

NP-complete problems

SAT

Instance: A boolean formula φ

Question: Is φ satisfiable?

3SAT

Instance: A boolean formula φ in 3CNF

Question: Is φ satisfiable?

SUBSET-SUM

Instance: $s_1, \dots, s_n, t \in \mathbb{Z}$

Question: Does there exist a subset $S \subseteq \{1, \dots, n\}$ such that $\sum_{i \in S} s_i = t$?

PARTITION

Instance: $s_1, \dots, s_n \in \mathbb{Z}$

Question: Does there exist $S \subseteq \{1, \dots, n\}$ such that $\sum_{i \in S} s_i = \sum_{j \notin S} s_j$?

KNAPSACK

Instance: $w_1, \dots, w_n \in \mathbb{N}; p_1, \dots, p_n \in \mathbb{N}; W, P \in \mathbb{N}$

Question: Does there exist $S \subseteq \{1, \dots, n\}$ such that $\sum_{i \in S} w_i \leq W$ and $\sum_{i \in S} p_i \geq P$?

VERTEX-COVER

Instance: A graph $G = (V, E)$, and $k \in \mathbb{N}$

Question: Does G have a vertex cover of size k or smaller?

INDEPENDENT-SET

Instance: A graph $G = (V, E)$ and $k \in \mathbb{N}$

Question: Does G have an independent set of size k or larger?

CLIQUE

Instance: A graph $G = (V, E)$, and $k \in \mathbb{N}$

Question: Does G have a clique of size k or larger?

SET-COVER

Instance: Sets S_1, \dots, S_n , and $k \in \mathbb{N}$

Question: Does there exist $I \subseteq \{1, \dots, n\}$ such that $|I| \leq k$ and $\bigcup_{i \in I} S_i = \bigcup_{i=1}^n S_i$?

HAM-PATH

Instance: A directed graph $G = (V, E)$, and $s, t \in V$

Question: Does G have a Hamiltonian path from s to t ?

HAM-CYCLE

Instance: A directed graph $G = (V, E)$

Question: Does G have a Hamiltonian cycle?

HAM-PATH-EXISTS

Instance: A directed graph $G = (V, E)$

Question: Does G have a Hamiltonian path (between any two vertices)?

TSP

Instance: A distance function $d : \{1, \dots, n\} \times \{1, \dots, n\} \mapsto \mathbb{N}$, and $k \in \mathbb{N}$

Question: Does there exist a permutation $\varphi : \{1, \dots, n\} \mapsto \{1, \dots, n\}$ such that $\sum_{i=1}^{i-1} d(\varphi(i), \varphi(i+1)) + d(\varphi(n), \varphi(1)) \leq k$?