

CSC304 Fall'26

Tutorial 5

Q1 Variants of the Vickrey Auction

Recall the single-item Vickrey auction from class. There are n agents, and each agent i has a private value $v_i \geq 0$ for the item. Given the reported values $\tilde{v} = (\tilde{v}_1, \dots, \tilde{v}_n)$, the item is given to an agent $i^* \in \arg \max_i \tilde{v}_i$, who pays $\max_{j \neq i^*} \tilde{v}_j$, and the other agents pay nothing. In class, we saw that the Vickrey auction is strategyproof. In this question, we consider two variants of the Vickrey auction. As in the Vickrey auction, it does not matter how ties are broken between agents with tied maximum reported value, but for simplicity, you can assume that in such a case the item will be given to the agent with the lowest index (i.e., agent 1 before agent 2 before agent 3, and so on).

(a) The *third-price auction* (with $n \geq 3$ agents) gives the item to an agent $i^* \in \arg \max_i \tilde{v}_i$ as before, but charges her the third highest reported value; the other agents pay nothing. Is the third-price auction strategyproof? Prove it or provide a counterexample.

(b) The *Vickrey auction with reserve price* $r > 0$ gives the item to an agent $i^* \in \arg \max_i \tilde{v}_i$ only if $\tilde{v}_{i^*} \geq r$, in which case it charges her $\max(r, \max_{j \neq i^*} \tilde{v}_j)$. If $\tilde{v}_{i^*} < r$, the item is not sold. In either case, the other agents pay nothing. Is the Vickrey auction with reserve price r strategyproof? Is it welfare maximizing?

[Hint: For strategyproofness, imagine a fake agent $n+1$ who always reports value r (note that he loses all ties), and interpret agent $n+1$ winning the item as the item not being sold. Then, use the fact that the Vickrey auction is strategyproof, which we saw in class.]

Q2 Procurement Auction

A city wants to hire one of $n \geq 2$ contractors to build a bridge. Each contractor i has a private cost $c_i \geq 0$ for building the bridge, and incurs no cost if she is not hired. The city asks each contractor to report her cost, and given the reported costs $\tilde{c} = (\tilde{c}_1, \dots, \tilde{c}_n)$, decides which contractor to hire and how much to pay each contractor.

This fits into the framework we saw in class. The set of outcomes A consists of n outcomes, one for hiring each contractor. The valuation of contractor i is $v_i(a) = -c_i$ for the outcome a in which she is hired, and $v_i(a) = 0$ for every other outcome a . If the city pays an amount x to contractor i , then $p_i = -x$, i.e., contractor i pays $-x$ to the city.

(a) Design a mechanism that always hires a contractor with the lowest reported cost (note that this is welfare maximizing), and is strategyproof and individually rational. Prove that your mechanism satisfies both properties.

(b) Does your mechanism satisfy no payments to agents? Can any mechanism that always hires a contractor satisfy both individual rationality and no payments to agents?