CSC 411: Lecture 05: Nearest Neighbors

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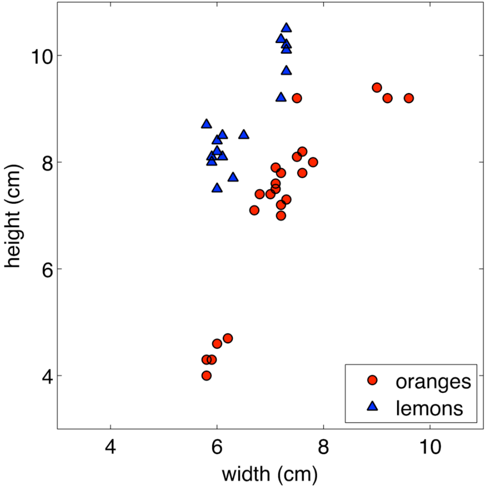
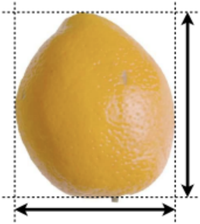
Sep 28, 2015

Today

- Non-parametric models
  - distance
  - non-linear decision boundaries

Classification: Oranges and Lemons





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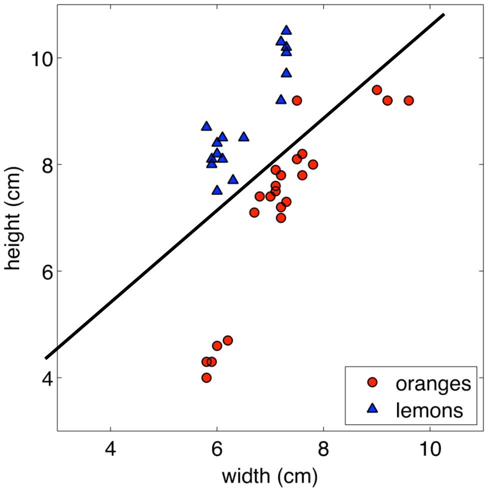
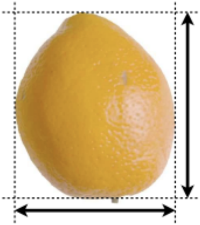


Can construct simple linear decision boundary:

$y = \text{sign}(w_0 + w_1x_1 + w_2x_2)$



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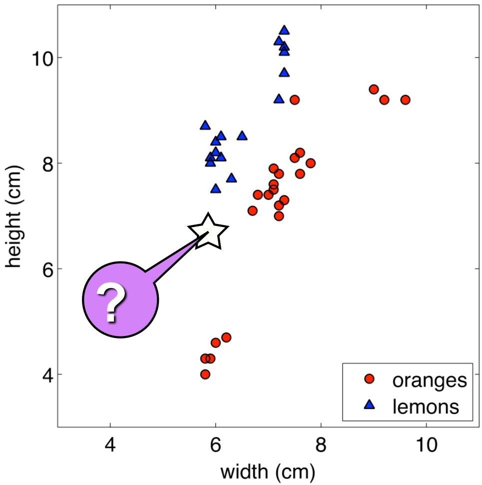
- What f have we seen so far in class?

Classification as Induction



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- Simple methods for approximating discrete-valued or real-valued target functions (classification or regression problems)
- Learning amounts to simply storing training data
- Test instances classified using similar training instances
- Embodies often sensible underlying assumptions:
  - Output varies smoothly with input
  - Data occupies sub-space of high-dimensional input space

