Computer Graphics

CSC 418/2504

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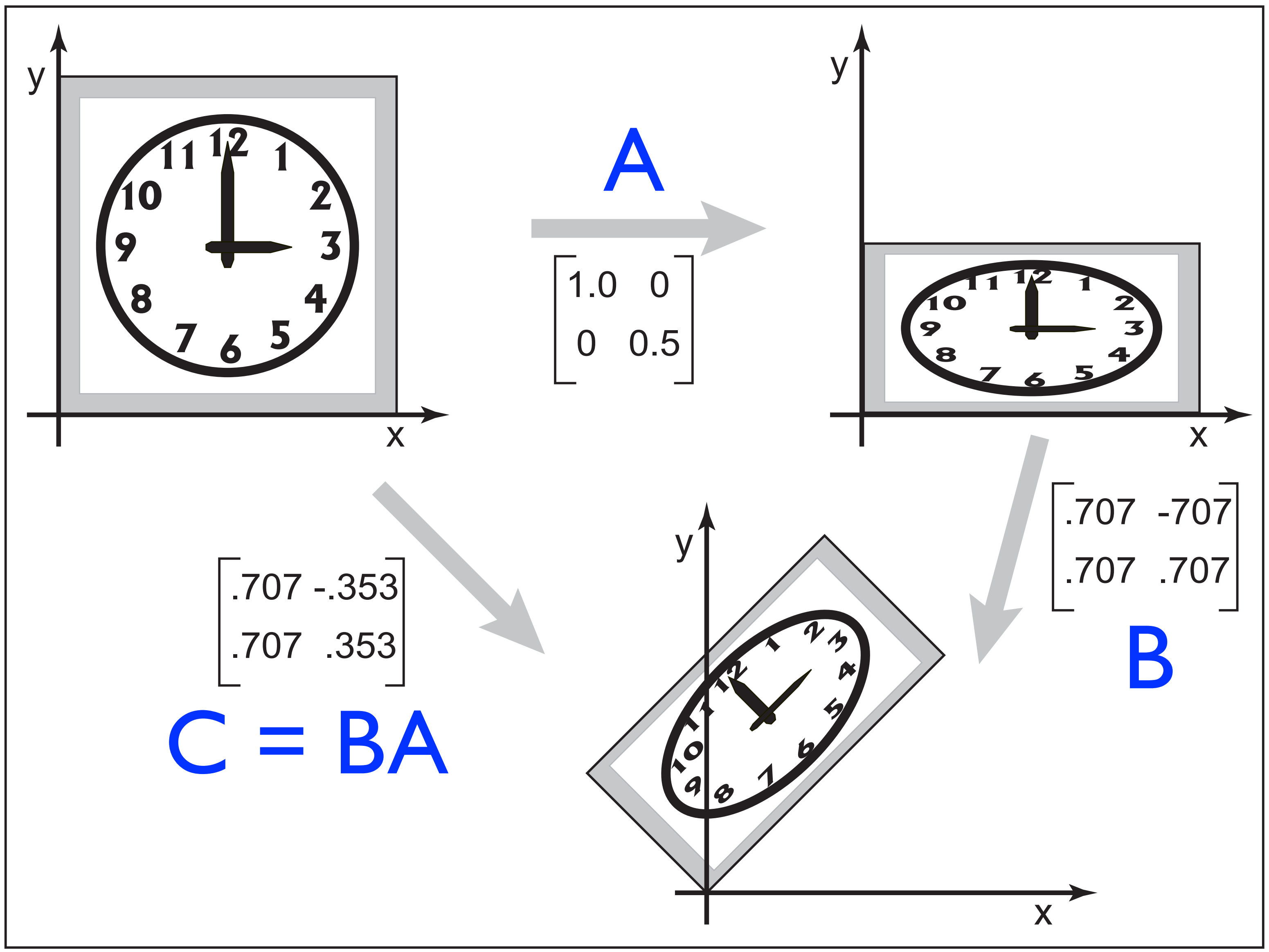
Coordinate-free geometry slides courtesy of Patrick Coleman
Composition figure courtesy of Peter Shirley

Today

- Homogeneous coordinates
- Coordinate-free geometry
- 3D geometric primitives

Composition



B(A**x**) = (BA)**x** = C**x**

Translation: Special Case

$$\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Homogeneous Coords

$$\begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Homogeneous Coords

$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$

Homogeneous Coords

$$\text{rotate}(\phi) = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Homogeneous Coords

$$\begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11}x + a_{12}y + t_x \\ a_{21}x + a_{22}y + t_y \\ 1 \end{bmatrix}$$

