Computer Graphics

CSC 418/2504

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September 21, 2011

2D curves and polygons slides courtesy of Patrick Coleman
2D transformation figures courtesy of Peter Shirley

Today

2D Curves & Polygons
2D Transformations

2D Lines and Curves

Explicit

$$y = f(x)$$

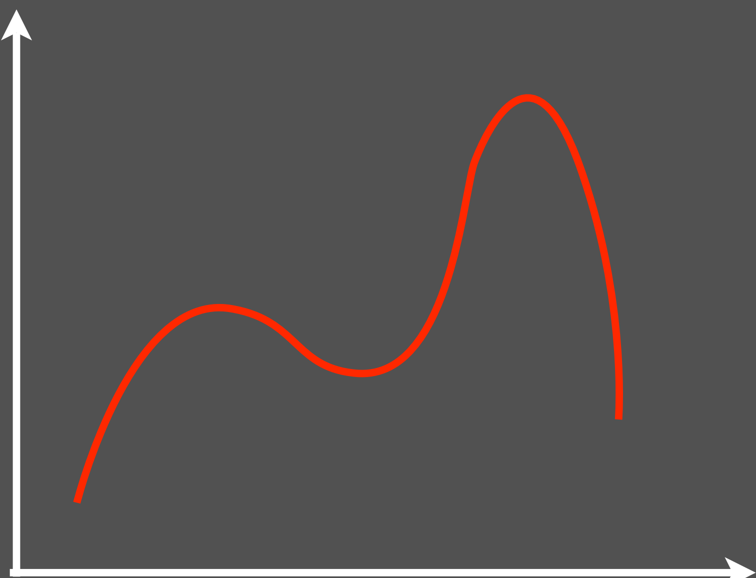
Parametric

$$\mathbf{p}(t) = (x(t), y(t))$$
$$t \in [0, 1]$$

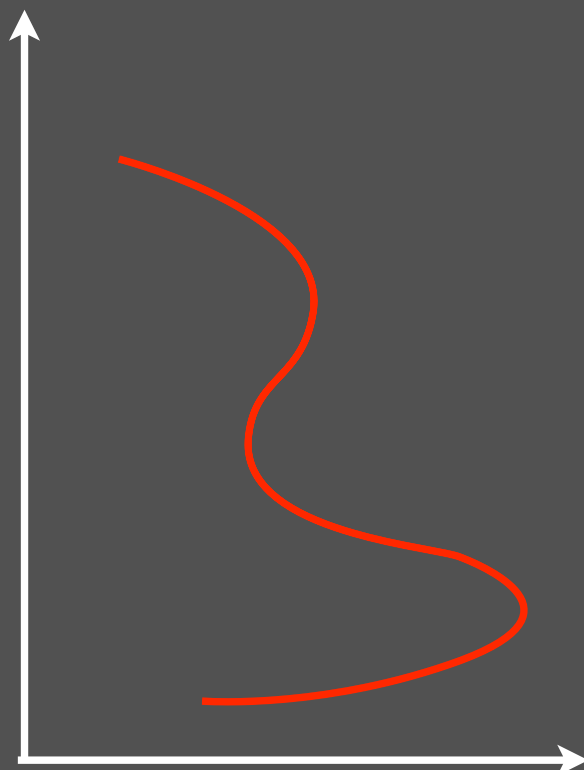
Implicit

$$f(x, y) = 0$$

Explicit Curves

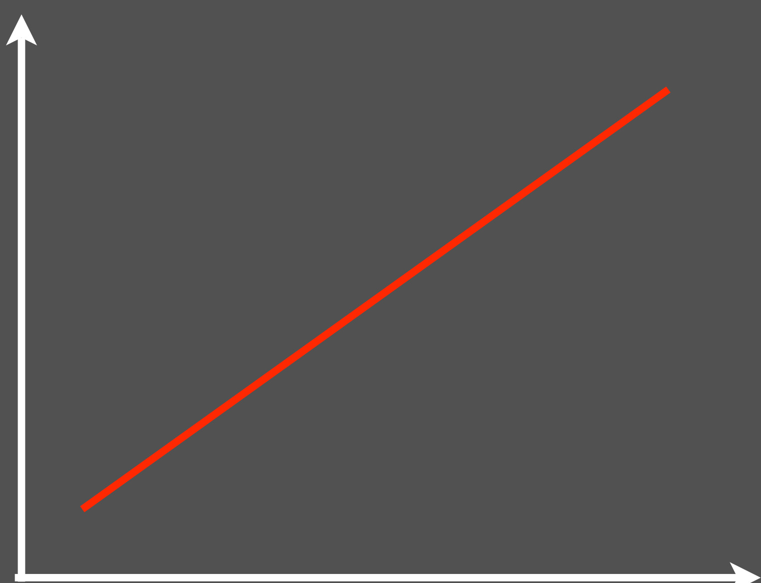


$y = f(x)$

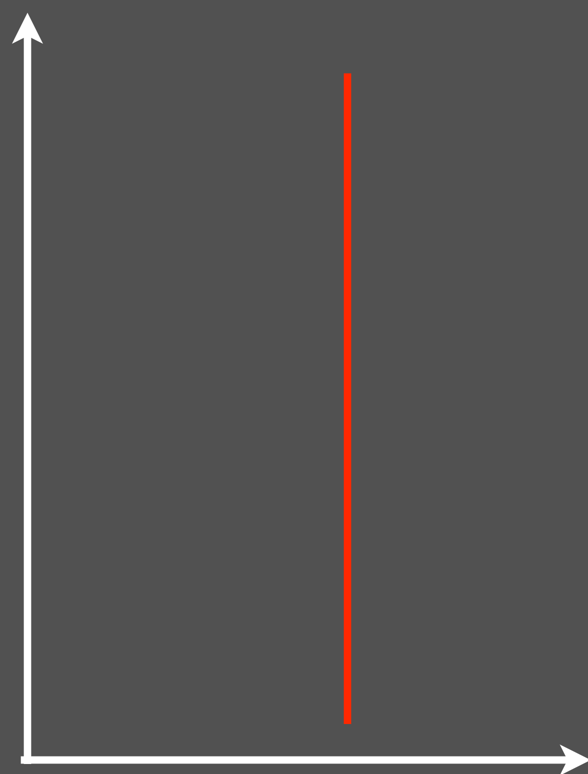


$x = f(y)$

Explicit Lines



$$y = mx + b$$



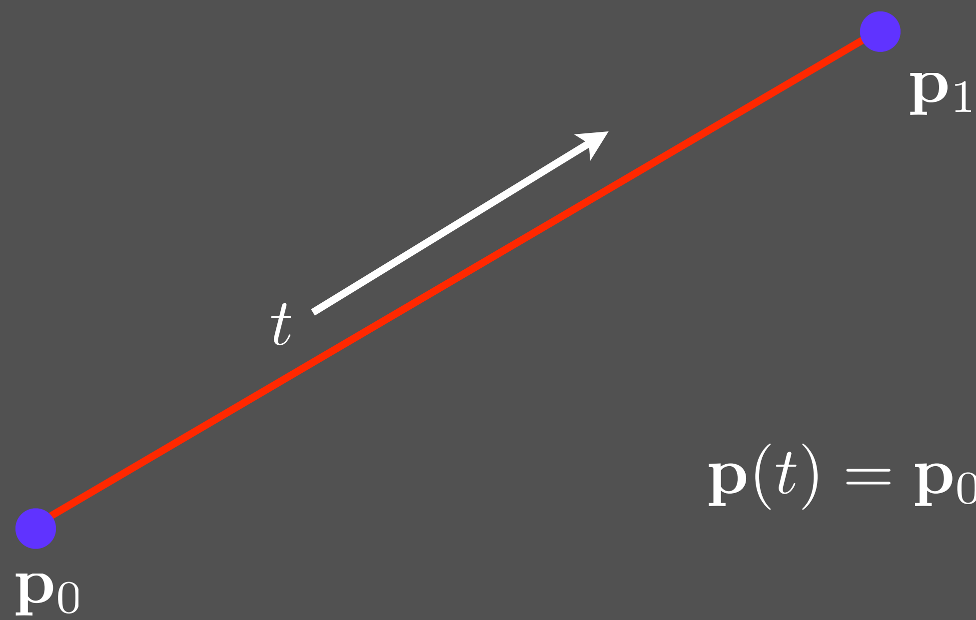
Vertical Lines?

$$x = C$$

Endpoint bounds?

Explicit?

Parametric Line Segments



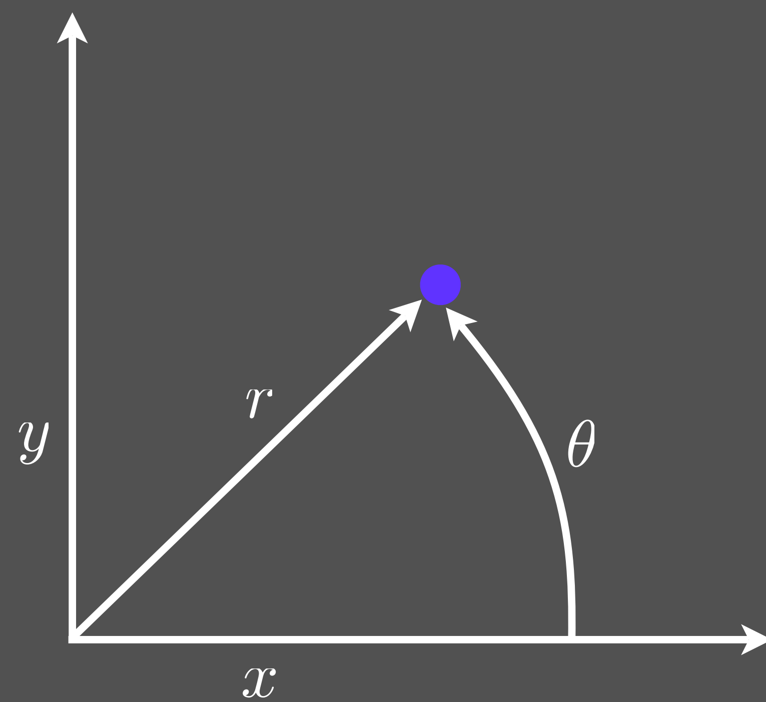
$$\mathbf{p}(t) = \mathbf{p}_0 + t(\mathbf{p}_1 - \mathbf{p}_0)$$
$$t \in [0, 1]$$

Notation: $\mathbf{p}_i = (x_i, y_i)$

$x(t)$ and $y(t)$?

