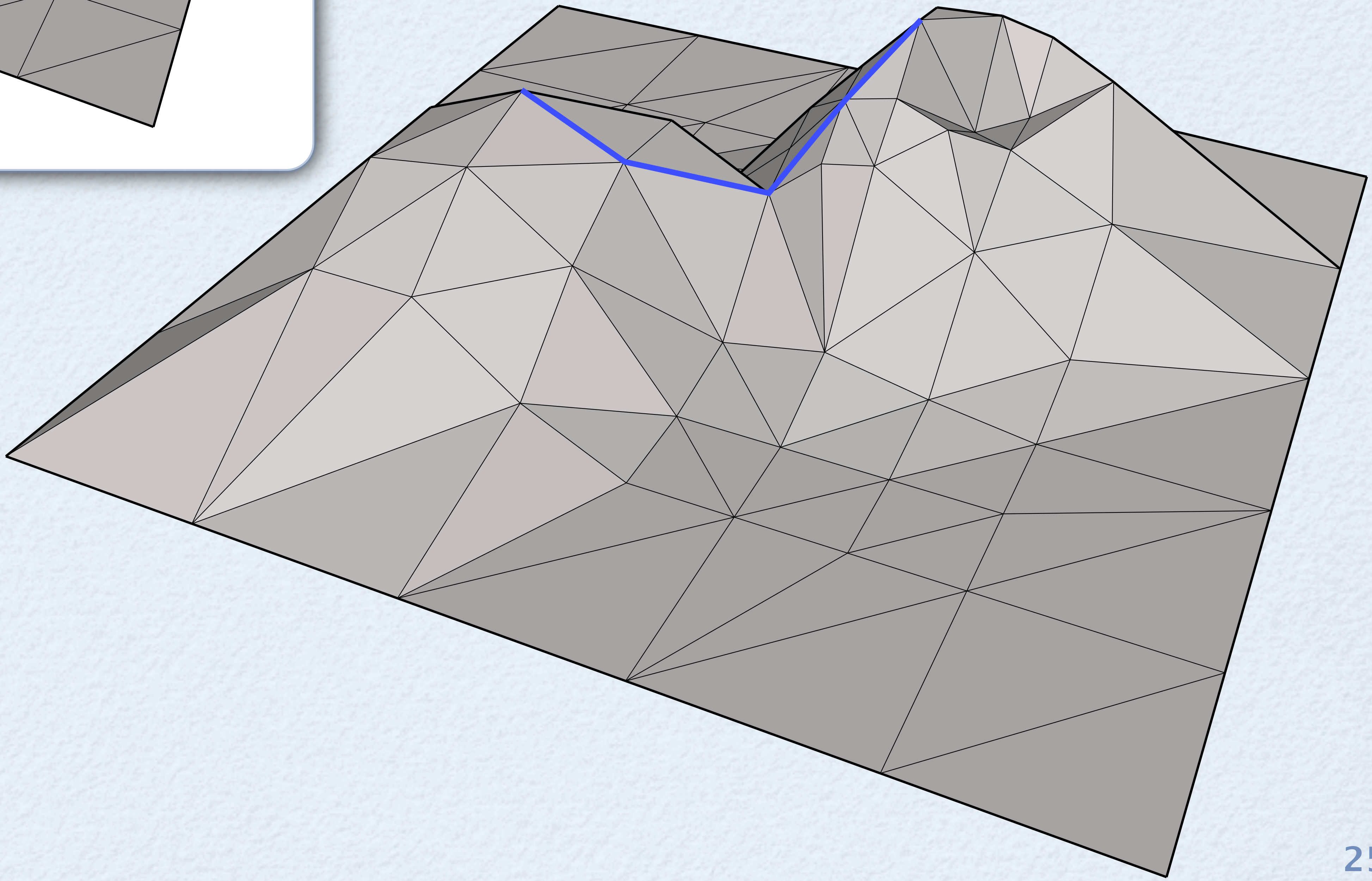

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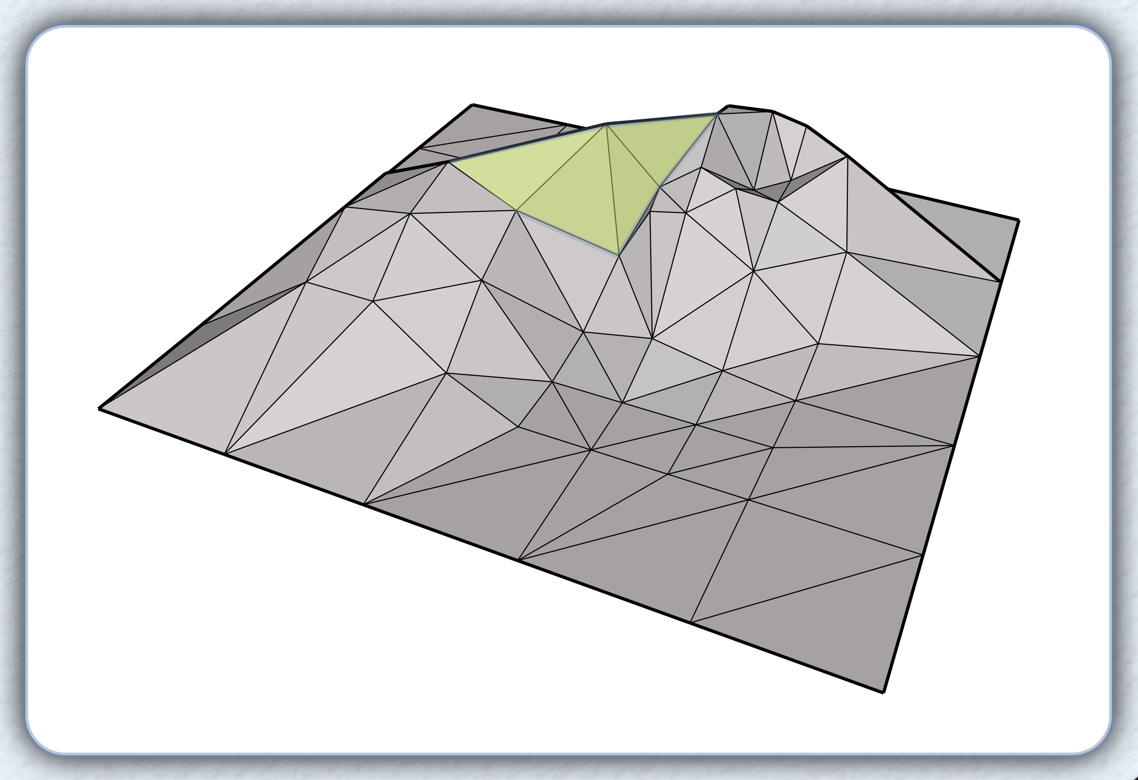