Coordinate Free Geometry

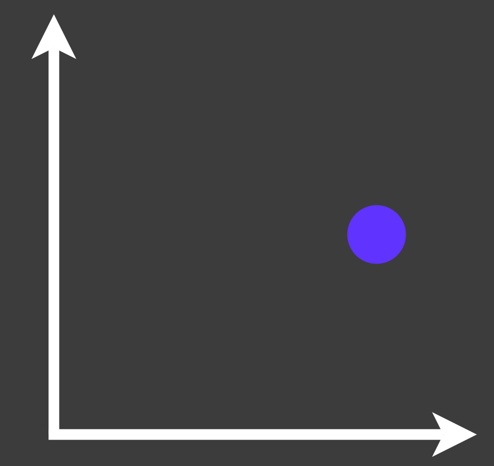
A style of expressing geometric computations
that avoids explicit use of coordinates

Easier derivations

Far simpler code

CFG Quantities

Point: location in space



Vector: direction in space

Scalar: real number

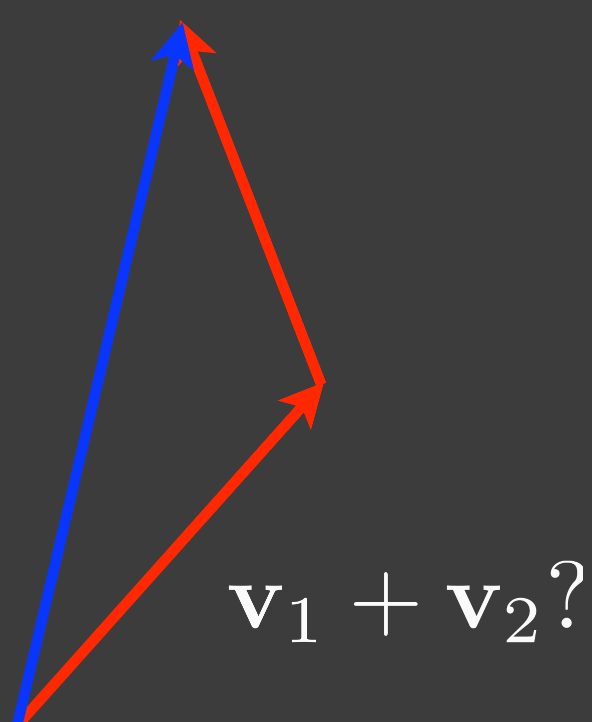
Points vs. Vectors

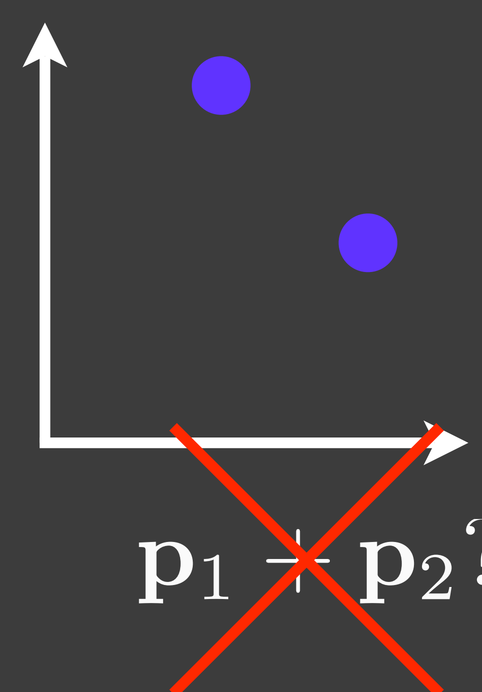
A point is not a vector.

A point is not a vector.

A point is not a vector.

Addition?





