1. (a) Define what it means for a set $A \subseteq \Sigma^*$ to be semi-decidable. (Sipser’s book calls this Turing- recognizable.)

(b) Recall that the certificate definition of $A$ to be semi-decidable is that there exists a decidable relation $R(x, y)$ such that $x \in A$ iff there exists $y$ such that $R(x, y)$ holds.

Show that the definition in (a) is equivalent to this definition.

(You may continue your solution on the next page.)
Continue your solution to Question 1b here.
2. Let $PAL$ be the set of even length palindromes. Thus

$$P AL = \{ww^R \mid w \in \Sigma^*\}$$

Let

$$A = \{\langle M \rangle \mid M \text{ is a TM and } L(M) \subseteq PAL\}$$

Is $A$ semi-decidable?
Is $\overline{A}$ semi-decidable?
Justify your answers.
You may continue your solution on the next page.
(Continue your solution from the previous page.)
Let $A$ be an infinite set of Turing machine descriptions such that for each $\langle M \rangle \in A$

(a) $M$ is a Turing machine that halts on all inputs, and
(b) $M$ computes a function $f_M : \Sigma^* \to \Sigma^*$.

Suppose that $A$ is semi-decidable. Use a diagonal argument to define a total computable function $g : \Sigma^* \to \Sigma^*$ such that $g \neq f_M$, for all $\langle M \rangle \in A$. (Make sure that $g(w)$ is defined for all strings $w \in \Sigma^*$.)

Suggestion: It might be helpful to use an enumerator for $A$. 
