

Imagine that you still work for the corporation from question 1. You have a new line of products  $P_1, \dots, P_n$  which you wish to have in stores as quickly as possible. To manufacture the products, you have at your disposal  $M$  factories in developing nations (think of the number of factories as being much smaller than the number of products). Each product  $P_i$  requires time  $t_i$  to manufacture, and this work cannot be split among factories. Your job is to find an allocation  $\sigma : \{1, \dots, n\} \mapsto \{1, \dots, M\}$  of products to factories such that all the products are ready after  $T$  units of time. A decision version of this problem, called PRODUCT-ALLOCATION, is formulated below.

**Instance:**  $t_1, \dots, t_n, M, T \in \mathbb{N}$

**Question:** Does there exist an allocation  $\sigma : \{1, \dots, n\} \mapsto \{1, \dots, M\}$  such that

$$\max_{1 \leq j \leq M} \left\{ \sum_{i: \sigma(i)=j} t_i \right\} \leq T$$

Prove that PRODUCT-ALLOCATION is **NP**-complete.

**Hint:** Try reducing from PARTITION.