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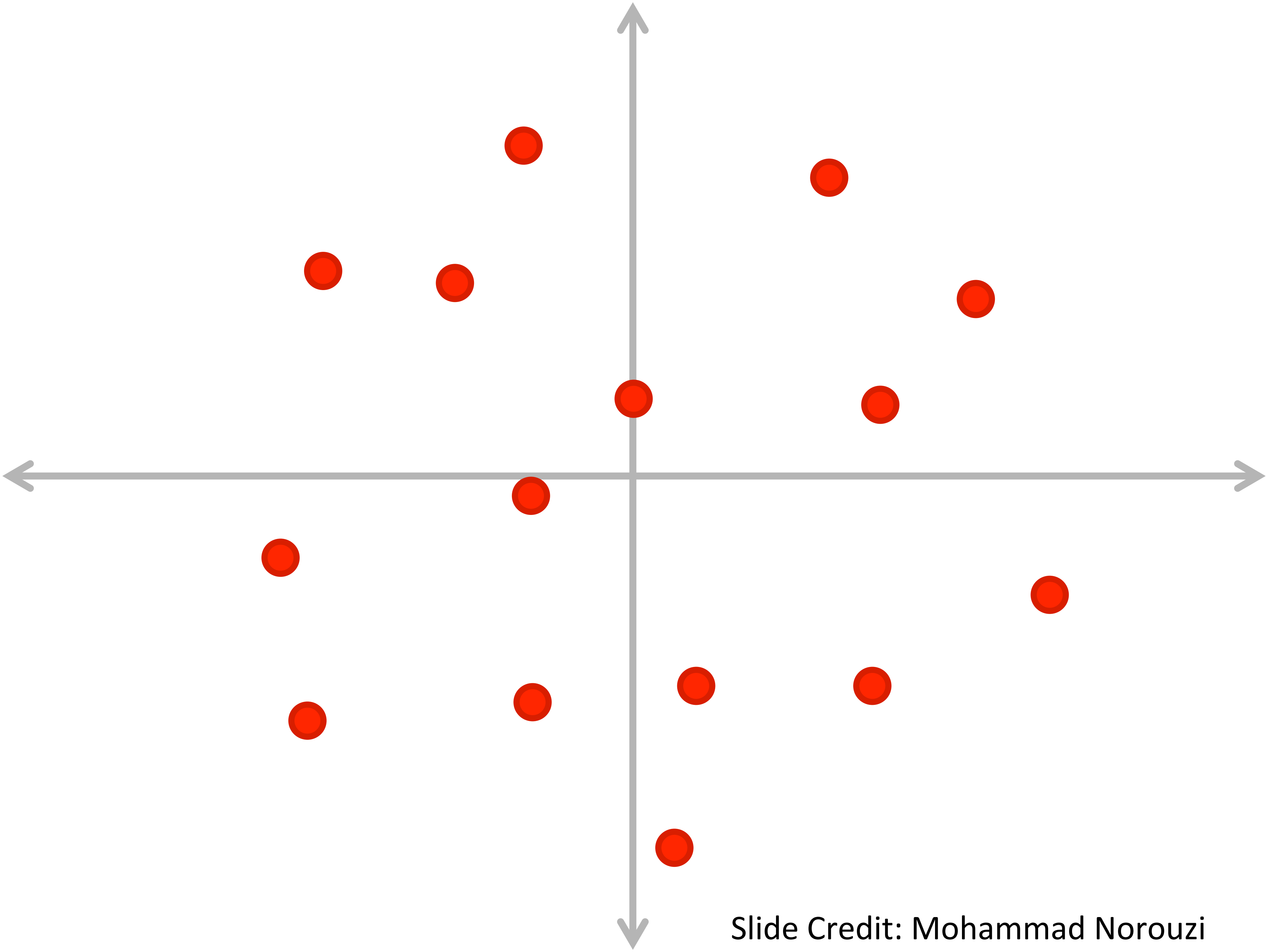
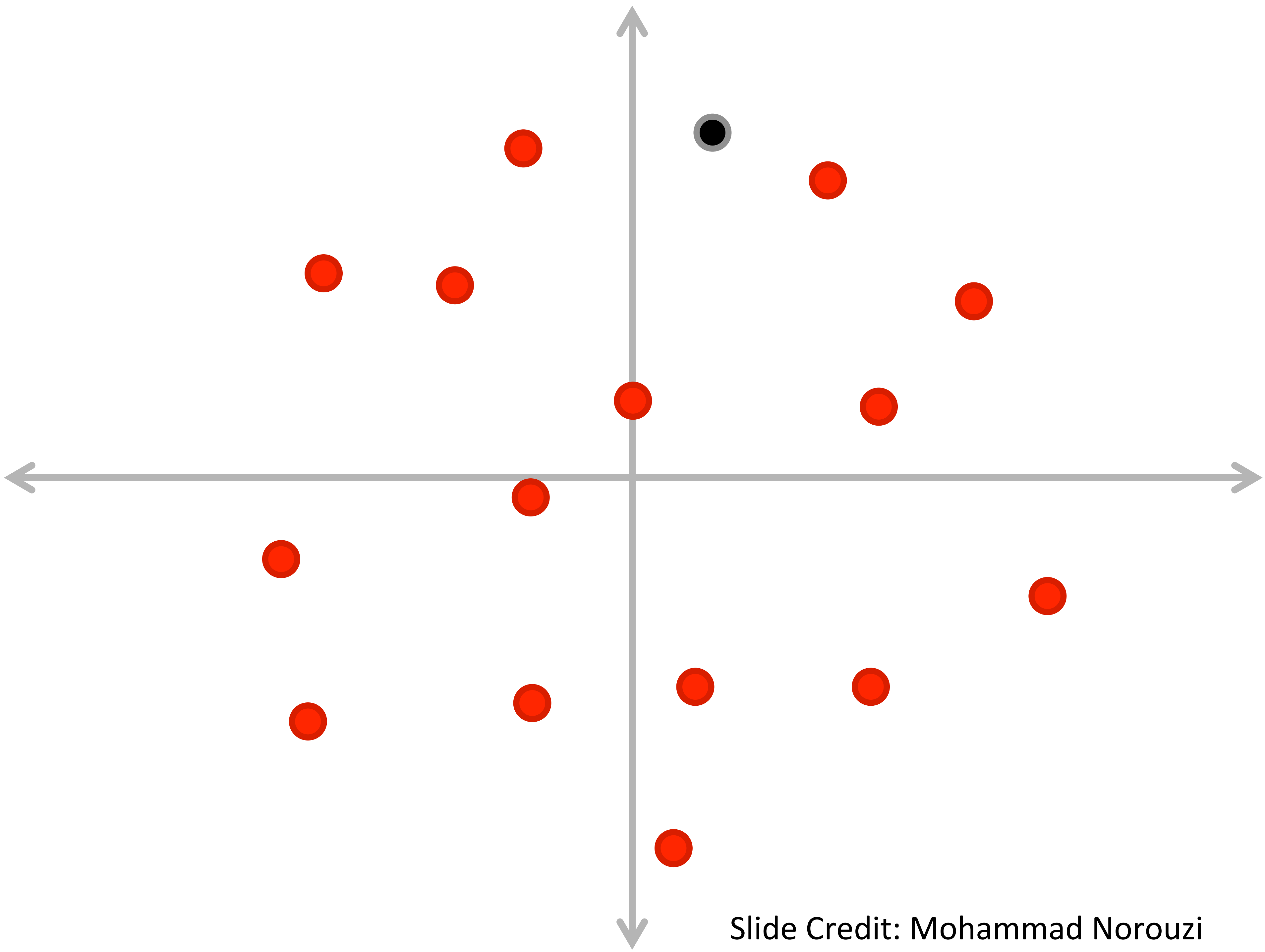
- Note: we don't need to compute the square root. Why?



Slide Credit: Mohammad Norouzi



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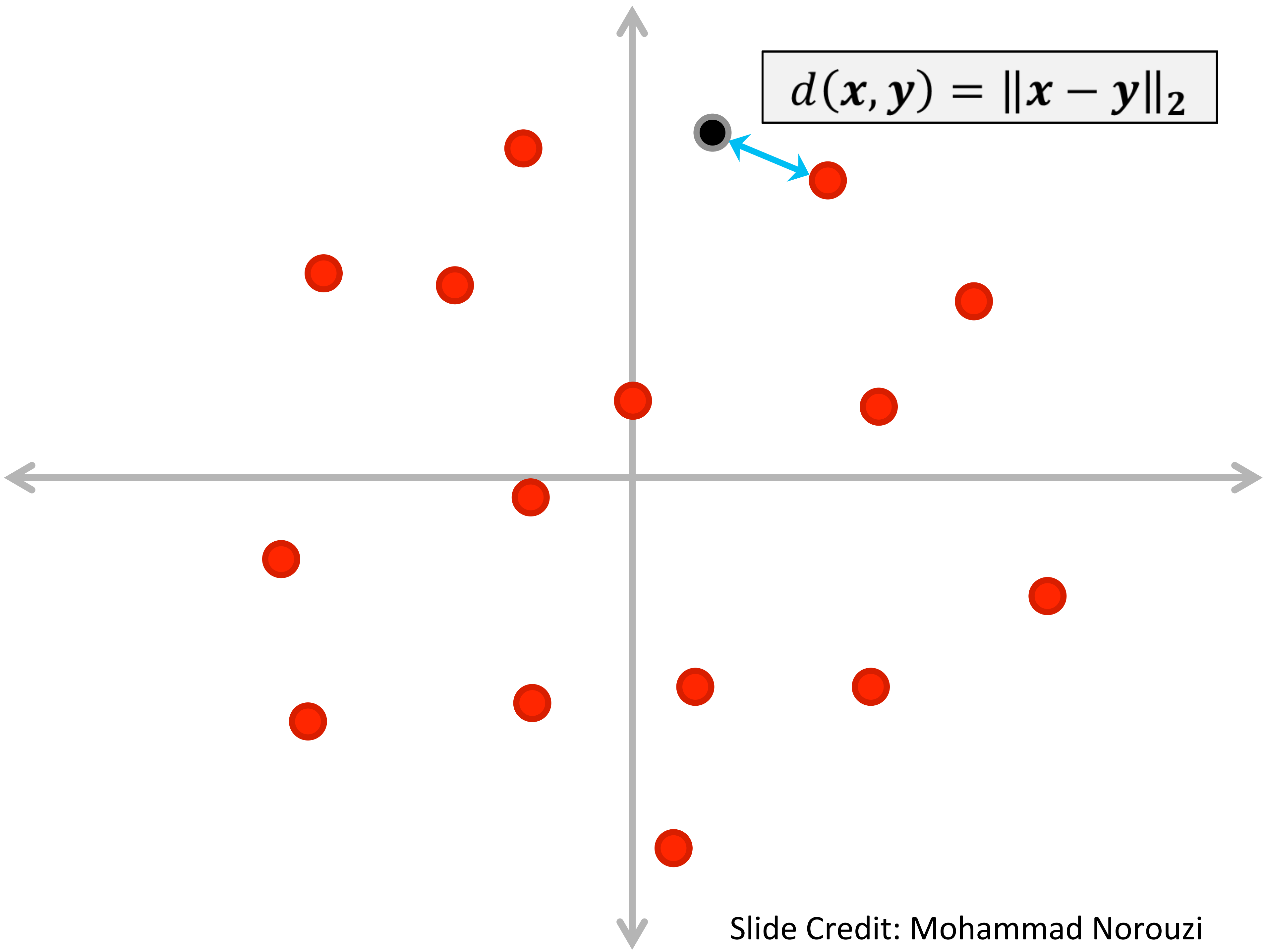