Operations on these Quantities

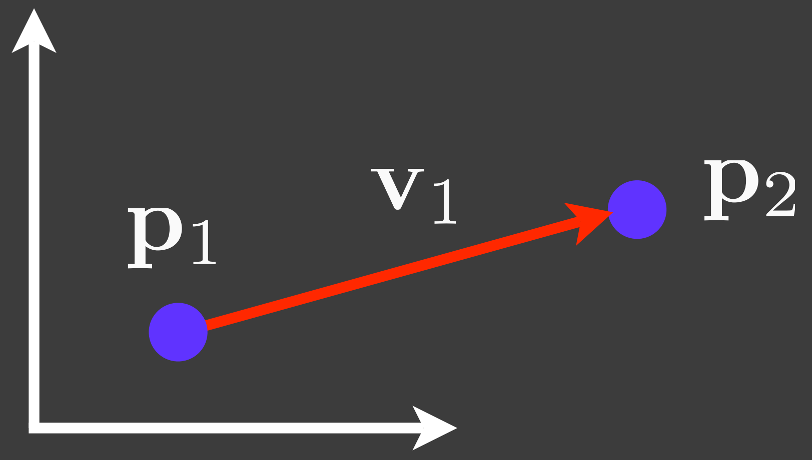
Magnitude of a vector

$$||\mathbf{v}||$$

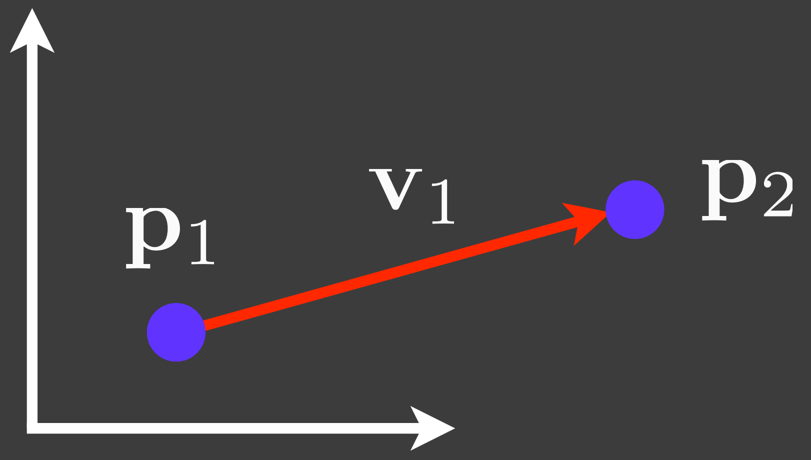
Operations on these Quantities

Point-vector addition

$$\mathbf{p}_1 + \mathbf{v}_1 = \mathbf{p}_2$$

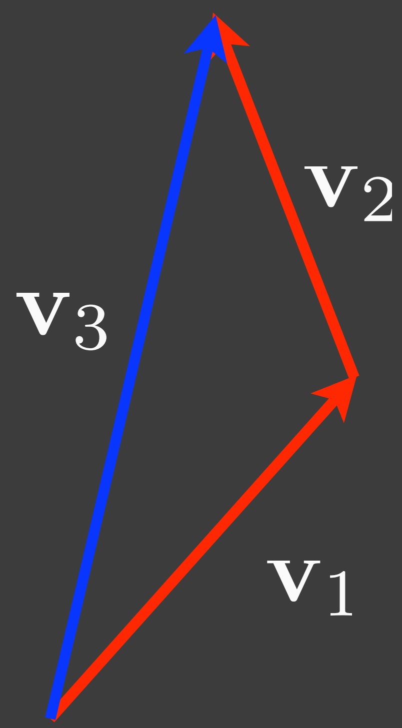


$$\mathbf{p}_2 - \mathbf{p}_1 = \mathbf{v}_1$$



Operations on these Quantities

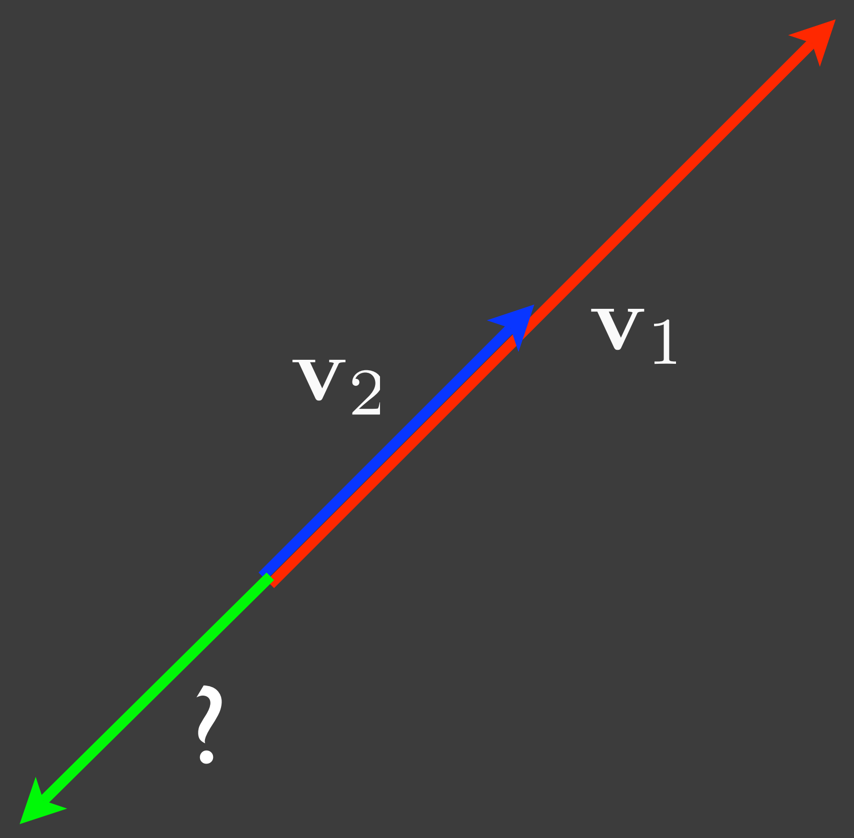
Vector addition $\mathbf{v}_1 + \mathbf{v}_2 = \mathbf{v}_3$



