Due: Friday, January 23, beginning of tutorial

NOTE: Each problem set counts 15% of your mark, and it is important to do your own work. You may consult with others concerning the general approach for solving problems on assignments, but you must write up all solutions entirely on your own. Copying assignments is a serious academic offense and will be dealt with accordingly.

1. Let

\[ A = \{ w \in \{0, 1\}^* \mid w \text{ has the same number of 0s as 1s} \} \]

Describe a Turing machine \( M \) whose accepted set is \( A \) (and \( M \) halts on all inputs).

Your description should start by specifying the tape alphabet of \( M \) (which can have extra symbols), and goes on to give a clear detailed English description of a TM, describing how the head moves and how the tape is modified on an arbitrary input string \( w \) in \( \{0, 1\}^* \).

You may also give a complete state diagram of \( M \), but your mark will be based on your English language description.

You may assume that \( M \) has a two-way infinite tape.

2. Recall that a decider is a Turing machine which halts on all inputs. Let

\[ A = \{ \langle M_1 \rangle, \langle M_2 \rangle, \ldots \} \]

be a set of decider descriptions. Use a diagonal argument to prove that if \( A \) is semidecidable, then there is a decidable set \( B \) such that \( B \neq L(M_i) \) for all \( \langle M_i \rangle \) in \( A \).

3. Suppose the function \( f : \mathbb{N} \to \mathbb{N} \) has the following property. For every Turing machine \( M \) with at most \( n \) states and at most \( n \) tape symbols (including \( b \) and 1), if \( M \) halts starting with a blank tape, then there are at most \( f(n) \) occurrences of 1 on its tape when it halts. (Assume that \( M \) has a two-way infinite tape.)

Prove that \( f \) is not a computable function.

(You may use the fact that HB (the set of Turing machines that halt on a blank tape) is not decidable. See the notes “Computability and Noncomputability” page 7.)