Due: Friday, March 11, beginning of tutorial

NOTE: Each problem set counts 15% of your mark, and it is important to do your own work. You may consult with others concerning the general approach for solving problems on assignments, but you must write up all solutions entirely on your own. Copying assignments is a serious academic offense and will be dealt with accordingly.

1. Recall that an undirected graph $G = (V, E)$ is 3-colorable iff there is a map $f : V \rightarrow \{\text{red, blue yellow}\}$ such that no edge is assigned the same color to both its end points.

   Define $3\text{COL} = \{\langle G \rangle \mid G \text{ is a 3-colorable graph}\}$.

   Give an explicit reduction showing $3\text{COL} \leq_p 3\text{SAT}$.

2. Let us say that the map $f$ defined in Problem 1 is a partial 3-coloring of $G = (V, E)$ if the domain of $f$ is a subset $V' \subseteq V$.

   Define the $\text{NP}$ problem Partial-3COL as follows:

   **Instance:**
   $\langle G, f \rangle$ where $f$ is a partial 3-coloring of the graph $G$.

   **Question:** Can $f$ be extended to a three-coloring of $G$?

   Give an explicit reduction showing that Partial-3COL $\leq_p 3\text{COL}$.

3. Consider the following decision problem:

   **Nice-SAT**

   **Instance:** $\langle \phi \rangle$, where $\phi$ is a CNF formula such that every clause either consists entirely of unnegated variables or entirely of negated variables.

   **Question:** Is $\phi$ satisfiable?

   Show that **Nice-SAT** is NP - complete. (You may use the fact that 3SAT is NP-complete.)