Operations on these Quantities

Vector scaling

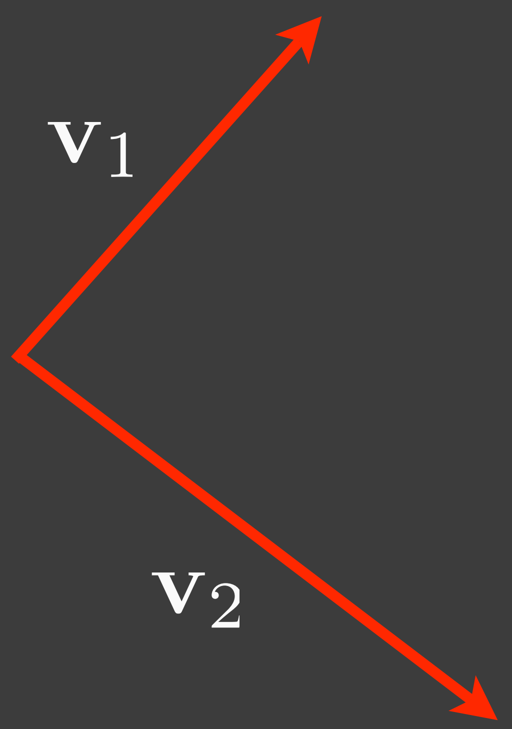
$$\alpha \mathbf{v}_1 = \mathbf{v}_2$$



Operations on these Quantities

Dot product

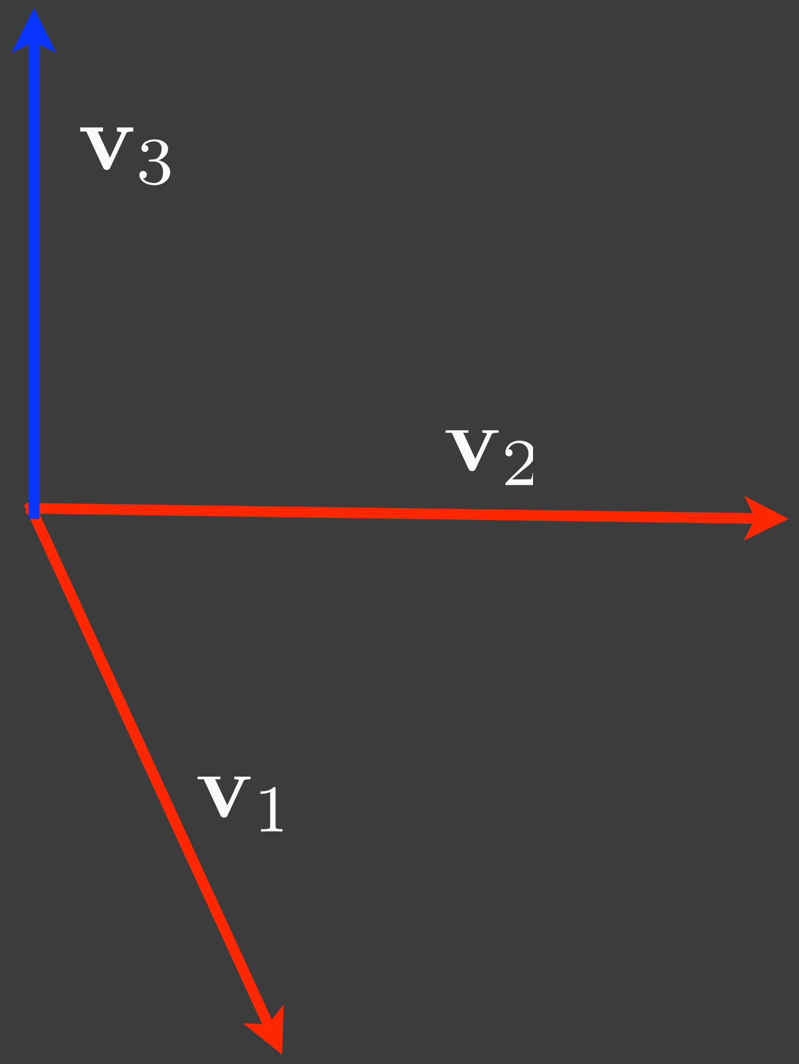
$$\mathbf{v}_1 \cdot \mathbf{v}_2$$



$$\mathbf{v}_1 \cdot \mathbf{v}_2 = \mathbf{v}_1^T \mathbf{v}_2 = ||\mathbf{v}_1||\ ||\mathbf{v}_2|| \cos(\theta)$$

