1. Give a sentence $A$ which has an infinite model, but no finite model. Your sentence should involve only one binary predicate symbol $P$, and no function symbols.

You should specify an infinite model for $A$, but you need not justify your claim that $A$ has no finite model.
2. Let $\mathcal{L}$ be a language consisting of an infinite set $\{c_1, c_2, \ldots\}$ of constant symbols, a binary predicate symbol $P$, and the equality symbol $\equiv$. Let $\Gamma$ be the set of sentences

$$\Gamma = \{c_i \neq c_j \mid i, j \in \mathbb{N}, i < j\}$$

Let $A$ be an $\mathcal{L}$-sentence such that $\Gamma \models A$. Prove that $A$ has a finite model.
3. Give an LK-Φ proof showing that $\Phi \models B$, where

$\Phi$ is $\{\forall x \exists y Pxy\}$

$B$ is $\exists x \exists y \exists z (Pxy \land Pyz)$

If you cannot give such a proof, then explain in English why $B$ is a logical consequence of $\Phi$, for part credit.
4. Give an LK proof of the sequent

\[ s_0 \neq ss_0 \rightarrow \forall x(x \neq s_0 \lor x \neq ss_0) \]

(Here \( t \neq u \) stands for \( \neg t = u \).)

You may leave out weakenings and exchanges.

Start by giving the specific instances of the LK equality axioms that you need in your proof.

Here are the LK equality axioms:

- **EL1:** \( \rightarrow t = t \)
- **EL2:** \( t = u \rightarrow u = t \)
- **EL3:** \( t = u, u = v \rightarrow t = v \)
- **EL4:** \( t_1 = u_1, \ldots, t_n = u_n \rightarrow ft_1 \ldots t_n = fu_1 \ldots u_n \), for each \( f \) in \( \mathcal{L} \), where \( f \) is an \( n \)-ary function symbol.
- **EL5:** \( t_1 = u_1, \ldots, t_n = u_n, Pt_1 \ldots t_n \rightarrow Pu_1 \ldots u_n \), for each \( P \) in \( \mathcal{L} \), where \( P \) is an \( n \)-ary predicate symbol.