Recap of different learning settings

So far the settings that you’ve seen imagine one learner or agent.

**Supervised**
Learner predicts labels.

**Unsupervised**
Learner organizes data.

**Reinforcement**
Agent maximizes reward.
Today

We will talk about learning in the context of a two-player game.

This lecture only touches a small part of the large and beautiful literature on game theory, multi-agent reinforcement learning, etc.
Game-playing in AI: Beginnings

- (1950) Claude Shannon proposes explains how games could be solved algorithmically via tree search
- (1953) Alan Turing writes a chess program
- (1956) Arthur Samuel writes a program that plays checkers better than he does
- (1968) An algorithm defeats human novices at Go

slide credit: Profs. Roger Grosse and Jimmy Ba
Game-playing in AI: Successes

• (1992) TD-Gammon plays backgammon competitively with the best human players
• (1996) Chinook wins the US National Checkers Championship
• (1997) DeepBlue defeats world chess champion Garry Kasparov
• (2016) AlphaGo defeats Go champion Lee Sedol.

slide credit: Profs. Roger Grosse and Jimmy Ba
Today

• Game-playing has always been at the core of CS.
  • Simple well-defined rules, but mastery requires a high degree of intelligence.

• We will study how to learn to play Go.
  • The ideas in this lecture apply to all zero-sum games with finitely many states, two players, and no uncertainty.
  • Go was the last classical board game for which humans outperformed computers.
  • We will follow the story of AlphaGo, DeepMind’s Go playing system that defeated the human Go champion Lee Sedol.

• Combines many ideas that you’ve already seen.
  • supervised learning, value function learning...
The game of Go: Start

- Initial position is an empty $19 \times 19$ grid.
The game of Go: Play

- 2 players alternate placing stones on empty intersections. Black stone plays first.
- \textbf{(Ko)} Players cannot recreate a former board position.
The game of Go: Play

- **(Capture)** Capture and remove a connected group of stones by surrounding them.
• **(Territory)** The winning player has the maximum number of occupied or surrounded intersections.
Outline of the lecture

To build a strong computer Go player, we will answer:

- What does it mean to play optimally?
- Can we compute (approximately) optimal play?
- Can we learn to play (somewhat) optimally?
Why is this a challenge?

- Optimal play requires searching over $\sim 10^{170}$ legal positions.
- It is hard to decide who is winning before the end-game.
  - Good heuristics exist for chess (count pieces), but not for Go.
- Humans use sophisticated pattern recognition.
Optimal play
Game trees

- Organize all possible games into a tree.
  - Each node contains a legal position.
  - Child nodes enumerate all possible actions taken by the current player.
  - Leaves are terminal states.
  - Technically board positions can appear in more than one node, but let’s ignore that detail for now.

- The Go tree is finite (Ko rule).
Game trees
Evaluating positions

• We want to quantify the utility of a node for the current player.

• Label each node $s$ with a value $v(s)$, taking the perspective of the black stone player.
  • $+1$ for black wins, $-1$ for black loses.
  • Flip the sign for white’s value (technically, this is because Go is zero-sum).

• Evaluations let us determine who is winning or losing.
Evaluating leaf positions

Leaf nodes are easy to label, because a winner is known.

- black stones win
- white stones win
Evaluating internal positions

- The value of internal nodes depends on the strategies of the two players.
- The so-called maximin value \( v^*(s) \) is the highest value that black can achieve regardless of white’s strategy.
- If we could compute \( v^* \), then the best (worst-case) move \( a^* \) is

\[
a^* = \arg \max_a \{ v^*(\text{child}(s, a)) \}
\]
Evaluating positions under optimal play
Evaluating positions under optimal play

\[ s \]

\[ \text{max} \]

\[ \text{min} \]

-1 +1 +1 +1 -1 -1 +1 +1 +1 -1 -1 -1 -1 -1 +1 -1

\[ \text{max} \]

\[ \text{min} \]

-1 +1 +1 +1 -1 -1 +1 +1 +1 -1 -1 -1 -1 -1 +1 -1

Intro ML (UofT)  CSC311-Lec12
Evaluating positions under optimal play

$\max \quad v^*(s) = +1$

$\min \quad +1 \quad -1$

$\max \quad +1 \quad +1 \quad -1 \quad -1 \quad -1 \quad -1$

$\min \quad -1 \quad +1 \quad -1 \quad +1 \quad -1 \quad -1 \quad -1 \quad -1 \quad +1 \quad -1$
Value function $v^*$

- $v^*$ satisfies the **fixed-point equation**

$$v^*(s) = \begin{cases} 
\max_a \{v^*(\text{child}(s, a))\} & \text{black plays} \\
\min_a \{v^*(\text{child}(s, a))\} & \text{white plays} \\
+1 & \text{black wins} \\
-1 & \text{white wins} 
\end{cases}$$

- Analog of the optimal value function of RL.
- Applies to other two-player games
  - Deterministic, zero-sum, perfect information games.
What is the maximin value $v^*(s)$ of the root?

