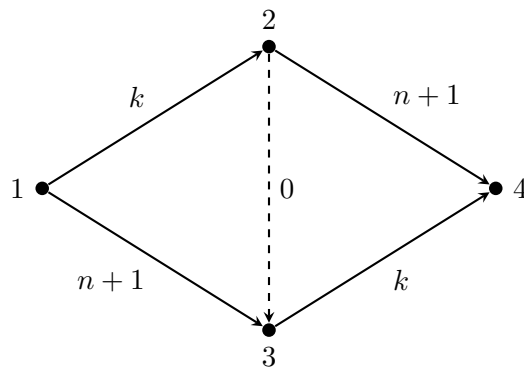


CSC304 Fall'26

Tutorial 4

Q1 Braess' Paradox

Consider the following variant of the Braess' paradox example from class. There are n players, where $n \geq 4$ is even, and each player wants to travel from node 1 to node 4 in the road network shown below. Roads $1 \rightarrow 2$ and $3 \rightarrow 4$ are congestible: if k players take such a road, each of them incurs a cost of k on that road. Roads $1 \rightarrow 3$ and $2 \rightarrow 4$ are not congestible: each player taking such a road incurs a cost of $n + 1$ on that road, regardless of how many players take it. The cost to a player is the total cost she incurs on the roads on her path.



- (a) First, suppose that the (dashed) road $2 \rightarrow 3$ does not exist. Find all pure Nash equilibria of this game and the cost to each player under them.
- (b) Now, suppose that the road $2 \rightarrow 3$, with a cost of 0 to each player taking it, is added to the network. Show that the resulting game has a unique Nash equilibrium, and find the cost to each player under it.
- (c) By what factor does adding the road $2 \rightarrow 3$ increase the cost to each player in equilibrium? What does this factor converge to as $n \rightarrow \infty$?
- (d) For the network with the road $2 \rightarrow 3$, find the optimal social cost (i.e., the minimum total cost to all players) and the price of anarchy.

Q2 Zero-Sum Games

Consider the following two-player zero-sum game, where the numbers indicate the reward to the row player.

Row \ Column	L	R
T	5	-3
B	-4	2

- (a) What is the maximin value and a maximin strategy for the row player?
- (b) What is the minimax value and a minimax strategy for the column player?

[Note: You do not need to solve a complex linear program for this. E.g., if you assume the row player's maximin strategy to be $p \cdot T + (1 - p) \cdot B$, you can compute the reward to the row player when the column player chooses L and when the column player chooses R . You want to find the value of p that maximizes the minimum of the two rewards. This can be found by plotting both rewards using any online graphing tool such as Desmos.]

Q3 No, You Go First!

Consider the following normal-form representation of a game. The row player John and the column player Marina each have three actions: L , M , and R .

John \ Marina	L	M	R
L	$(0, 1)$	$(-2, -1)$	$(-\frac{3}{2}, \frac{2}{3})$
M	$(-1, -2)$	$(-1, 0)$	$(-3, -1)$
R	$(1, 0)$	$(-2, -1)$	$(-2, \frac{1}{2})$

- (a) What is the (unique) pure Nash equilibrium of this game? What are John and Marina's rewards in this case?
- (b) If John can move first and commit to a pure strategy, which pure strategy should he commit to, and what would be the outcome? Explain.
- (c) If Marina can move first and commit to a pure strategy, which pure strategy should she commit to, and what would be the outcome? Explain.
- (d) Compare the rewards of John and Marina across the three possibilities (simultaneous moves, John first, Marina first), and say something interesting.
- (e) Now, suppose that the player who moves first can commit to a *mixed* strategy. As in class, if the player who moves second has multiple best responses, assume that she breaks ties in favor of the player who moves first. If John moves first, which mixed strategy should he commit to, and what would be the outcome? Explain.
- (f) If Marina moves first and can commit to a mixed strategy, which mixed strategy should she commit to, and what would be the outcome? Explain.
- (g) Redo the comparison from part (d) with mixed commitments. What changes?

[Hint: As in class, for each action of the player who moves second, find the best mixed strategy for the player who moves first subject to this action being a best response of the player who moves second.]