Motivation

- Emergence of private infrastructure, e.g., in microgrids, creates an organization problem on the electricity grid.
- The problem can be addressed using the tools of game theory and optimization.
- Want to coordinate the distribution of locally-generated and main-grid power of varying cost across private and public infrastructure.
- We assume that we do not control private infrastructure—we must incentivize its owners to use it in the most efficient way.

Formulation as a Cooperative/Competitive Game

- Players decide what coalition (trading group) to join.
- Coalition chooses a strategy—the strategies available depend on the actions of other coalitions.
- Payoff distributed to members of coalition.
- Example of network infrastructure:

```
Q = 3          Q = -1          Q = -1
       Q = 1          Q = -1
       Q = -1          Q = 1
```

- Transmission losses proportional to square of the amount distributed.
- Closest known model is the Market Game (Shapley and Shubik, 1975).
- Each agent has an endowment, utility function.
- Core always exists, is easy to find.
- No natural generalization to non-independent losses.
- Open question as to whether supporting payments always exist, but examples to date always have them.

Coalitions Are Not Independent

```
Q = -1
```

Models of Agent Behavior

- Different models represent different control assumptions about agents.
- All models calculable or closely approximable efficiently in CPLEX. Because of quadratic losses, they require quadratically-constrained quadratic programs (QCQP).
- The behavior of each model is described on the 8-node example.

Four Models of Agent Behavior

Ad hoc
- Agents trade with other nearby agents to heuristically maximize profit.
- Trades with nearer agents occur first to model limited knowledge of trading agents.
- Flow on public edges constrained by physics.
- Private trades on example: \((A_i, A_j): 1\) unit and \((A_i, A_j): >1\) unit.
- Public trades: remaining requirements of \(A_i, A_j, A_k\) and \(A_l\) purchase leftovers from \(A_i\).

Private self-interested
- Groups of agents trade to collectively maximize their profits.
- Flow on public edges constrained by physics.
- Private trades on example: \((A_i, A_j): 1\) unit, \((A_i, A_j): 2\) units, and \((A_i, A_j): 1\) unit.
- Public trades: remaining requirements of \(A_i, A_j, A_k\) and \(A_l\) purchase leftovers from \(A_i\).

Cooperative
- Groups of agents cooperate with the grid to minimize the overall cost of supplying electricity.
- Flow on public edges constrained by physics.
- Private trades on example: \((A_i, A_j): >1\) unit and \((A_i, A_j): >0.5\) units.
- Public trades: Remaining demands met, but \(A_i\) receives half from \(A_j\) and half from the public grid.

Integrated
- Same as cooperative model, but do not restrict public flow to physics.
- Trades: same as cooperative, but \(A_i\) receives 1 unit from \(A_j\).

Effect of Coordination on Efficiency

- Use IEEE 300-bus system as public network.
- Generate private network as Erdős-Rényi random graph.
- Total losses in distribution: lower = better.

Table showing edge density on private network for different models and node densities.

Conclusion

Contributions
- Calculate optimal flow and payments in idealized model.
- Open problem: Market Games with non-independent losses.
- Importance of coordination.

Future Work
- Richer agent preference spaces—time-based decisions, trading off comfort vs. cost.
- New game type—representation as partition function game.