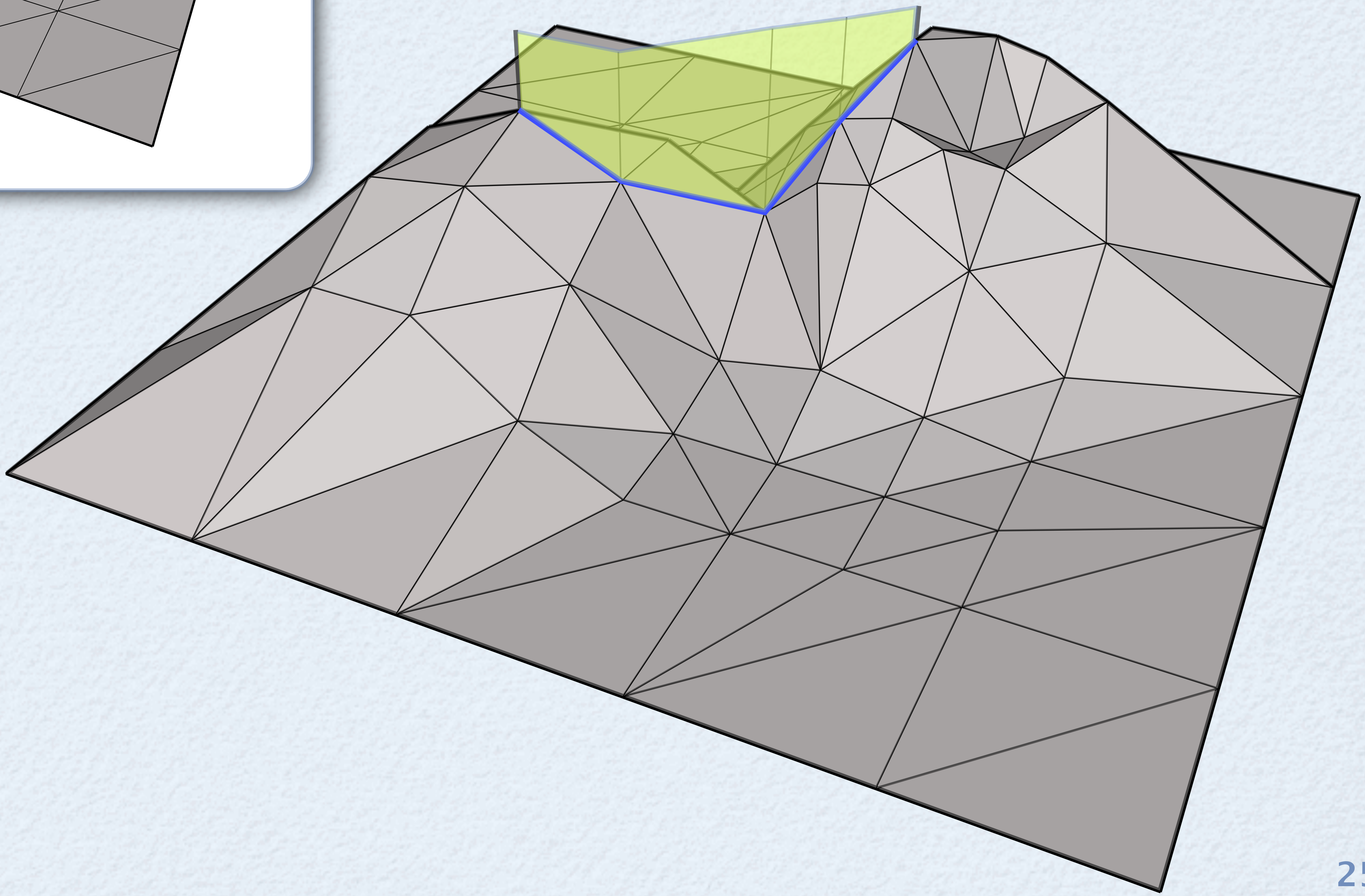

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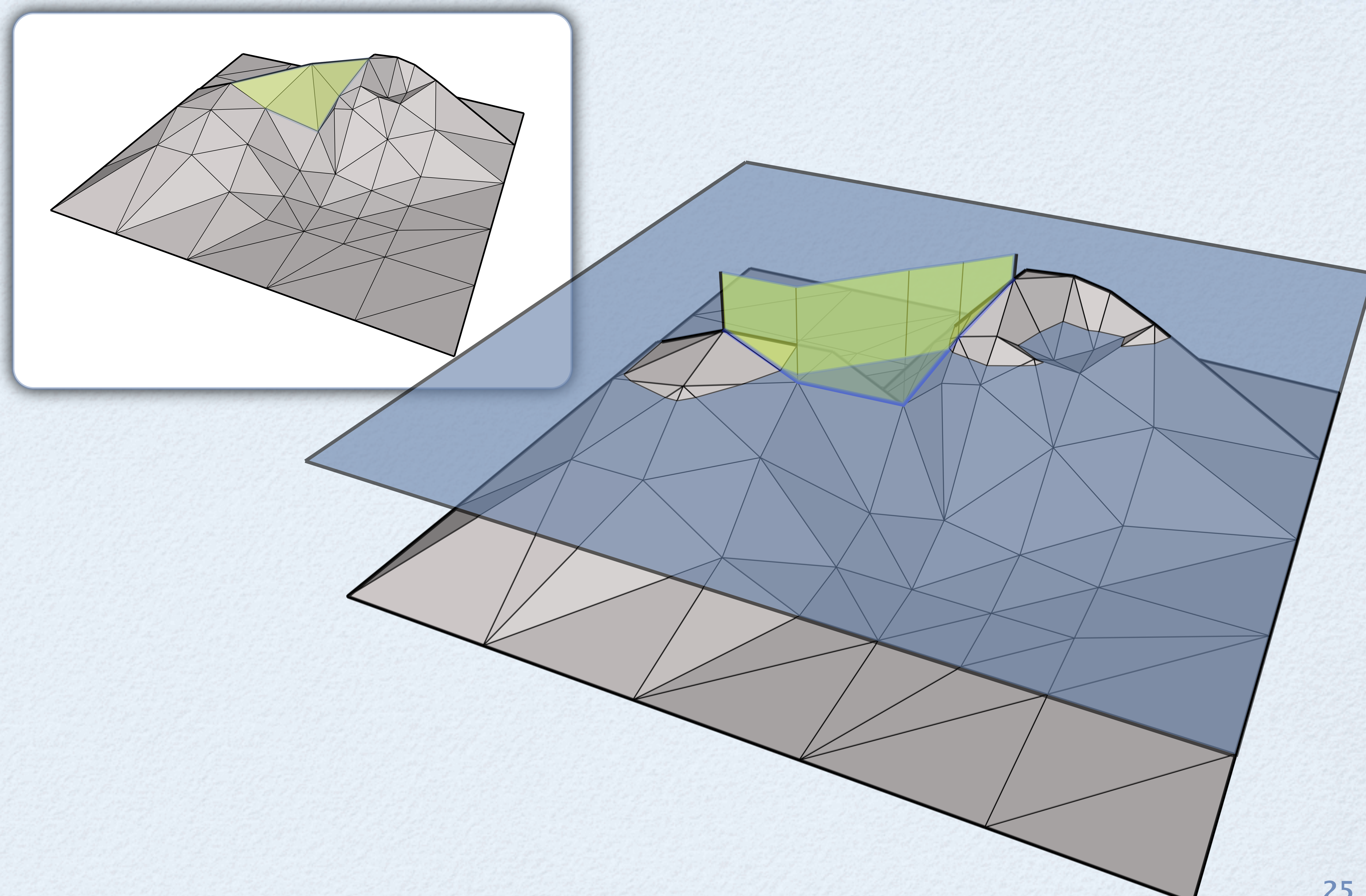

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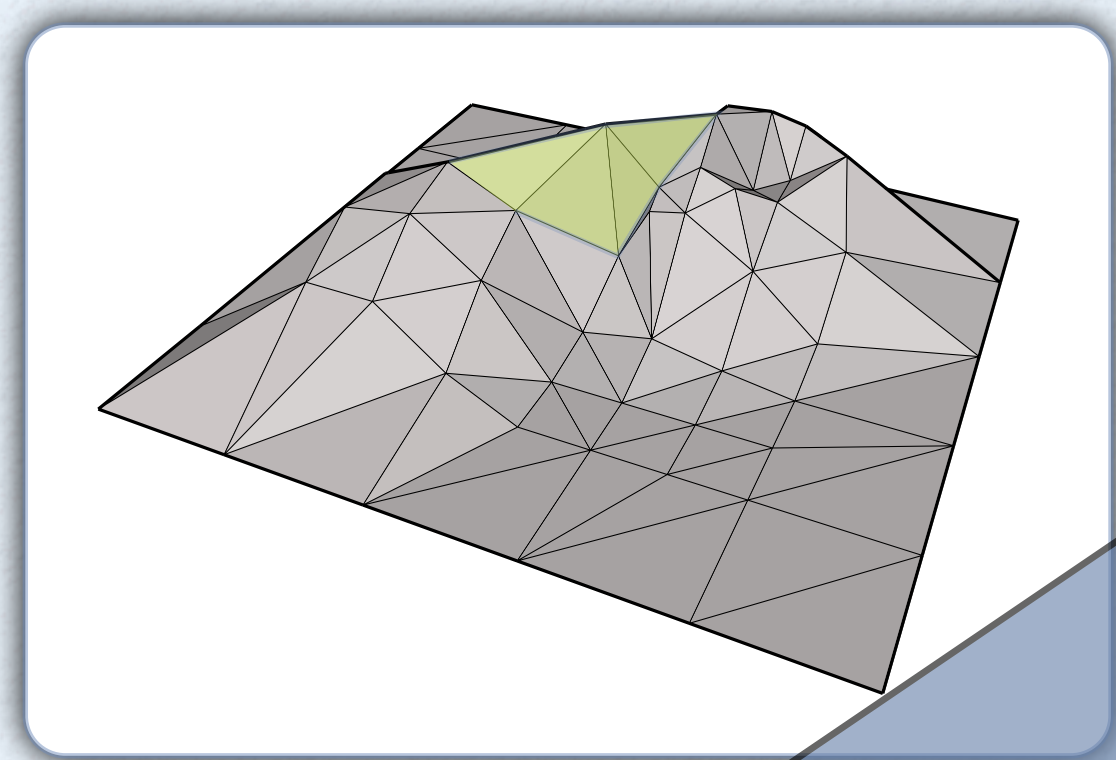
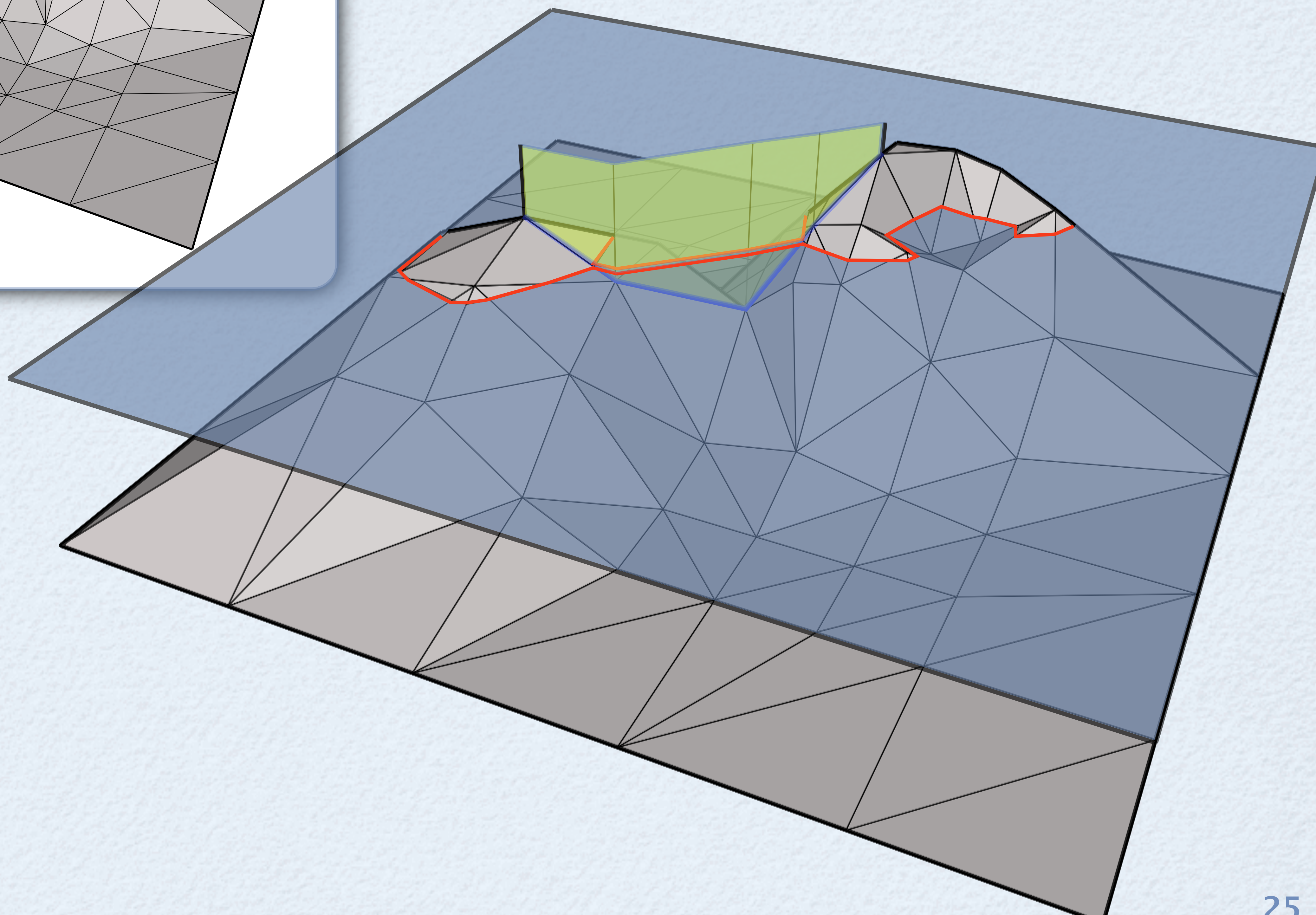
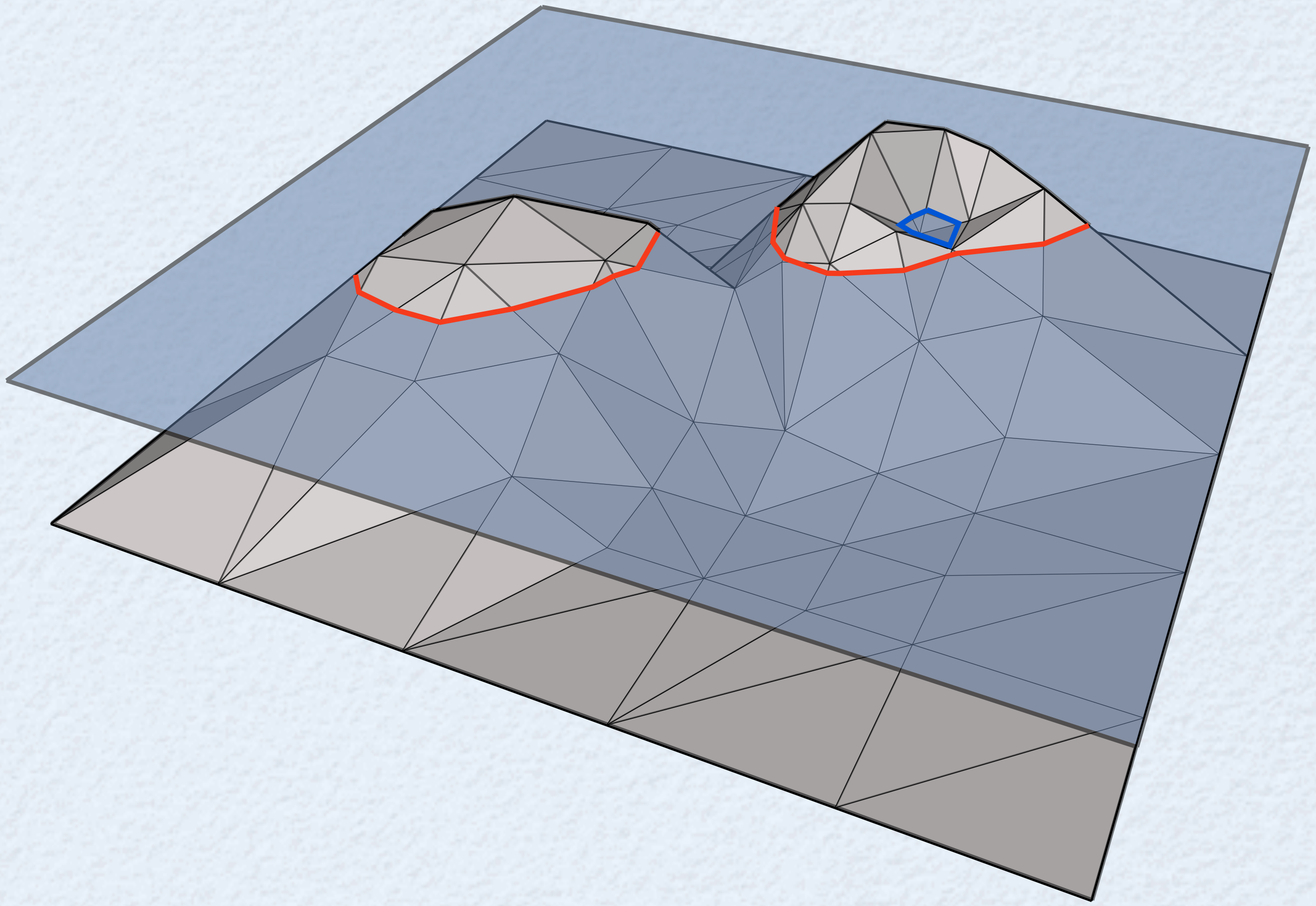




