Imagine your job is to plan a route for a delivery-person to make deliveries to n different clients $C_1, \ldots, C_n$. For each pair of clients $i \neq j$, $t(i, j)$ is the time required to travel from client i to client j. A schedule is a one-to-one, onto mapping $\sigma : [n] \mapsto [n]$, where $[n]$ denotes the set $\{1, \ldots, n\}$, corresponding to visiting the clients in the order $\sigma(1), \sigma(2), \ldots, \sigma(n)$. The wait time for client i in a schedule σ is the time it takes the delivery-person to reach client i:

$$W_\sigma(i) = \sum_{j=1}^{k-1} t(\sigma(j), \sigma(j+1))$$

where k is such that $\sigma(k) = i$. The objective is to minimize the average wait time among the clients, or equivalently, to minimize the total wait time of all clients (the average and total wait times differ by a factor of n, so minimizing one is equivalent to minimizing the other). A decision version of this problem, called MIN-WAIT-TIME, is formulated below.

Instance: A time function $t : [n]^2 \mapsto \mathbb{N}$, and $W \in \mathbb{N}$
Question: Does there exist a schedule $\sigma : \{1, \ldots, n\} \mapsto \{1, \ldots, n\}$ such that

$$\sum_{i=1}^{n} W_\sigma(i) \leq W$$

Prove that MIN-WAIT-TIME is **NP**-complete.

Hint: Try reducing from HAM-PATH.