1. Design a DFA\(^1\) for the following language over the alphabet \{a, b\}:

\[
\{ x \mid x \text{ contains an equal number of occurrences of } ab \text{ and } ba \}.
\]

So, for example, the string \(aba \in D\) since it has one occurrence each of \(ab\) and \(ba\); but the string \(baba \notin D\) since it has two occurrences of \(ba\) but only one occurrence of \(ab\).

2. Suppose that both \(A, B \subseteq \{0, 1\}^*\) are regular languages. Show that the language

\[
A \setminus B = \{ x \mid x \in A, x \notin B \}
\]

is regular.

3. Give non-deterministic finite automata for the following two languages over \{0, 1\}:

(a) \(\{00\}\) (that is, the language that contains only the string 00).

(b) \(\{x \mid \text{The second-last character of } x \text{ is a 0}\}\).

(Extra Practice: For the second language, also give a DFA. What can you say about the DFA vs. the NFA?)

4. Let \(\Sigma\) be an alphabet. Let \(x\) and \(y\) be strings over \(\Sigma\) and let \(L \subseteq \Sigma^*\) be a language over \(\Sigma\). We say that \(x\) and \(y\) are distinguishable by \(L\) if there is a string \(z\) such that exactly one of the strings \(xz, yz\) is in \(L\). If \(x\) and \(y\) are not distinguishable by \(L\) then we write \(x \equiv_L y\).

\(^1\)You just need to submit the state diagram.
(a) Consider the language \( L = \{ x \mid x \text { contains the string } 010 \} \) over \( \{0, 1\}^* \). Give two distinct strings \( x \neq y \) which are distinguishable by \( L \), and two distinct strings \( x' \neq y' \) which are not distinguishable by \( L \). Explain your answer.

(b) Let \( x = 000 \) and \( y = 111 \). Give an infinite regular language \( L \subseteq \{0, 1\}^* \) such that \( x \) and \( y \) are distinguishable by \( L \), and another infinite regular language \( L' \) such that \( x \) and \( y \) are not distinguishable by \( L' \subseteq \{0, 1\}^* \). Explain your answer.

(c) Show that for any language \( L, \equiv_L \) is an equivalence relation (that is, it is reflexive, symmetric, and transitive).

5. (Bonus, not marked.) Go somewhere you haven’t gone before and take a picture of it. Why did you go where you went? What made you go there now?