**Energy-Based Model Framework**

**GOAL:** make $F(Y;W)$ lower around areas of high data density
Energy-Based Model Framework

GOAL: make $F(Y;W)$ lower around areas of high data density

ENERGY BEFORE TRAINING
Energy-Based Model Framework

INPUT $Y$ \rightarrow \text{MODEL $W$} \rightarrow \text{ENERGY $F(Y;W)$}

**GOAL:** make $F(Y;W)$ lower around areas of high data density

ENERGY AFTER TRAINING
Energy-Based Model Framework

INPUT $Y$ → MODEL $W$ → ENERGY $F(Y;W)$

**GOAL:** make $F(Y;W)$ lower around areas of high data density

**WANT TO AVOID** FLAT ENERGY
Energy-Based Model Framework

**GOAL:** make $F(Y;W)$ lower around areas of high data density

Train the parameters of the model by minimizing a loss.
**Energy-Based Model Framework**

**GOAL:** make \( F(Y;W) \) lower around areas of high data density

Train the parameters of the model by minimizing a loss

\[
L(W) = F(Y;W)
\]
Energy-Based Model Framework

INPUT $Y$ → MODEL $W$ → ENERGY $F(Y;W)$

**GOAL:** make $F(Y;W)$ lower around areas of high data density

Train the parameters of the model by minimizing a **loss**

$$L(W) = F(Y;W)$$

NOT A GOOD LOSS IN GENERAL!!
Energy-Based Model Framework

Use contrastive loss

\[ L(W) = L(F(Y; W), F(\bar{Y}; W)) \]

\( L(a,b) \): increasing fn of a, decreasing fn of b
Energy-Based Model Framework

INPUT Y $\rightarrow$ MODEL $W$ $\rightarrow$ ENERGY $F(Y;W)$

Use contrastive loss

$L(W) = L(F(Y;W), F(\bar{Y};W))$
Energy-Based Model Framework

Use contrastive loss

\[ L(W) = L(F(Y; W), F(\bar{Y}; W)) \]
Each Stage is Trained as an Estimator of the Input Density

- **Probabilistic View:**
  - Produce a probability density function that:
  - has high value in regions of high sample density
  - has low value everywhere else (integral = 1).

- **Energy-Based View:**
  - produce an energy function $E(Y, W)$ that:
  - has low value in regions of high sample density
  - has high(er) value everywhere else
$P(Y|W) = \frac{e^{-\beta E(Y,W)}}{\int_y e^{-\beta E(y,W)}}$

$E(Y,W) \propto - \log P(Y|W)$
The Intractable Normalization Problem

Example: Image Patches

Learning:
- Make the energy of every “natural image” patch low
- Make the energy of everything else high!

\[ P(Y, W) = \frac{e^{-\beta E(Y, W)}}{\int_y e^{-\beta E(y, W)}} \]
Training an Energy-Based Model to Approximate a Density

Maximizing $P(Y|W)$ on training samples

$$P(Y|W) = \frac{e^{-\beta E(Y,W)}}{\int_y e^{-\beta E(y,W)}}$$

Minimizing $-\log P(Y,W)$ on training samples

$$L(Y, W) = E(Y, W) + \frac{1}{\beta} \log \int_y e^{-\beta E(y,W)}$$
Training an Energy-Based Model with Gradient Descent

Gradient of the negative log-likelihood loss for one sample $Y$:

$$\frac{\partial L(Y, W)}{\partial W} = \frac{\partial E(Y, W)}{\partial W} - \int_y P(y|W) \frac{\partial E(y, W)}{\partial W}$$

Gradient descent:

$$W \leftarrow W - \eta \frac{\partial L(Y, W)}{\partial W}$$

Pushes down on the energy of the samples

Pulls up on the energy of low-energy $Y$'s
Probabilistic unsupervised learning is hard

- Pushing up on the energy of every points in regions of low data density is often impractical.

**Solution 1: contrastive divergence [Hinton 2000]**

- Only push up on points that are not too far from the training samples, and only on those points that have low energy. These points are obtained from the training samples through MCMC.
- This makes a “groove” in the energy surface around the data manifold.