Aside: Polar Coordinate Review

$$x = r\cos(\theta)$$
$$y = r\sin(\theta)$$
$$r = \sqrt{x^2 + y^2}$$
$$\theta = \tan^{-1}(y/x)$$



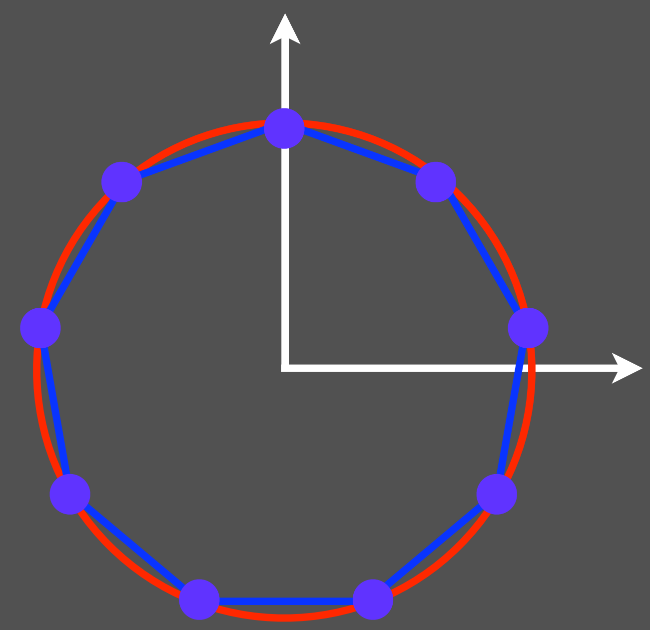
Parametric Circles

Center: origin
Radius: r

$$\mathbf{p}(t) = (r\cos(t), r\sin(t))$$
$$t \in [0, 2\pi]$$

$t \in [0, 1]$?

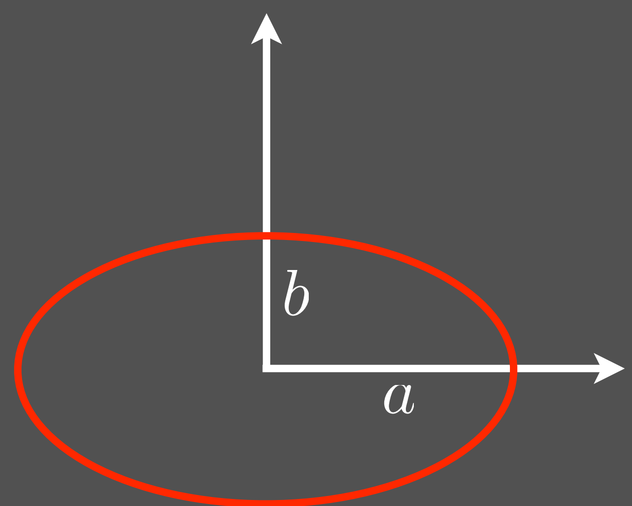
$$\mathbf{p}(t) = (r\cos(2\pi t), r\sin(2\pi t))$$



Point samples on the circle?

Parametric Ellipses

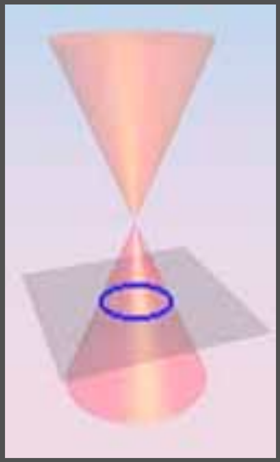
Center: origin
Major Axis: a
Minor Axis: b



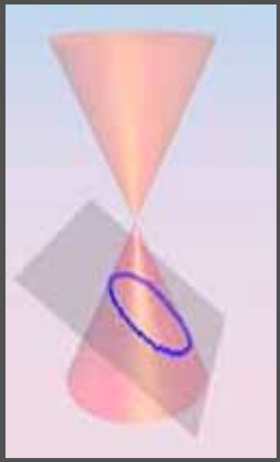
$$\mathbf{p}(t) = (a\cos(t), b\sin(t))$$
$$t \in [0, 2\pi]$$

Conic Sections

Intersection between a cone and a plane

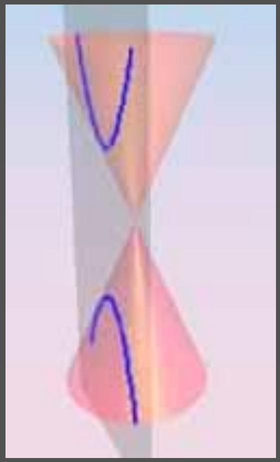


Circle



Ellipse

Parabola



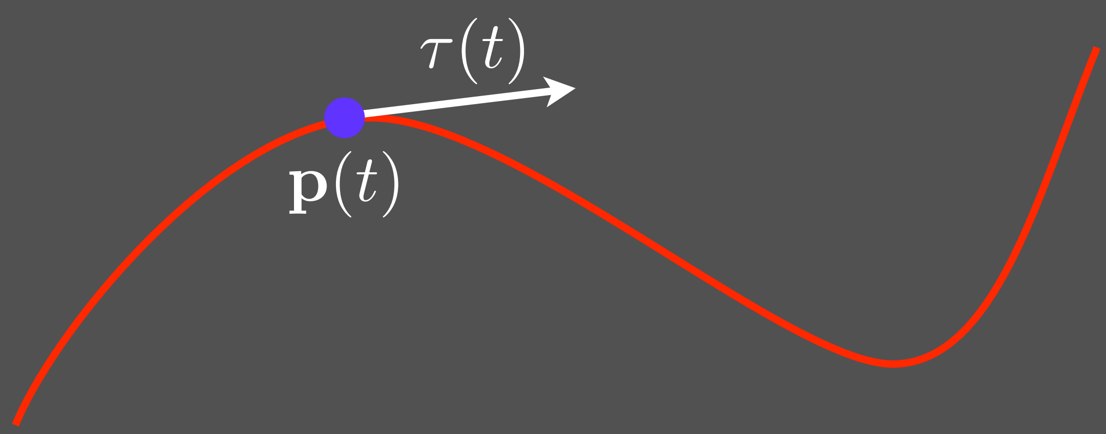
Hyperbola

Also points and lines

Parametric Tangents

Instantaneous direction of curve:

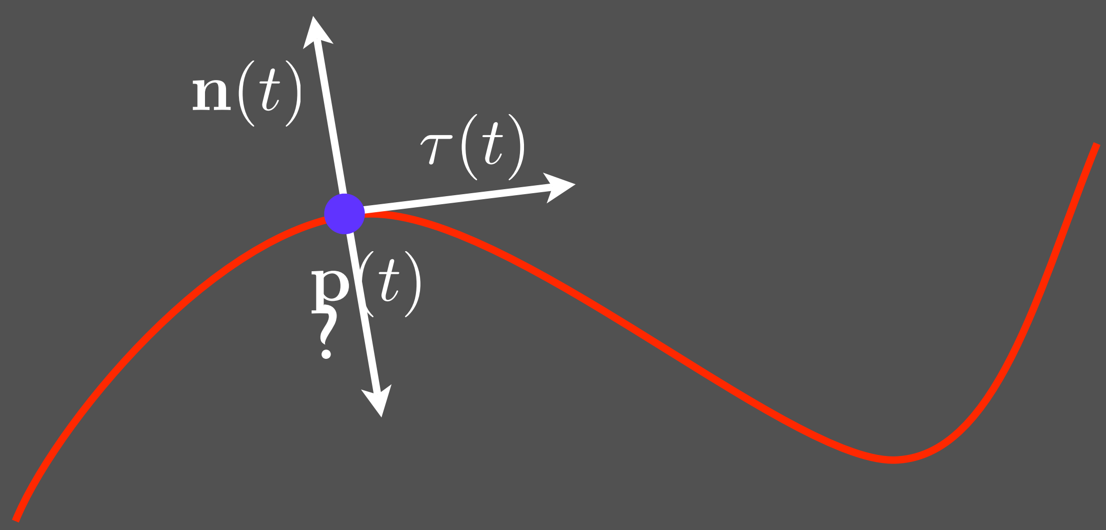
$$\tau(t) = \frac{d\mathbf{p}}{dt} = \left(\frac{dx(t)}{dt}, \frac{dy(t)}{dt} \right)$$



Parametric Normals

Perpendicular to curve:

$$\mathbf{n}(t) = \left(-\frac{dy(t)}{dt}, \frac{dx(t)}{dt} \right)$$



Unit Tangents and Unit Normals

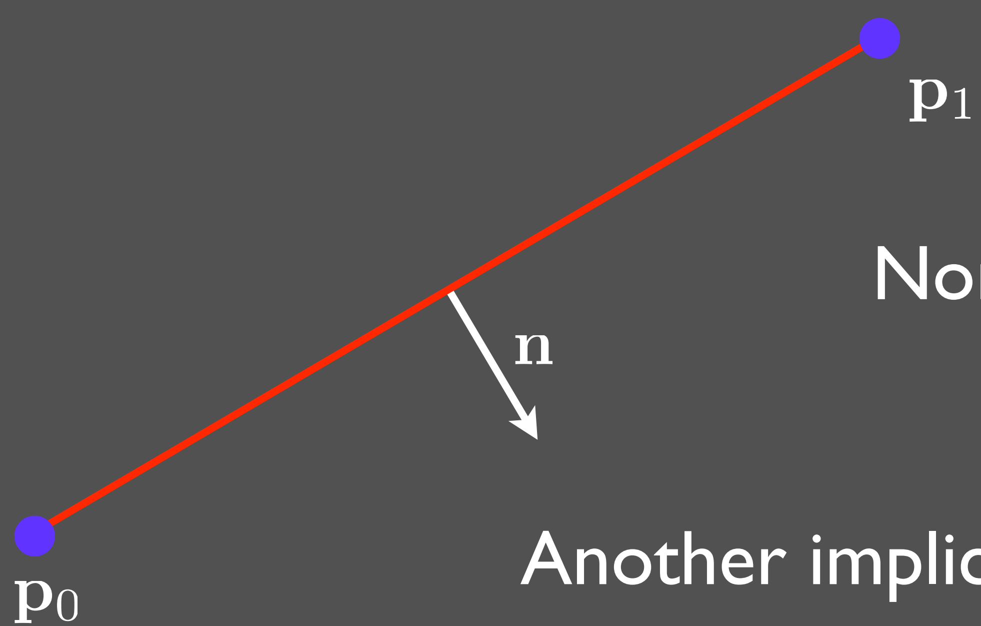
Set the length to one:

