Due: Wednesday, April 1, beginning of lecture

NOTE: Each problem set counts 15% of your mark, and it is important to do your own work. You may consult with others concerning the general approach for solving problems on assignments, but you must write up all solutions entirely on your own. Copying assignments is a serious academic offense and will be dealt with accordingly.

1. Let $3TQBF$ be the same as $TQBF$ except that the part of the input formula following the quantifiers is 3CNF. Give a reduction showing

$$TQBF \leq_p 3TQBF$$

Prove that your reduction is correct.

You may use the fact that the reduction showing $SAT \leq_p 3SAT$ transforms (in polynomial time) a Boolean formula $\varphi$ to a Boolean formula $\varphi'$, such that $\varphi$ is satisfiable iff $\varphi'$ is satisfiable (but $\varphi'$ may have extra variables). In fact, you need more information about that proof. Namely, a new variable is introduced for certain subformulas of $\varphi$, and every assignment satisfying $\varphi$ can be extended to an assignment satisfying $\varphi'$.

2. Recall that $\text{EXPTIME} = \sum_k \text{DTIME}(2^{n^k})$ Prove that the complexity class $\text{DSPACE}(2^n)$ is not the same as $\text{EXPTIME}$. It will be helpful to look at the solution to problem 9.15 in the text.

3. Prove that $PATH \leq_L 2SAT$. It will be helpful to use the fact that $\text{NL} = \text{coNL}$.

Remark: It is also true that $2SAT \leq_L PATH$, but you need not prove this.