Red and Blue Contours

A contour is red (blue) if "locally" the sublevel set is in its outside (inside).



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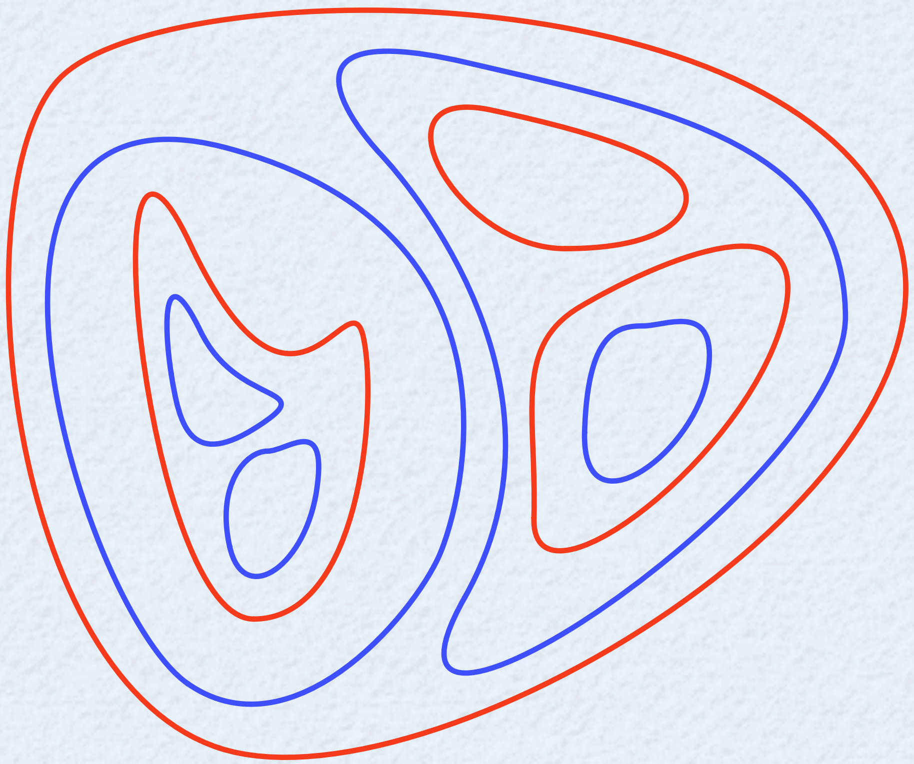

