Hint for Question 5

This is a hint to help you come up with a loop invariant. Suppose that a is a natural number represented in decimal by $a_{n-1} \cdots a_0$, and b is a natural number represented in decimal by $b_{n-1} \cdots b_0$. By the definition of "decimal representation",

$$a = \sum_{i=0}^{n-1} a_i \cdot 10^i$$

$$b = \sum_{i=0}^{n-1} b_i \cdot 10^i$$

Adding the two equations together, we have

$$(a + b) = \sum_{i=0}^{n-1} (a_i + b_i) \cdot 10^i$$

Notice that, if $0 \leq (a_i + b_i) \leq 9$ for all i, then the sequence $(a_{n-1} + b_{n-1}) \cdots (a_0 + b_0)$ is in fact the decimal representation of the sum $(a + b)$. Now suppose just that $0 \leq (a_0 + b_0) \leq 9$. Then the digit z_0 computed by the algorithm is equal to $a_0 + b_0$, and so we have

$$(a + b) = \sum_{i=1}^{n-1} (a_i + b_i) \cdot 10^i + z_0 \cdot 10^0$$

In general, we cannot assume that $(a_i + b_i)$ is between 0 and 9, which is where the carry c comes in. In trying to formulate a loop invariant predicate $L(i)$, I would suggest trying to come up with an expression which includes the first i computed digits $z_{i-1} \cdots z_0$, as well as the remaining terms $(a_{n-1} + b_{n-1}) \cdots (a_i + b_i)$, and the carry bit c, so that the expression is equal to $a + b$ (like in the example above).