$$\hat{\tau}(t) = \frac{\tau(t)}{||\tau(t)||}$$

$$\hat{\mathbf{n}}(t) = \frac{\mathbf{n}(t)}{||\mathbf{n}(t)||}$$

Implicit Lines

$$f(x, y) = (x - x_0)(y_1 - y_0) - (y - y_0)(x_1 - x_0) = 0$$



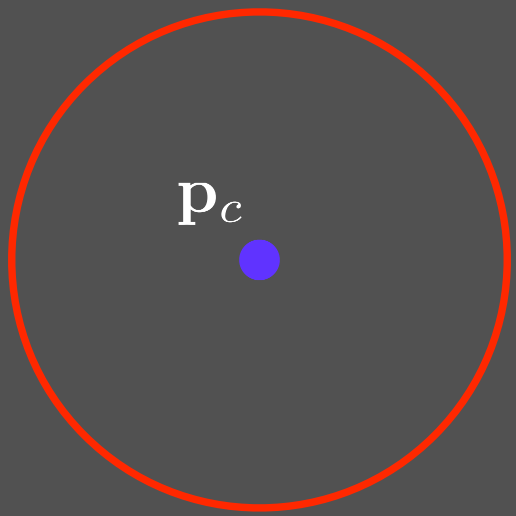
Normal:

$$\mathbf{n} = (y_1 - y_0, x_0 - x_1)$$

Another implicit form:

$$(\mathbf{p} - \mathbf{p}_0) \cdot \mathbf{n} = 0$$

Implicit Circles



$$(x - x_c)^2 + (y - y_c)^2 - r^2 = 0$$

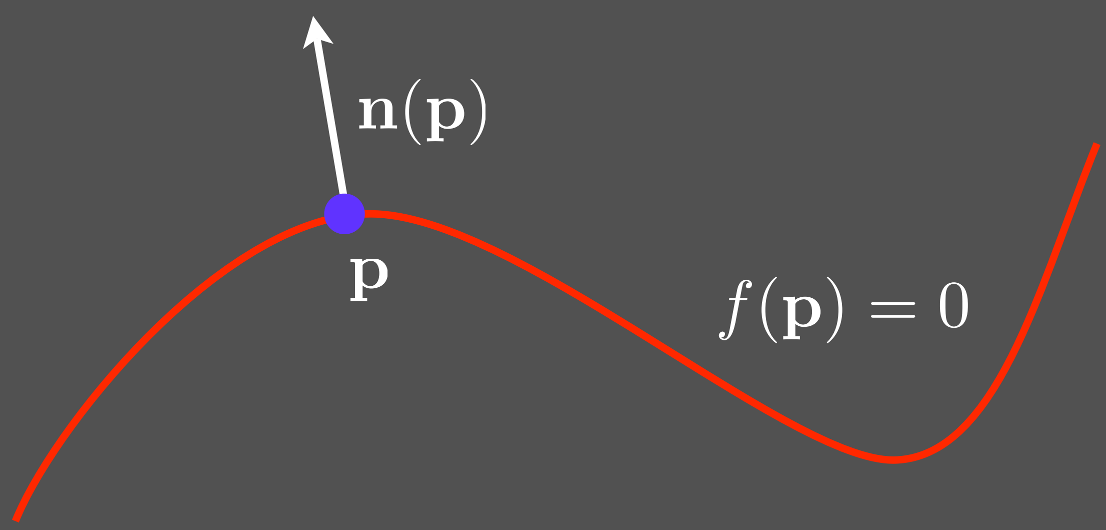
General Implicit Conics

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

$B^2 - 4AC < 0$	Ellipse, circle, point, or none
$B^2 - 4AC = 0$	Parabola, two parallel lines, one line, or none
$B^2 - 4AC > 0$	Hyperbola or two intersecting lines

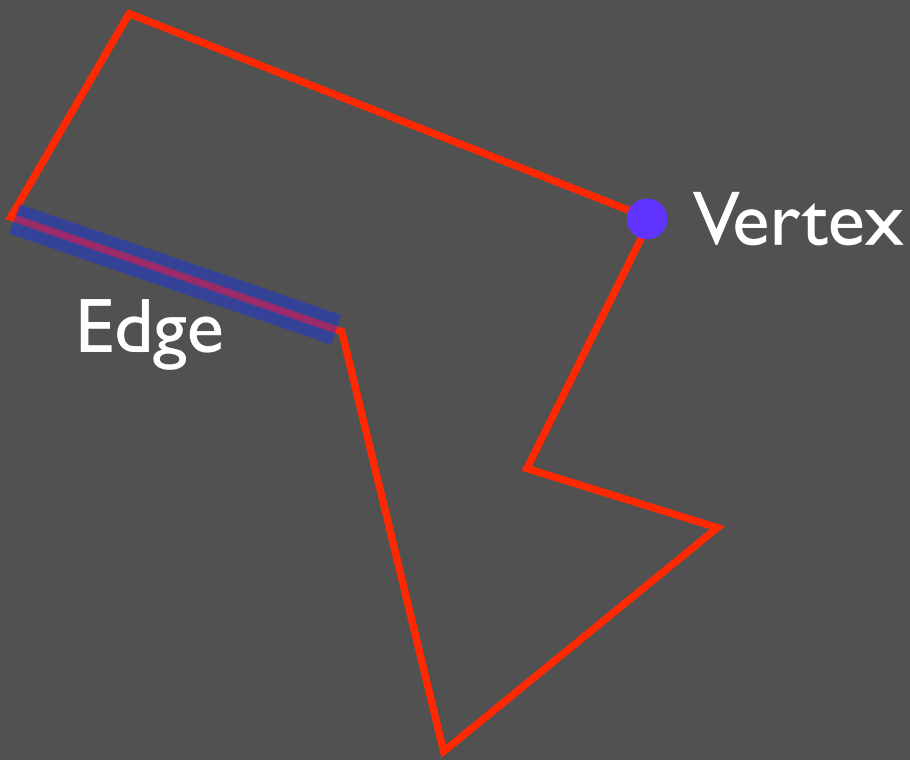
Normals from the Implicit Form

$$\mathbf{n}(\mathbf{p}) = \nabla f(\mathbf{p}) = \left(\frac{\partial f(x,y)}{\partial x}, \frac{\partial f(x,y)}{\partial y} \right)$$

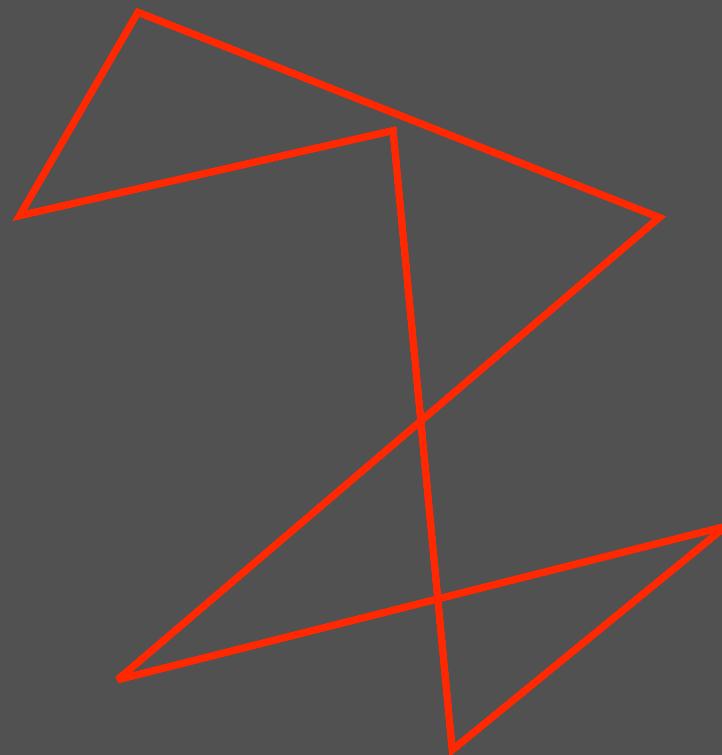


Polygons

Piecewise linear
Continuous
Closed
Planar



Polygon?



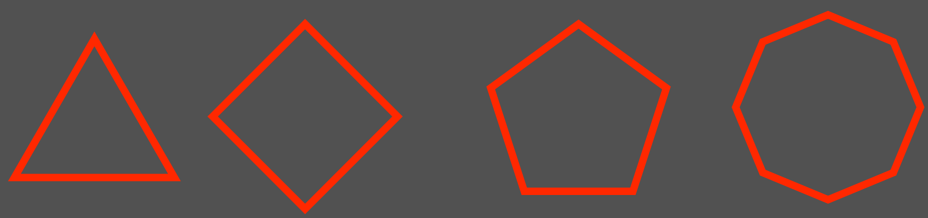
Piecewise linear?
Continuous?
Closed?
Planar?

Yes, but not *simple*

Simple: no self intersections

Regular Polygons

Simple
Equilateral
Equiangular



Parametric vertex representation:

$$\mathbf{p}_i = r(\cos \frac{2\pi}{n} i, \sin \frac{2\pi}{n} i)$$

Translate?