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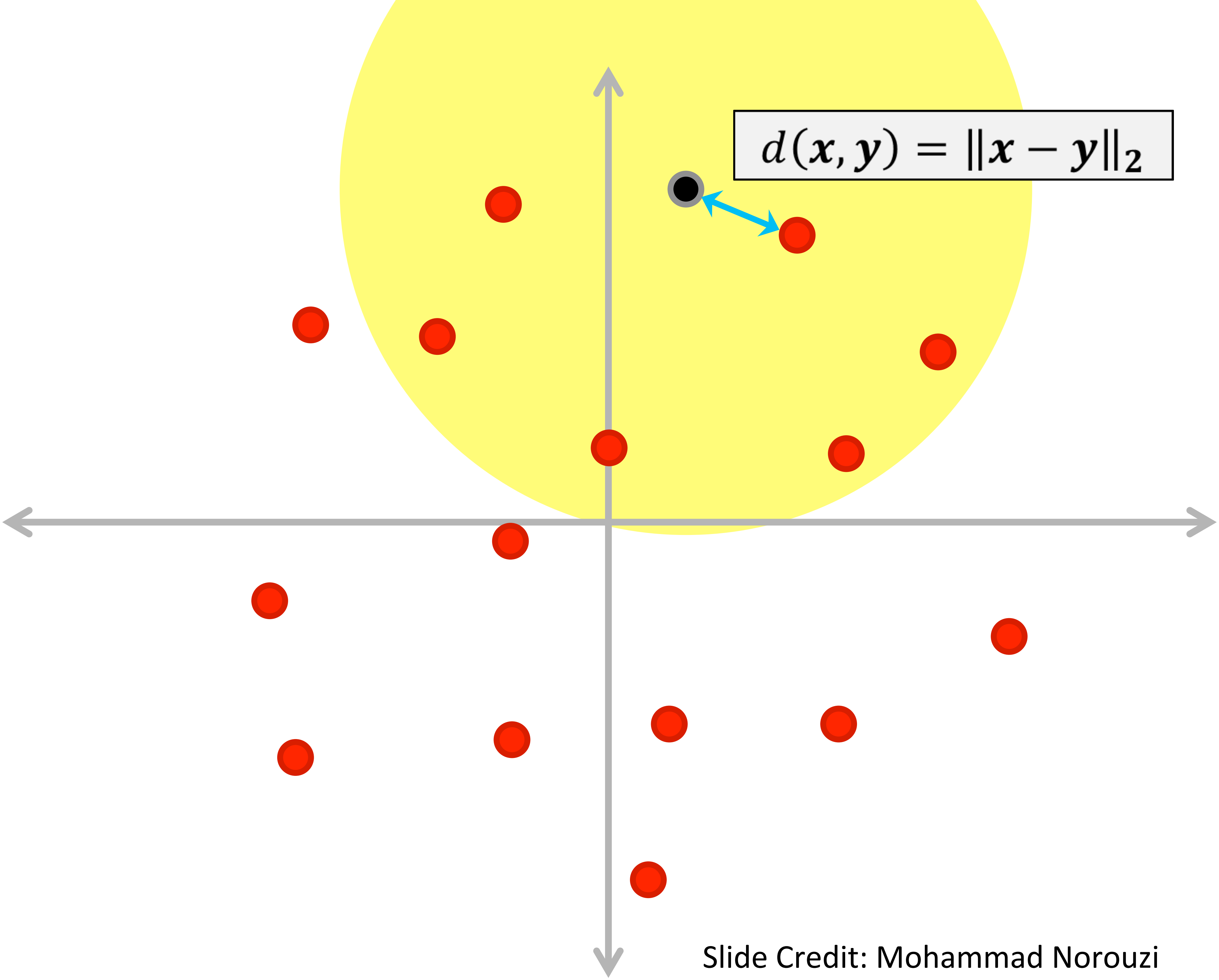

