University of Toronto
Department of Computer Science

Course instructor:
Periklis Papakonstantinou

Assignment 2

Print this COVER PAGE and complete the information as indicated. Attach this page to your report. Note that without this page (completed and signed) your assignment will not be marked.

Student data

Name (First, Last): ______________________

Student number: ______________________

E-mail: ______________________

Collaborators: ______________________

(If you did not discuss this assignment with others then write Collaborators: none)

References: ______________________

(Cite any sources you have used - apart from the textbook. If you did not use any other sources then write References: none)

I certify that this assignment is my own work, and that I have acknowledged every person I discussed it with and I have cited every source of information used in its completion.

Signature: ______________________

Problem	1	2	3	4	Total
Points:	100	50	50	150	350
Score:					

Assignment 2

Due date: December 2, 2009 (beginning of class)

We adopt the notation of the text.

Problem 1 (100 points) Give a simplified proof of a weaker version of Shannon's Fundamental Theorem for Binary Symmetric Channels. This proof must follow the outline of the proof presented in class. You are allowed to make the assumption $R < \frac{C}{3}$ (or any other reasonable assumption). In other words: modify the outline of the proof presented in pp. 88-90 of your text. Carry on the averaging arguments the way mentioned in lecture. If you oversimplify things by making the assumption that everything is concetrated at $n\bar{P}$ (recall that $\bar{P} = Q$) then you get at most 90/100. To get the remaining 10 points you must apply the Law of Large Numbers (or concetration results, e.g. Chernoff bounds).

Problem 2 (50 points) Prove the following statement: A decoder Δ for a code $\mathcal{C}$ with minimum distance $d_{\min} > 2t$ corrects every error pattern of $\leq t$ errors **if and only if** all received messages $v \in S_t(u)$, $u \in \mathcal{C}$, are mapped to u; i.e. $\Delta(v) = u$.

Problem 3 (50 points) Suppose that we want to find a $[n, k]$ binary block linear code that has the property that it can correct any single bit error.

(a) (10 points) What is the smallest permissible Hamming distance between any two codewords?

(b) (15 points) If $n = 2^m - 1$, what is the maximum number of codewords we can have in such a code?

(c) (10 points) With this maximum number of codewords, what is the rate of the code (again, assuming $n = 2^m - 1$).

(d) (15 points) When $n = 7$, construct such a code (ie with the maximum possible rate for that n). List all the codewords.

Problem 4 (150 points)

(a) (50 points) What is the advantage of syndrome decoding? Make up an example (i.e. give an example not contained in your text or in any other source) showing the advantage of syndrome decoding.

(b) (100 points) Research about decoding techniques not considered in this course. Don't use wikipedia as a source. Give examples and provide a comparative study. Add references.