1. -1?
2. +1?

Recall: black plays first and is trying to maximize, whereas white is trying to minimize.
What is the maximin value \( v^*(s) \) of the root?

\begin{itemize}
\item[1] -1?
\item[2] +1?
\end{itemize}

Recall: black plays first and is trying to maximize, whereas white is trying to minimize.
In a perfect world

- So, for games like Go, all you need is $v^*$ to play optimally in the worst case:

$$a^* = \arg \max_a \{ v^*(\text{child}(s, a)) \}$$

- Claude Shannon (1950) pointed out that you can find $a^*$ by recursing over the whole game tree.

- Seems easy, but $v^*$ is wildly expensive to compute...
  - Go has $\sim 10^{170}$ legal positions in the tree.
Approximating optimal play
Depth-limited Minimax

- In practice, recurse to a small depth and back off to a static evaluation $\hat{v}^*$.
  - $\hat{v}^*$ is a heuristic, designed by experts.
  - Other heuristics as well, e.g. pruning.
  - For Go (Müller, 2002).
Progress in Computer Go

Minimax search for Go

adapted from Sylvain Gelly & David Silver, Test of Time Award ICML 2017
Expected value functions

• Designing static evaluation of $v^*$ is very challenging, especially so for Go.
  • Somewhat obvious, otherwise search would not be needed!
• Depth-limited minimax is very sensitive to misevaluation.
• Monte Carlo tree search resolves many of the issues with Minimax search for Go.
  • Revolutionized computer Go.
  • To understand this, we will introduce \textbf{expected value functions}. 
Expected value functions

If players play by rolling fair dice, outcomes will be random.

This is a decent approximation to very weak play.
Expected value functions

Averaging many random outcomes $\rightarrow$ expected value function.

$v(s) = -1/9$

Contribution of each outcome depends on the length of the path.
Consider two players that pick their moves by flipping a fair coin, what is the expected value $v(s)$ of the root?

1. $1/3$?
2. $1/2$?
Consider two players that pick their moves by flipping a fair coin, what is the expected value $v(s)$ of the root?

1. 1/3?
2. 1/2?
Expected value functions

• Noisy evaluations $v_n$ are cheap approximations of expected outcomes:

$$v_n(s) = \frac{1}{n} \sum_{i=1}^{n} o(s_i')$$

$$\approx \mathbb{E}[o(s') := v(s)]$$

$o(s) = \pm 1$ if black wins / loses.

• Longer games will be underweighted by this evaluation $v$, but let’s ignore that.
Monte Carlo tree search

• Ok expected value functions are easy to approximate, but how can we use $v_n$ to play Go?
  • $v_n$ is not at all similar to $v^*$.
  • So, maximizing $v_n$ by itself is probably not a great strategy.
  • Minimax won’t work, because it is a pure exploitation strategy that assumes perfect leaf evaluations.

• Monte Carlo tree search (MCTS; Kocsis and Szepesvári, 2006; Coulom, 2006; Browne et al., 2012) is one way.
  • MCTS maintains a depth-limited search tree.
  • Builds an approximation $\hat{v}^*$ of $v^*$ at all nodes.
Monte Carlo tree search

Selection → Expansion → Simulation → Backpropagation

- **Select** an existing leaf or **expand** a new leaf.
- Evaluate leaf with Monte Carlo **simulation** $v_n$.
- Noisy **values** $v_n$ are backed-up the tree to improve approximation $\hat{v}^*$.

(Browne et al., 2012)
Monte Carlo tree search

- Selection strategy greedily descends tree.
- MCTS is robust to noisy misevaluation at the leaves, because the selection rule balances exploration and exploitation:

\[
    a^* = \arg \max_a \left\{ \hat{v}^*(\text{child}(s, a)) + \sqrt{\frac{2 \log N(s)}{N(\text{child}(s, a))}} \right\}
\]

- \(\hat{v}^*(s) = \) estimate of \(v^*(s)\), \(N(s)\) number of visits to node \(s\).
- MCTS is forced to visit rarely visited children.
- Key result: MCTS approximation \(\hat{v}^* \rightarrow v^*\) (Kocsis and Szepesvári, 2006).
Progress in Computer Go

Monte Carlo tree search for Go

adapted from Sylvain Gelly & David Silver, Test of Time Award ICML 2017
Scaling with compute and time

- The strength of MCTS bots scales with the amount of compute and time that we have at play-time.
- But play-time is limited, while time outside of play is much more plentiful.
- How can we improve computer Go players using compute when we are not playing? Learning!
  - You can try to think harder during a test vs. studying more beforehand.
Learning to play Go
This is where I come in

• 2014 Google DeepMind internship on neural nets for Go.
  • Working with Aja Huang, David Silver, Ilya Sutskever, I was responsible for designing and training the neural networks.
  • Others came before (e.g., Sutskever and Nair, 2008).
• Ilya Sutskever’s (Chief Scientist, OpenAI) argument in 2014: expert players can identify a good set of moves in 500 ms.
  • This is only enough time for the visual cortex to process the board—not enough for complex reasoning.
  • At the time we had neural networks that were nearly as good as humans in image recognition, thus we thought we would be able to train a net to play Go well.

