

# Intro to RL & Policy Gradient

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# Outline:

- Brief intro to RL
- Policy Gradient
  - The log-derivative trick
  - Practical fixes: baseline & temporal structure
- OpenAI Gym
- Example: policy gradient on Gym environments
- References

Slides on intro & policy gradient are from / inspired by the Deep RL Bootcamp Lecture 4A: Policy Gradients by Pieter Abbeel [https://www.youtube.com/watch?v=S\\_gwYj1Q-44](https://www.youtube.com/watch?v=S_gwYj1Q-44)

# Brief Intro to RL

Represent agent with stochastic policy  $\pi_{\theta}(a|s)$

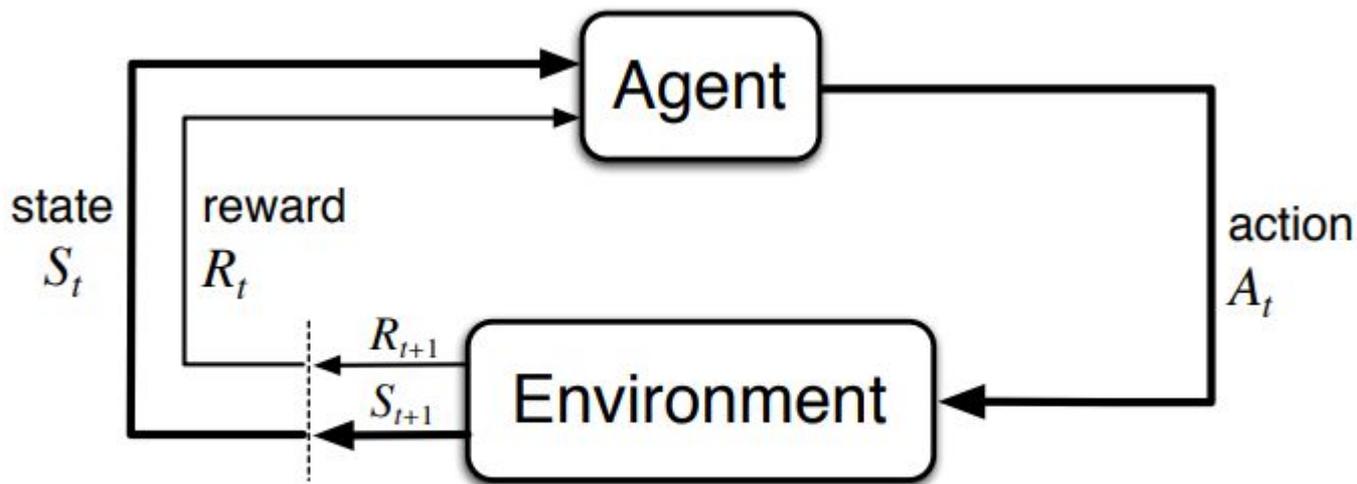
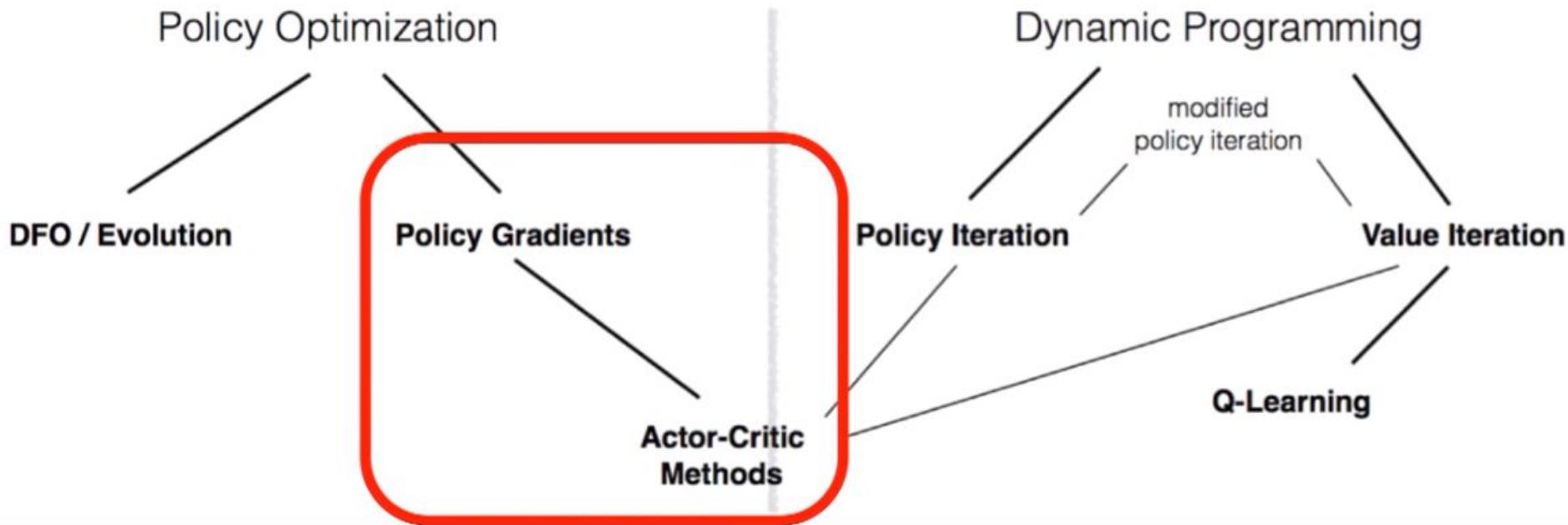