Operations on these Quantities

Cross product (3D)



$$\mathbf{v}_1 \times \mathbf{v}_2 = \begin{vmatrix} \mathbf{x} & \mathbf{y} & \mathbf{z} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

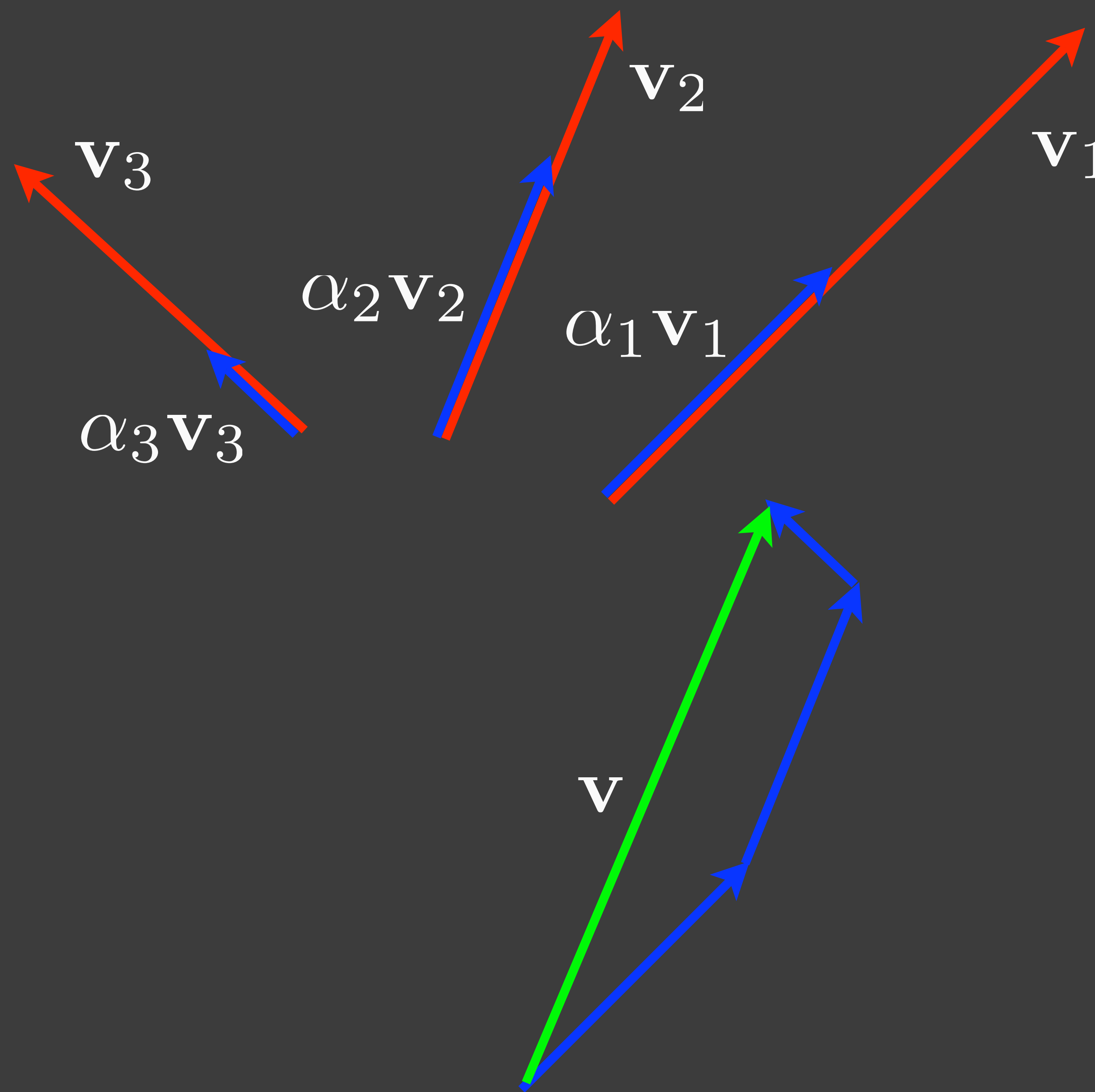
$$||\mathbf{v}_1 \times \mathbf{v}_2|| = ||\mathbf{v}_1||\ ||\mathbf{v}_2|| \sin(\theta)$$

