Strategy-Proofness in the Stable Matching Problem with Couples

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Stable Matching Problem (SMP)

• Two-sided matching problem

• *Stable matching*: no resident-hospital pair prefers each other to their current matching

• Polynomial-time algorithm: “deferred acceptance” (Gale and Shapley, 1962)
Stable Matching Problem with Couples (SMP-C)

• Same objective as before, but couples can apply together

• NP-Complete
Significance of SMP-C

- United States National Resident Matching Program (NRMP): 34,905 residents, 6% in couples
- Smaller markets in Canada, Israel, Scotland...
Contributions

• Use satisfiability (SAT) encoding for SMP-C to analyze strategic properties of SMP-C
  • Analyze a conjecture and result from SMP

• Some new theory relevant to strategy-proofness in SMP-C

• Implement a mechanism for SMP-C with good strategic properties
Strategic Concerns in the NRMP

• NRMP algorithm redesigned in 90s
• New algorithm designed to make manipulation by residents as hard as possible
• Study of manipulations has focused on *truncations*
Truncation Example

Rankings:
1: Doctor > Hospital
2: Doctor > Hospital
3: Doctor > Hospital
4: Doctor > Hospital

Stable Matchings:
\[ \mu_1 \]
\[ \mu_2 \]
Truncations in NRMP

• Roth and Peranson (1999): at most 0.01% of residents and 0.1% of hospitals have an incentive to truncate
  • Very few opportunities for truncating on either side
  • Roth and Peranson conjectured that market size plays a role
Market Size and Strategy-Proofness in SMP

• Let $n$ be the market size
• Let $k$ be the preference list length
• Roth and Peranson (1999): “even when preferences are uncorrelated, as $k/n$ becomes small, the set of stable matchings becomes small.”
• Immorlica and Mahdian (2005) proved that, for SMP, expected fraction of residents with more than one stable hospital approaches zero as $n$ approaches infinity (for fixed $k$)
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Outline

• Introduction and Contributions ✓
• Theory of Strategy-Proofness
• Finding Stable Matchings in SMP-C
• Empirical Results
• Conclusion
Why Truncations?

• In SMP, truncations are sufficient for manipulation (Roth and Vande Vate, 1991)
• Out of all manipulations, truncations can be identified with the least information about others’ prefs (Roth and Rothblum, 1999)
• Easy to check empirically if a resident can benefit by truncating
Definitions: Resident Preferred

- $\mu$ is *resident preferred* ($\succeq_R$) to $\mu'$ if, for each resident or couple $a$, $\mu(a) \succeq_a \mu'(a)$
  - All residents and couples at least as well off
Definitions: Resident Optimal

• $\mu$ is resident optimal ($R_{opt}$) if, for all $\mu', \mu \succeq_R \mu'$
  • No resident or couple can do better in a stable matching

• Theorem (this paper): in SMP-C, residents can’t benefit by truncating in an $R_{opt}$ matching
Definitions: Resident Pareto Optimal

• New, but natural extension
• $\mu$ is resident Pareto optimal ($\mathcal{RP}_{\text{opt}}$) if there is no $\mu'$ such that $\mu' \succeq_R \mu$
  • Always exists in SMP and SMP-C
• All $\mathcal{R}_{\text{opt}}$ matchings are $\mathcal{RP}_{\text{opt}}$
Strategy in SMP-C: Resident Pareto Optimal Matchings

• Theorem (this paper): no stable mechanism is strategy-proof against resident truncations
  • WLOG, mechanism chooses $\mu_1$
  • Some residents prefer $\mu_2$
Strategy in SMP-C: Random Stable Matchings

• Theorem (this paper): random stable mechanism may be strategy-proof when $\mathcal{RP}_{opt}$ mechanism is not
  • Suppose $\mu_1 >_r \mu_2 >_r \mu_3 >_r \mu_4$
  • $r$ truncates below $\mu_1(r)$
  • Truncating increases chance of being unmatched
  • Depends on utility values of ranked programs
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Solvers for SMP-C

• NRMP uses “deferred acceptance” alg. (based on Gale-Shapley)
  • Relies on low % of couples (Drummond et al., 2015)
    • With low % couples, can solve large instances very fast
• Drummond et al. (2015) develop a satisfiability (SAT) encoding for SMP-C
  • Best scaling results of any complete solver
Advantages of SAT

• Can quickly find $\mathcal{RP}_{opt}$ or $\mathcal{R}_{opt}$ matchings
• Can also enumerate all stable matchings
  • Could be used to implement randomized mechanisms
• Can implement an $\mathcal{RP}_{opt}$ mechanism
  • Guaranteed to return an $\mathcal{R}_{opt}$ matching if one exists
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Preference Models

• Impartial culture with geography (IC-GEOG) - Kojima et al. (2013)
  • Uniformly distributed (uncorrelated) preferences, couples only apply to hospitals in same region
• Scottish Foundation Allocation Scheme with geography (SFAS-GEOG) - Biró et al. (2013)
  • Geography plus Plackett-Luce
    • Hospitals and residents have varying popularity
Performance of Deferred Acceptance Algorithms

• Return $RP_{opt}$ matching 90-100% of the time, i.e., 0-10% failure rate
• Also sometimes fails to find existing stable matching
Effect of Market Size

• Not affected by market size:
  • # stable matchings
  • # $\mathcal{R}P_{opt}$ matchings
  • % of instances with $\mathcal{R}_{opt}$ matching
  • % instances with at least one stable matching
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Avg. # of Add’l Stable Hospitals per Resident

• Immorlica and Mahdian’s result appears to hold for SMP-C
Avg. # of Residents with Incentive to Manipulate Under Truncations

- There will always be some residents with incentive to truncate under an $RP_{opt}$ mechanism
Conclusions and Future Work

• Use SAT encoding for SMP-C to show:
  • Roth and Peranson’s conjecture appears false for SMP-C
  • Immorlica and Mahdian’s result appears true for SMP-C
• New theory for study of strategic behavior in SMP-C
• Provide implementation of $\mathcal{RP}_{opt}$ mechanism
• Future work
  • Proofs possible?
  • Study more general class of manipulations—reorderings
  • Use of randomization for greater strategy-proofness
Thank You! Questions?

• Poster tomorrow (Thursday)
• Code available online at git.io/vwlXq or link from website
A Caveat: Reorderings

Informally, for truncations, only need to look at set of stable matchings under true preferences (Roth and Vande Vate, 1991) (analogue for SMP-C proved in this paper)

In SMP-C, reorderings can create stable matchings that are not stable under true preferences (Biró and Klijn, 2011)

Reorderings hard to analyze computationally
  • May also be hard for manipulators to find
Add’l $RP_{OPT}$ Hospitals, % of Instances with Stable Matching
Resident Optimal Matchings as Market Size Grows

• Not affected by market size varying between 250 and 30,000 residents
  • $R_{OPT}$ exists 90-95% with 10% couples
  • $R_{OPT}$ exists 60-70% with 30% couples
• TODO: insert graph
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  • Always exist for SMP, but not SMP-C
• Theorem (this paper): in SMP-C, residents can’t benefit by truncating in an $R_{OPT}$ matching