1. Recall that a Hamiltonian cycle in a graph $G = (V, E)$ is a cycle that visits each vertex exactly once. In other words, if $n = |V|$, a Hamiltonian cycle is a permutation $\varphi(1), \ldots, \varphi(n)$ of $V$ such that $(\varphi(i), \varphi(i+1)) \in E$ for $1 \leq i < n$ and $(\varphi(n), \varphi(1)) \in E$. The decision problem HAM-CYCLE is described below.

**Instance:** A directed graph $G = (V, E)$
**Question:** Does $G$ have a Hamiltonian cycle?

Prove that HAM-CYCLE is **NP**-complete.

2. The traveling salesman decision problem (TSP) is defined below.

**Instance:** A distance function $d : [n]^2 \mapsto \mathbb{R}^+$, and $D \in \mathbb{R}^+$
**Question:** Does there exist a permutation $\varphi$ of $\{1, \ldots, n\}$ such that

$$\sum_{i=1}^{n-1} d(\varphi(i), \varphi(i+1)) + d(\varphi(n), \varphi(1)) \leq D$$

Prove that TSP is **NP**-complete.