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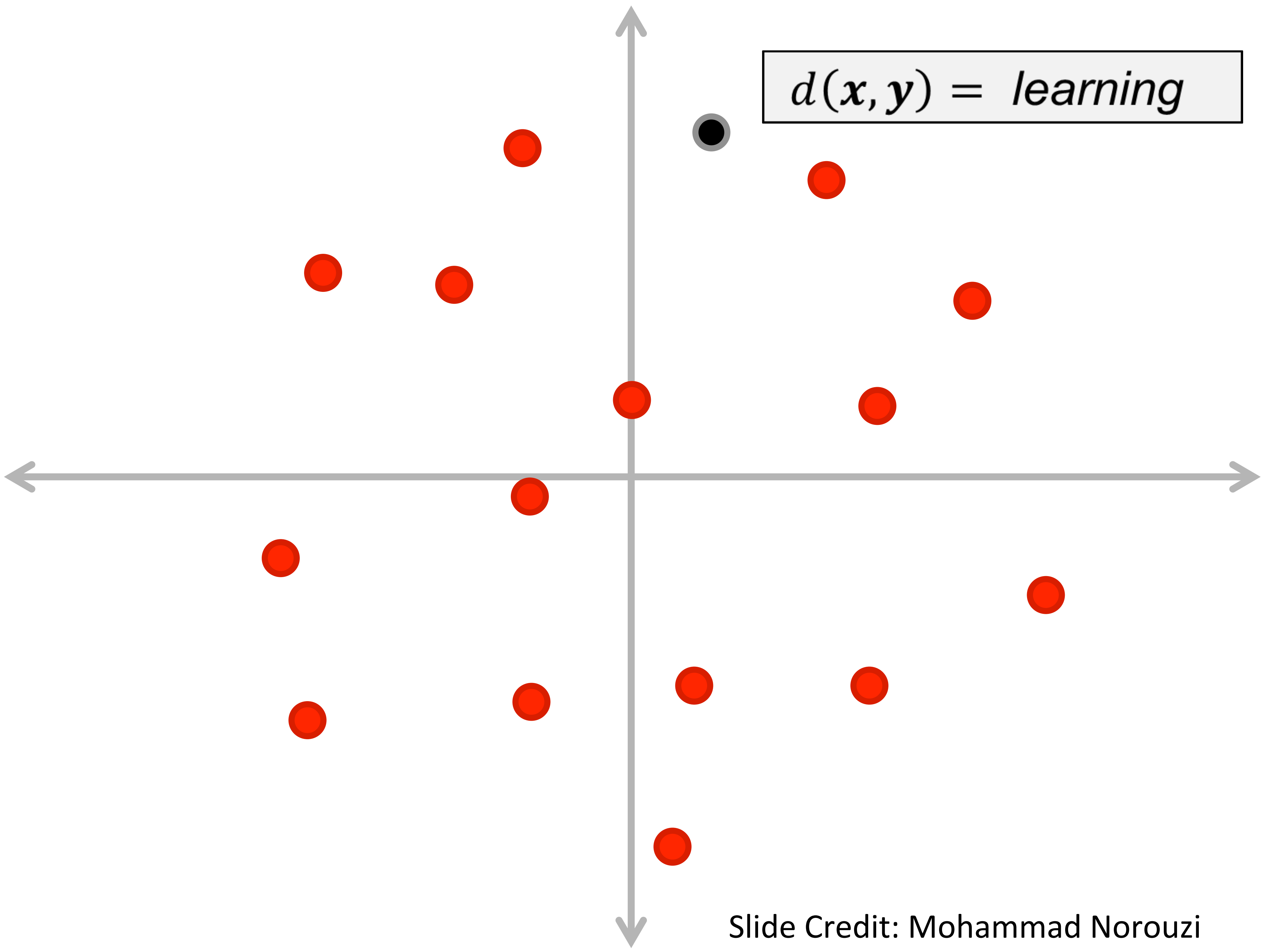
$d(\boldsymbol{x}, \boldsymbol{y}) =$ *learning*
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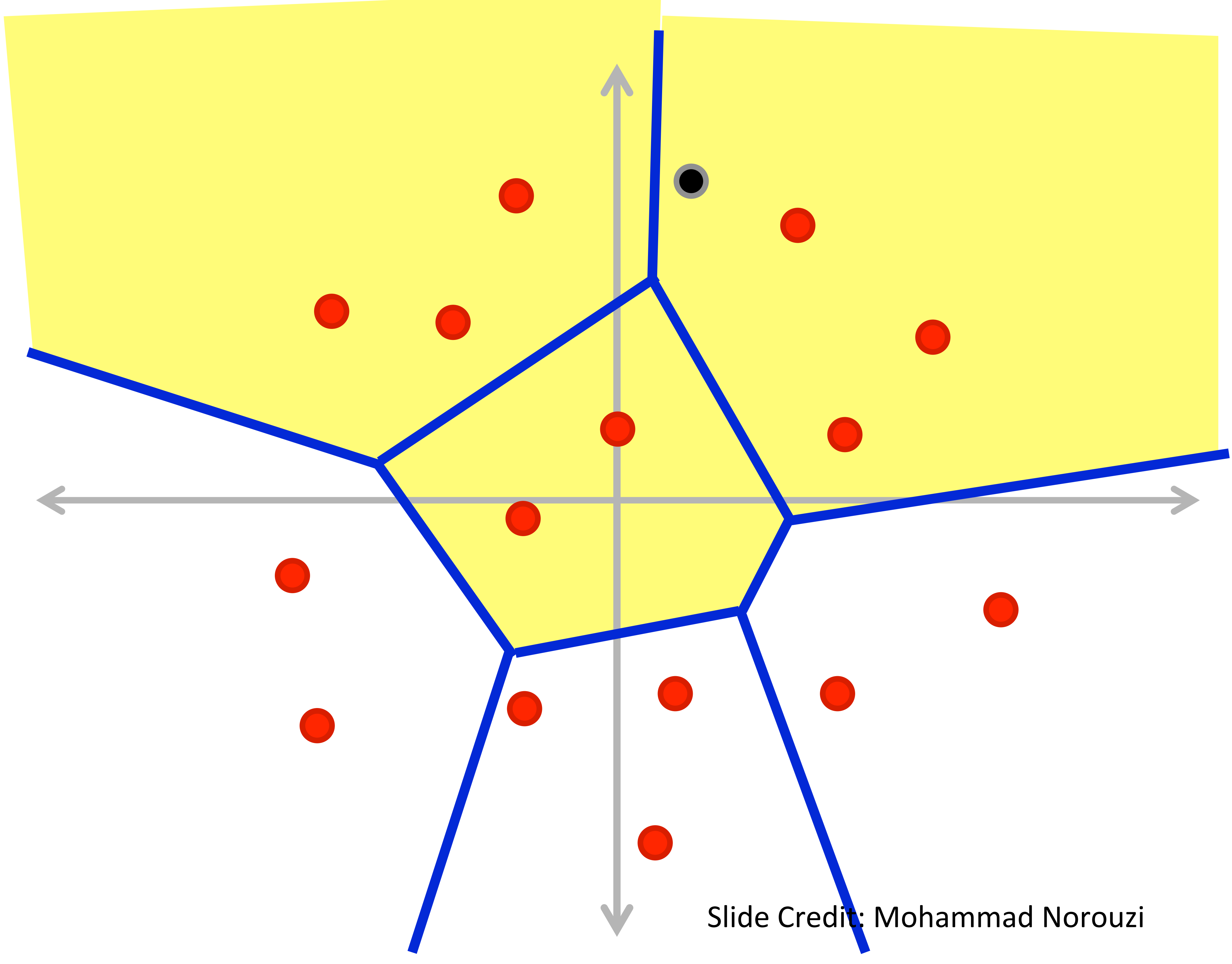

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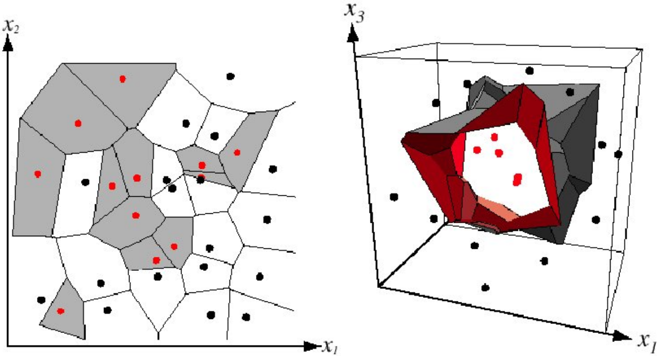
Nearest Neighbors Decision Boundaries

- Nearest neighbor algorithm does not explicitly compute decision boundaries, but these can be inferred
- Decision boundaries: Voronoi diagram visualization
  - show how input space divided into classes
  - each line segment is equidistant between two points of opposite classes