**Solution 2: MAIN INSIGHT! [Ranzato, ..., LeCun AI-Stat 2007]**

- **Restrict the information content of the code (features) Z**
- If the code Z can only take a few different configurations, only a correspondingly small number of Ys can be perfectly reconstructed
- Idea: impose a sparsity prior on Z
- This is reminiscent of sparse coding [Olshausen & Field 1997]
Contrastive Divergence Trick [Hinton 2000]

- push down on the energy of the training sample \( Y \)
- Pick a sample of low energy \( Y' \) near the training sample, and pull up its energy
  - this digs a trench in the energy surface around the training samples

\[
W \leftarrow W - \eta \frac{\partial E(Y, W)}{\partial W} + \eta \frac{\partial E(Y', W)}{\partial W}
\]

Pushes down on the energy of the training sample \( Y \)
Pulls up on the energy \( Y' \)
Contrastive Divergence Trick [Hinton 2000]

- **push down** on the energy of the training sample \( Y \)

- **Pick a sample of low energy** \( Y' \) near the training sample, and **pull up its energy**
  - this digs a trench in the energy surface around the training samples

\[
W \leftarrow W - \eta \frac{\partial E(Y, W)}{\partial W} + \eta \frac{\partial E(Y', W)}{\partial W}
\]

Pushes down on the energy of the training sample \( Y \)

Pulls up on the energy \( Y' \)
Energy-Based Model Framework

Use contrastive loss

- e.g. maximum likelihood learning
- generally intractable and expensive in high dimensions

\[ L(W) = L(F(Y;W), F(\bar{Y};W)) \]
Energy-Based Model Framework

Restrict the information content of internal representation

- assume that input is reconstructed from code
- inference determines the value of $Z$ and $F(Y;W)$
Restrict information content of internal representation

- assume that input is reconstructed from code
- inference determines the value of Z and F(Y;W)
Restrict information content of internal representation

- assume that input is reconstructed from code
- inference determines the value of Z and F(Y;W)

If Z is constrained, we can simply train by minimizing the energy loss over the training set:

\[ L(W) = F(Y;W) \]
Each stage is composed of

- an encoder that produces a feature vector from the input
- a decoder that reconstructs the input from the feature vector

PCA is a special case (linear encoder and decoder)

[Hinton 05, Bengio 06, LeCun 06, Ng 07]
Train each stage one after the other

1. Train the first stage
Deep Learning: Stack of Encoder/Decoders

- Train each stage one after the other
- 2. Remove the decoder, and train the second Stage
Train each stage one after the other

3. Remove the 2nd stage decoder, and train a supervised classifier on top

4. Refine the entire system with supervised learning
   e.g. using gradient descent / backprop
Define the Energy $F(Y)$ as the reconstruction error
- Example: $F(Y) = || Y - \text{Decoder}(\text{Encoder}(Y)) ||^2$

Probabilistic Training, given a training set $(Y_1, Y_2, \ldots)$
- Interpret the energy $F(Y)$ as a $-\log P(Y)$ (unnormalized)
- Train the encoder/decoder to maximize the prob of the data

Train the encoder/decoder so that:
- $F(Y)$ is small in regions of high data density (good reconstruction)
- $F(Y)$ is large in regions of low data density (bad reconstruction)
E(X,Z) = Dist[X,Dec(Z)] + Dist[Z,Enc(X)] + Reg(Z)

F(X) = MIN_z E(X,Z)  or  F(X) = -log SUM_z exp(-E(X,z))

RBM is a special case:
Enc(X) = W.X,  Dist(Z,W.X) = Z'W.X
Dec(Z) = W'Z,  Dist(X,W'X) = X'W.Z
If the information content of the feature vector is limited (e.g. by imposing sparsity constraints), the energy MUST be large in most of the space.

Pulling down on the energy of the training samples will necessarily make a groove.

The volume of the space over which the energy is low is limited by the entropy of the feature vector.

Input vectors are reconstructed from feature vectors.

If few feature configurations are possible, few input vectors can be reconstructed properly.
Why Limit the Information Content of the Code?

- Training sample
- Input vector which is NOT a training sample
- Feature vector
Why Limit the Information Content of the Code?

- Training sample
- Input vector which is NOT a training sample
- Feature vector

*Training based on minimizing the reconstruction error over the training set*
Why Limit the Information Content of the Code?

- Training sample
- Input vector which is NOT a training sample
- Feature vector

BAD: machine does not learn structure from training data!!

It just copies the data.
Why Limit the Information Content of the Code?

- Training sample
- Input vector which is NOT a training sample
- Feature vector

IDEA: reduce number of available codes.
Why Limit the Information Content of the Code?

