Last Name ____________________ First Name & Initial ____________________

Student No. ____________________

NO AIDS ALLOWED. Answer ALL questions on test paper. Use backs of sheets for scratch work.
DO NOT USE RICE'S THEOREM.

Total Marks: 42

[2] 1. (a) Define what it means for a set $A \subseteq \Sigma^*$ to be *semi-decidable*. (Sipser's book calls this *Turing-recognizable*.)

[15] (b) Recall that the *certificate definition* of A to be semi-decidable is that there exists a decidable relation $R(x, y)$ such that $x \in A$ iff there exists y such that $R(x, y)$ holds.

Show that the definition in (a) is equivalent to this definition.

(You may continue your solution on the next page.)

Continue your solution to Question 1b here.

[15] 2. Let PAL be the set of even length palindromes. Thus

$$PAL = \{ww^R \mid w \in \Sigma^*\}$$

Let

$$A = \{\langle M \rangle \mid M \text{ is a TM and } L(M) \subseteq PAL\}$$

Is A semi-decidable?
Is $\overline{A}$ semi-decidable?
Justify your answers.

You may continue your solution on the next page.

(Continue your solution from the previous page.)

[10] 3. Let A be an infinite set of Turing machine descriptions such that for each $\langle M \rangle \in A$

(a) M is a Turing machine that halts on all inputs, and

(b) M computes a function $f_M : \Sigma^* \to \Sigma^*$.

Suppose that A is semi-decidable. Use a diagonal argument to define a total computable function $g : \Sigma^* \to \Sigma^*$ such that $g \neq f_M$, for all $\langle M \rangle \in A$. (Make sure that $g(w)$ is defined for all strings $w \in \Sigma^*$.)

Suggestion: It might be helpful to use an enumerator for A.