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- Algorithm:
  1. find k examples $\{\mathbf{x}^{(i)}, t^{(i)}\}$ closest to the test instance $\mathbf{x}$
  2. classification output is majority class

$$y = arg \max_{t^{(z)}} \sum_{r=1}^{k} \delta(t^{(z)}, t^{(r)})$$

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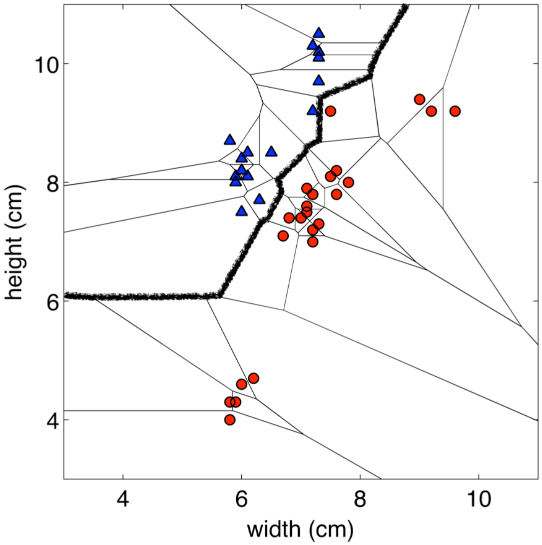
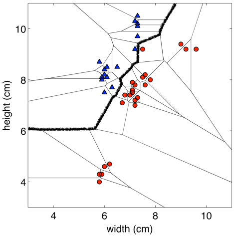
     - Form efficient search tree (kd-tree), use Hashing (LSH), etc

Decision Boundary K-NN



K-NN Summary



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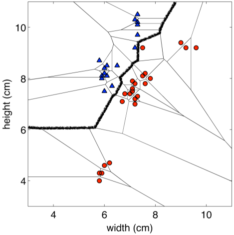

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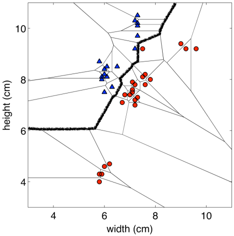

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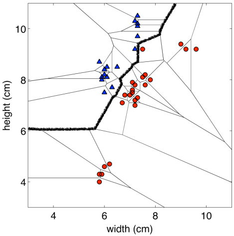

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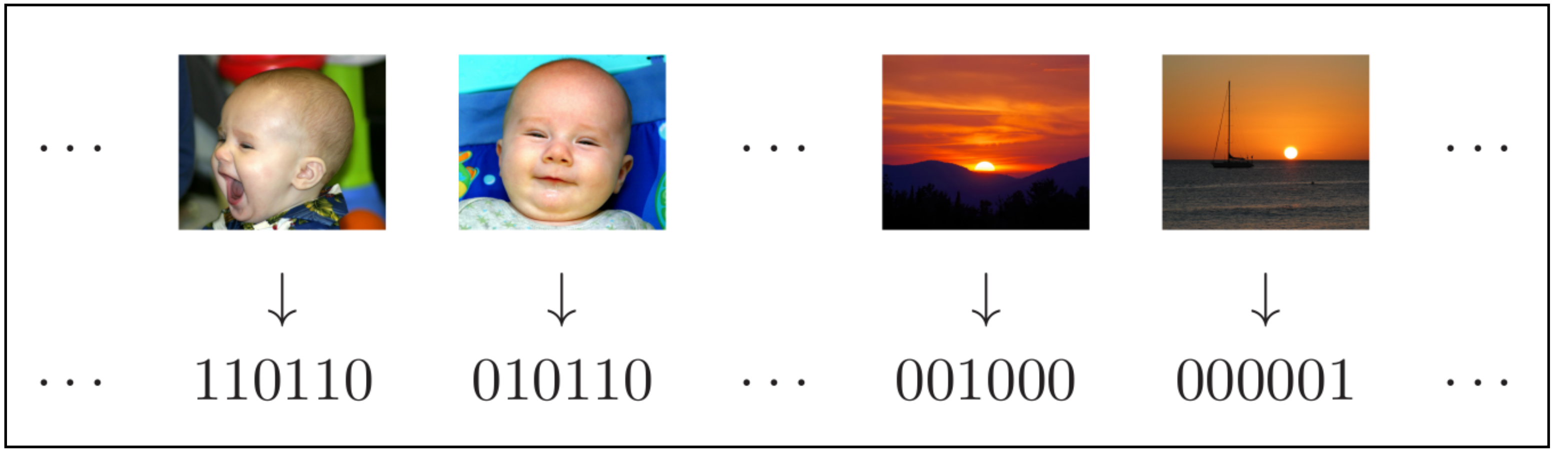
- Inductive Bias: What kind of decision boundaries do we expect to find?



- Similar data points map to nearby codes
- Dissimilar data points map to distant codes

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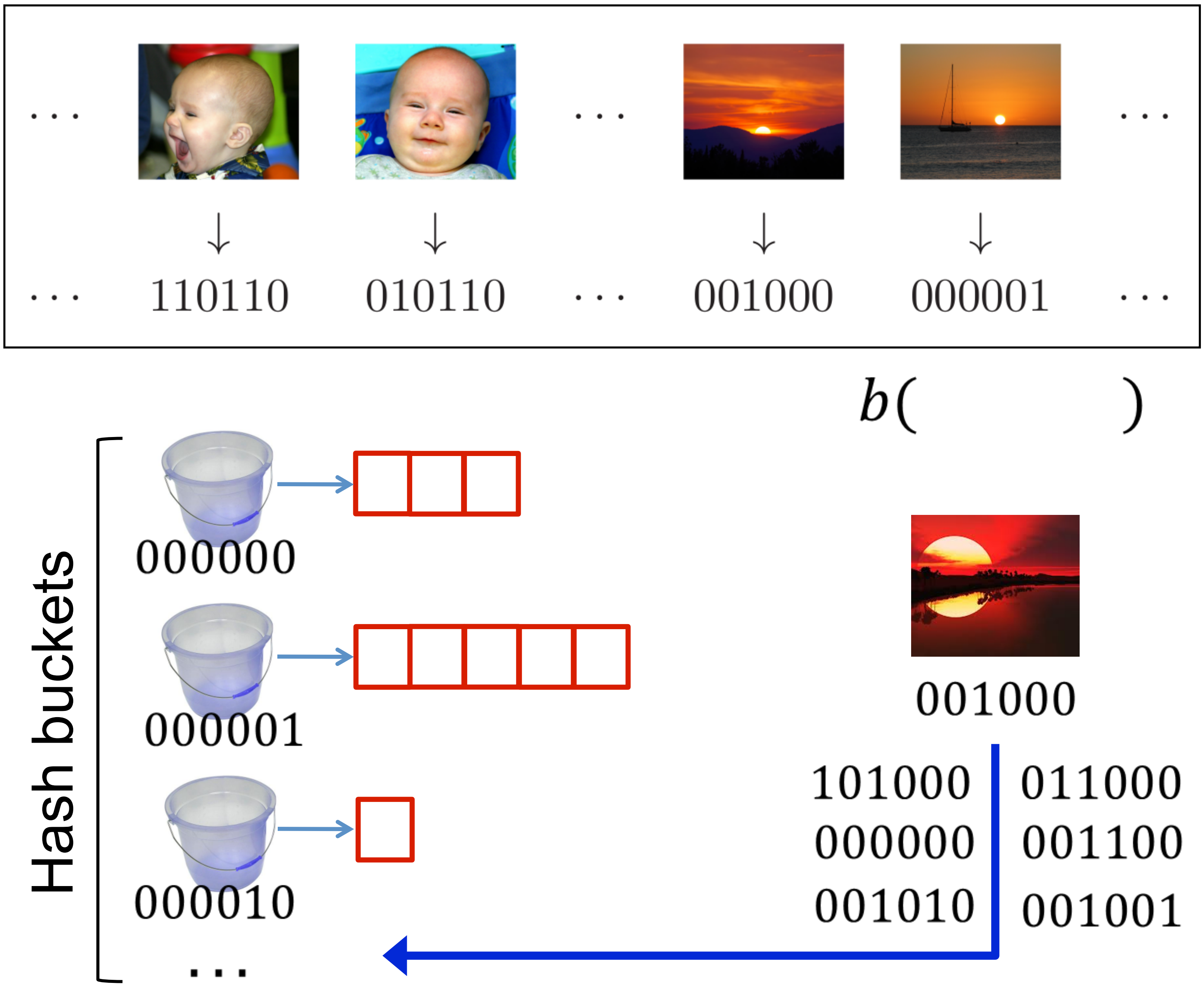