- Training sample
- Input vector which is NOT a training sample
- Feature vector

*IDEA: reduce number of available codes.*
Why Limit the Information Content of the Code?

- Training sample
- Input vector which is NOT a training sample
- Feature vector

**IDEA:** reduce number of available codes.
2 dimensional toy dataset
- Mixture of 3 Cauchy distrib.
- Visualizing energy surface
  (black = low, white = high)
- 2 dimensional toy dataset spiral
- Visualizing energy surface (black = low, white = high)

<table>
<thead>
<tr>
<th>Method</th>
<th>Encoder</th>
<th>Decoder</th>
<th>Energy</th>
<th>Loss</th>
<th>Pull-up</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCA</td>
<td>$W^T_Y$</td>
<td>$WZ$</td>
<td>$|Y - WZ|^2$</td>
<td>$F(Y)$</td>
<td>dimens.</td>
</tr>
<tr>
<td>autoencoder</td>
<td>$\sigma(W_e Y)$</td>
<td>$W_d Z$</td>
<td>$|Y - WZ|^2$</td>
<td>$F(Y)$</td>
<td>dimens.</td>
</tr>
<tr>
<td>sparse coding</td>
<td>$\sigma(W_e Z)$</td>
<td>$W_d Z$</td>
<td>$|Y - WZ|^2$</td>
<td>$F(Y)$</td>
<td>sparsity</td>
</tr>
<tr>
<td>K-Means</td>
<td>-</td>
<td>$WZ$</td>
<td>$|Y - WZ|^2$</td>
<td>$F(Y)$</td>
<td>1-of-N code</td>
</tr>
</tbody>
</table>
We are going to impose a sparsity penalty on the code to restrict its information content.

We will allow the code to have higher dimension than the input.

Categories are more easily separable in high-dim sparse feature spaces.

- This is a trick that SVM use: they have one dimension per sample.

Sparse features are optimal when an active feature costs more than an inactive one (zero).

- e.g. neurons that spike consume more energy.
- The brain is about 2% active on average.
Sparse Decomposition with Linear Reconstruction

[Olshausen and Field 1997]

**Energy**\((\text{Input}, \text{Code}) = \| \text{Input} - \text{Decoder}(\text{Code}) \|^2 + \text{Sparsity}(\text{Code})\)

**Energy**\((\text{Input}) = \text{Min_over_Code}[\text{Energy}(\text{Input}, \text{Code})]\)

\[
\begin{align*}
E(Y^i, Z^i; W) &= \|Y^i - W_d Z^i\|^2 + \lambda \sum_j |z^i_j| \\
F(Y^i; W) &= \text{min}_z E(Y^i, z; W)
\end{align*}
\]

**Energy:** minimize to infer \(Z\)

**Loss:** minimize to learn \(W\) (the columns of \(W\) are constrained to have norm 1)

\[
L(W) = \sum_i F(Y^i; W) = \sum_i (\text{min}_{Z^i} E(Y^i, Z^i; W))
\]
Problem with Sparse Decomposition: It's slow

- **Inference:** Optimal Code = $\text{Arg\_Min\_over\_Code}[\text{Energy}(\text{Input}, \text{Code})]

$$
E (Y^i, Z^i ; W) = \|Y^i - W_d Z^i\|^2 + \lambda \sum_j |z^i_j|
$$

$$
F (Y^i ; W) = \min_z E (Y^i, z ; W)
$$

$$
Z^i = \text{argmin}_z E (Y^i, z ; W)
$$

- For each new $Y$, an optimization algorithm must be run to find the corresponding optimal $Z$

- This would be very slow for large scale vision tasks

- Also, the optimal $Z$ are very unstable:
  - A small change in $Y$ can cause a large change in the optimal $Z$
**Solution: Predictive Sparse Decomposition (PSD)**

- **Prediction** the optimal code with a trained encoder

- **Energy** = reconstruction_error + code_prediction_error + code_sparsity

\[
E(Y^i, Z^i; W) = \| Y^i - W_d Z^i \|^2 + \| Z^i - f_e(Y^i) \|^2 + \lambda \sum_j |z^i_j|
\]

\[
f_e(Y^i) = D \tanh(W_e Y)
\]

---

- **Observed Input** \( Y \)
- **Latent Code** \( Z \)
- **Reconstructed Input** \( Y - f_d(Z) \)
- **Prediction of code** \( f_e(Y) = D \tanh(W_e Y) \)
- **Predicted Code** \( Z - f_e(Y) \)
PSD: Inference

Inference by gradient descent starting from the encoder output

\[ E(Y^i, Z^i; W) = \|Y^i - W_d Z^i\|^2 + \|Z^i - f_e(Y^i)\|^2 + \lambda \sum_j |z_j^i| \]

\[ Z^i = \arg \min_z E(Y^i, z; W) \]
Learning by minimizing the average energy of the training data with respect to $W_d$ and $W_e$.