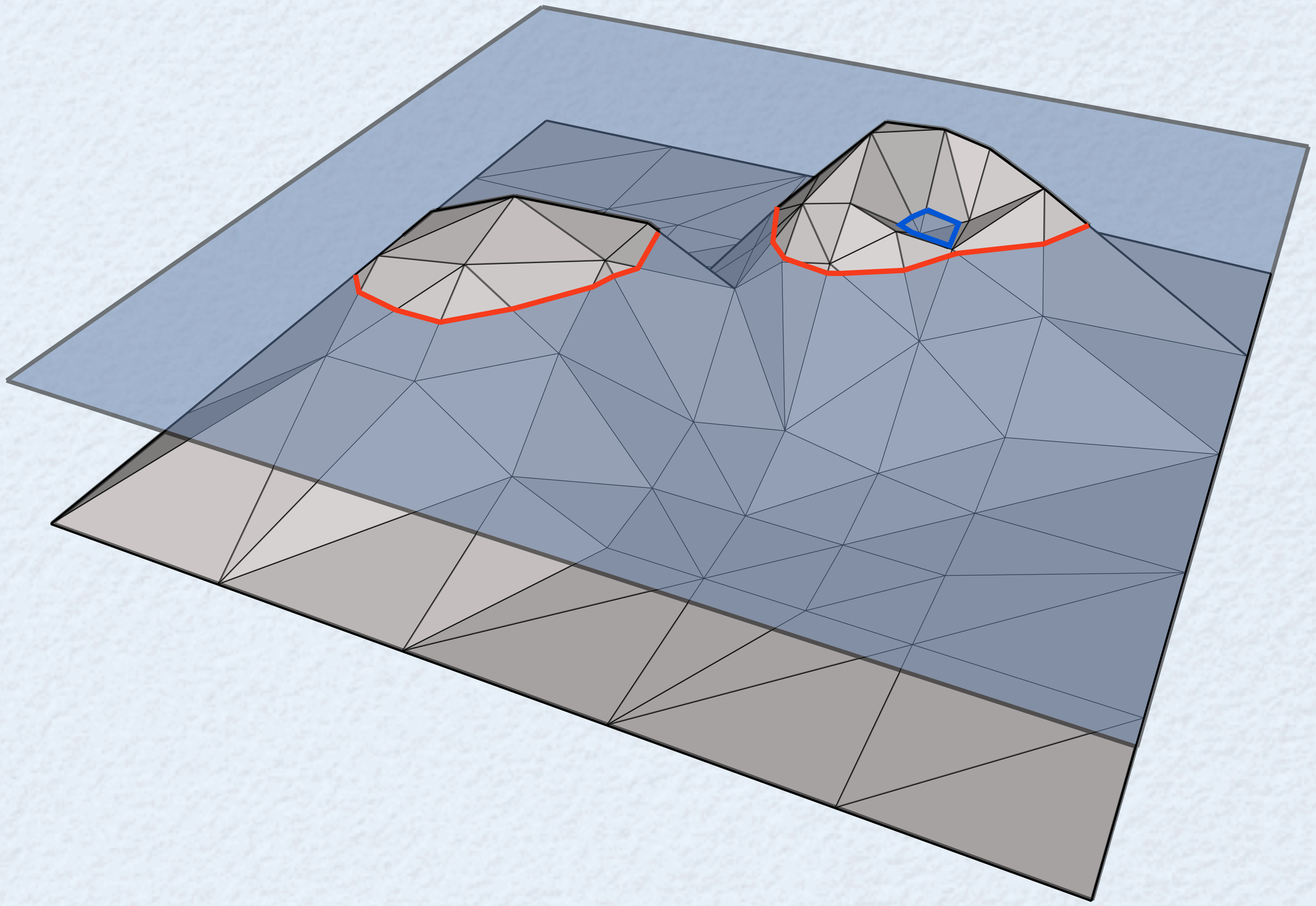

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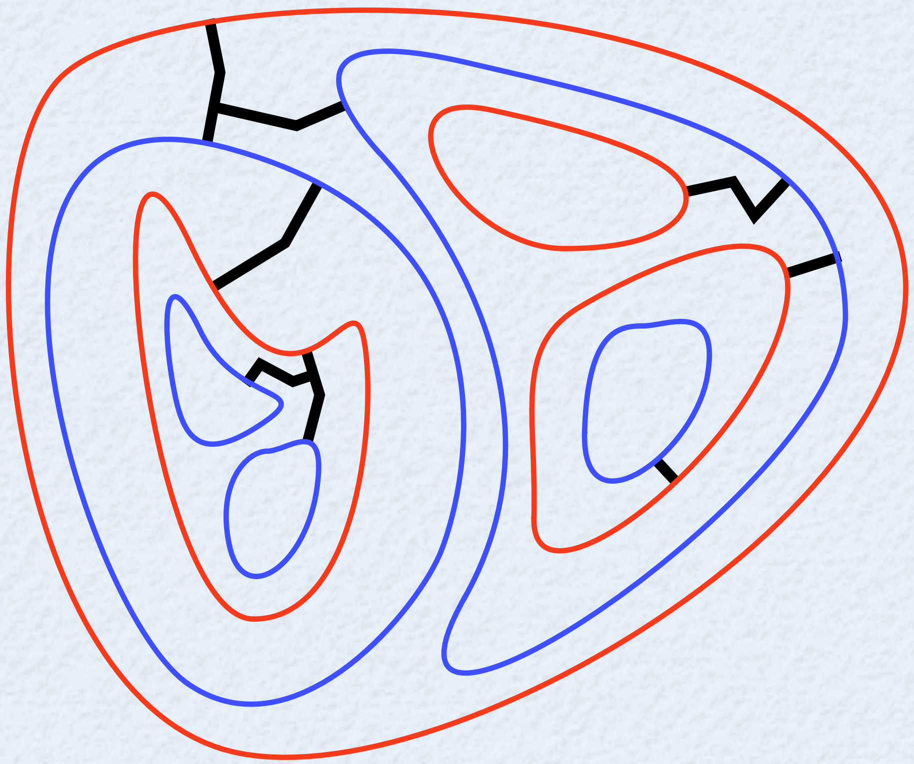

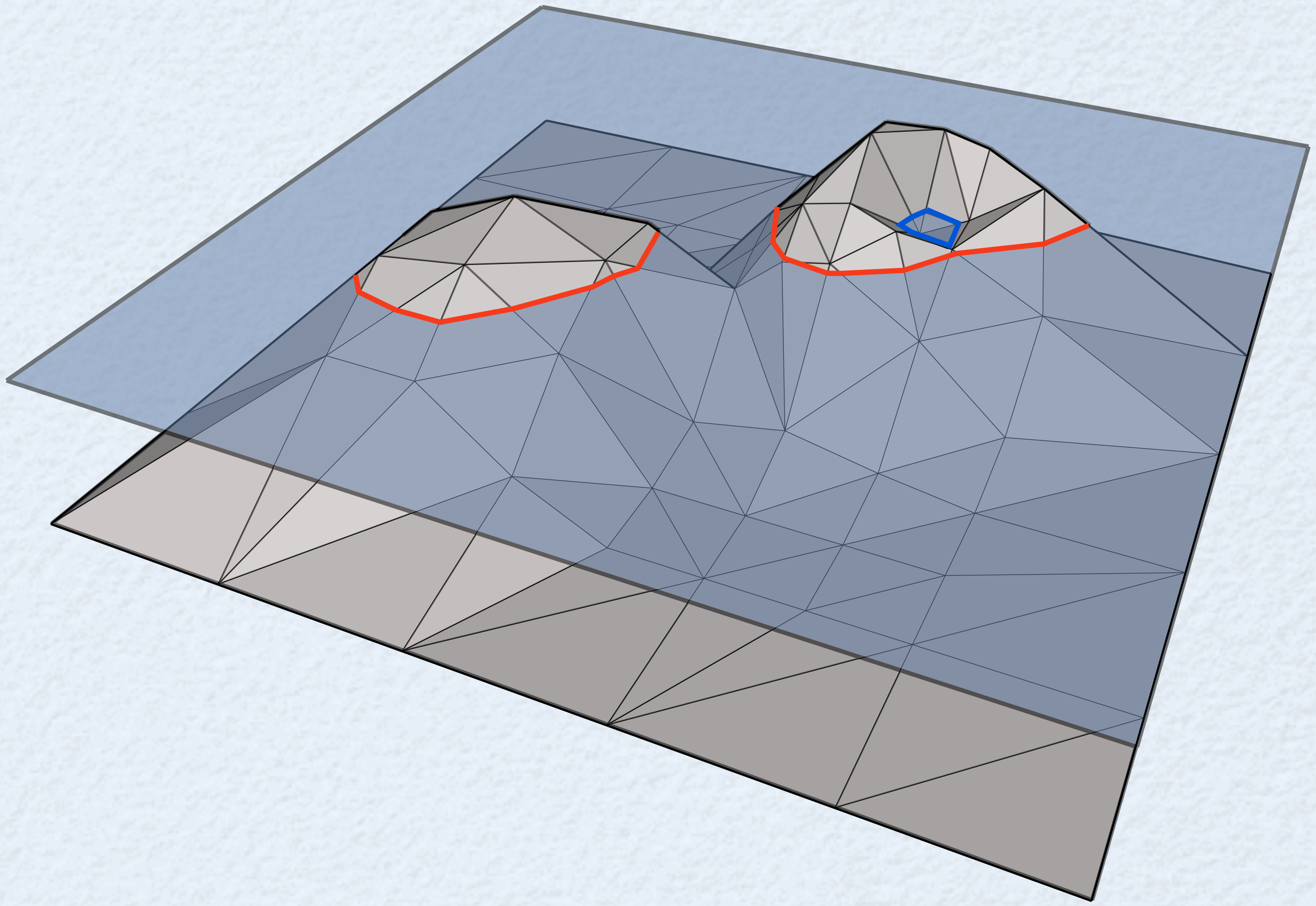

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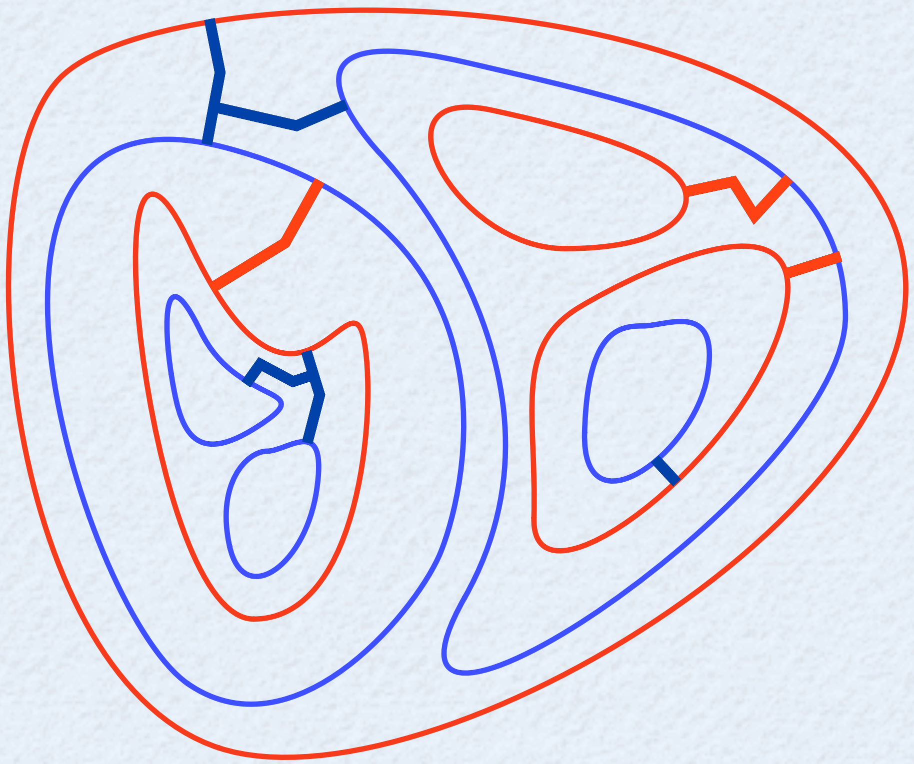

