

Imagine that you are the chief strategist for a multinational chain of retail stores. You have a set  $\mathcal{L}$  of locations at which you might open a new store, and a set  $\mathcal{C}$  of cities which you wish to serve. For each city  $c \in \mathcal{C}$  and each location  $\ell \in \mathcal{L}$ ,  $d(c, \ell)$  is the distance from  $c$  to  $\ell$ . To open a store at location  $\ell \in \mathcal{L}$  will require you to pay a bribe  $b(\ell)$  to the local government to obtain the necessary permission. You have determined that a customer is likely to shop at your store if there is a location at distance  $D$  or less. Your task is to find a set  $S \subseteq \mathcal{L}$  of locations to open, such that every city has a location at distance  $D$  or less, which minimizes the cost of the bribes you will need to pay.

Below is a decision version of this problem, called STORE-OPENING. Prove that STORE-OPENING is **NP**-complete.

**Instance:** A set  $\mathcal{C}$  of cities, a set  $\mathcal{L}$  of locations, a distance function  $d : \mathcal{C} \times \mathcal{L} \mapsto \mathbb{N}$ , a cost function  $b : \mathcal{L} \mapsto \mathbb{N}$ , and  $B, D \in \mathbb{N}$ .

**Question:** Does there exist a set  $S \subseteq \mathcal{L}$  such that  $\sum_{\ell \in S} b(\ell) \leq B$ , and for all  $c \in \mathcal{C}$ ,

$$\min_{\ell \in S} \{d(c, \ell)\} \leq D$$

**Hint:** Try reducing from SET-COVER, so that locations correspond to sets.