The job-interval scheduling problem (JISP) is defined below.

Instance: $(s_1, f_u, \mu_1), \ldots, (s_n, f_n, \mu_n)$, where $s_i, f_i, \mu_i \in \mathbb{N}$ for $1 \leq i \leq n$
Solution: A subset $I \subseteq \{1, \ldots, n\}$ such that

1. $[s_i, f_i) \cap [s_j, f_j) = \emptyset$ for all $i \neq j \in I$
2. $\mu_i \neq \mu_j$ for all $i \neq j \in I$

Objective: Maximize $|I|$

Intuitively, all the intervals with the same μ-value represent possible times when the same job could be scheduled. The goal is to schedule as many jobs as possible.

Below is an algorithm called EARLIEST-FINISH-TIME (EFT) for JISP.

sort intervals so that $f_1 \leq f_2 \leq \cdots \leq f_n$
$S \leftarrow \emptyset$
$t \leftarrow 0$
for $i \leftarrow 1, \ldots, n$ **do**
  if $\mu_i \neq \mu_j$ for all $j \in S$ and $s_i \geq t$ **then**
    $S \leftarrow S \cup \{i\}$
    $t \leftarrow f_i$
  end if
end for

1. Give an example of an input where EFT achieves an approximation ratio of exactly 1/2.

2. Show that EFT is a 1/2-approximation algorithm for JISP.