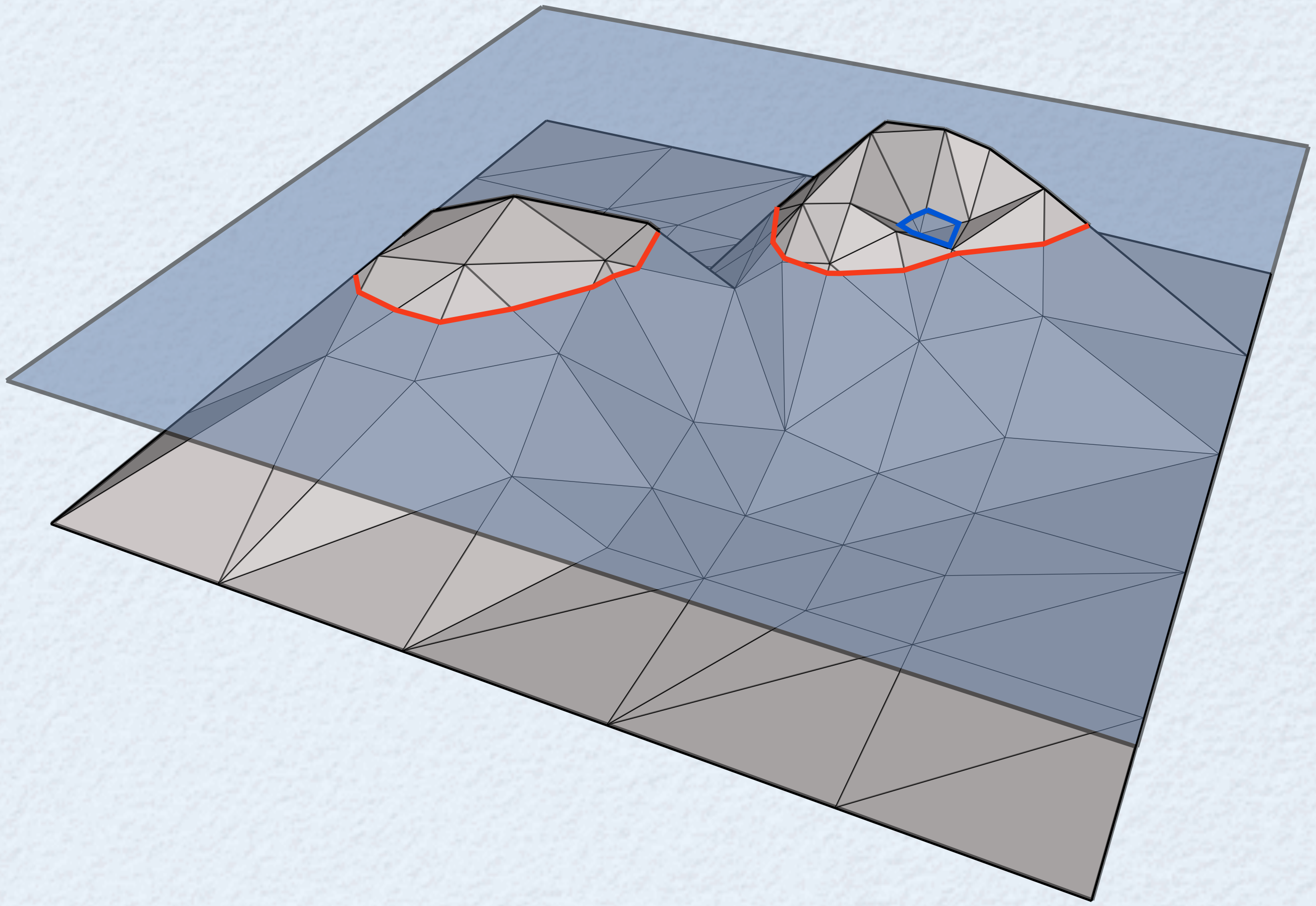

Theorem. Only the red tree connects a blue contour and its red children.

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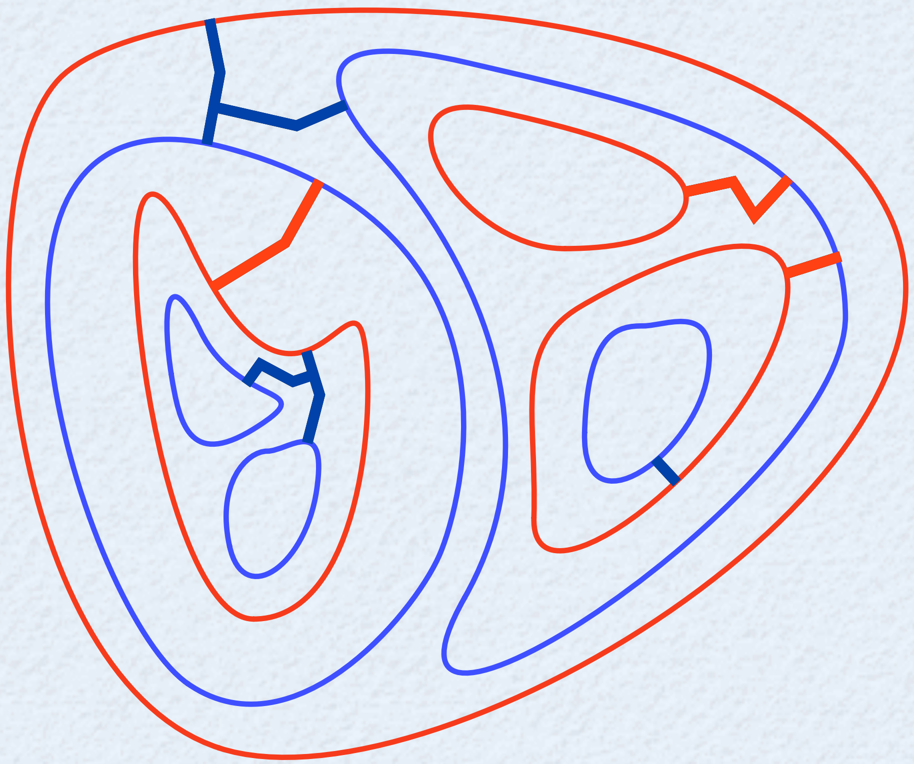

