
CSC236 / Introduction to the Theory of Computation

Tutorial Exercises

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1. Below is a recursive definition of a set of strings. Some notation: if x and y are strings, then $|x|$ denotes the length of x , and for $1 \leq i \leq |x|$, x_i denotes the i -th symbol in x ; xy denotes the concatenation of x and y .

Let S be the smallest set such that

- $0, 1 \in S$
- If $x, y \in S$, and $x_{|x|} \leq y_1$, then $xy \in S$

Let us call a binary string x *sorted* if $x_i \leq x_j$ for every $1 \leq i \leq j \leq |x|$.

- (a) Use induction to prove that for every sorted binary string x , and every $1 \leq i \leq |x|$, $x_1 \leq x_i \leq x_{|x|}$
- (b) Use structural induction to prove that every string in S is sorted.