Figure 3.1: The agent–environment interaction in a Markov decision process.

# Policy Optimization in the RL Landscape



## Policy Optimization

- Conceptually:

Optimize what you care about

- Empirically:

More compatible with rich architectures (including recurrence)

More versatile

More compatible with auxiliary objectives

## Dynamic Programming

Indirect, exploit the problem structure, self-consistency

More compatible with exploration and off-policy learning

More sample-efficient when they work

# Policy Gradient

Suppose we have a trajectory:  $\tau = (s_0, a_0, s_1, a_1, \dots, s_{H-1}, a_{H-1}, s_H)$

And represent the reward for the whole trajectory:  $R(\tau) = \sum_{t=0}^{H-1} R(s_t, a_t)$

The expected reward under policy  $\pi_\theta$  (utility function):

$$U(\theta) = \mathbb{E}\left[\sum_{t=0}^H R(s_t, a_t); \pi_\theta\right] = \sum_{\tau} P(\tau; \theta) R(\tau)$$

The goal is to find the optimal parameters to max the utility function.

$$\max_{\theta} U(\theta) = \max_{\theta} \sum_{\tau} P(\tau; \theta) R(\tau)$$

# Policy Gradient: the log-derivative trick

Take the gradient:

$$\begin{aligned}\nabla_{\theta}U(\theta) &= \nabla_{\theta} \sum_{\tau} P(\tau; \theta)R(\tau) \\ &= \sum_{\tau} \nabla_{\theta}P(\tau; \theta)R(\tau) \\ &= \sum_{\tau} \frac{P(\tau; \theta)}{P(\tau; \theta)} \nabla_{\theta}P(\tau; \theta)R(\tau) \\ &= \sum_{\tau} P(\tau; \theta) \frac{\nabla_{\theta}P(\tau; \theta)}{P(\tau; \theta)} R(\tau) \\ &= \sum_{\tau} P(\tau; \theta) \nabla_{\theta} \log P(\tau; \theta) R(\tau)\end{aligned}$$

# Policy Gradient: approximate gradient with samples

Now we can approximate the gradient using Monte Carlo Samples!

$$\begin{aligned}\nabla_{\theta}U(\theta) &= \sum_{\tau} P(\tau; \theta) \nabla_{\theta} \log P(\tau; \theta) R(\tau) \\ &\approx \frac{1}{m} \sum_{i=1}^m \nabla_{\theta} \log P(\tau^{(i)}; \theta) R(\tau^{(i)})\end{aligned}$$

Where  $\tau^{(i)}$  are sample rollout trajectories under policy  $\pi_{\theta}$

# Policy Gradient: approximate gradient with samples

Take a moment to appreciate this:

$$\nabla_{\theta} U(\theta) \approx \frac{1}{m} \sum_{i=1}^m \nabla_{\theta} \log P(\tau^{(i)}; \theta) R(\tau^{(i)})$$

This gradient approximation is valid even when:

- The reward is discontinuous / unknown
- Sample space is a discrete set

# Policy Gradient: intuition

$$\nabla_{\theta} U(\theta) \approx \frac{1}{m} \sum_{i=1}^m \nabla_{\theta} \log P(\tau^{(i)}; \theta) R(\tau^{(i)})$$

The gradient tries to:

- Increase probability of paths with positive rewards
- Decrease probability of paths with negative rewards

Does NOT try to change the paths themselves.

