

1. The problem SET-COVER asks, given a collection  $S_1, \dots, S_n$  of sets whose union is  $U$ , and a natural number  $k$ , does there exist a subcollection of at most  $k$  of the sets whose union is  $U$ ?

**Instance:** Sets  $S_1, \dots, S_n$ , and  $k \in \mathbb{N}$

**Question:** Does there exist  $C \subseteq \{1, \dots, n\}$  s.t.  $|C| \leq k$  and  $\bigcup_{i \in C} S_i = \bigcup_{i=1}^n S_i$ ?

Prove that SET-COVER is **NP**-complete.

2. An independent set in a graph  $G = (V, E)$  is a subset  $I \subseteq V$  of vertices such that there are no edges between vertices in  $I$ , i.e. for all  $u, v \in I$ ,  $\{u, v\} \notin E$ . The INDEPENDENT-SET decision problem is defined below.

**Instance:** A graph  $G = (V, E)$ , and  $k \in \mathbb{N}$

**Question:** Does  $G$  have an independent set of size  $k$  or larger?

Prove that INDEPENDENT-SET is **NP**-complete.

3. Prove that CLIQUE is **NP**-complete.