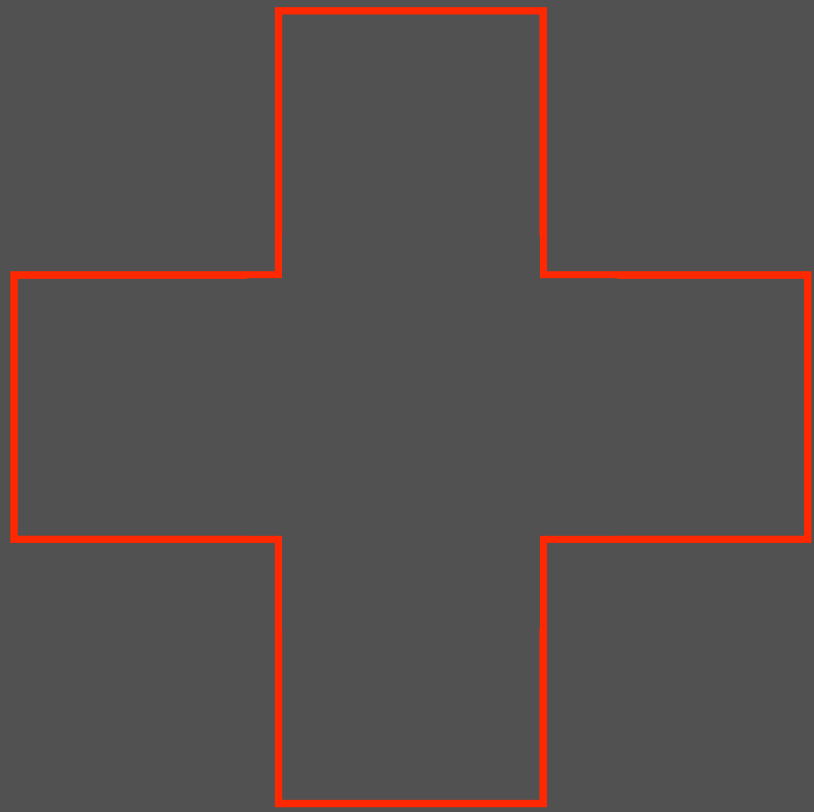
Add (x_t, y_t)

Rotate?

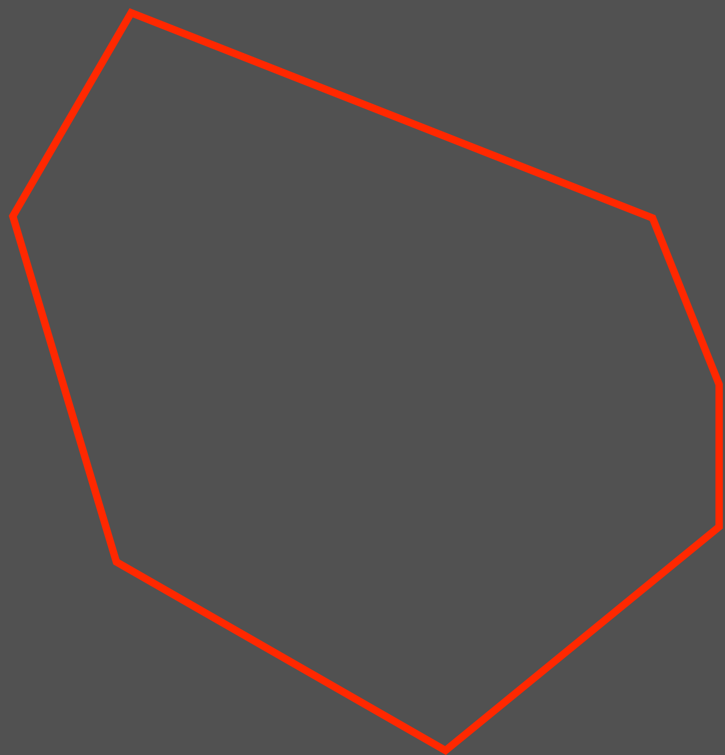
Add θ to trigonometric arguments

Regular?

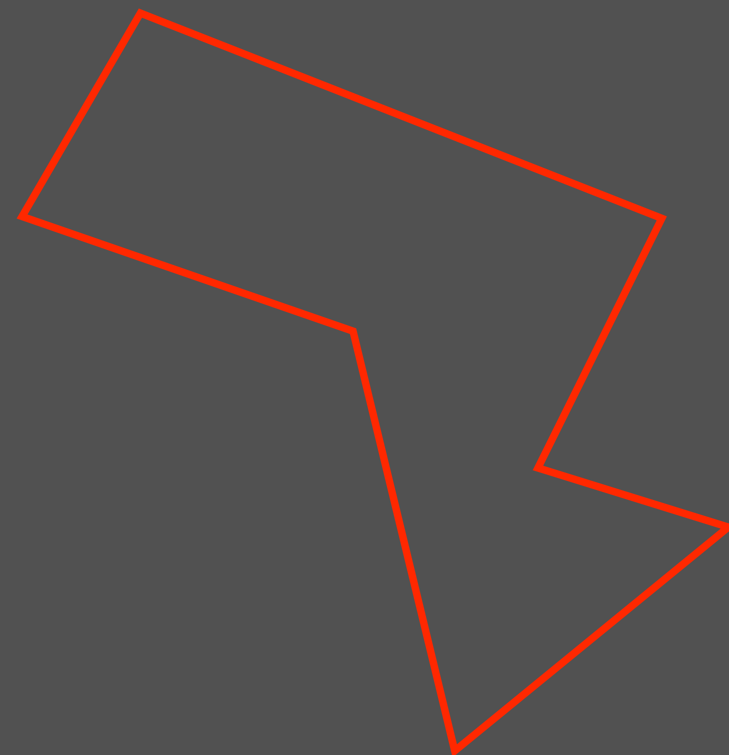
Simple?
Equilateral?
Equiangular?



Convexity

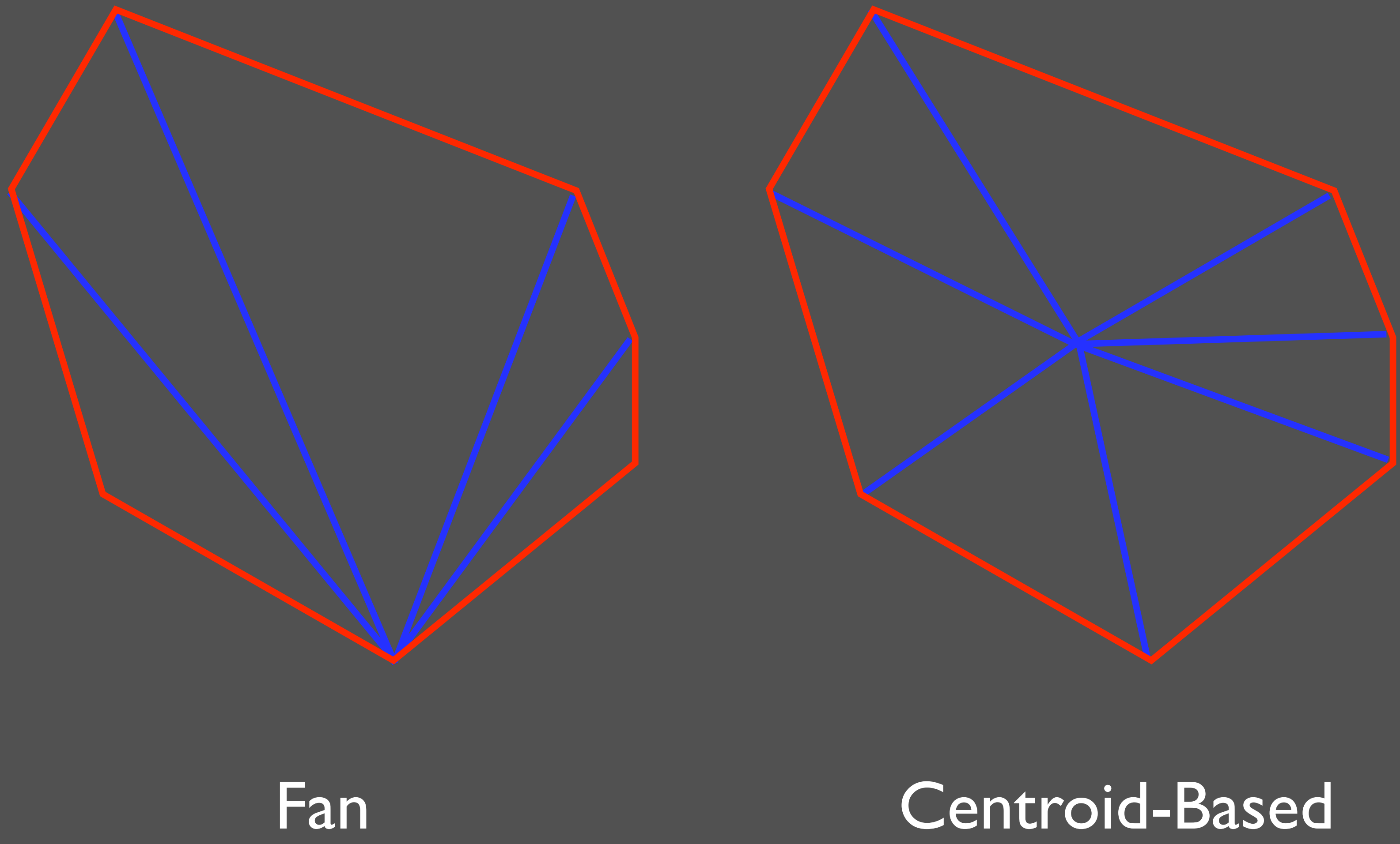


Convex

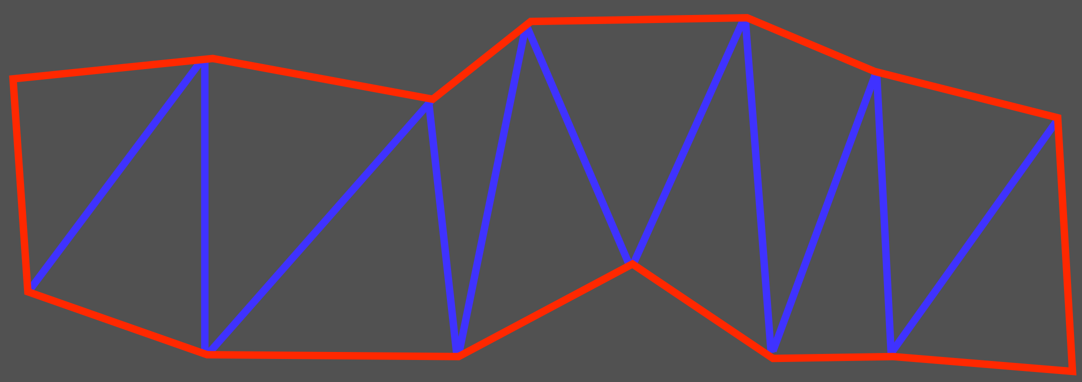


Not Convex

Triangulation (Tessellation)