**Loss function:**

$$ L(W) = \sum_i F(Y^i; W) $$

$$ F(Y^i; W) = \min_z E(Y^i, z; W) $$

---

The diagram illustrates the process of prediction and reconstruction. The observed input $Y$ is processed through $f_e(Y) = D\tanh(W_e Y)$ to predict the code $Z$. The reconstruction error is calculated as $\|Y - f_d(Z)\|_2^2$. The latent code $Z$ is further transformed by $f_d(Z) = W_d Z$ to produce the final output $|Z|_1$. The image also shows the minimization of energy with respect to the code $z$. 

---

**Yann LeCun**
1. Initialize $Z = \text{Encoder}(Y)$
2. Find $Z$ that minimizes the energy function
3. Update the Decoder basis functions to reduce reconstruction error
4. Update Encoder parameters to reduce prediction error

Repeat with next training sample
Decoder Basis Functions on MNIST

- PSD trained on handwritten digits: decoder filters are “parts” (strokes).

- Any digit can be reconstructed as a linear combination of a small number of these “parts”.

![MNIST dataset images]
Basis functions are like Gabor filters (like receptive fields in V1 neurons)

- 256 filters of size 12x12
- Trained on natural image patches from the Berkeley dataset
- Encoder is linear-tanh-diagonal
Classification Error Rate on MNIST

Supervised Linear Classifier trained on 200 trained sparse features

Red: linear-tanh-diagonal encoder; Blue: linear encoder
Classification Error Rate on MNIST

- Supervised Linear Classifier trained on 200 trained sparse features

Graphs showing error rate versus RMSE for different sample sizes (10, 100, 1000) and different feature representations (Raw pixels, PCA, RBM, Sparse Features).
Learned Features on natural patches: V1-like receptive fields
Learned Features: V1-like receptive fields

- 12x12 filters
- 1024 filters
How well do PSD features work on Caltech-101?

Recognition Architecture

- Filter Bank
- Non-Linearity
- Spatial Pooling
- Classifier

Yann LeCun
1. Pre-process images
   - remove mean, high-pass filter, normalize contrast

2. Train encoder-decoder on 9x9 image patches

3. use the filters in a recognition architecture
   - Apply the filters to the whole image
   - Apply the tanh and D scaling
   - Add more non-linearities (rectification, normalization)
   - Add a spatial pooling layer

4. Train a supervised classifier on top
   - Multinomial Logistic Regression or Pyramid Match Kernel SVM
64 filters on 9x9 patches trained with PSD

with Linear-Sigmoid-Diagonal Encoder

weights \(-0.2828 - 0.3043\)
Feature Extraction

Convolution/sigmoid layer: filter bank? Learning, fixed Gabors?
Feature Extraction

Convolution/sigmoid layer: filter bank? Learning, fixed Gabors?

Pinto, Cox and DiCarlo, PloS 08
Feature Extraction

- Convolution/sigmoid layer: filter bank? Learning, fixed Gabors?
- Rectification layer: needed?

Pinto, Cox and DiCarlo, PloS 08
Feature Extraction

- C Convolution/sigmoid layer: filter bank? Learning, fixed Gabors?
- Abs Rectification layer: needed?

Pinto, Cox and DiCarlo, PloS 08
Feature Extraction

- **C** Convolution/sigmoid layer: filter bank? Learning, fixed Gabors?
- **Abs** Rectification layer: needed?

\[
\frac{x - \mu}{\max(t, \sigma)}
\]

Pinto, Cox and DiCarlo, PloS 08
Feature Extraction

- Convolution/sigmoid layer: filter bank? Learning, fixed Gabors?
- Rectification layer: needed?
- Normalization layer: needed?

\[ \text{Local Contrast} \]

\[ \frac{x - \mu}{\max(t, \sigma)} \]

Pinto, Cox and DiCarlo, PloS 08
Feature Extraction

- **C** Convolution/sigmoid layer: filter bank? Learning, fixed Gabors?
- **Abs** Rectification layer: needed?
- **N** Normalization layer: needed?

