Due: Friday, March 31, beginning of tutorial

NOTE: Each problem set counts 10% of your mark, and it is important to do your own work. You may consult with others concerning the general approach for solving problems on assignments, but you must write up all solutions entirely on your own. Copying assignments is a serious academic offense and will be dealt with accordingly.

1. Let 2SAT be like 3SAT except that the given formula has exactly two literals (involving two distinct variables) per clause. The purpose of this question is to show that 2SAT is NL-complete.

   Recall that a literal has one of the forms $p, \bar{p}$, where p is a variable. If ℓ is $\bar{p}$, then $\bar{\ell}$ is p.

   Given a 2CNF formula φ we associate a directed graph $G_\varphi = (V, E)$, where V is the set of all literals ℓ such that either ℓ or $\bar{\ell}$ occurs in φ, and for every clause $(\ell_1 \vee \ell_2)$ in φ we put the directed edges $(\bar{\ell_1}, \ell_2)$ and $(\bar{\ell_2}, \ell_1)$ in E.

   (The idea is that if a truth assignment τ satisfies the clause $(\ell_1 \vee \ell_2)$, then if τ makes ℓ_1 false then ℓ_2 must be true, and if τ makes ℓ_2 false then ℓ_1 must be true.)

   (a) Show that given any literals ℓ_1 and ℓ_2, if there is a directed path from ℓ_1 to ℓ_2 in G_φ then there is a directed path from $\bar{\ell_2}$ to $\bar{\ell_1}$, and every truth assignment to φ which satisfies φ and ℓ_1 also satisfies ℓ_2.

   (b) Use part (a) to show that φ is unsatisfiable iff G_φ has a directed cycle which includes both p and $\bar{p}$, for some variable p.

   (c) Use part (b) to show that 2SAT is NL-complete. (Use the fact that PATH is NL-complete.)

2. Use a padding argument to show that if EXP is a proper subset of NEXP, then P $\neq$ NP. (See exercises 9.13 and 9.14 in the text).