1. a) Show that $\oplus$ (Parity) is $AC^0$-reducible to binary multiplication. Here binary multiplication is the function $MULT: \{0, 1\}^n \times \{0, 1\}^n \rightarrow \{0, 1\}^{2n}$ is defined by $MULT(x, y) = z$ if $z$ if $a \cdot b = c$ where strings $x, y, z$ represent natural numbers $a, b, c$ in binary.

To show the $AC^0$ reduction, define an $AC^0$ function $f$ such that $f(x) = (y, z, i)$, where the string $x$ has an odd number of 1’s iff the $i$th bit of $MULT(y, z)$ is 1. Do this as follows: Specify a number $k$, and let $y$ be $x$ with each of its consecutive bits separated by $k$ 0’s, and let $z$ be constructed similarly. (The idea is to remove the carries that would arise from normal long-hand multiplication of $x$ times $y$.) Show that this is an $AC^0$ reduction.

b) Conclude that $MULT$ is not an $AC^0$ function.

2. The point of this exercise is to show that Theorem 14.4, page 291 (Razborov-Smolensky) generalizes to the case that $p$ and $q$ are positive powers of distinct primes. (We use $AC^0(m)$ for $ACC0(m)$.)

(a) Show that if $a, b > 1$ then $MOD_a$ is in $AC^0(ab)$.
(b) Show that if $a, b > 1$ then $MOD_{ab}$ is in $AC^0(a, b)$.
(c) Conclude from the above that if $i, j > 0$ and $m > 1$, then $MOD_m$ is in $AC^0(m^j)$.

3. Let $\#MATCHINGS(G)$ be the number of matchings of a bipartite graph $G$. (A matching is any set of edges of $G$ such that no two edges in the set share a common vertex.) Let $\#MON2SAT(\varphi)$ be the number of satisfying assignments of a monotone 2CNF formula $\varphi$.

Use the fact that $\#MATCHINGS(G)$ is $\#P$ complete to show that $\#MON2SAT(\varphi)$ is $\#P$ complete.

4. Show that $P^{PP} = P^{#P}$. (See page 345 for the definition of $PP$.)