CSC463S **Problem Set 4** **Winter, 2018**

Due: Monday, April 2, beginning of lecture

NOTE: Each problem set counts 10% of your mark, and it is important to do your own work. You may consult with others concerning the general approach for solving problems on assignments, but you must write up all solutions entirely on your own. Copying assignments is a serious academic offense and will be dealt with accordingly.

1. By definition, **EXPTIME** $= \sum_k \text{DTIME}(2^{n^k})$.

   Prove that **DSPACE**(2^n) is not equal to **EXPTIME**.

   Hint: See problem 9.15 in the text, and its solution.

2. A directed graph is *doubly connected* if every two vertices are connected by a directed path in each direction. Let

   $$DCG = \{\langle G \rangle \mid G \text{ is a doubly connected graph}\}$$

   Prove that DCG is **NL**-complete. (You may use the fact that PATH is **NL**-complete.)

3. Consider the problem $FixedLengthPath$

   $FixedLengthPath$
   Instance
   $\langle G, s, t, d \rangle$, where G is an undirected graph, s and t are nodes in G, and d is a positive integer.

   Question: Is there a path from s to t of length d, and no shorter such path?

   (a) Show that $FixedLengthPath \in$ **NL**.

   (b) Show that $FixedLengthPath$ is **NL**-complete.

   Hint: Show that $PATH \leq_L FixedLengthPath$. Start with a directed graph G. Construct an undirected graph G' by making n copies of G. Each edge in G' should go from copy i to copy $i + 1$.