ASYMPTOTIC NOTATIONS

Definition Notation for upper bound (**Big-O**):

$$f(n) = \mathcal{O}(g(n))$$

if *there are* a constant $c > 0$ and a "threshold" n_0 so that

$$\text{for all } n > n_0 : \qquad f(n) \leq cg(n)$$

Example: $1000n^2 = \mathcal{O}(n^2)$

$$1000n^2 = \mathcal{O}(n^3) \qquad \text{but} \qquad n^3 \neq \mathcal{O}(n^2)$$

$$n^{200} = \mathcal{O}(2^n) \qquad \text{but} \qquad 2^n \neq \mathcal{O}(n^{200})$$

$$2^n = \mathcal{O}(5^n) \qquad \text{but} \qquad 5^n \neq \mathcal{O}(2^n)$$

$$n\log(n) = \mathcal{O}(n^2) \qquad \text{but} \qquad n^2 \neq \mathcal{O}(n\log(n))$$

Definition Notation for upper bound (**little-o**):

$$f(n) = o(g(n))$$

if *for all* $c > 0$, *there is* a "threshold" n_0 so that

$$\text{for all } n > n_0 : \qquad f(n) \leq cg(n)$$

Example:

$$n \log(n) = o(n^2)$$

$$n^{200} = o(2^n)$$

$$2^n = o(5^n)$$

Definition Notation for lower bound (**Big-Omega**):

$$f(n) = \Omega(g(n))$$

if *there are* a constant $c > 0$ and a "threshold" n_0 so that

$$\text{for all } n > n_0 : \qquad f(n) \geq cg(n)$$

Example

$$n^3 + 4n^2 + 12 = \Omega(n^3)$$

$$2^n = \Omega(n^{100})$$

$$n^2 \log(n) = \Omega(n^2)$$

Definition Notation for lower bound (**little-omega**):

$$f(n) = \omega(g(n))$$

if *for all* $c > 0$ *there is* a “threshold” n_0 so that

$$\text{for all } n > n_0 : \qquad f(n) \geq cg(n)$$

Example

$$n^3 + 4n^2 + 12 \neq \omega(n^3)$$

$$2^n = \omega(n^{100})$$

$$n^2 \log(n) = \omega(n^2)$$

Definition Notation for exact order (**Theta**):

$$f(n) = \Theta(g(n))$$

if both

$$f(n) = \mathcal{O}(g(n)) \qquad \text{and} \qquad f(n) = \Omega(g(n))$$

i.e., there are constants $c_1, c_2 > 0$ and a "threshold" n_0 so that

$$\text{for all } n > n_0 : \qquad c_1 g(n) \leq f(n) \leq c_2 g(n)$$

Example: $2^n + n^{100} = \Theta(2^n)$

$$6n^5 + 100n^4 + 15n^3 + 28n^2 - 5n + 18 = \Theta(n^5)$$

$$n \log(n) + 20n = \Theta(n \log(n))$$