Right hand rule

Operations on these Quantities

Linear combination of vectors

$$\sum_i \alpha_i \mathbf{v}_i = \mathbf{v}$$

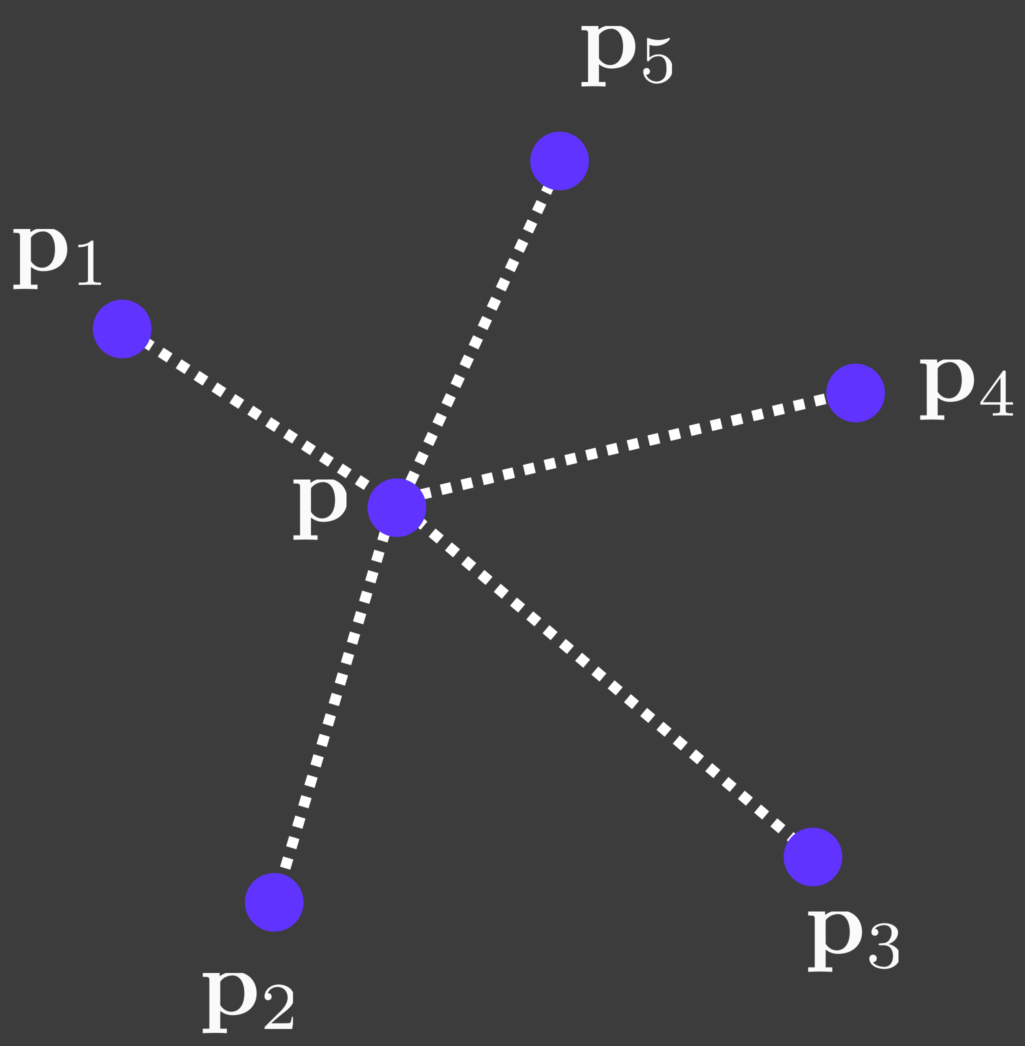


Operations on these Quantities

Affine combination
of points

$$\sum_i \alpha_i \mathbf{p}_i = \mathbf{p}$$