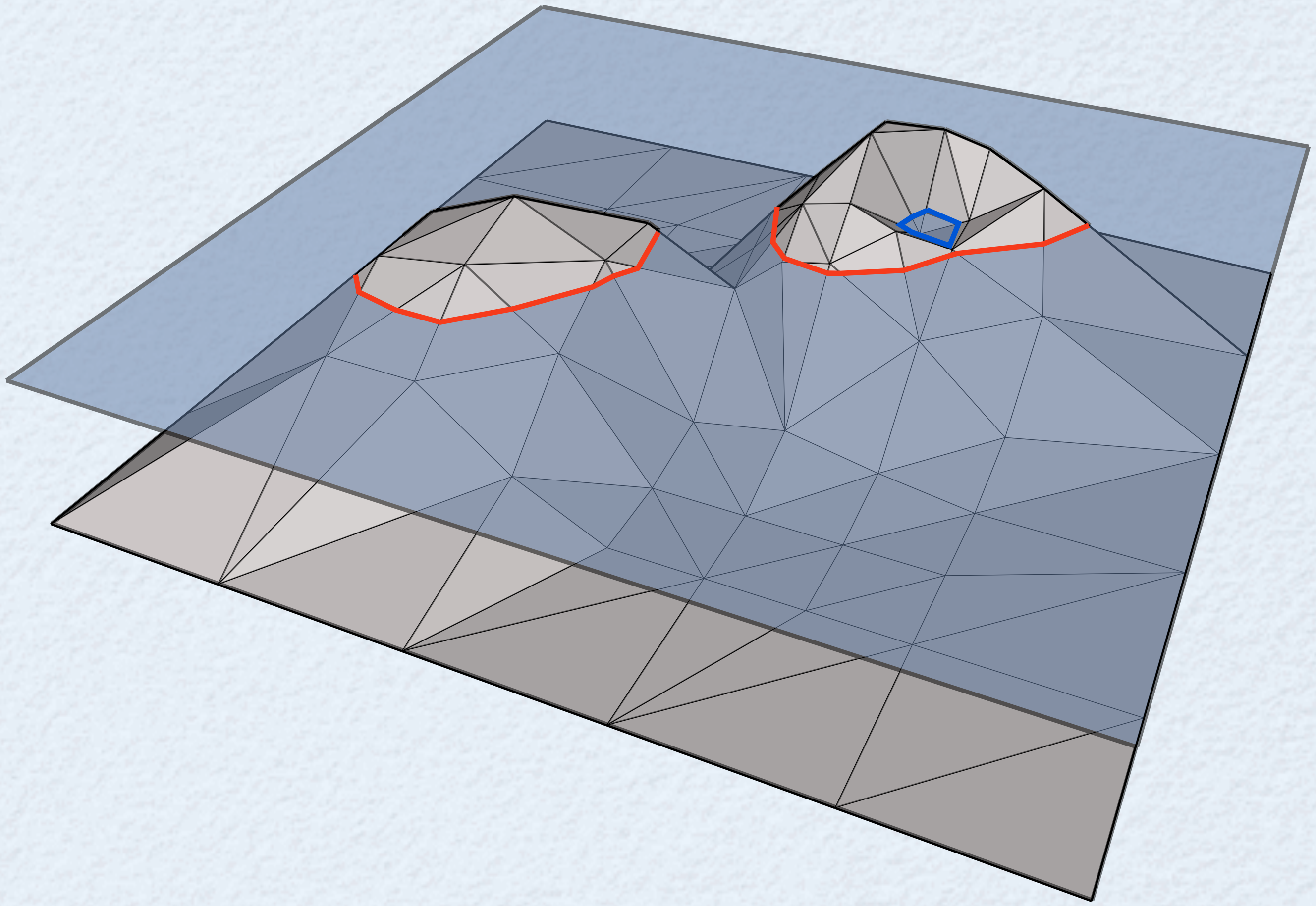

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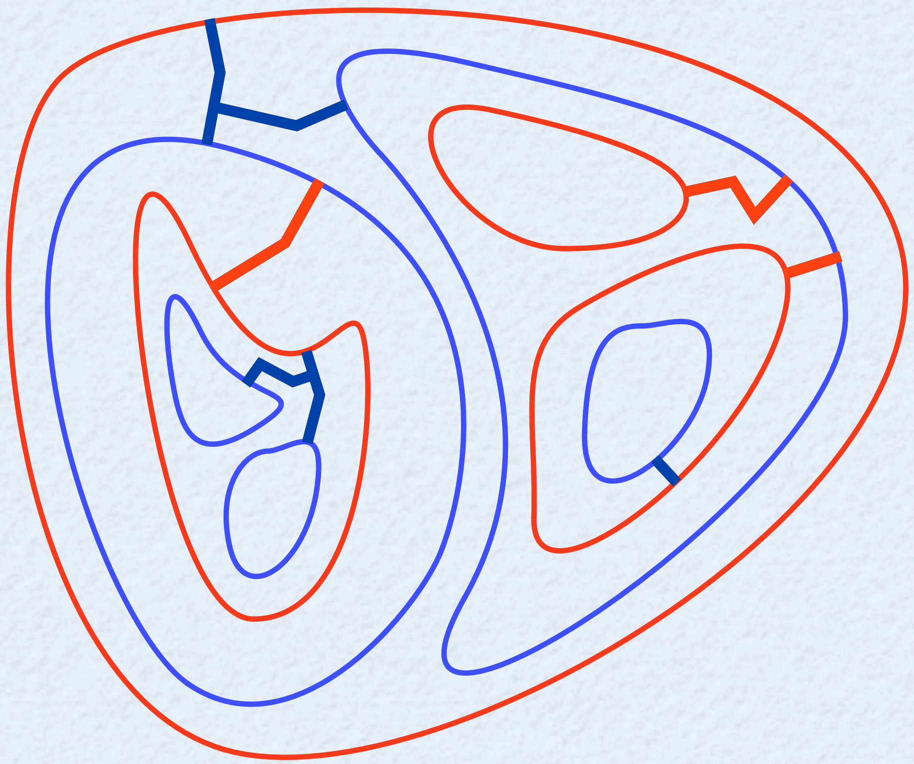

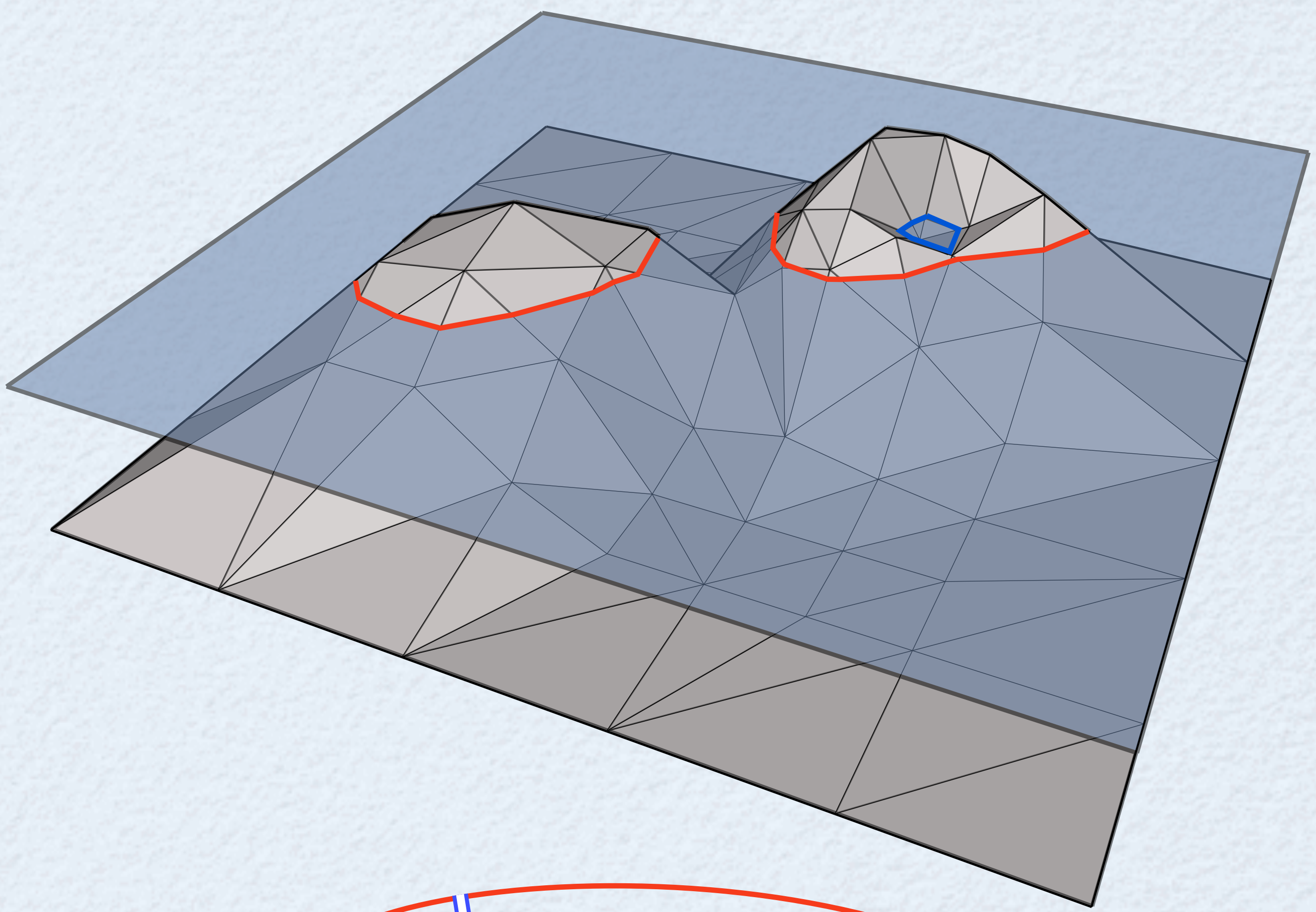

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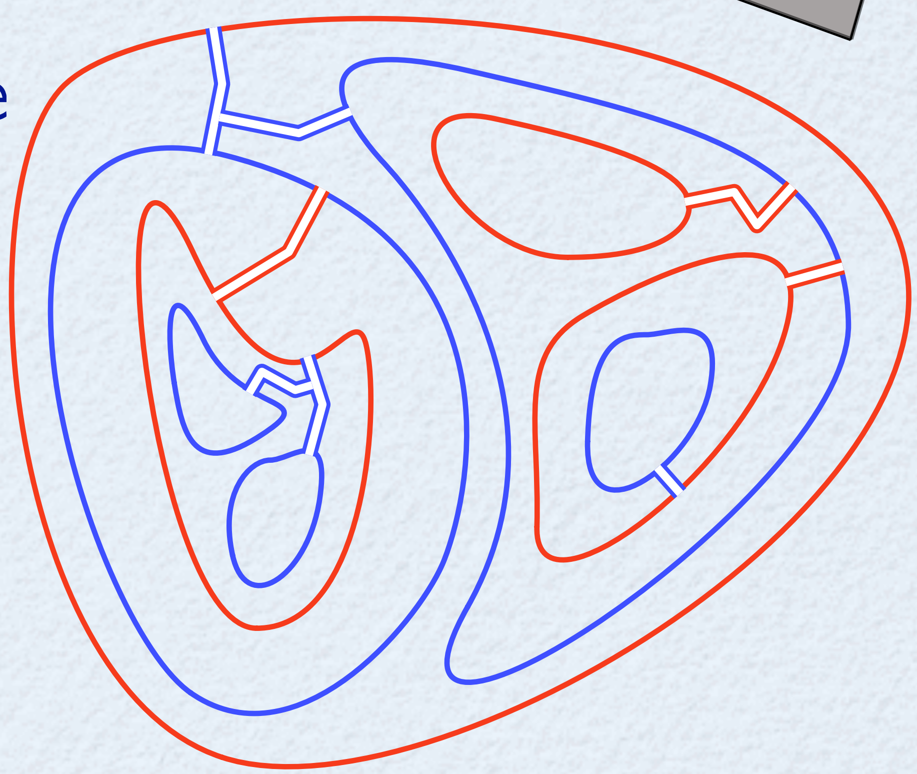
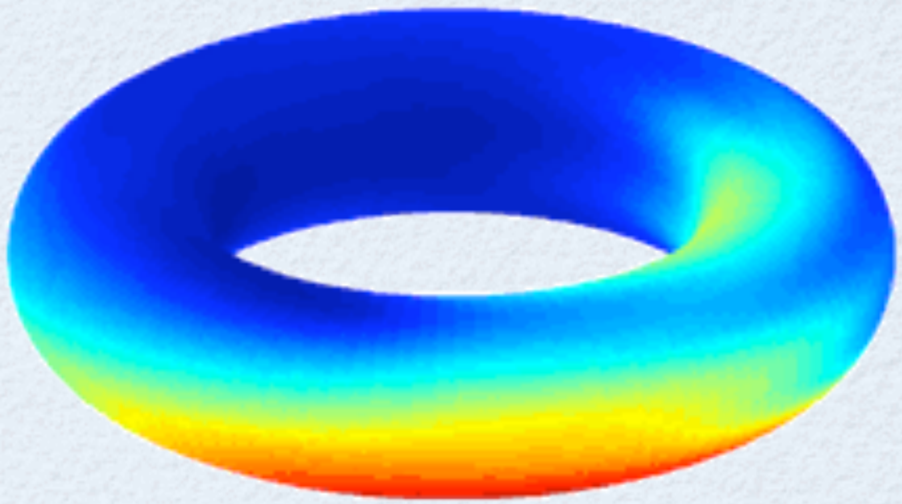


Remarks and Open Problems

The embedding of the terrain is unimportant: the set of triangles that intersect a level set only depends on function value on vertices.

