Last Name ______________________ First Name & Initial ______________________

Student No. ______________________

NO AIDS ALLOWED. Answer ALL questions on test paper. Use backs of sheets for scratch work.

Total Marks: 35

[10] 1. Let $B \subseteq \Sigma^*$ be a decidable set. For each $y \in \Sigma^*$ let

$$B_y = \{x \in \Sigma^* \mid \langle x, y \rangle \in B\}$$

Show that there is a decidable language $C \subseteq \Sigma^*$ such that $C \neq B_y$ for all $y \in \Sigma^*$.

[15] 2. Let

$$A = \{\langle M \rangle : M \text{ is a TM and } \mathcal{L}(M) \text{ has at least two strings}\}$$

Is A semidecidable? Is $\overline{A}$ semidecidable?

Justify your answers.

DO NOT USE RICE'S THEOREM.

(You may continue your solution on the next page.)

(Continue your solution to Question 2 here.)

[10] 3. Prove that every infinite semidecidable set has an infinite decidable subset.

Hint: Consider enumerators.