where $\sum_i \alpha_i = 1$



Operations on these Quantities

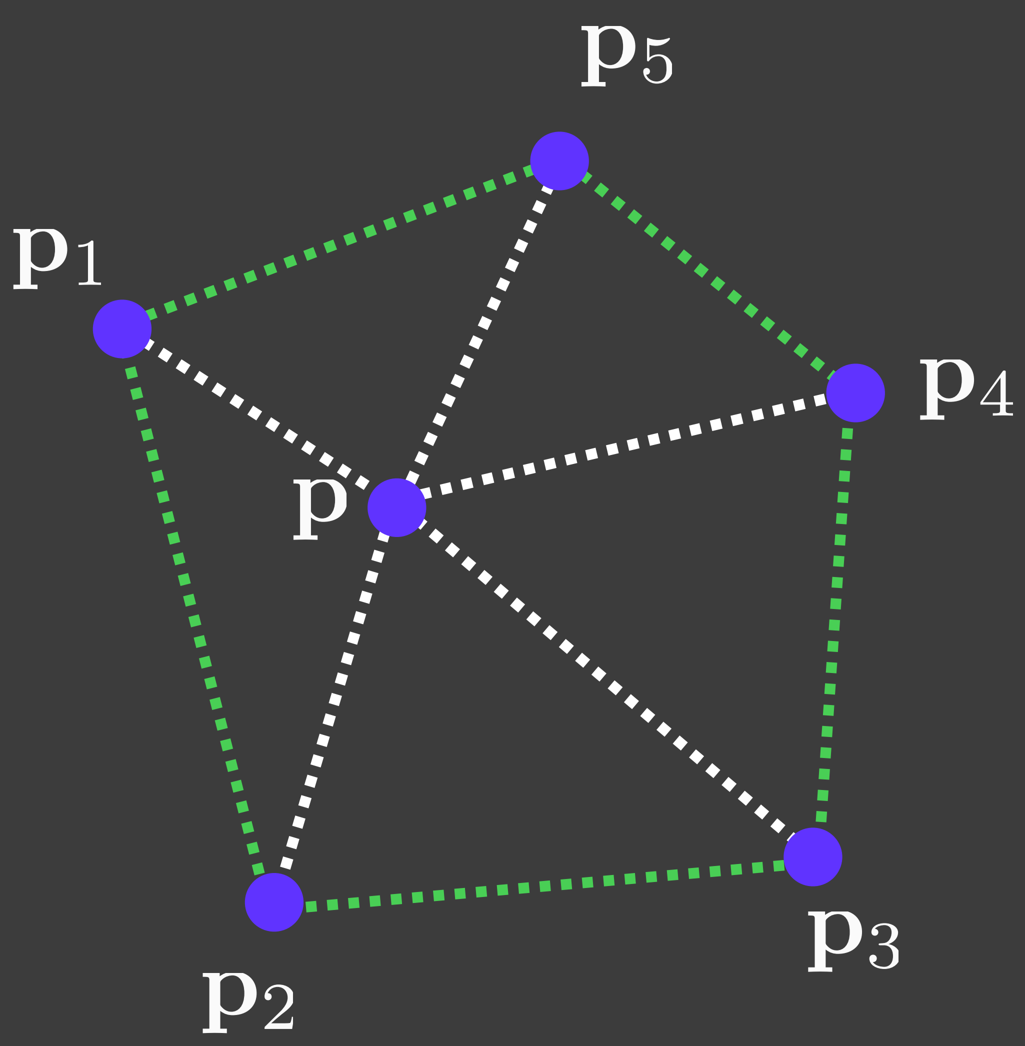
Affine combination
of points

$$\sum_i \alpha_i \mathbf{p}_i = \mathbf{p}$$

where $\sum_i \alpha_i = 1$

$\alpha_3 = 1$?

$\alpha_i >= 0$?