What about surfaces of genus ≥ 1? (Orientable or not)



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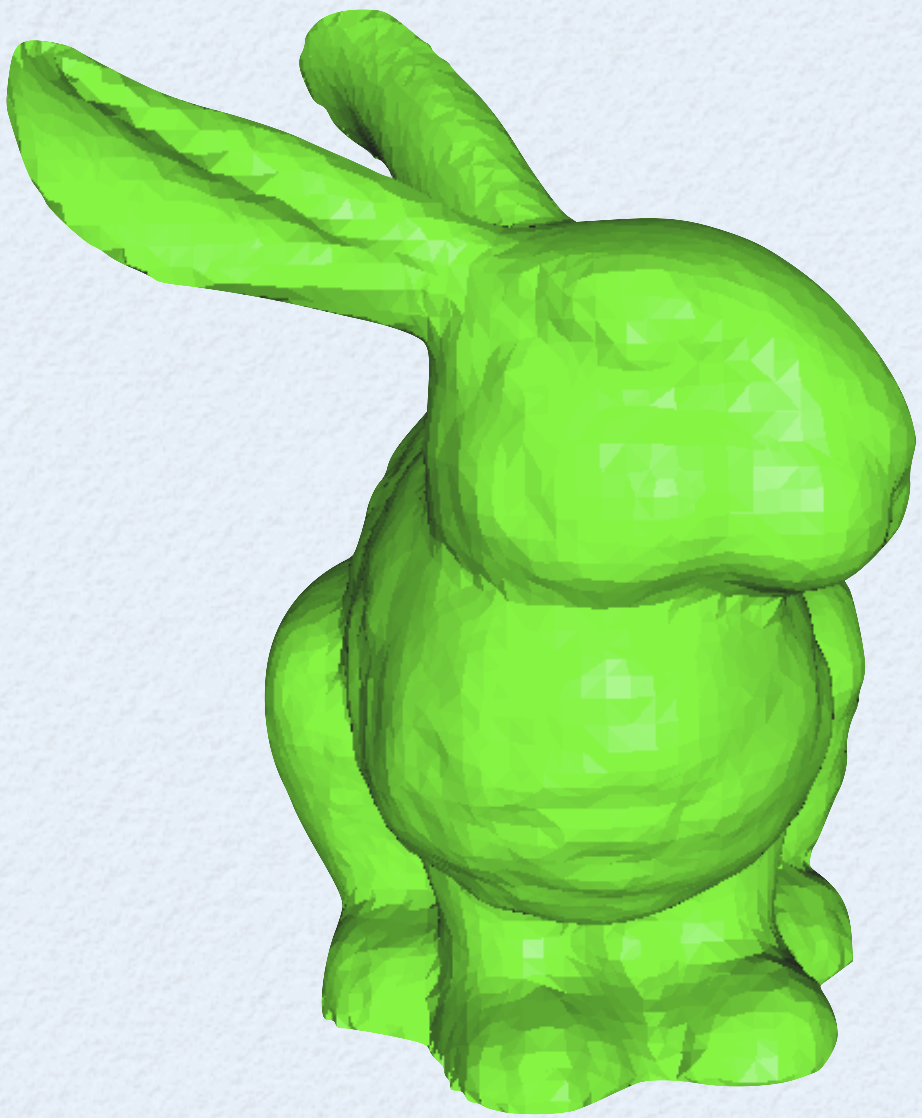

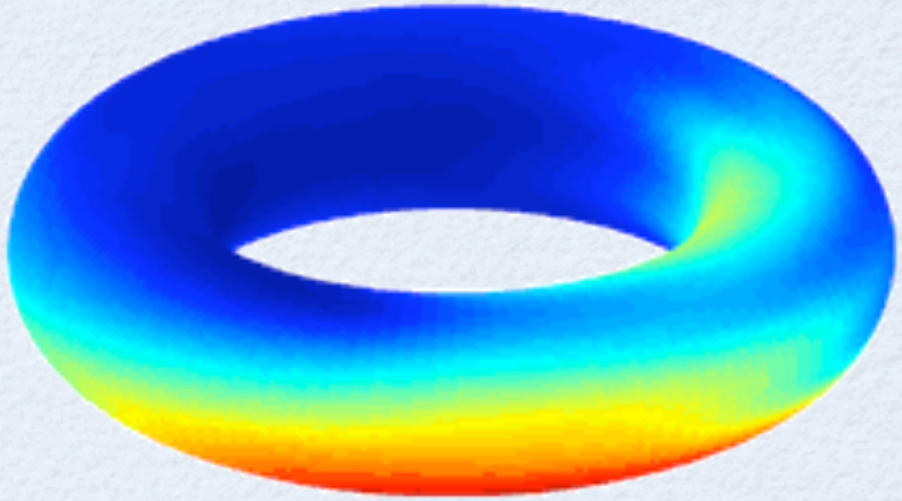

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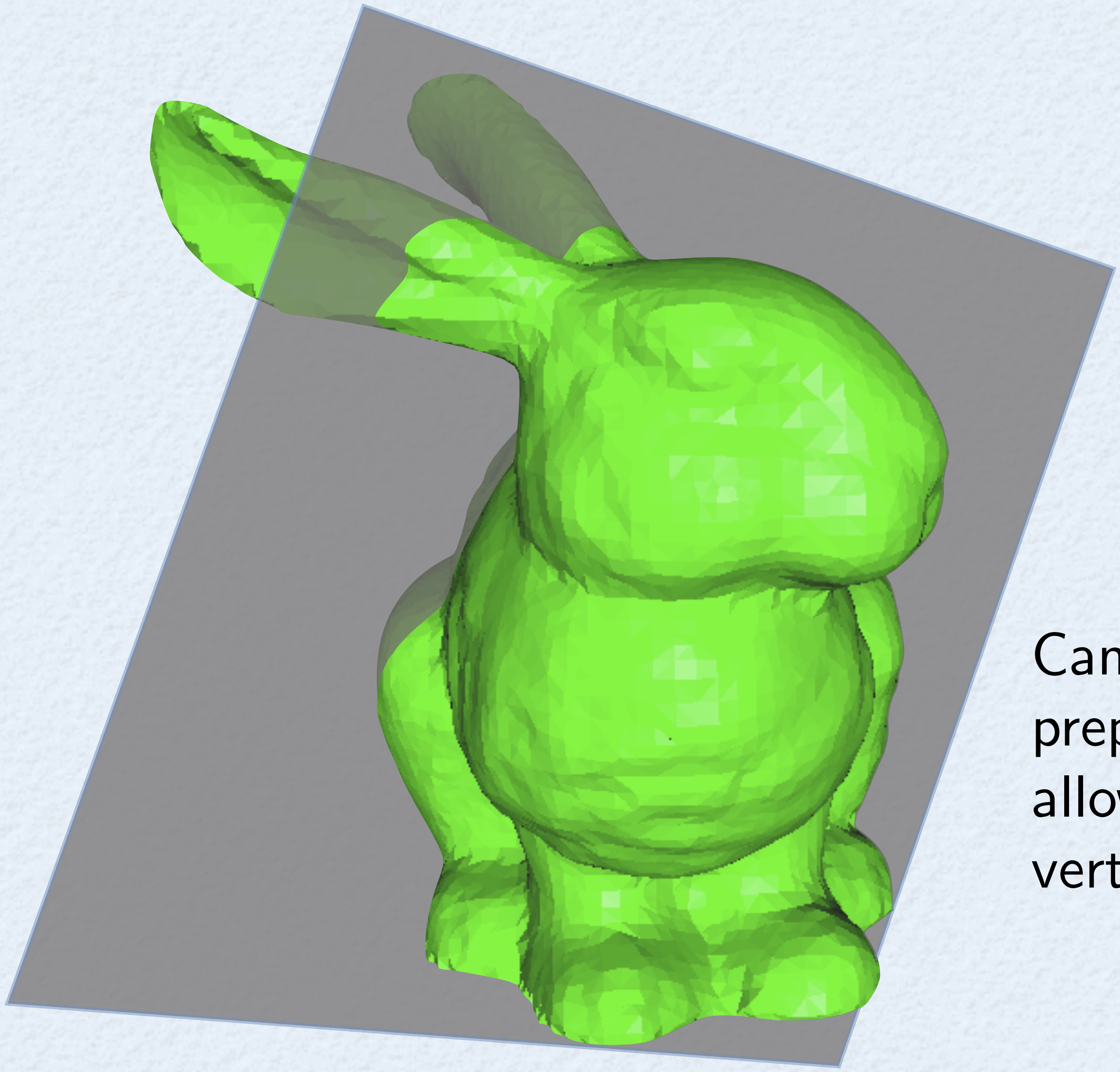

