Math Puzzles

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February 1, 2020

Abstract

I hope, but do not necessarily expect, to update this list indefinitely with new math puzzles that I come up with (and solve). For now, here are two that were easy to extract from likely dead ends in a research problem I'm working on. Certain mathematical knowledge[1] is likely to be useful. Solutions are available here for Problem 1 and here for Problem 2.

1. Let $A \in \{0,1\}^{n \times n}$, and let σ be a uniform random permutation of $[n]$.[2] Prove that for all $t \geq 0$,

$$P\left(\sum_{j=1}^{n}\left(A_{\sigma(j),j} - \frac{1}{n}\sum_{i=1}^{n} A_{i,j}\right) \geq t\right) \leq \exp\left(-t^2/O(n)\right)$$

   and

$$P\left(\sum_{j=1}^{n}\left(A_{\sigma(j),j} - \frac{1}{n}\sum_{i=1}^{n} A_{i,j}\right) \leq -t\right) \leq \exp\left(-t^2/O(n)\right).$$

2. Given $s \in [2^n]$, find, up to a constant factor, the maximum value of $\left\|\frac{1}{s}\sum_{x \in A} x\right\|_2^2$ over all sets $A \subseteq \{\pm 1\}^n$ of size s.[3]

[1] A subset of this, more or less. Nothing too obscure.
[2] $[n] = \{1, \ldots, n\}$
[3] Thanks to Deeksha Adil, Lily Li and Ian Mertz for feedback on the wording of this.