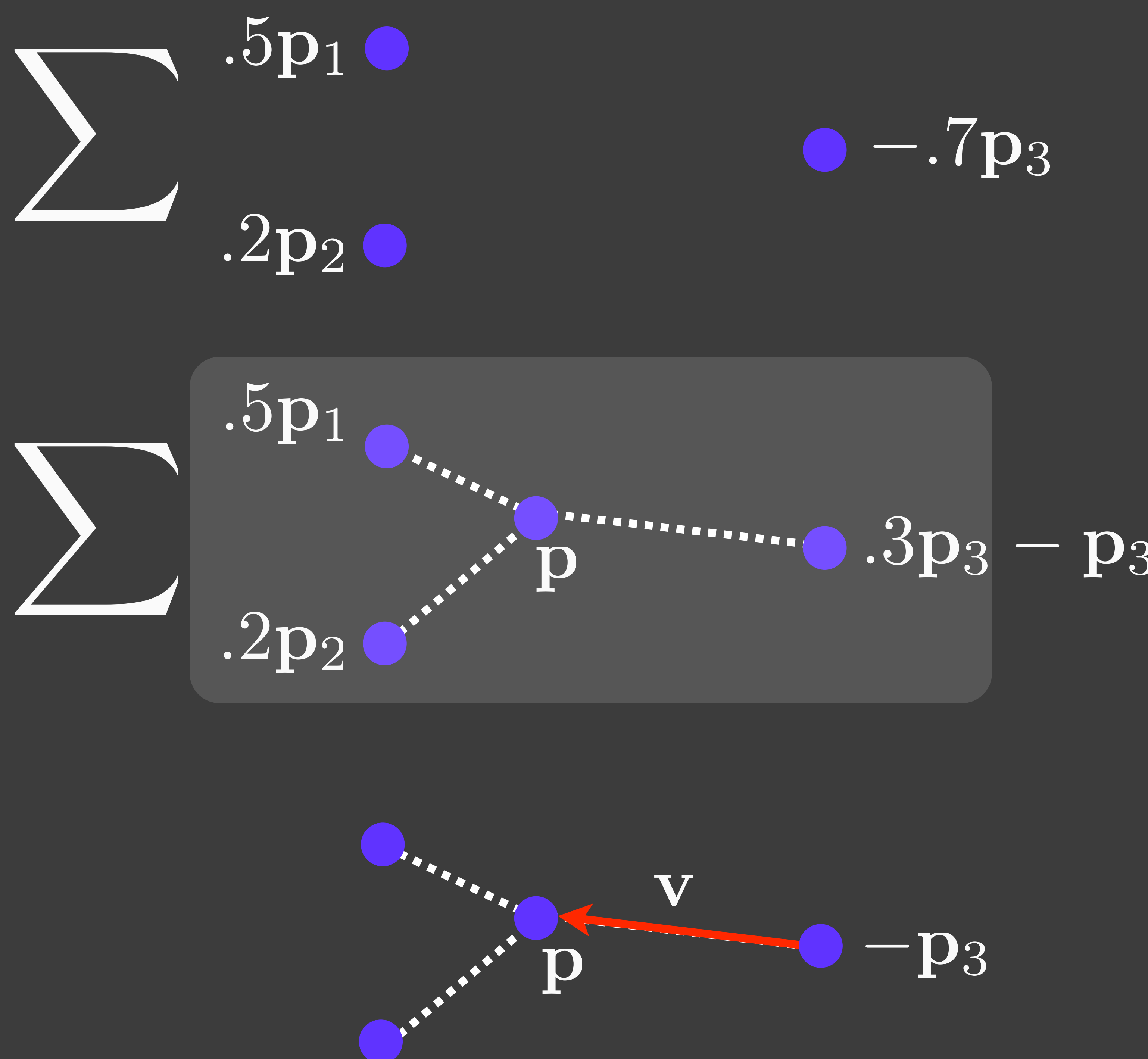
Operations on these Quantities

Vectors as restricted point combination

$$\sum_i \alpha_i \mathbf{p}_i = \mathbf{v}$$

where $\sum_i \alpha_i = 0$

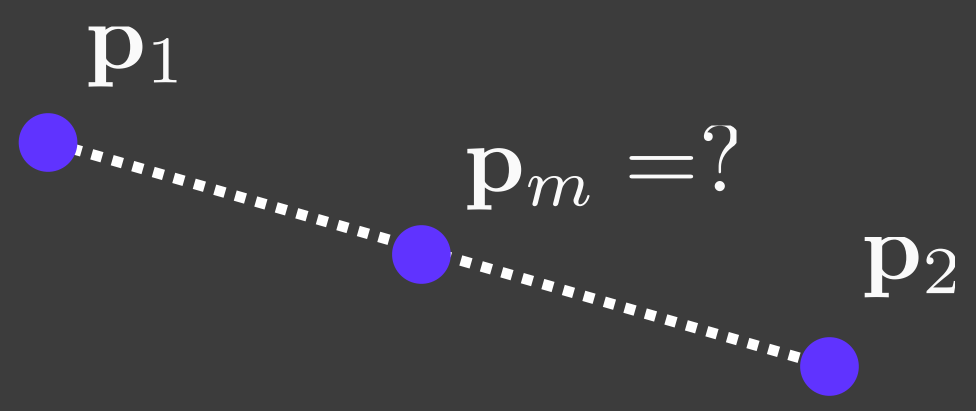
Example...



Everything Else

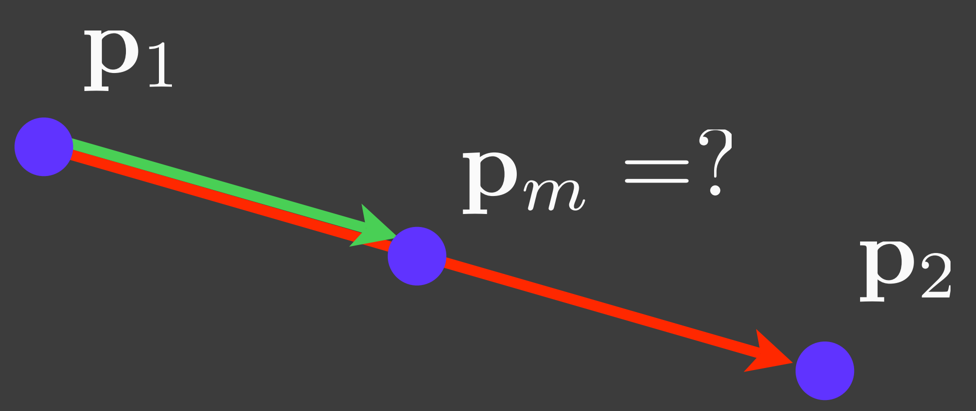
Undefined

CFG Reasoning Example



$$\mathbf{p}_m = \frac{1}{2}\left(\mathbf{p}_1 + \mathbf{p}_2\right)$$

$$\mathbf{p}_m = \frac{1}{2}\mathbf{p}_1 + \frac{1}{2}\mathbf{p}_2$$



$$\mathbf{p}_m = \mathbf{p}_1 + \frac{1}{2}\left(\mathbf{p}_2 - \mathbf{p}_1\right)$$

3D Geometric Primitives

To the Board