Triangulation (Tesselation)



Strip

2D Transforms

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a_{11}x + a_{12}y \\ a_{21}x + a_{22}y \end{bmatrix}$$

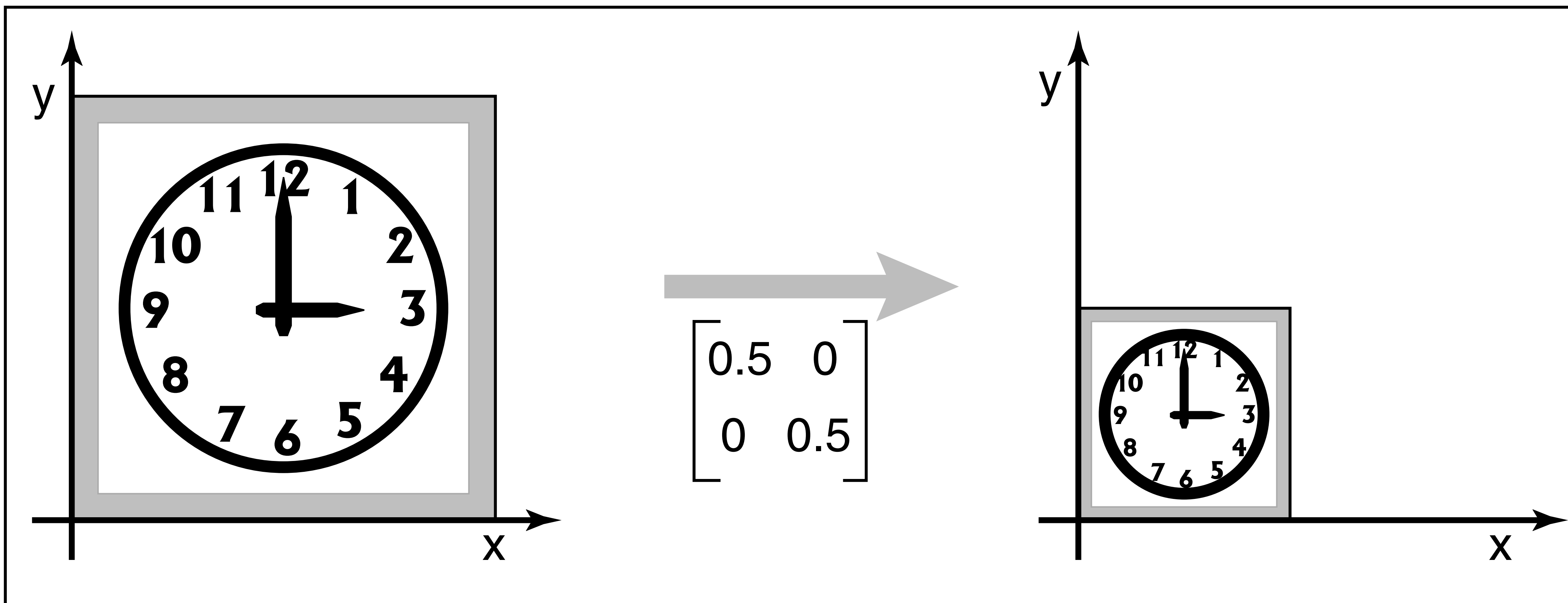
(m by n) times (n by o) results in (m by o)
(2 by 2) times (2 by 1) results in (2 by 1)

Scaling

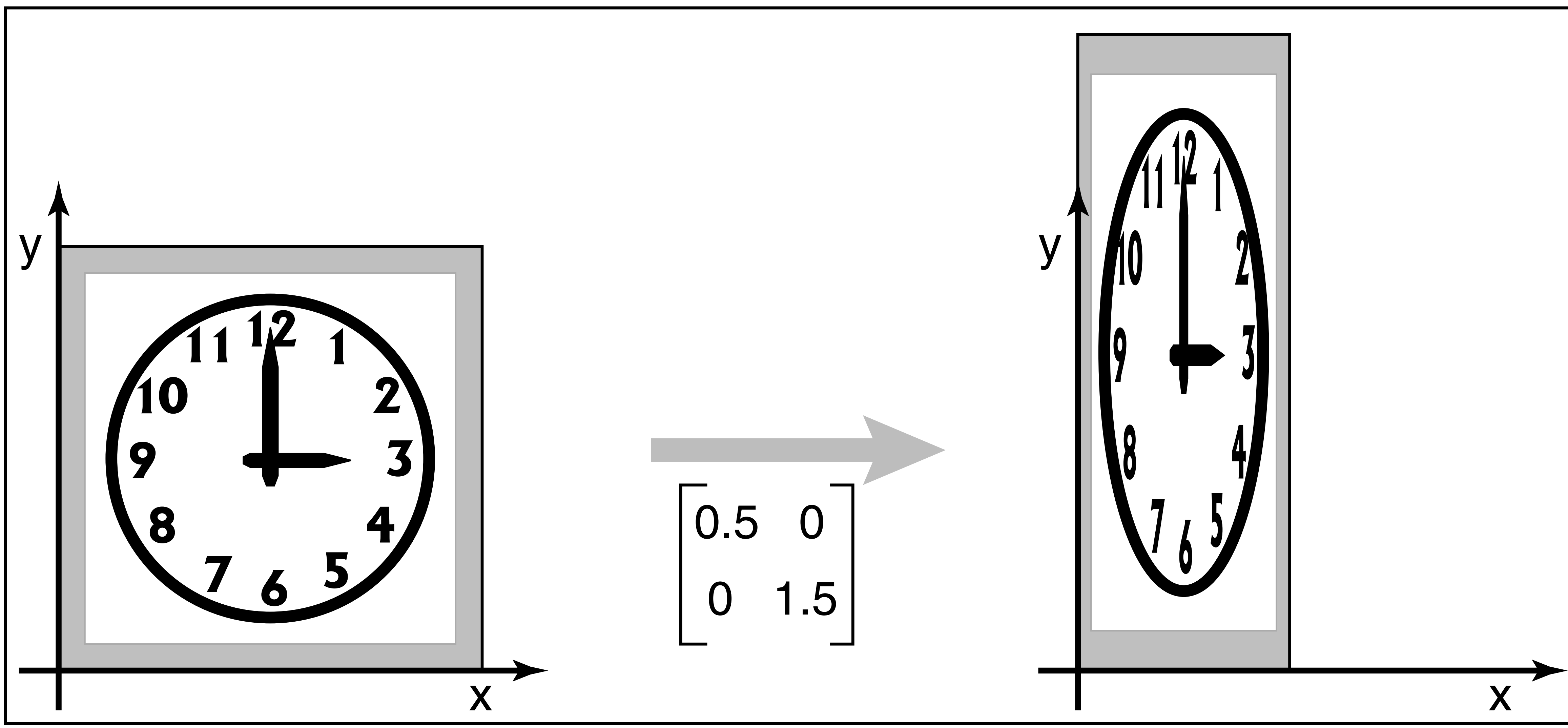
$$\text{scale}(s_x, s_y) = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$$

$$\begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} s_x x \\ s_x y \end{bmatrix}$$

$$\text{scale}(.5, .5) = \begin{bmatrix} .5 & 0 \\ 0 & .5 \end{bmatrix}$$



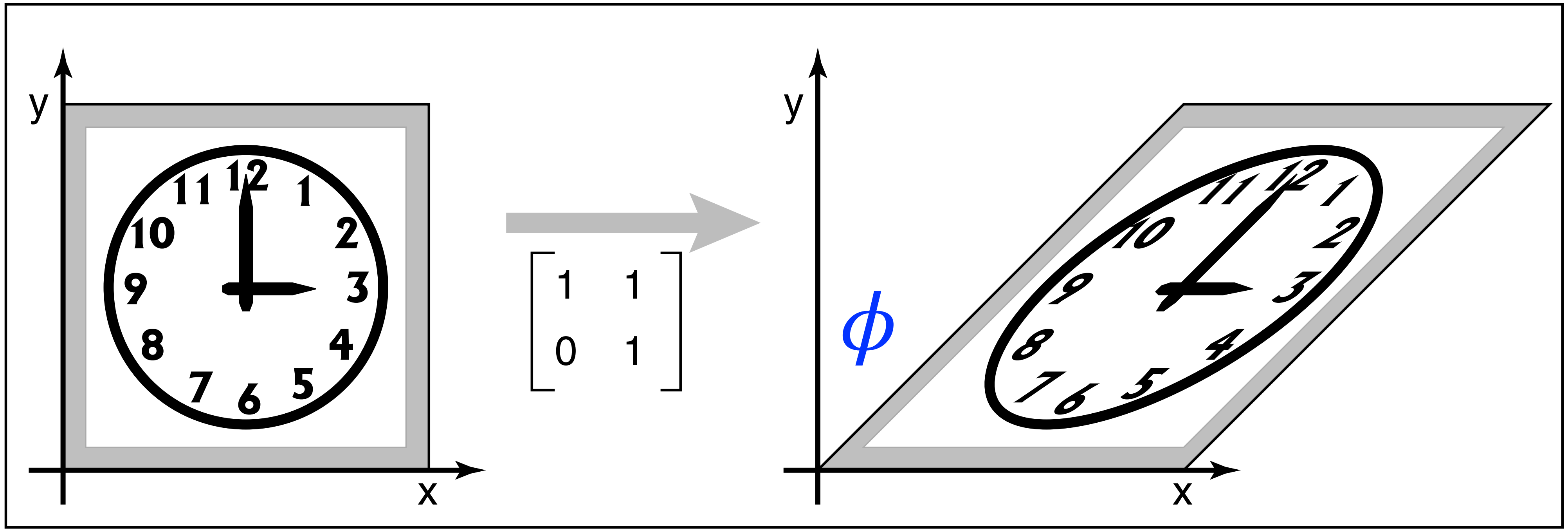
$$\text{scale}(.5, 1.5) = \begin{bmatrix} .5 & 0 \\ 0 & 1.5 \end{bmatrix}$$

