Due: Friday, March 16, beginning of tutorial

NOTE: Each problem set counts 10% of your mark, and it is important to do your own work. You may consult with others concerning the general approach for solving problems on assignments, but you must write up all solutions entirely on your own. Copying assignments is a serious academic offense and will be dealt with accordingly.

1. Let

$$A = \{\langle G, s, t \rangle \mid G \text{ is an undirected graph with a Hamiltonian path from } s \text{ to } t\}$$

   and
$$B = \{\langle G \rangle \mid G \text{ is an undirected graph with a Hamiltonian cycle}\}$$

   Give a reduction showing that $A \leq_p B$. Prove that your reduction is correct.

2. Let CNF-SAT be the set of all satisfiable propositional formulas in conjunctive normal form.

   Let Independent-Set be the set of all pairs (G, k) such that G is an undirected graph with an independent set of size k.

   Give an explicit reduction showing that Independent-Set $\leq_p$ CNF-SAT.

3. Show that the following problem is NP-complete:

   SS
   Instance:
   $\langle S_1, \ldots, S_m, k \rangle$, where each $S_i \subseteq \{1, \ldots, n\}$ for some $n \in \mathbb{N}$, and k is a positive integer.

   Question:
   Is there $C \subseteq \{1, ..., m\}$, $|C| = k$, such that $S_i \cap S_j = \varnothing$ for all $i, j \in C, i \neq j$.

   (Try using Independent-Set, which is NP-complete.)

4. Show that the following problem is NP-complete.

   RESTRICTED-CNF
   Instance:
   $\langle \varphi \rangle$, where φ is a CNF-Formula with at most three occurrences of every variable
   Question:
   Is φ satisfiable?

   (Try using 3SAT.)