CSC165 Assignment 1, Summer 2011

Due on June 15, 2011

This assignment is meant to give you some practice with some of the logical expressions in Chapters 1–3 of the course notes.

1. (20 marks) Suppose $P$ is the set of students. $S(p)$ means that a student is smart, and $C(p)$ means that a student that likes the sound of cello. For each of the following, write the original statement (the one within quotation marks) in symbolic form. Is it possible to write the related statement asked for both as a precise English sentence and in symbolic form? If not, why?

(a) The negation of “Only smart students like the sound of cello.”
(b) The converse of “There is a smart student that does not like the sound of cello.”
(c) The contrapositive of “Some students like the sound of cello, if they are not smart.”
(d) The negation of “There is a smart student that is {\bf either} smart and does not like the sound of cello {\bf or} does like the sound of cello and is not smart.”

2. (20 marks) In number theory, a smooth number is an integer which factors completely into small prime numbers. A positive integer is called B-smooth if none of its prime factors are greater than $B$. For example, 75 has prime factorization $3 \times 5^2$; therefore 75 is 5-smooth because none of its prime factors is greater than 5. 5-smooth numbers are also called regular numbers. A number of important practical applications of smooth numbers and regular numbers are for fast Fourier transform (FFT) algorithms, in music theory and in cryptography.

Suppose $P$ is the set of natural numbers. Consider the following statement:

$$(S1) \text{ Every element in } P \text{ is a regular number.}$$

Which of the following statements are necessary conditions for $S1$, in other words they are implied by $S1$? Which are sufficient for $S1$, in other words they imply $S1$? Explain your answers carefully in precise, grammatical English.

(a) $P$ is empty.
(b) 33 is not a member of $P$ or 42 is not a member of $P$.
(c) $P = \{2, 6, 10, 12, 25\}$
(d) Every element in $P$ is 7-smooth.

3. (20 marks) **The Venn of Statements**- Let $S$ represent the set of students, and

(a) Let $A(s)$ represent “$s$ is antisocial.”
(b) Let $B(s)$ represent “$s$ is brave.”
(c) Let \( C(s) \) represent “\( s \) is caring.”

Consider this Venn diagram, where shaded regions are exactly those containing one or more students.

Which of the following are true, which are false? Justify your answers using the Venn diagram. If you are giving an example from one region in the diagram, show exactly which region.

(a) \( \forall s \in S, B(s) \Rightarrow A(s) \).
(b) \( \exists s \in S, C(s) \Rightarrow A(s) \).
(c) Every brave student is antisocial.
(d) Some student is both antisocial and brave only if she/he is caring.
(e) Any student who is brave must be antisocial and caring.

4. (20 marks) Express the following statements in precise, grammatical English. Do you believe each statement? Explain why, or why not. (You may want to consult the sheet of prerequisites.)

- \( Sa: \exists n \in \mathbb{N}, n \mid 14 \wedge n \mid 35 \wedge (\forall m \in \mathbb{N}, [m \mid 14 \wedge m \mid 35] \Rightarrow m \leq n) \)
- \( Sb: \forall n \in \mathbb{N}, (n \mid 140 \wedge n \mid 50) \Rightarrow (5 \mid n \wedge 2 \mid n) \)
- \( Sc: \exists n \in \mathbb{N}, n < 0 \Rightarrow n^2 > 11 \)
- \( Sd: \forall n \in \mathbb{N}, n \mid 3 \Rightarrow (\exists m \in \mathbb{N}, m \mid 5 \vee m + 1 \mid 7) \)

5. (20 marks) Each of the sentences below consists of one or more sentences that express an implication of the form “\( P \), then \( Q \).” For each case (i) identify the antecedent \( P \) and the consequent \( Q \) for each sentence, and (ii) express a counterexample to the whole sentence (even if such a counterexample doesn’t exist).

(a) If you want to get a good job, you need to finish school.
(b) You should go home, unless you’re ready to study.
(c) Babies can’t talk or water can be cold.
(d) If it doesn’t rain, I will walk back home, unless, I have sore feet.