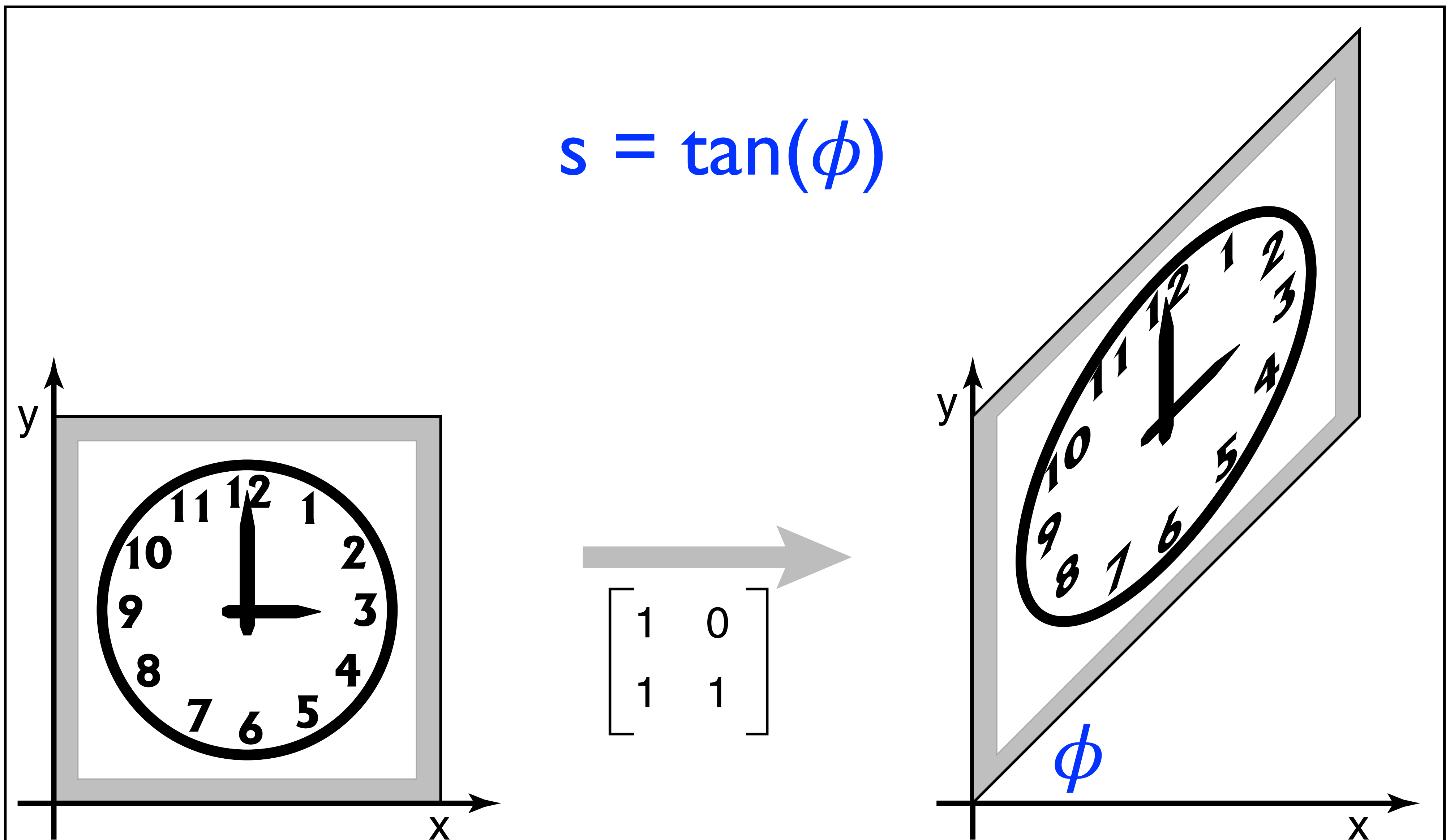


Shearing

$$\text{scale-x}(s) = \begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}$$

$$\text{scale-y}(s) = \begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}$$


y
x
1 1
0 1
y
x
φ

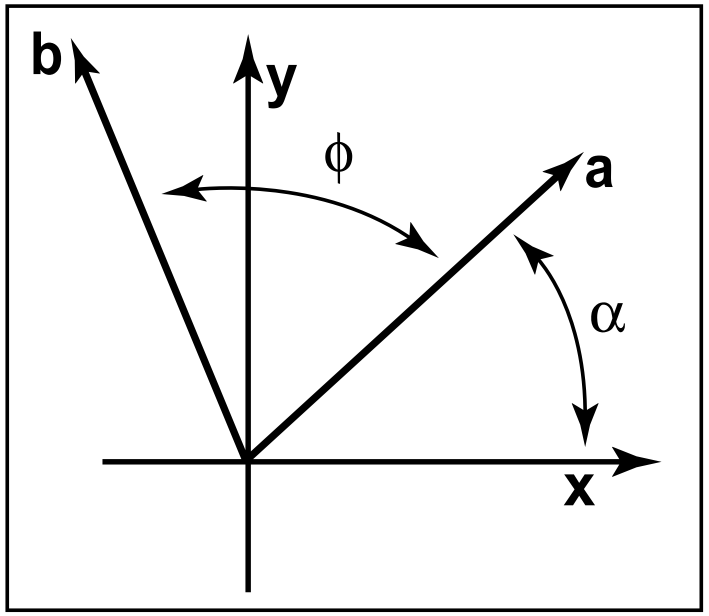

s = tan(φ)
y
x
1 0
1 1
y
x
φ

Rotation

$\mathbf{a} = (x_a, y_a)$

$\mathbf{b} = (x_b, y_b)$

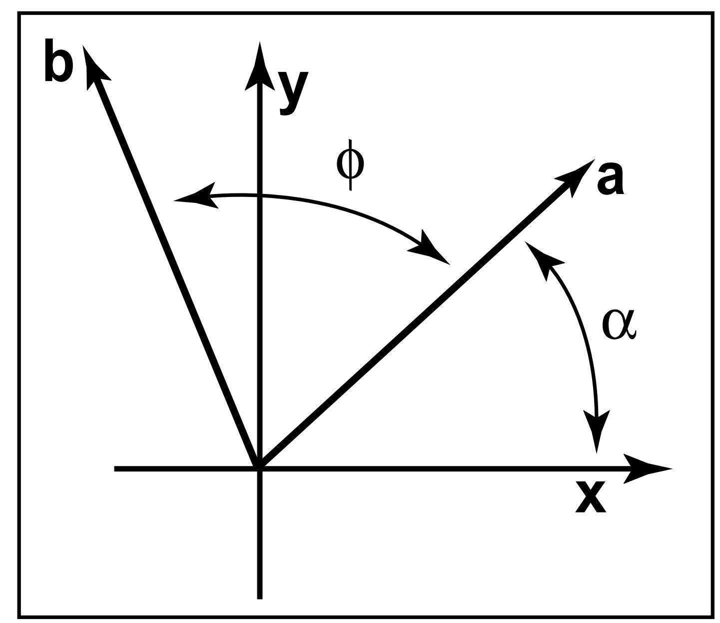
$\mathbf{b} = \text{rotate}(\phi)\mathbf{a}$



Rotation

$$x_a = r \cos \alpha$$
$$y_a = r \sin \alpha$$

$$x_b = r \cos (\alpha + \phi)$$
$$y_b = r \sin (\alpha + \phi)$$



Rotation

$$x_b = r \cos(\alpha + \phi) = r \cos\alpha \cos\phi - r \sin\alpha \sin\phi$$

$$y_b = r \sin(\alpha + \phi) = r \sin\alpha \cos\phi + r \cos\alpha \sin\phi$$

Rotation

$$x_a = r \cos \alpha$$
$$y_a = r \sin \alpha$$

$$x_b = r \cos(\alpha + \phi) = r \cos \alpha \cos \phi - r \sin \alpha \sin \phi$$
$$y_b = r \sin(\alpha + \phi) = r \sin \alpha \cos \phi + r \cos \alpha \sin \phi$$

$$x_b = r \cos(\alpha + \phi) = x_a \cos \phi - y_a \sin \phi$$
$$y_b = r \sin(\alpha + \phi) = y_a \cos \phi + x_a \sin \phi$$