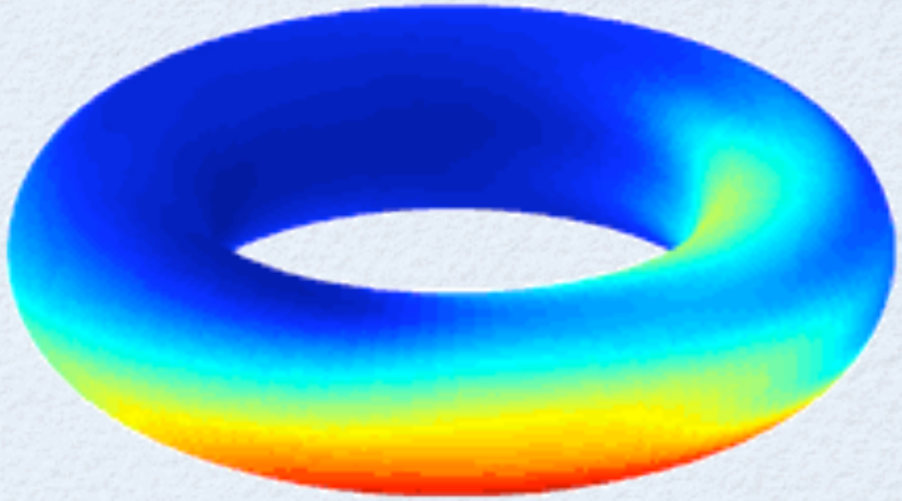

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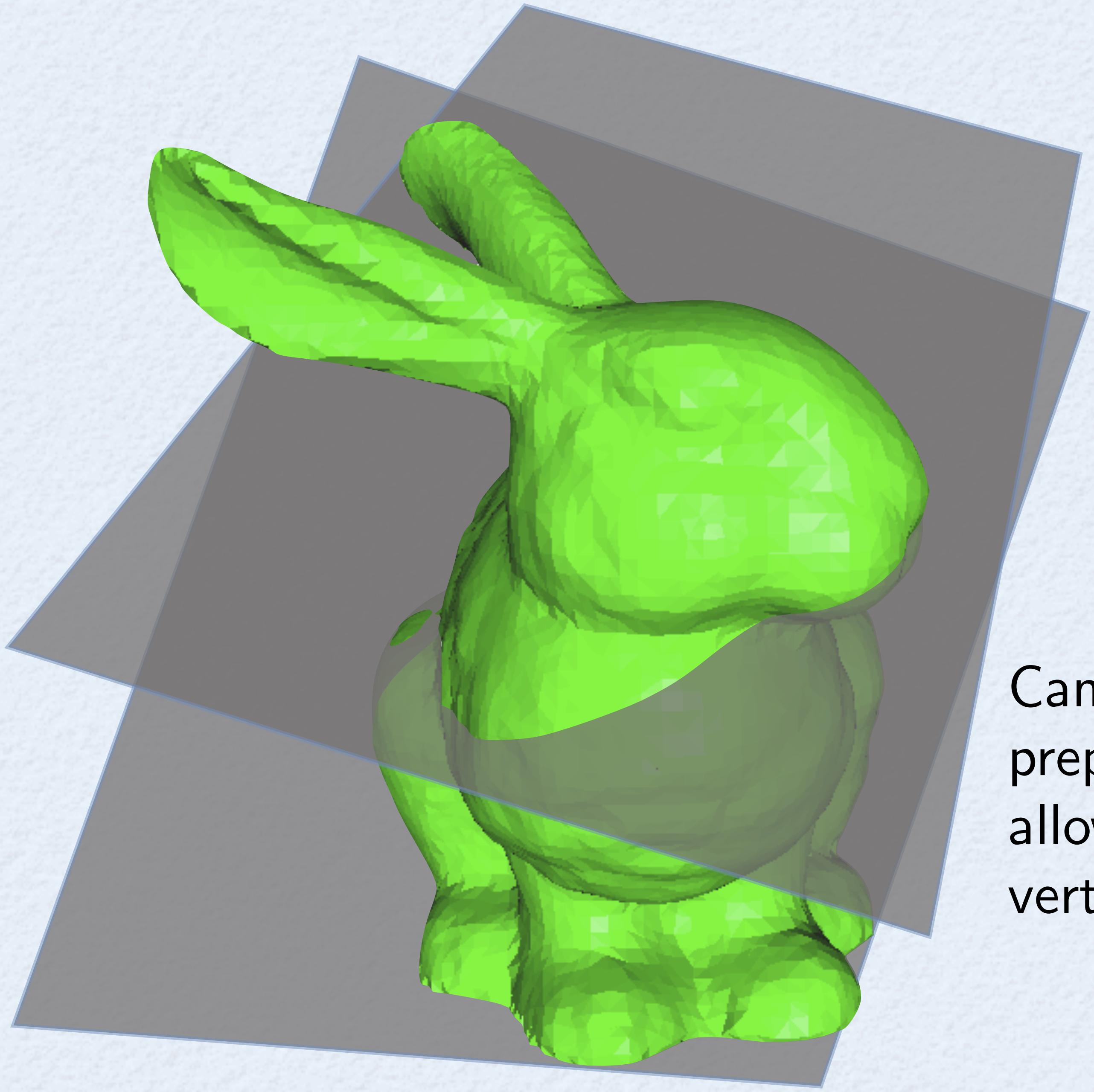

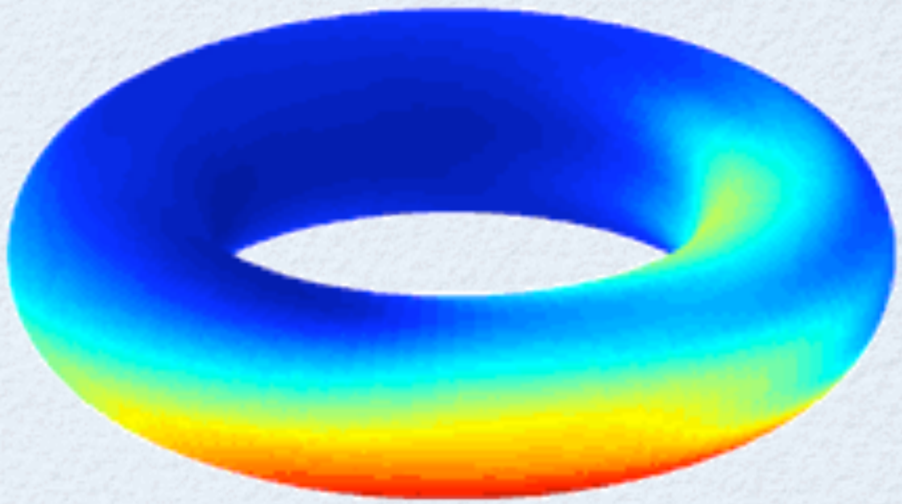

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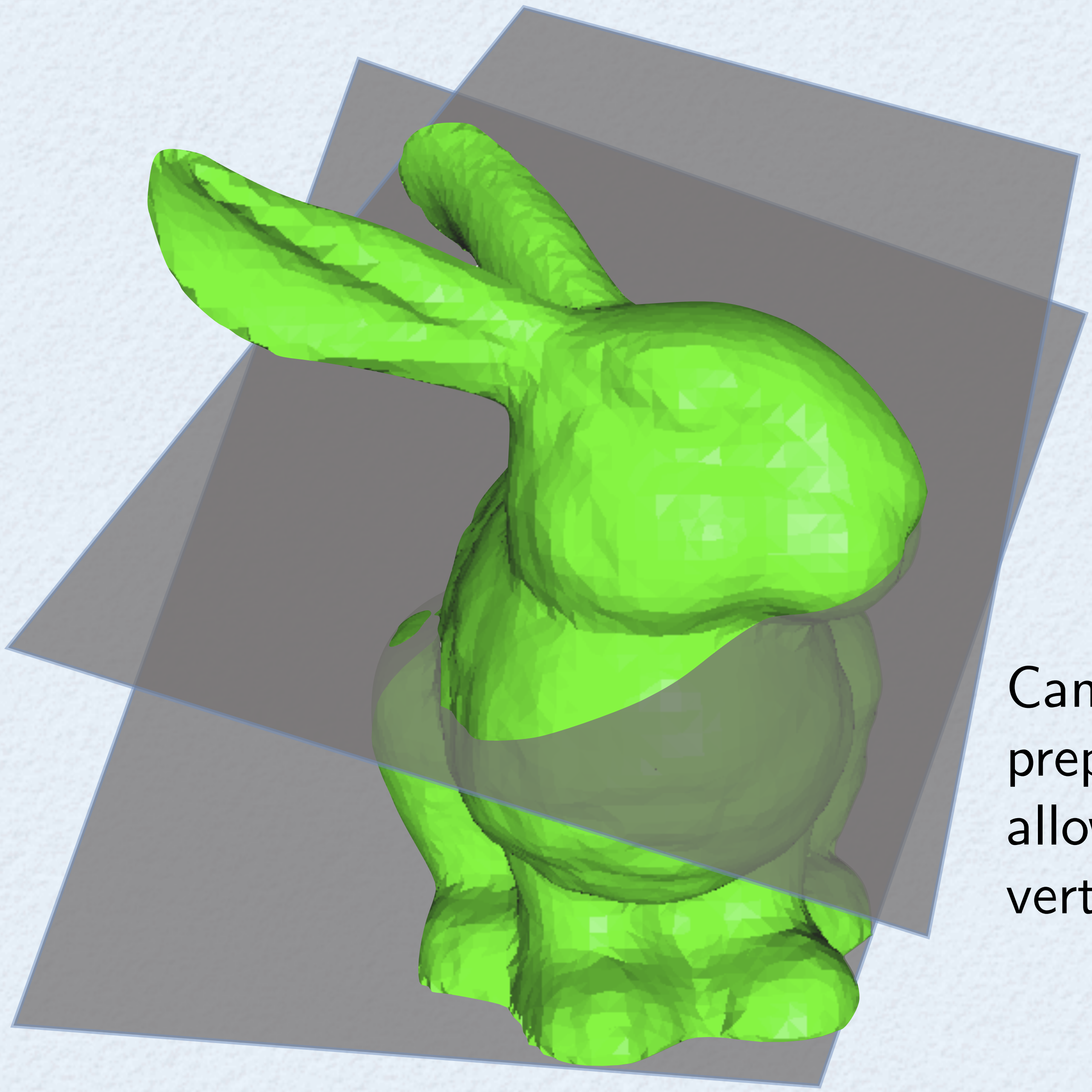

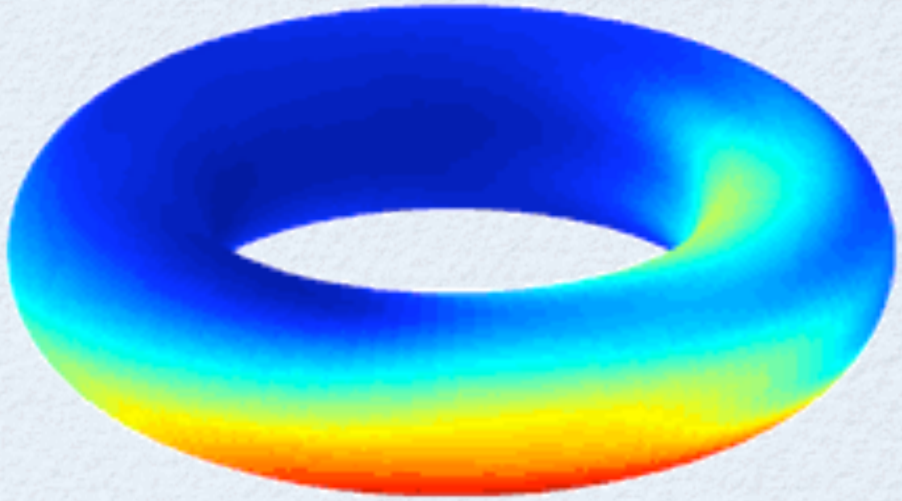

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Thank You!