CSC236 / Introduction to the Theory of Computation
Homework #2
Due date: August 8

Part 1

1. For each of the following languages over the alphabet $\Sigma = \{0, 1\}$, define a regular expression that accepts the language.

   (a) $L_1 = \{x \mid x \text{ contains the substring } 01001\}$

   (b) $L_2 = \{x \mid x \text{ contains an even number of 0's and an even number of 1's}\}$

   (c) $L_3 = \{x \mid x \text{ contains a substring of length at most 5 that has at least three 1's}\}$

2. For each language in question 1, define a deterministic finite state automaton that accepts the language.

3. For a language L over an alphabet Σ, let $L^{(2)}$ denote the following language over Σ^2:

$$L^{(2)} = \{(x_1, y_1) \cdots (x_n, y_n) \mid \text{either } x_1 \cdots x_n \in L \text{ or } y_1 \cdots y_n \in L\}$$

   Prove that if L is a regular language, then $L^{(2)}$ is a regular language.

Part 2

4. In this question we consider deterministic finite automata which are augmented with a finite "memory" which, for some constant $m \in \mathbb{N}$, stores the last m input symbols that were read. We shall call such an automaton a DFSA-M.

   A DFSA-M is a tuple $(Q, \Sigma, \delta, s_0, F, m, M, \{f_q \mid q \in M\})$, where $Q, \Sigma, \delta, s_0, F$ are as in the definition of a DFSA, and

   - $m \in \mathbb{N}$ is the size of the memory
   - $M \subseteq Q$ is a set of special "function" states
   - For each function state $q \in M$ there is a function $f_q : \Sigma^m \mapsto Q$

   The computation of a DFSA-M is the same as the computation of a DFSA, with the following exception: if the DFSA-M enters a function state $q \in M$, and $x_1, \ldots, x_m \in \Sigma$ are the last m symbols read by the automaton, then the automaton immediately enters the state $f_q(x_1, \ldots, x_m)$. If the automaton has not yet read m symbols, then computation ends and the input is rejected.

   Prove that the class of languages accepted by a DFSA-M is exactly the class of regular languages.

5. Prove that the following languages over the alphabet $\{0, 1\}$ are not regular.

   (a) $L_4 = \{xx \mid x \in \{0, 1\}^*\}$

   (b) $L_5 = \{x \mid x \text{ contains more 1's than 0's}\}$