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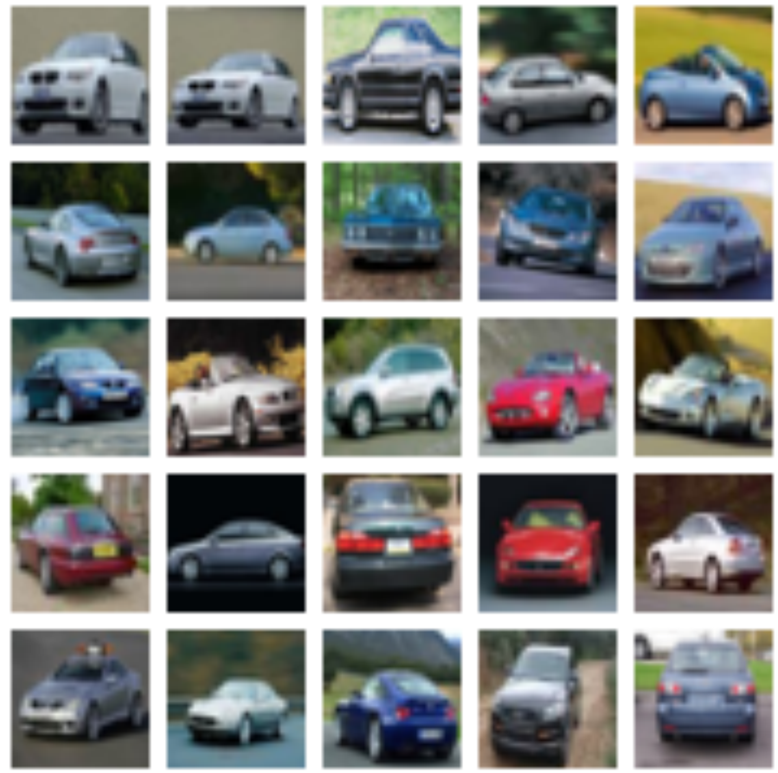
Query

Euclidean NNs



Hamming NNs

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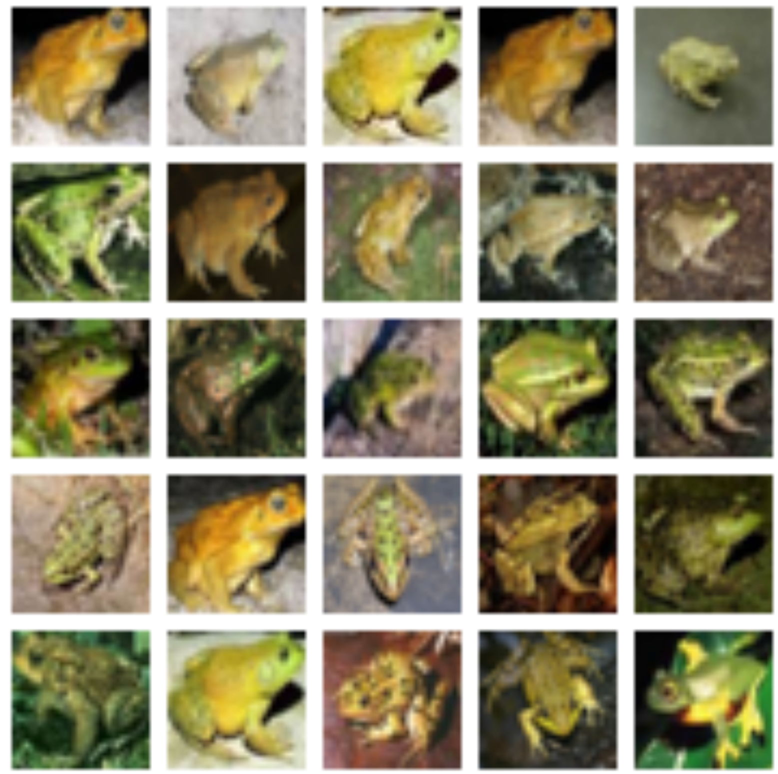

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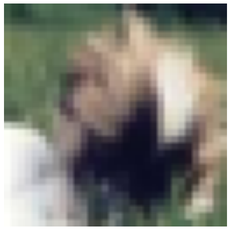

