
CSC236 / Introduction to the Theory of Computation

Tutorial Exercises

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1. Consider the following first-order language \mathcal{L} .

Predicate symbols: A (arity 1)
 B (arity 2)
 C (arity 3)

Function symbols: c (arity 0)
 f (arity 1)
 g (arity 2)

Here is a structure S for \mathcal{L} :

$$\begin{aligned} \mathcal{D} &= \{\square, \triangle\} \\ A^S(x) &= \begin{cases} 1, & \text{if } x = \triangle \\ 0, & \text{if } x = \square \end{cases} \\ B^S(x, y) &= \begin{cases} 1, & \text{if } x = \triangle \text{ or } y = \square \\ 0, & \text{otherwise} \end{cases} \\ C^S(x, y, z) &= \begin{cases} 1, & \text{if } x = \square \text{ or } y = z = \triangle \\ 0, & \text{otherwise} \end{cases} \\ c^S &= \square \\ f^S(x) &= \begin{cases} \triangle, & \text{if } x = \square \\ \square, & \text{otherwise} \end{cases} \\ g^S(x, y) &= \begin{cases} \square, & \text{if } x = \square \text{ and } y = \triangle \\ \triangle, & \text{otherwise} \end{cases} \end{aligned}$$

Here is a valuation σ for \mathcal{L} :

$$\begin{aligned} \sigma(x) &= \square \\ \sigma(y) &= \square \\ \sigma(z) &= \triangle \end{aligned}$$

For each of the following first-order formulas over \mathcal{L} , state whether the formula is true or false in the interpretation $I = (S, \sigma)$.

- (a) $\forall x B(x, f(x))$
- (b) $\exists z \neg C(x, y, z)$
- (c) $\forall x \exists y \forall z \left((A(z) \wedge B(x, z)) \rightarrow C(x, y, z) \right)$