Theorem 5.15 in the above paper is correct, but the proof needs a small correction. The statement of the theorem gives a tight lower bound on the number of states in a nondeterministic thrifty branching program solving the tree evaluation problem for binary trees of height 4. Precisely

**Theorem 1** Every nondeterministic thrifty branching program solving $BT^4_2(k)$ has $\Omega(k^3)$ states.

The proof considers YES inputs $I$ in a certain set $E^{r,s}$ of inputs to a nondeterministic thrifty branching program $B$, and associates a thrifty accepting computation $C(I)$ with each such $I$. The proof also associates a tuple\[ U(I) = (u, \gamma^I, \delta^I, x_1, x_2, x_3, x_4) \]with $I$, where $\gamma^I$ and $\delta^I$ are states in the computation $C(I)$. We use $(\gamma^I, \delta^I)$ to denote the segment of $C(I)$ between $\gamma^I$ and $\delta^I$.

The tag $u \in \{1, 2, 3\}$ in $U(I)$ specifies a partition of the middle nodes \{v_2, v_3, v_4, v_5, v_6, v_7\} of the input tree into disjoint sets $S_1$ and $S_2$ with the following properties:

- Every middle node queried during the computation segment $(\gamma^I, \delta^I)$ is in $S_1$.
- $S_1$ has at most four nodes, and the values of all nodes in $S_1$ are specified by $x_1, x_2, x_3, x_4$ in $U(I)$.
- The parent of every node in $S_2$ is queried during $(\gamma^I, \delta^I)$.

Near the end of the proof is the following claim:

**Claim:** If $I, J \in E^{r,s}$ and $U(I) = U(J)$, then $I = J$.

The claim is correct, but the proof of the claim is wrong, since it states that if $U(I) = U(J)$ then input $I$ is consistent with the segment $(\gamma^I, \delta^I)$ of the computation $C(J)$. (A thrifty query for $J$ might be nonthrifty for $I$, so the two answers could be different.)

To fix the proof, define a new input $I'$ as follows: For each non-leaf node $v_i$, let $f_i^I(x, y) = f_i^J(x, y)$ if $x, y$ are the correct values for the children of node $v_i$ in input $I$, and otherwise let $f_i^I(x, y) = f_i^J(x, y)$. Let the values of the leaf nodes of $I'$ be $r$ or $s$, as in $E^{r,s}$.

Thus the node values for $I$ and $I'$ are the same, but some of the functions associated with $I$ and $I'$ are different. This input $I'$ may not be in the set $E^{r,s}$, but this does not matter, because we assume that $B$ is a nondeterministic thrifty branching program which runs correctly on all inputs.

The state sequence for the computation $C(I)$ is also a possible state sequence for $B$ on input $I'$, because $C(I)$ only makes thrifty queries. We construct a different accepting computation $C'$ for the input $I'$ as follows: $C'$ coincides with $C(I)$ until $\gamma^I$, then follows $C(J)$ from $\gamma^I$ to $\delta^I$. 

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and finally follows $C(I)$ to the accept state. This is possible, because every query made by $C(J)$ during the segment $(\gamma^J, \delta^J)$ is either thrifty for $I$ as well as for $J$, (so the answer is specified by $(x_1, x_2, x_3, x_4)$ in $U(I) = U(J)$, and is the same for all three inputs $I, I', J$), or it is not thrifty for $I$, so by construction of $I'$ the answer is the same for inputs $I'$ and $J$.

Suppose $I \neq J$. Given that both $I$ and $J$ are in $E_{r,s}$ and $U(I) = U(J)$, it follows that $I$ and $J$ (and hence $I'$ and $J$) differ on the value of some middle node $v$ in the set $S_2$ specified by the tag $u$. Thus by the stated property of $S_2$, the computation segment $(\gamma^J, \delta^J)$ queries the parent of $v$, and this query cannot be thrifty for both $J$ and $I'$. But all accepting computations of a thrifty branching program must make only thrifty queries.

This proves the claim.

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