Let $\alpha \geq 1$ be given. We describe an $\alpha$-gap-introducing reduction from HAM-PATH-EXISTS to MIN-WAIT-TIME-OPT.

The reduction is specified by the pair of functions $(f, g)$. The function $f$ is defined by $f(\langle G = (V, E)\rangle) = \langle d \rangle$, where $d : [n]^2 \mapsto \mathbb{R}^+$ ($n = |V|$) is given by

$$d(i,j) = \begin{cases} 1, & \text{if } (i,j) \in E \\ \alpha \cdot \frac{1}{2} \cdot n(n-1) + 1, & \text{otherwise} \end{cases}$$

The function $g$ is defined by $g(\langle G = (V, E)\rangle) = \frac{1}{2} \cdot n(n-1)$. Suppose that $G$ has a Hamiltonian path. Then the same path (viewed as a permutation of $[n]$) has a total wait time of $1 + 2 + 3 + \cdots + (n-1) = n(n-1)/2$, and thus

$$\text{OPT}(f(\langle G \rangle)) \leq \frac{1}{2} \cdot n(n-1) = g(\langle G \rangle)$$

On the other hand, if $G$ has no Hamiltonian path, then every permutation must traverse a non-edge of $G$, and thus the optimal value for the instance $\langle d \rangle$ of MIN-WAIT-TIME-OPT is

$$\text{OPT}(f(\langle G \rangle)) \geq \alpha \cdot \frac{1}{2} \cdot n(n-1) + 1 > \alpha \cdot g(\langle G \rangle)$$

Both $f$ and $g$ are easily seen to be computable in polynomial time: $f$ can be computed in roughly linear time by considering each pair of vertices and outputting one of two possible values, depending on whether there is an edge present; and $g$ can also be computed in roughly linear time by counting the number $n$ of vertices in the input graph and computing the value $n(n-1)/2$.