CSC 373: Assignment 1

Worth 10%. Due June 8 at the beginning of tutorial (8pm).

The work you submit must be your own. You may discuss problems with each others; however, you should prepare written solutions alone. Copying assignments is a serious academic offence, and will be dealt with accordingly.

Question 1 [Problem 5 (Chapter 4, page 190 in the text) with explicit proof of correctness.]

Let's consider a long, quiet country road with houses scattered very sparsely along it. (We can picture the road as a long line segment, with an eastern endpoint and a western endpoint.) Further, let's suppose that despite the bucolic setting, the residents of all these houses are avid cell phone users. You want to place cell phone base stations at certain points along the road, so that every house is within four miles of one of the base stations.

a) Give an efficient algorithm that achieves this goal, using as few base stations as possible.

b) Prove that your algorithm is correct.

Question 2 Suppose that there are n video streams to be sent over a communication link provided by a service provider. Each stream i consists of b_i bits in total, and needs to be sent (with a constant rate) over a time interval of length $t_i > 0$. Suppose also that two streams cannot be sent at the same time, and there cannot be any delay between the end of one stream and the next. (Thus the total time for sending all streams is $\sum_{i=1}^{n} t_i$.)

The problem is to minimize the bandwidth in the following sense. The timing for the transmission starts at 0. (So when all streams are sent, the time is $\sum_{i=1}^{n} t_i$.) We need to minimize the integer parameter r that satisfies the following condition:

$$\text{at any time } t \text{ during the transmission,}$$
$$\text{the total number of bits sent over the time interval from 0 to } t \text{ is} \le rt. \tag{1}$$

The service provider suggests the following value for r:

$$r^{\star} = \left\lceil \frac{\sum_{i=1}^{n} b_i}{\sum_{i=1}^{n} t_i} \right\rceil$$

Your task in this question is to show that $r^{\star}$ is actually the optimal value for r.

a) Show that any integer r that satisfies (1) must be $\ge r^{\star}$.

b) Give a greedy algorithm for sending the streams that satisfies condition (1) with $r = r^{\star}$.

c) Prove the correctness of your algorithm.

Question 3 Consider the following scheduling problem. There are n jobs $J_1, J_2, \ldots, J_n$ that need to be processed by a single processor. Each job J_i has processing time t_i. A schedule is an ordering of the jobs so that $J_{\pi(1)}$ is processed first, then $J_{\pi(2)}$, etc. Notice that the completion time for the job $J_{\pi(i)}$ is

$$C_{\pi}(i) = \sum_{k=1}^{i} t_{\pi(k)}$$

The objective is to minimize the average completion time of all jobs, i.e., give the ordering π with smallest value of

$$\frac{1}{n}\sum_{i=1}^{n} C_{\pi}(i)$$

a) Give a greedy algorithm that produces an optimal schedule.

b) Prove that your algorithm is correct.

Question 4 For part a) of this question, you can use the following Lemma, which shows one way of obtaining a spanning tree from an existing one:

Lemma Let T be a spanning tree of a graph $G = (V, E)$, $E' \subset T$ and $e \in E \setminus T$ so that $E' \cup \{e\}$ does not contain a cycle. Then there is an edge $e' \in T \setminus E'$ so that

$$T' = (T \setminus \{e'\}) \cup \{e\}$$

is a spanning tree of G.

Now you are asked to show that if

$$\text{the edges of } G \text{ has distinct weights} \tag{2}$$

then

$$G \text{ has a unique minimum spanning tree.} \tag{3}$$

This can be proved in several ways. One way is to prove the following statement: Let E_i' be the partial solution obtained after iteration i in the Kruskal's algorithm, then

$$E_i' \text{ is contained in } \textit{every} \text{ minimum spanning tree of } G. \tag{4}$$

a) Assume (2), prove (4) by induction on i.

b) Conclude that (2) implies (3).