• Key goal: can we train a net to understand Go?
Neural nets for Go

Neural networks are powerful parametric function approximators.

board $s$

parameters $x$

Idea: map board position $s$ (input) to a next move or an evaluation (output) using simple convolutional networks.
Neural nets for Go

• We want to train a neural policy or neural evaluator, but how?
• Existing data: databases of Go games played by humans and other compute Go bots.
• The first idea that worked was learning to predict expert’s next move.
  • Input: board position $s$
  • Output: next move $a$
Policy Net (Maddison et al., 2015)

- **Dataset**: KGS server games split into board / next-move pairs \((s_i, a_i)\)
  - 160,000 games \(\rightarrow\) 29 million \((s_i, a_i)\) pairs.
- **Loss**: negative log-likelihood,
  \[
  - \sum_{i=1}^{N} \log \pi_{\text{net}}(a_i | s_i, x).
  \]
- Use trained net as a Go player:
  \[
  a^* = \arg \max_a \{ \log \pi_{\text{net}}(a | s, x) \}.
  \]

(Silver et al., 2016)
Like learning a better traversal

As supervised accuracy improved, searchless play improved.
Progress in Computer Go

Progress in my internship

- Traditional Search
- MCTS

Amateur Dan
- 9 dan
- 8 dan
- 7 dan
- 6 dan
- 5 dan
- 4 dan
- 3 dan
- 2 dan
- 1 dan

Amateur Kyu
- 1 kyu
- 2 kyu
- 3 kyu
- 4 kyu
- 5 kyu
- 6 kyu
- 7 kyu
- 8 kyu

Professional Dan
- 1 dan
- 2 dan
- 3 dan
- 4 dan
- 5 dan
- 6 dan
- 7 dan
- 8 dan
- 9 dan

RL net 12 layer
- SL net 12 layer
- SL net 10 layer
- SL net 6 layer
- SL net 3 layer

adapted from Sylvain Gelly & David Silver, Test of Time Award ICML 2017
Can we improve MCTS with neural networks?

• These results prompted the formation of big team inside DeepMind to combine MCTS and neural networks.

• To really improve search, we needed strong evaluators.
  • **Recall**: an evaluation function tells us who is winning.
  • $\pi_{\text{net}}$ rollouts would be a good evaluator, but this is too expensive.

• Can we learn one?
Value Net (Silver et al., 2016)

Failed attempt.

- **Dataset**: KGS server games split into board / outcome pairs $(s_i, o(s_i))$
- **Loss**: squared error,

$$\sum_{i=1}^{N} (o(s_i) - v_{net}(s_i, x))^2.$$

- **Problem**: Effective sample size of 160,000 games was not enough.
**Value Net** (Silver et al., 2016)

Successful attempt.

- Use Policy Net playing against itself to **generate millions of unique games**.
- **Dataset**: Board / outcome pairs \((s_i, o(s_i))\), each from a unique self-play game.
- **Loss**: squared error,

\[
\sum_{i=1}^{N} (o(s_i) - v_{\text{net}}(s_i, x))^2.
\]

(Silver et al., 2016)
**AlphaGo** (Silver et al., 2016)

- The Value Net was a very strong evaluator.

- The final version of AlphaGo used rollouts, Policy Net, and Value Net together.
  - Rollouts and Value Net as evaluators.
  - Policy Net to bias the exploration strategy.
Progress in Computer Go

AlphaGo Team (Silver et al., 2016)

AlphaGo (Lee Sedol)

AlphaGo (Nature)

MCTS

Neural nets

Traditional Search

adapted from Sylvain Gelly & David Silver, Test of Time Award ICML 2017
Impact
Go is not just a game

- Go originated in China more than 2,500 years ago. Reached Korea in the 5th century, Japan in the 7th.
- In the Tang Dynasty, it was one of the four arts of the Chinese scholar together with calligraphy, painting, and music.
- The aesthetics of Go (harmony, balance, style) are as essential to top-level play as basic tactics.
2016 Match—AlphaGo vs. Lee Sedol

• Best of 5 matches over the course of a week.
• Most people expected AlphaGo to lose 0-5.
• AlphaGo won 4-1.
Human moments

Lee Sedol is a titan in the Go world, and achieving his level of play requires a life of extreme dedication.

It was humbling and strange to be a part of the AlphaGo team that played against him.
Game 2, Move 37
Thanks!

I played a key role at the start of AlphaGo, but the success is owed to a large and extremely talented team of scientists and engineers.

Course Evals

Use this time to finish course evals.


