

In class, we said that a Turing machine has a tape which is infinite in both directions.

In the textbook, Turing machines are defined to have a tape which is only infinite in one direction; that is, the tape has a left end but is infinite to the right. The input is written on consecutive cells, starting from the left-most cell. Additionally, if the tape-head is scanning the left-most cell and attempts to move to the left, it remains over the left-most cell.

Prove that Turing machines with a doubly infinite tape decide and recognize the same class of languages as Turing machines with a singly infinite tape. That is, show how to simulate a doubly-infinite tape TM with a singly-infinite tape TM, and vice-versa.