[See any problems here?](#)

# Decomposing the paths into states & actions

$$\nabla_{\theta} U(\theta) \approx \frac{1}{m} \sum_{i=1}^m \nabla_{\theta} \log P(\tau^{(i)}; \theta) R(\tau^{(i)})$$

$$\nabla_{\theta} \log P(\tau^{(i)}; \theta) = \nabla_{\theta} \log \left[ \prod_{t=0}^{H-1} P(s_{t+1}^{(i)} | s_t^{(i)}, a_t^{(i)}) \pi_{\theta}(a_t^{(i)} | s_t^{(i)}) \right]$$

$$= \nabla_{\theta} \left[ \sum_{t=0}^{H-1} \log P(s_{t+1}^{(i)} | s_t^{(i)}, a_t^{(i)}) \right] + \sum_{t=0}^{H-1} \log \pi_{\theta}(a_t^{(i)} | s_t^{(i)})$$

$$= \nabla_{\theta} \left[ \sum_{t=0}^{H-1} \log \pi_{\theta}(a_t^{(i)} | s_t^{(i)}) \right]$$

Not a function of  $\theta$

# Policy Gradient: problems and fixes

The vanilla policy gradient estimator is unbiased, but very noisy.

- Requires lots of samples to make it work

## **Fixes:**

- Baseline
- Temporal Structure
- Other (e.g. KL trust region)

# Policy Gradient: baseline

$$\nabla_{\theta} U(\theta) \approx \frac{1}{m} \sum_{i=1}^m \nabla_{\theta} \log P(\tau^{(i)}; \theta) (R(\tau^{(i)}) - b)$$

Subtract the reward with a baseline ( $b$ ) does not change the optimization problem.

- The gradient estimation is still unbiased, but with lower variance

**Intuition:** we want to adjust path probabilities based on how the path reward compares to the **average**, not the path reward itself.

- Increase probability if the path reward is higher than average
- Decrease probability if the path reward is lower than average

# Policy Gradient: temporal structure

Put together  
what we have:

$$\begin{aligned}\nabla_{\theta} U(\theta) &\approx \frac{1}{m} \sum_{i=1}^m \nabla_{\theta} \log P(\tau^{(i)}; \theta) (R(\tau^{(i)}) - b) \\ &= \frac{1}{m} \sum_{i=1}^m \nabla_{\theta} \left[ \sum_{t=0}^{H-1} \log \pi_{\theta}(a_t^{(i)} | s_t^i) \right] \left[ \sum_{t=0}^{H-1} R(s_t^{(i)}, a_t^{(i)}) - b \right] \\ &= \frac{1}{m} \sum_{i=1}^m \nabla_{\theta} \left[ \sum_{t=0}^{H-1} \log \pi_{\theta}(a_t^{(i)} | s_t^i) \left( \sum_{k=0}^{H-1} R(s_t^{(i)}, a_t^{(i)}) - b \right) \right] \\ &= \frac{1}{m} \sum_{i=1}^m \nabla_{\theta} \left[ \sum_{t=0}^{H-1} \log \pi_{\theta}(a_t^{(i)} | s_t^i) \left( \sum_{k=0}^{t-1} R(s_t^{(i)}, a_t^{(i)}) + \sum_{k=t}^{H-1} R(s_t^{(i)}, a_t^{(i)}) - b \right) \right] \\ &= \frac{1}{m} \sum_{i=1}^m \nabla_{\theta} \left[ \sum_{t=0}^{H-1} \log \pi_{\theta}(a_t^{(i)} | s_t^i) \left( \sum_{k=t}^{H-1} R(s_t^{(i)}, a_t^{(i)}) - b \right) \right]\end{aligned}$$

Past reward does not affect current action

# OpenAI Gym

<https://gym.openai.com/>

Widely-used testing platform for RL algorithms.

- `pip install gym`

Different kinds of environments, including discrete / continuous control, pixel-input Atari games, etc.

You can also create your own environments, following the Gym interface.

# OpenAI Gym environments

Create an environment:

- `env = gym.make("<environment_name>")` ← e.g. `gym.make("CartPole-v1")`

Env methods you will need the most:

- `state = env.reset()`
- `next_state, reward, done, info = env.step(action)`
- `env.seed(seed=None)`
- `env.close()`

Useful attributes:

- `env.observation_space`
- `env.action_space`

More documentation at <https://gym.openai.com/docs/>

# Example: Policy Gradient in PyTorch on a Gym Environment (CartPole-v1)

# References

- Sutton & Barto “Reinforcement Learning: An Introduction”, 1998
- Williams, “Simple Statistical Gradient-Following Algorithms for Connectionist Reinforcement Learning”, 1992
- Sutton et al, “Policy Gradient Methods for Reinforcement Learning with Function Approximation”, 1999
- Pieter Abbeel, Deep RL Bootcamp Lecture 4A: Policy Gradients [https://www.youtube.com/watch?v=S\\_gwYj1Q-44](https://www.youtube.com/watch?v=S_gwYj1Q-44)