---

Pinto, Cox and DiCarlo, PloS 08
Feature Extraction

- **C** Convolution/sigmoid layer: filter bank? Learning, fixed Gabors?
- **Abs** Rectification layer: needed?
- **N** Normalization layer: needed?

Pooling Down-Sampling Layer
Feature Extraction

- **C** Convolution/sigmoid layer: filter bank? Learning, fixed Gabors?
- **Abs** Rectification layer: needed?
- **N** Normalization layer: needed?
- **P** Pooling down-sampling layer: average or max?

Diagram:
- Input image
- Convolution (C)
- Absolute value (Abs)
- Normalization (N)
- Pooling Down-Sampling Layer
Feature Extraction

- \( C \)  Convolution/sigmoid layer: filter bank? Learning, fixed Gabors?
- \( \text{Abs} \)  Rectification layer: needed?
- \( \text{N} \)  Normalization layer: needed?
- \( \text{P} \)  Pooling down-sampling layer: average or max?

**Diagram:**
- C → Abs → N → P
Feature Extraction

- **C** Convolution/sigmoid layer: filter bank? Learning, fixed Gabors?
- **Abs** Rectification layer: needed?
- **N** Normalization layer: needed?
- **P** Pooling down-sampling layer: average or max?

THIS IS ONE STAGE OF FEATURE EXTRACTION
Training Protocol

Training

- Logistic Regression on Random Features: \( R \)
- Logistic Regression on PSD features: \( U \)
- Refinement of whole net from random with backprop: \( R^+ \)
- Refinement of whole net starting from PSD filters: \( U^+ \)

Classifier

- Multinomial Logistic Regression or Pyramid Match Kernel SVM
Using PSD Features for Recognition

\[
[64 \cdot F^{9 \times 9}_{\text{CSG}} - R/N/P^{5 \times 5}] - \log\_\text{reg}
\]

<table>
<thead>
<tr>
<th>R/N/P</th>
<th>(R_{\text{abs}} - N - P_A)</th>
<th>(R_{\text{abs}} - P_A)</th>
<th>N - (P_M)</th>
<th>N - (P_A)</th>
<th>(P_A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>U(^+)</td>
<td>54.2%</td>
<td>50.0%</td>
<td>44.3%</td>
<td>18.5%</td>
<td>14.5%</td>
</tr>
<tr>
<td>R(^+)</td>
<td>54.8%</td>
<td>47.0%</td>
<td>38.0%</td>
<td>16.3%</td>
<td>14.3%</td>
</tr>
<tr>
<td>U</td>
<td>52.2%</td>
<td>43.3(\pm 1.6)%</td>
<td>44.0%</td>
<td>17.2%</td>
<td>13.4%</td>
</tr>
<tr>
<td>R</td>
<td>53.3%</td>
<td>31.7%</td>
<td>32.1%</td>
<td>15.3%</td>
<td>12.1(\pm 2.2)%</td>
</tr>
</tbody>
</table>

\[
[64 \cdot F^{9 \times 9}_{\text{CSG}} - R/N/P^{5 \times 5}] - \text{PMK}
\]

| U     | 65.0%                          |

\[
[96 \cdot F^{9 \times 9}_{\text{CSG}} - R/N/P^{5 \times 5}] - \text{PCA - lin\_svm}
\]

| U     | 58.0%                          |

96.Gabors - PCA - lin\_svm (Pinto and DiCarlo 2006)

| Gabors | 59.0%                          |

SIFT - PMK (Lazebnik et al. CVPR 2006)

| Gabors | 64.6%                          |
Using PSD Features for Recognition

- **Rectification makes a huge difference:**
  - 14.5% -> 50.0%, without normalization
  - 44.3% -> 54.2% with normalization

- **Normalization makes a difference:**
  - 50.0 -> 54.2

- **Unsupervised pretraining makes small difference**

- **PSD works just as well as SIFT**

- **Random filters work as well as anything!**
  - If rectification/normalization is present

- **PMK_SVM classifier works a lot better than multinomial log_reg on low-level features**
  - 52.2% → 65.0%
Approximated Sparse Features Predicted by PSD give better recognition results than Optimal Sparse Features computed with Feature Sign!