Rotation

$$x_b = x_a \cos\phi - y_a \sin\phi$$

$$y_b = y_a \cos\phi + x_a \sin\phi$$

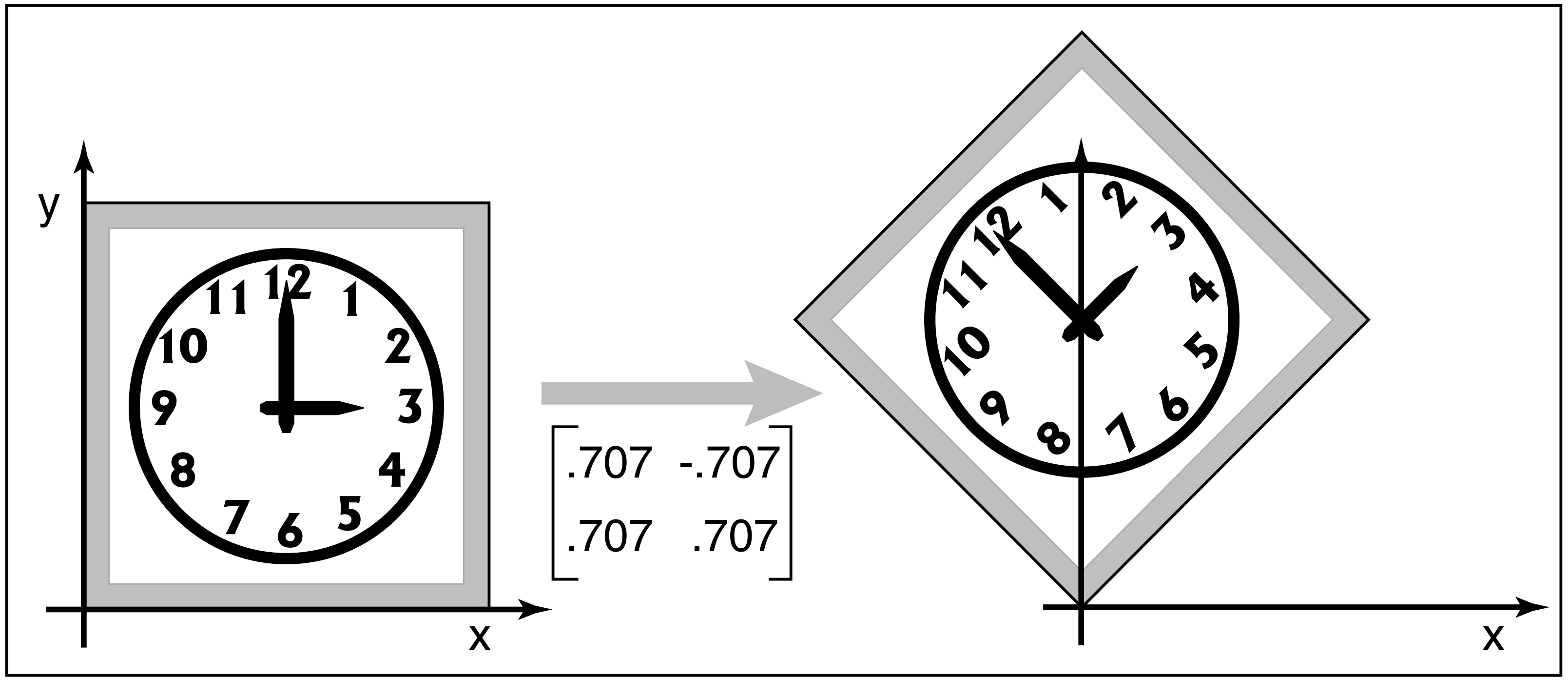
$$x_b = x_a \cos\phi - y_a \sin\phi$$

$$y_b = x_a \sin\phi + y_a \cos\phi$$

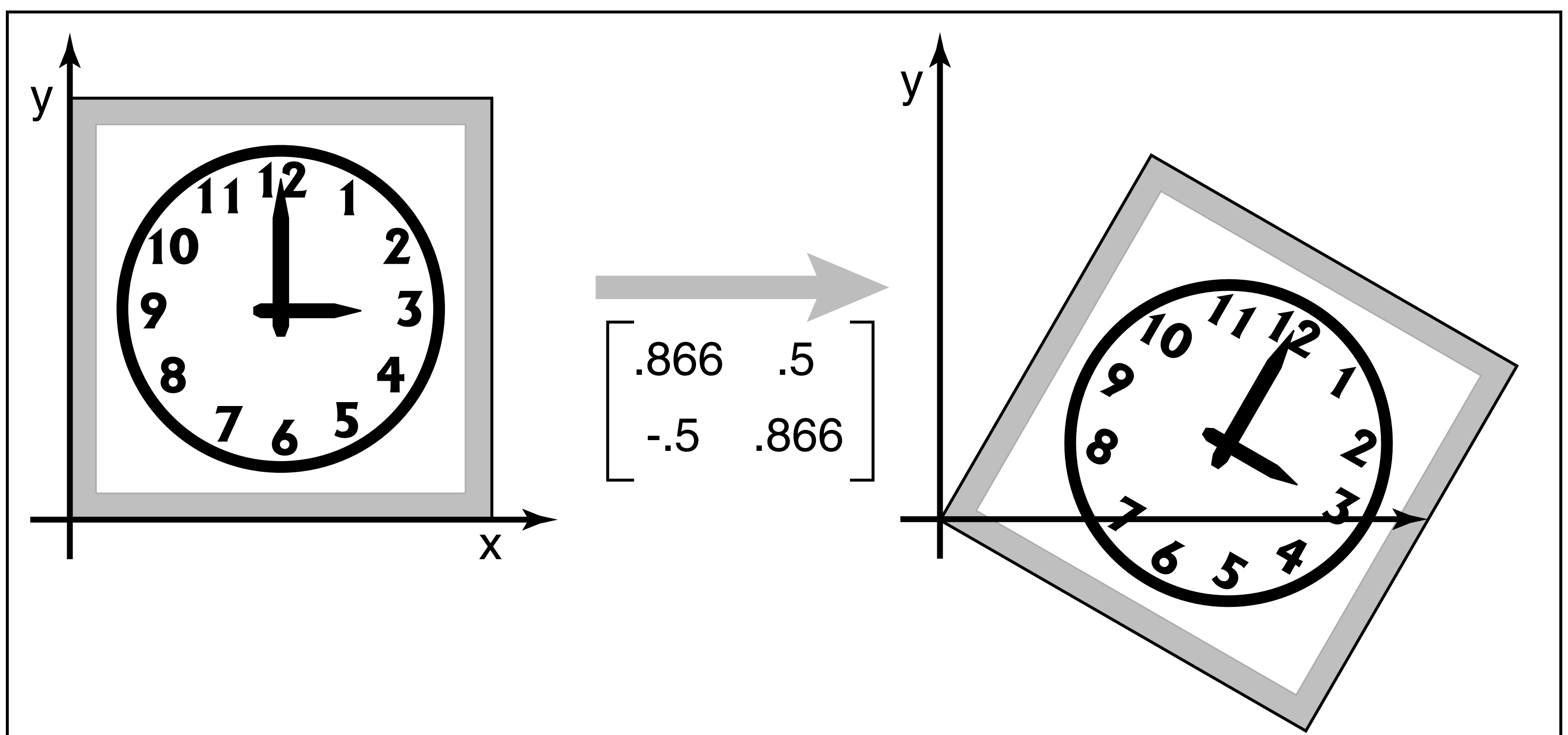
$$\begin{bmatrix} x_b \\ y_b \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} x_a \\ y_a \end{bmatrix}$$