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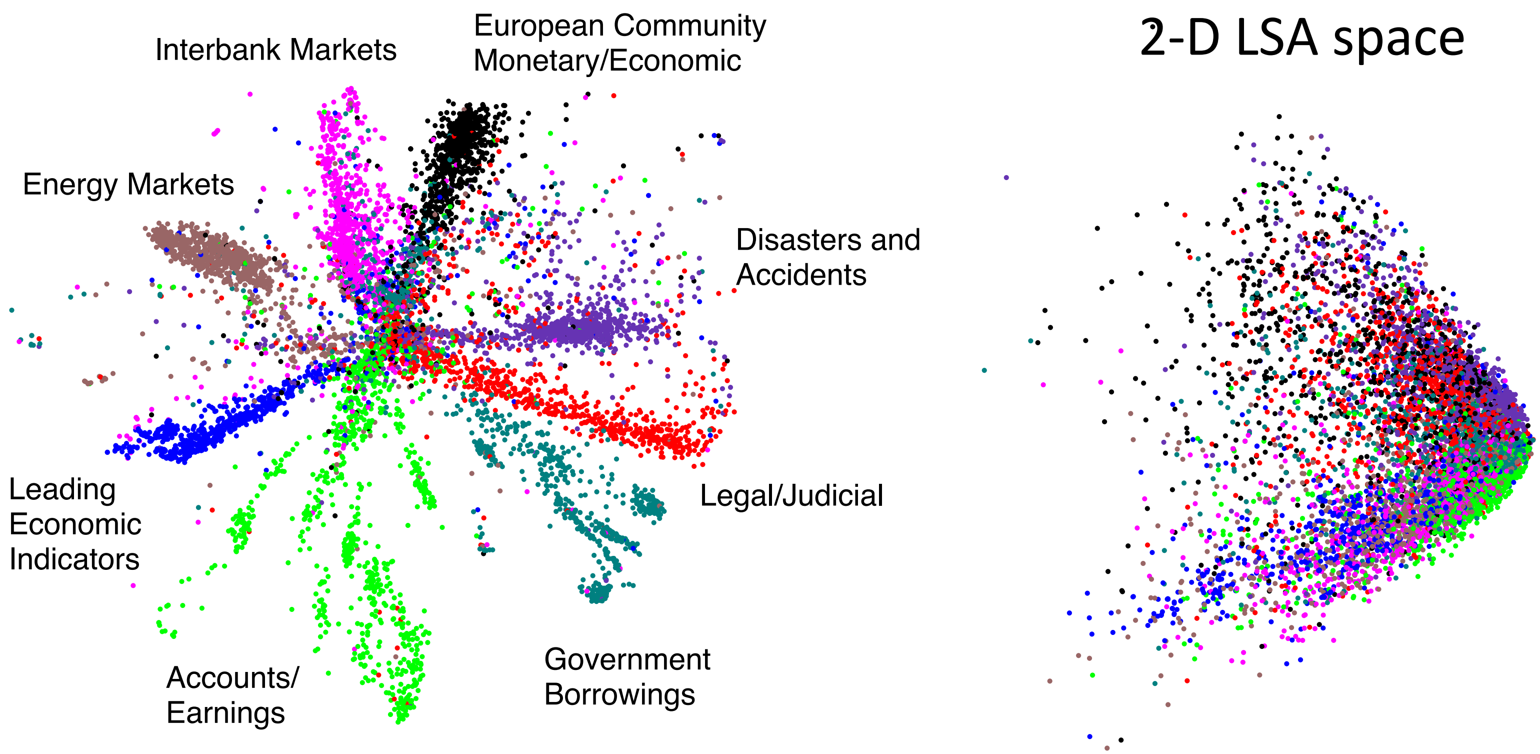
Examples of using Nearest Neighbor Approaches

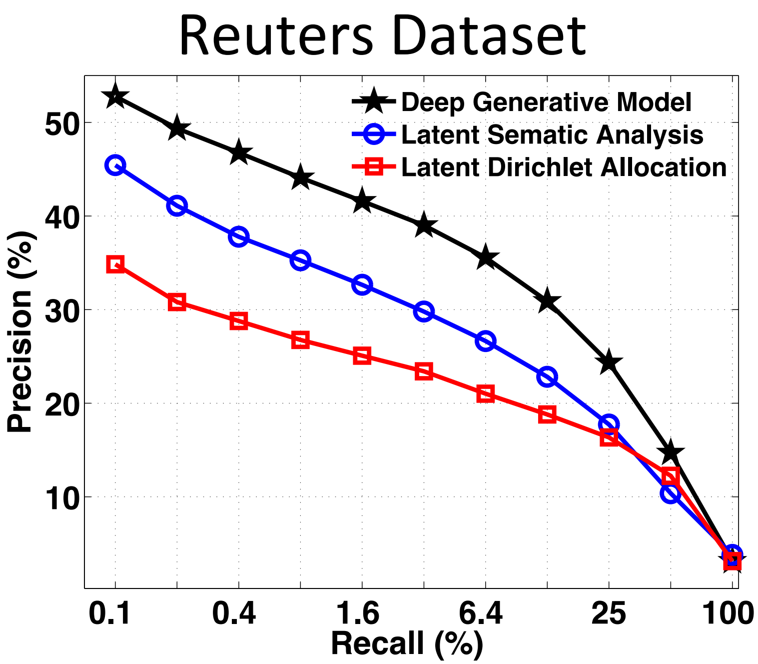
Information Retrieval using NN



• The Reuters Corpus Volume II contains 804,414 newswire stories (randomly split into **402,207 training** and **402,207 test).**

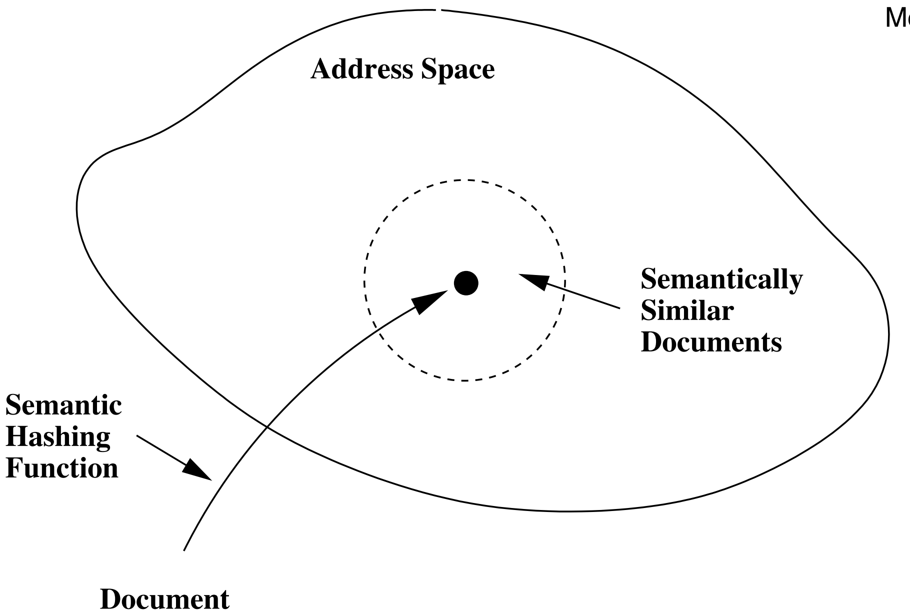
• "Bag-of-words" representation: each article is represented as a vector containing the counts of the most frequently used 2000 words in the training set.

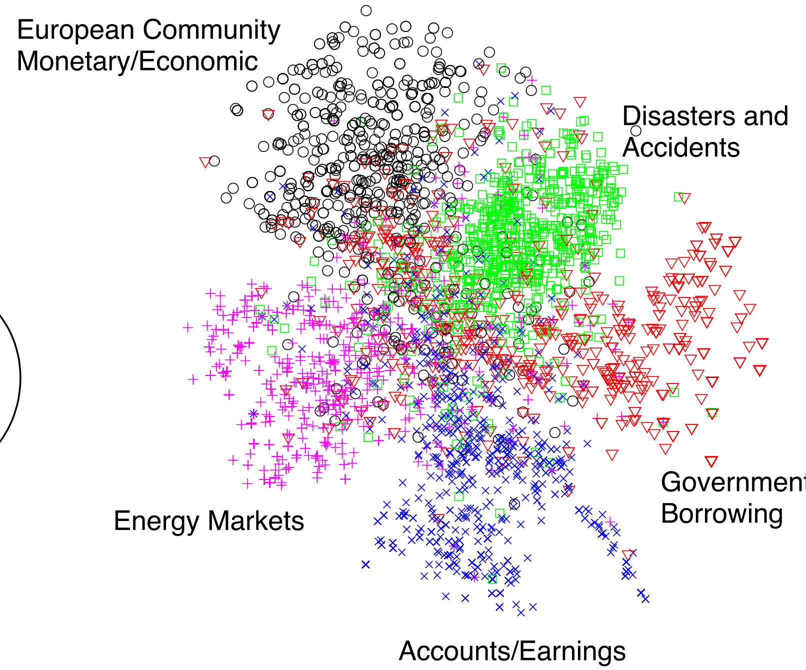
Information Retrieval



Reuters dataset: 804,414 newswire stories.