- PSD features are more stable.

Feature Sign (FS) is an optimization methods for computing sparse codes [Lee...Ng 2006]
Approximated Sparse Features Predicted by PSD give better recognition results than Optimal Sparse Features computed with Feature Sign!

Because PSD features are more stable. Feature obtained through sparse optimization can change a lot with small changes of the input.

<table>
<thead>
<tr>
<th>Feature Sign</th>
<th>PSD</th>
<th>PSD Random</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(0</td>
<td>0) 0.39</td>
<td>P(0</td>
</tr>
<tr>
<td>P(-</td>
<td>) 0.60</td>
<td>P(-</td>
</tr>
<tr>
<td>P(+</td>
<td>+) 0.59</td>
<td>P(+</td>
</tr>
<tr>
<td>P(0</td>
<td>+) 0.41</td>
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<td>P(0</td>
<td>-) 0.40</td>
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</tr>
<tr>
<td>P(+</td>
<td>0) 0.01</td>
<td>P(+</td>
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<tr>
<td>P(</td>
<td>0) 0.01</td>
<td>P(</td>
</tr>
<tr>
<td>P(+</td>
<td>+) 0.00</td>
<td>P(+</td>
</tr>
<tr>
<td>P(-</td>
<td>+) 0.00</td>
<td>P(-</td>
</tr>
</tbody>
</table>

How many features change sign in patches from successive video frames (a,b), versus patches from random frame pairs (c).
PSD features are much cheaper to compute

Computing PSD features is hundreds of times cheaper than Feature Sign.
How Many 9x9 PSD features do we need?

- Accuracy increases slowly past 64 filters.
1. Train stage-1 filters with PSD on patches from natural images
2. Compute stage-1 features on training set
3. Train state-2 filters with PSD on stage-1 feature patches
4. Compute stage-2 features on training set
5. Train linear classifier on stage-2 features
6. Refine entire network with supervised gradient descent

What are the effects of the non-linearities and unsupervised pretraining?
Multistage Hubel-Wiesel Architecture on Caltech-101

- **INPUT**: 3@140x140
- **CONVOLUTIONS** (9x9):
  - 32@132x132
- **MAX/SUBSAMPLING** (4x4):
  - 32@33x33
- **CONVOLUTIONS** (9x9):
  - 64@25x25
- **MAX/SUBSAMPLING** (5x5):
  - 64@5x5

Yann LeCun
Multistage Hubel-Wiesel Architecture

**Image Preprocessing:**
- High-pass filter, local contrast normalization (divisive)

**First Stage:**
- Filters: 64 9x9 kernels producing 64 feature maps
- Pooling: 10x10 averaging with 5x5 subsampling

**Second Stage:**
- Filters: 4096 9x9 kernels producing 256 feature maps
- Pooling: 6x6 averaging with 3x3 subsampling
- Features: 256 feature maps of size 4x4 (4096 features)

**Classifier Stage:**
- Multinomial logistic regression

**Number of parameters:**
- Roughly 750,000
## Multistage Hubel-Wiesel Architecture on Caltech-101

\[
\begin{bmatrix}
64.F_{\text{CSG}}^{9\times9} - \frac{R}{N/P}^{5\times5} \\
256.F_{\text{CSG}}^{9\times9} - \frac{R}{N/P}^{4\times4}
\end{bmatrix}
\quad \text{log_reg}
\]

<table>
<thead>
<tr>
<th>R/N/P</th>
<th>(R_{\text{abs}} - N - P_A)</th>
<th>(R_{\text{abs}} - P_A)</th>
<th>(N - P_M)</th>
<th>(N - P_A)</th>
<th>(P_A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(U^+U^+)</td>
<td>65.5%</td>
<td>60.5%</td>
<td>61.0%</td>
<td>34.0%</td>
<td>32.0%</td>
</tr>
<tr>
<td>(R^+R^+)</td>
<td>64.7%</td>
<td>59.5%</td>
<td>60.0%</td>
<td>31.0%</td>
<td>29.7%</td>
</tr>
<tr>
<td>UU</td>
<td>63.7%</td>
<td>46.7%</td>
<td>56.0%</td>
<td>23.1%</td>
<td>9.1%</td>
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<td>33.7((\pm 1.5)%</td>
<td>37.6((\pm 1.9))%</td>
<td>19.6%</td>
<td>8.8%</td>
</tr>
<tr>
<td>GT</td>
<td>X</td>
<td></td>
<td></td>
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</table>

\[
\begin{bmatrix}
64.F_{\text{CSG}}^{9\times9} - \frac{R}{N/P}^{5\times5} \\
256.F_{\text{CSG}}^{9\times9} - \frac{R}{N}
\end{bmatrix}
\quad \text{PMK}
\]

