Use the proof structure from this course to prove or disprove. You may assume that all functions map natural numbers to non-negative real numbers.

1. \( (f \in \Omega(g) \land f \in \Omega(g)) \Rightarrow f \in \Omega(g + h) \)
2. \( (f \in \Omega(g) \land f \in \Omega(h)) \Rightarrow f \in \Omega(g \cdot h) \)
3. \( (f \in O(g) \land g \in \Omega(h)) \Rightarrow f \in O(h) \)
4. \( (f \in O(g) \land f \in O(h)) \Rightarrow f \in O(\min(g, h)) \)
SOLUTIONS

These are not fully written out, but contain the gist of each solution.

1) The claim is true. If asymptotically $f(n) \geq c_1 g(n)$ and $f(n) \geq c_2 h(n)$, then set $c = \min(c_1, c_2)$, or something similar.

2) The claim is false. A counterexample is $f(n) = g(n) = h(n) = n$.

3) The claim is false. A counterexample is $f(n) = 1, g(n) = n^2, h(n) = n$.

4) The claim is true. If asymptotically $f(n) \leq c_1 g(n)$ and $f(n) \leq c_2 h(n)$, then set $c = \max(c_1, c_2)$. Notice that $\max(c_1, c_2) \times \min(g(n), h(n)) \geq \min(c_1 g(n), c_2 h(n)) \geq f(n)$.