CSC 373H (2007): Assignment 2

Worth 5%. Due June 21 at 6pm in lecture.

The work you submit must be your own. You may discuss problems with each others; however, you should prepare written solutions alone. Copying assignments is a serious academic offence, and will be dealt with accordingly.

Question 1 [Dynamic Programming]

Given n points $p_1, p_2, \ldots, p_n$ lying clockwise on a circle in that order (here $n \geq 3$). We want to form exactly $(n-2)$ triangles whose vertices are $p_1, p_2, \ldots, p_n$ by joining exactly $(2n-3)$ pairs (p_i, p_j) so that there is no crossing between any two pairs. (Note that $(n-2)$ is the maximum number of triangles can be obtained this way, and that among $(2n-3)$ pairs, the n pairs $(p_1, p_2), (p_2, p_3), \ldots, (p_{n-1}, p_n), (p_n, p_1)$ are always present.)

The cost of joining a pair (p_i, p_j) is $c(i, j)$, and we want to minimize the total cost. Give a dynamic programming algorithm that, on input the costs $c(i, j)$ (for $1 \leq i < j \leq n$), output the set of pairs to be joined that satisfies our requirement and that incurs the least possible cost. What is the running time of your algorithm (state your answer using Θ notation)?

Question 2 [Dynamic Programming]

Consider the alphabet $A = \{a, b, c\}$. The elements of A have the following multiplication table, where the rows show the left-hand symbol and the columns show the right-hand symbol.

	a	b	c
a	b	b	a
b	c	b	a
c	a	c	c

For example, $ab = b, ba = c$, and so on. Note that the multiplication defined by this table is neither commutative nor associative.

Find a dynamic programming algorithm that examines a string $X[1 \ldots n]$ of characters of A and decides whether or not it is possible to parenthesize X in such a way that the value of the resulting expression is a. For instance, if $X = bbbba$, your algorithm should return "yes" because $(b(bb))(ba) = a$. This expression is not unique, e.g., $(b(b(b(ba)))) = a$ as well. Note also that different parenthesizing may have different value. For example, for $X = aba$: $(ab)a = c$ but $a(ba) = a$. State the running time of your algorithm in terms of n using Θ notation.

Question 3 [Network Flow]

Consider a set of mobile computing *clients* in a certain town who each need to be connected to one of several possible *base stations*. There are n clients $C_1, C_2, \ldots, C_n$, with the position of each client specified by its (x, y) coordinates in the plane. There are also k base stations $B_1, B_2, \ldots, B_k$; the position of each of these is specified by its (x, y) coordinates as well.

For each client, we wish to connect it to exactly one of the base stations. Our choice of connections is constrained in the following ways:

- There is a *range parameter* r: A client can only be connected to a base station that is within distance r.
- There is also a *load parameter* L: no more than L clients can be connected to any single base station.

Given the positions of a set of clients and a set of base stations as well as the range and load parameters, decide whether every client can be connected simultaneously to a base station subject to the constraints above.