Rotation

$$\text{rotate}(\phi) = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$$



ϕ = 45 deg = π/4

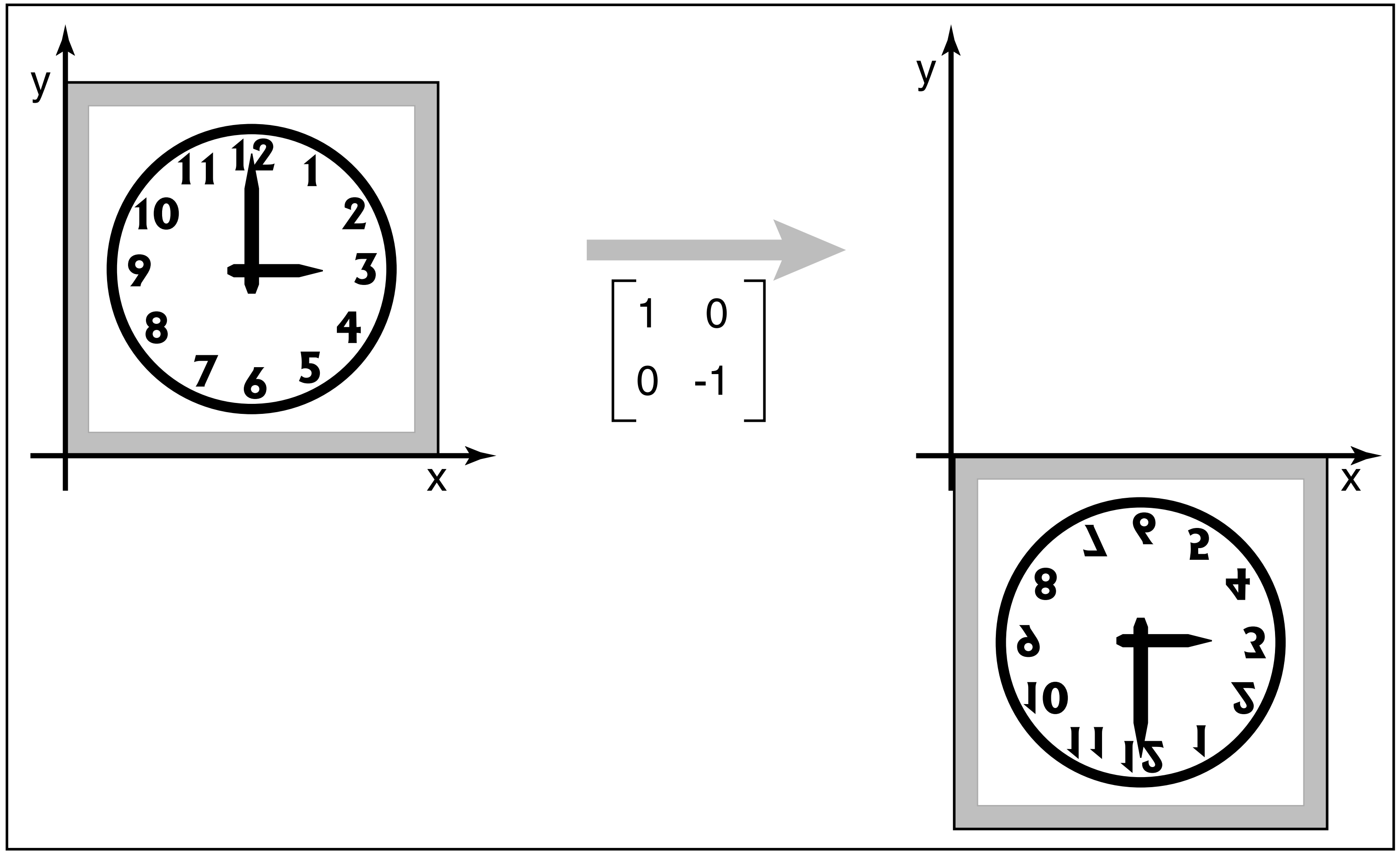


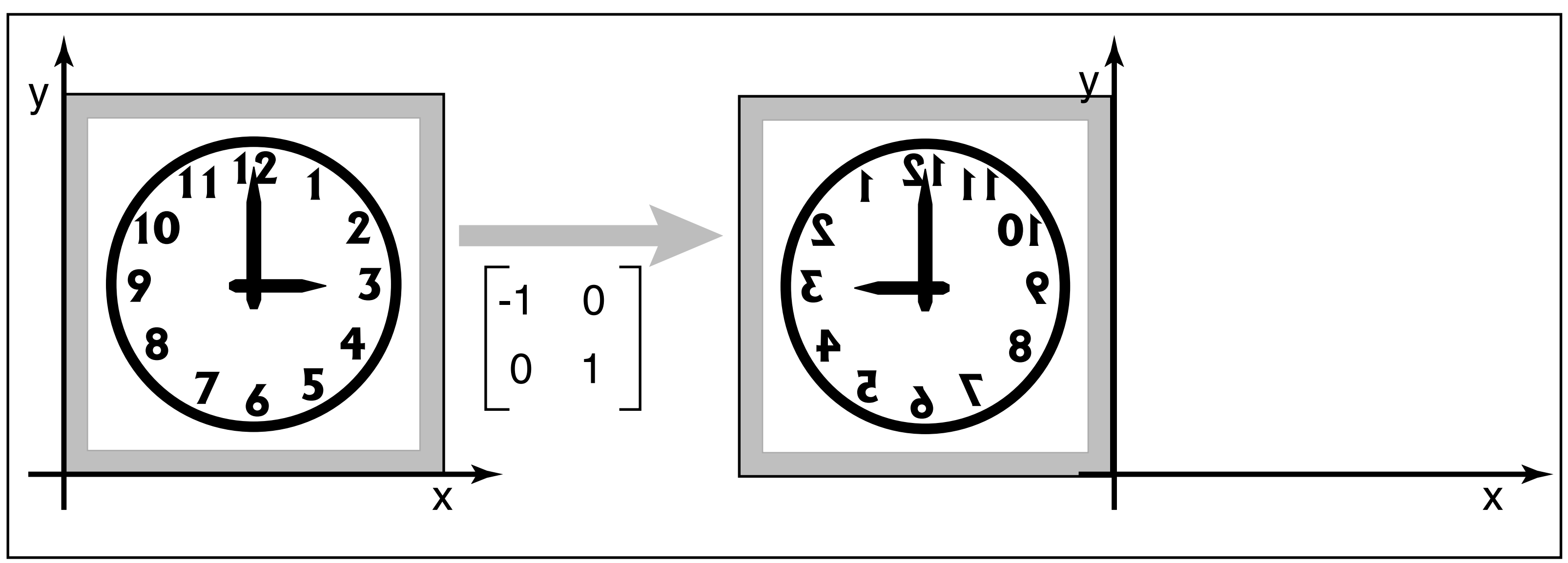
ϕ = -30 deg = π/6

Reflection

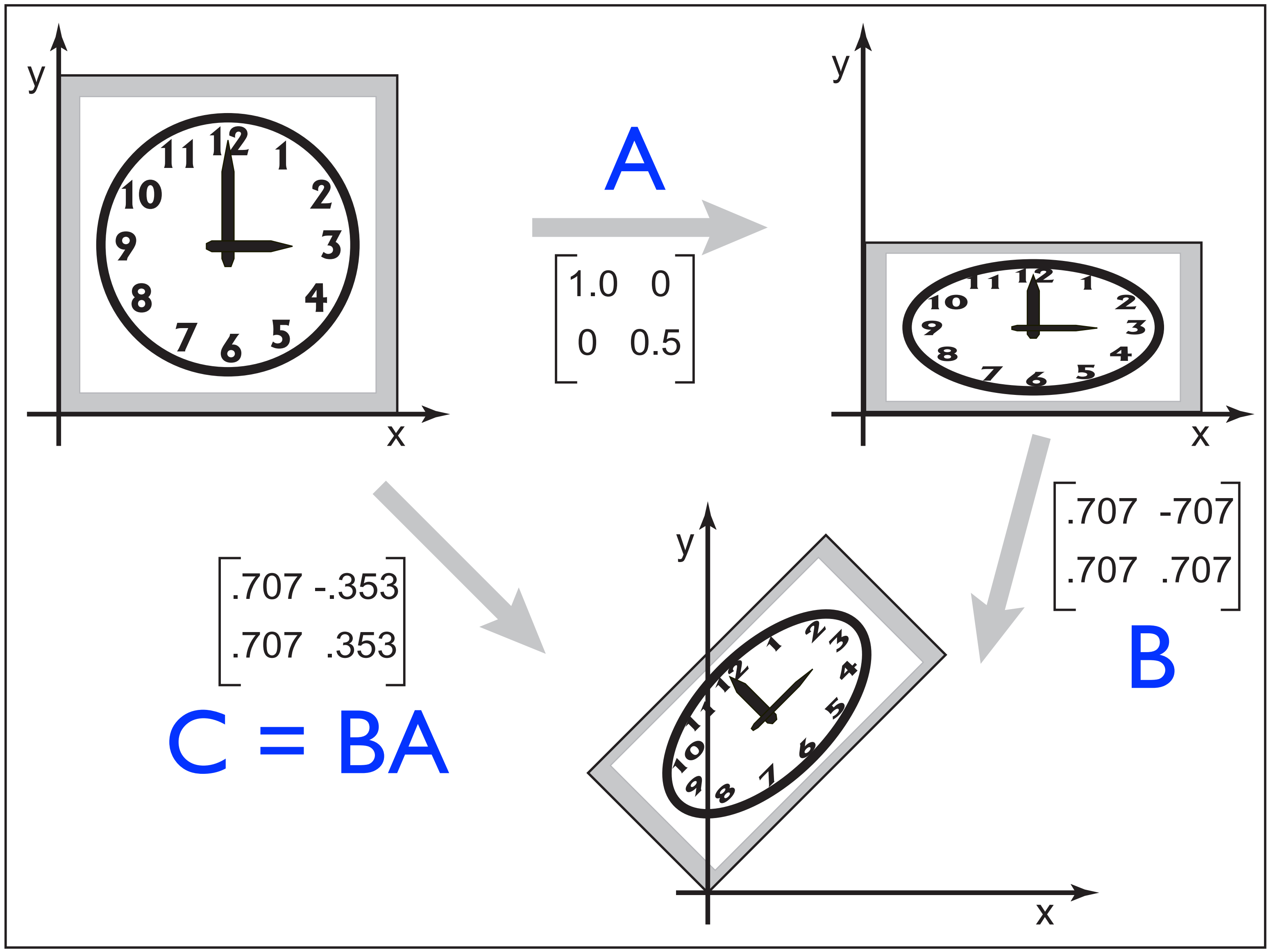
$$\text{reflect-y} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{reflect-x} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$


y
x
1 0
0 -1
y
x

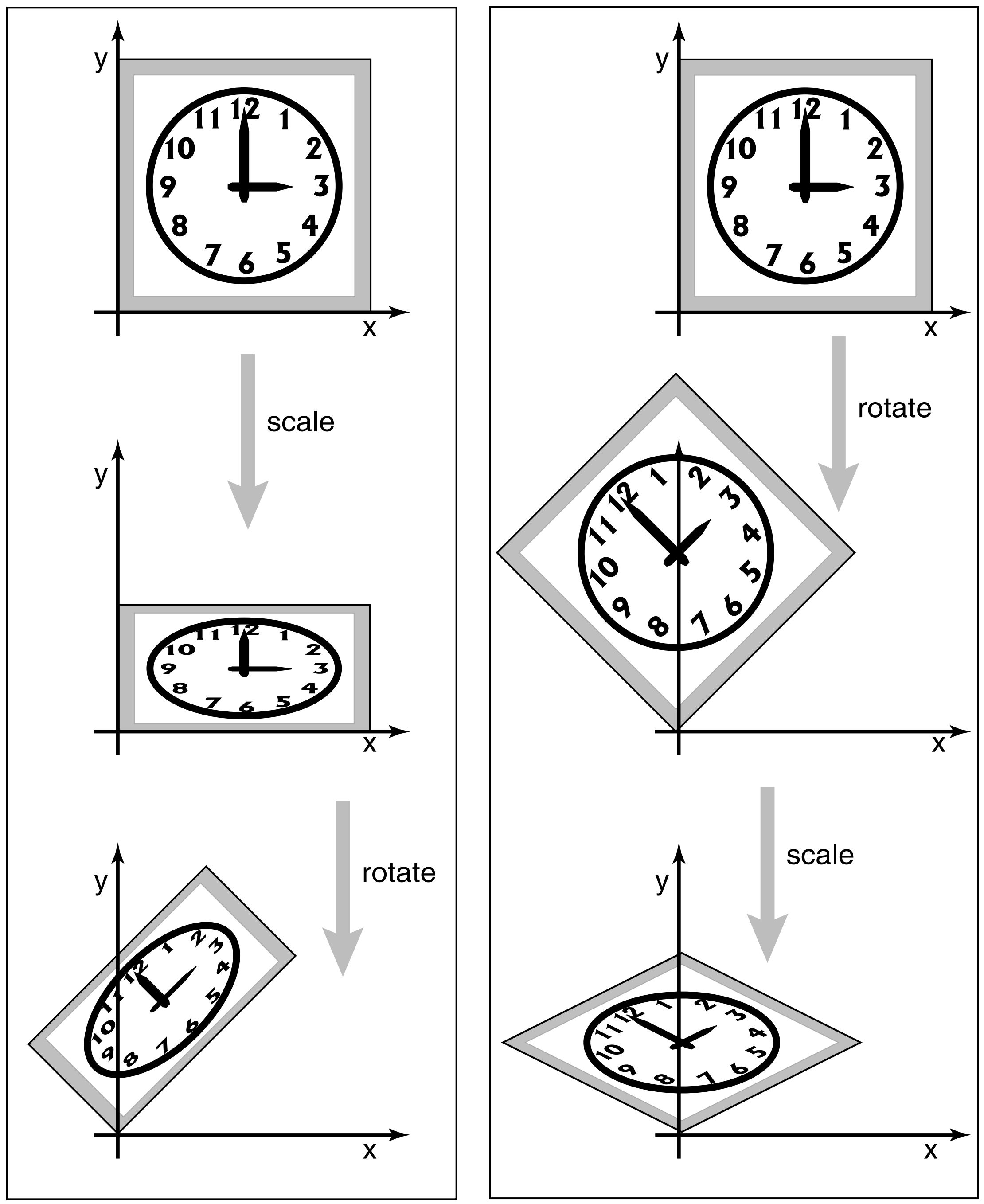

y
x
-1 0
0 1
y
x

Composition



$$B(A\mathbf{x}) = (BA)\mathbf{x} = C\mathbf{x}$$

Non commutativity



Order matters!