Semantic Hashing (using Hamming Distance)



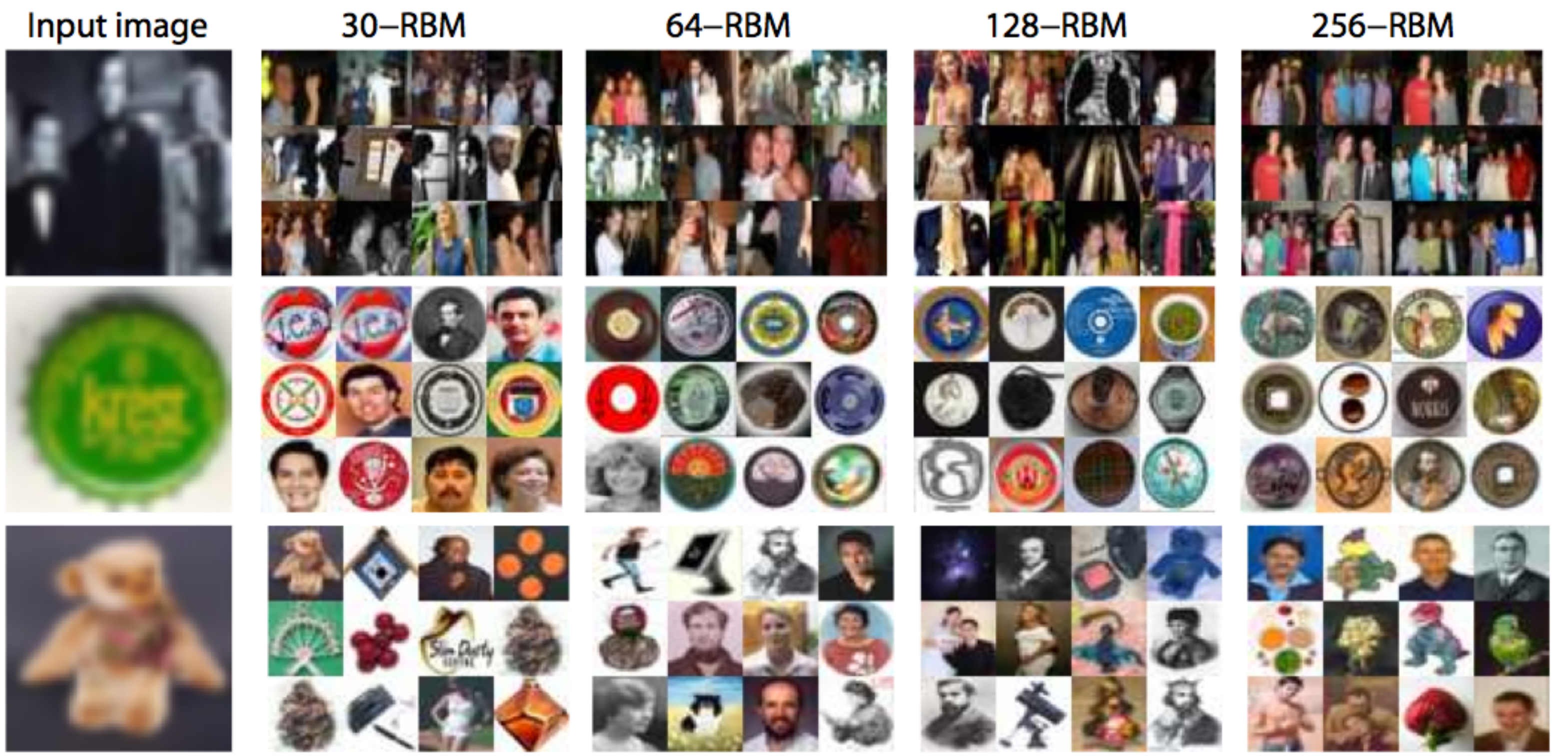


- Learn to map documents into **semantic 20-D binary codes.**
- Retrieve similar documents stored at the nearby addresses **with no search at all.**

(Salakhutdinov and Hinton, SIGIR 2007)

Searching Large Image Database using Binary Codes

- Map images into binary codes for fast retrieval.



- Small Codes, Torralba, Fergus, Weiss, CVPR 2008
- Spectral Hashing, Y. Weiss, A. Torralba, R. Fergus, NIPS 2008
- Kulis and Darrell, NIPS 2009, Gong and Lazebnik, CVPR 20111
- Norouzi and Fleet, ICML 2011,

Retrieval using Nearest Neighbors

fluffy

delicious

Retrieving Sentences using 1-NN

The dogs are in the snow in front of a fence .

Four men playing basketball , two from each team .

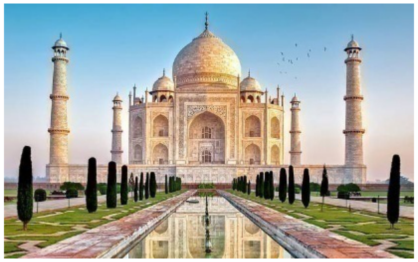
A boy skateboarding

Two men and a woman smile at the camera .

Women participate in a skit onstage .

A man is doing tricks on a bicycle on ramps in front of a crowd .

Tagging and Retrieval using NN



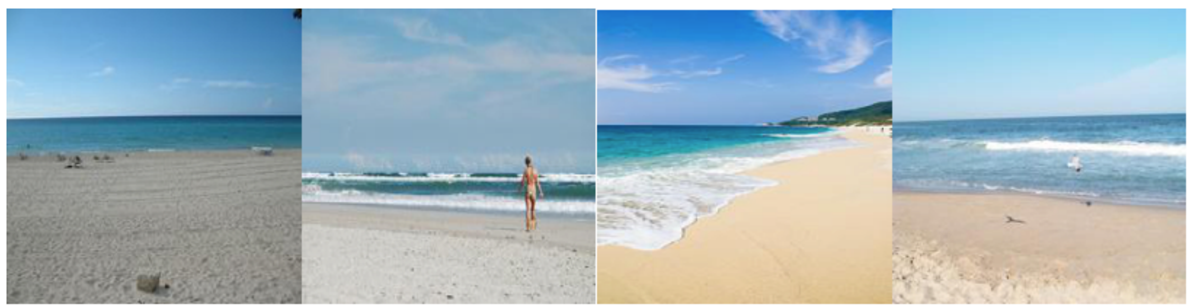
mosque, tower, building, cathedral, dome, castle

ski, skiing, skiers, skiiers, snowmobile

kitchen, stove, oven, refrigerator, microwave

bowl, cup, soup, cups, coffee

beach



snow