UU | 52.8% |

**HMAX:** [Gabor-R/P_M]-[Templates-R/P_M]-lin_svm (Serre 2005)(Mutch-Lowe 2006)

| GT | 56.0% |
Two-Stage Result Analysis

- Second Stage + logistic regression = PMK_SVM
- Unsupervised pre-training doesn't help much :-(
- Random filters work amazingly well with normalization
- Supervised global refinement helps a bit
- The best system is really cheap
- Either use rectification and average pooling or no rectification and max pooling.
Multistage Hubel-Wiesel Architecture: Filters

Stage 1

- After PSF
  - Weights: -0.2232 - 0.2075

Stage 2

- After supervised refinement
  - Weights: -0.2828 - 0.3043

- Weights: -0.0778 - 0.064

- Weights: -0.0989 - 0.0784
Why Random Filters Work?
Small NORB dataset

- 5 classes and up to 24,300 training samples per class
Small NORB dataset

**Architecture**

- Two Stages

**Error Rate (log scale)** VS. **Number Training Samples (log scale)**
Unsupervised PSD ignores the spatial pooling step.

Could we devise a similar method that learns the pooling layer as well?

Idea [Hyvarinen & Hoyer 2001]: sparsity on pools of features

- Minimum number of pools must be non-zero
- Number of features that are on within a pool doesn't matter
- Pools tend to regroup similar features

\[
\sum_j \left( \sqrt{\sum_{i \in P_j} Z_i^2} \right)
\]
Using an idea from Hyvarinen: topographic square pooling (subspace ICA)

1. Apply filters on a patch (with suitable non-linearity)
2. Arrange filter outputs on a 2D plane
3. Square filter outputs
4. Minimize sqrt of sum of blocks of squared filter outputs

Units in the code $Z$
Define pools and enforce sparsity across pools
Learning the filters and the pools

The filters arrange themselves spontaneously so that similar filters enter the same pool.

The pooling units can be seen as complex cells.

They are invariant to local transformations of the input.

For some it's translations, for others rotations, or other transformations.
Pinwheels?
Invariance Properties Compared to SIFT

- Measure distance between feature vectors (128 dimensions) of 16x16 patches from natural images
  - Left: normalized distance as a function of translation
  - Right: normalized distance as a function of translation when one patch is rotated 25 degrees.

- Topographic PSD features are more invariant than SIFT
Learning Invariant Features

**Recognition Architecture**
- HPF/LCN
- Filters
- tanh
- square
- Pooling
- square
- Classifier

- Block pooling plays the same role as rectification
**Recognition Accuracy on Caltech 101**

- A/B Comparison with SIFT (128x34x34 descriptors)
- 32x16 topographic map with 16x16 filters
- Pooling performed over 6x6 with 2x2 subsampling
- 128 dimensional feature vector per 16x16 patch
- Feature vector computed every 4x4 pixels (128x34x34 feature maps)
- Resulting feature maps are spatially smoothed

<table>
<thead>
<tr>
<th>Method</th>
<th>Av. Accuracy/Class (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>local norm(<em>{5\times5}) + boxcar(</em>{5\times5}) + PCA(_{3060}) + linear SVM</strong></td>
<td></td>
</tr>
<tr>
<td>IPSD (24x24)</td>
<td>50.9</td>
</tr>
<tr>
<td>SIFT (24x24) (non rot. inv.)</td>
<td>51.2</td>
</tr>
<tr>
<td>SIFT (24x24) (rot. inv.)</td>
<td>45.2</td>
</tr>
<tr>
<td>Serre et al. features [25]</td>
<td>47.1</td>
</tr>
</tbody>
</table>

| **local norm\(_{9\times9}\) + Spatial Pyramid Match Kernel SVM**       |                       |
| SIFT [11]                                                              | 64.6                   |
| IPSD (34x34)                                                           | 59.6                   |
| IPSD (56x56)                                                           | 62.6                   |
| IPSD (120x120)                                                         | 65.5                   |