

CSC304 Fall'26

Tutorial 2

Q1 Iterated Elimination of Dominated Actions

Recall elimination of strictly/weakly dominated actions: we consider the given game, find a strictly/weakly dominated action s_i of player i , and remove it from the game. In case of iterated elimination of strictly/weakly dominated actions, we repeat this process until there are no strictly/weakly dominated actions left in the game.

- (a) Show that elimination of a strictly dominated action cannot add or remove Nash equilibria, i.e., the set of Nash equilibria is preserved. (Note: This is why in iterated elimination of strictly dominated actions, the order in which we remove strictly dominated actions does not matter.)
- (b) Show that elimination of a weakly dominated action cannot add Nash equilibria (prove this), but can remove existing Nash equilibria (give an example).
- (c) Produce an example game in which the order of elimination of weakly dominated actions can affect which Nash equilibria remain in the resulting game.

Q2 Computing Nash Equilibria

Consider the following normal-form game between two players. Each player has two actions. Payoffs for the row player P1 are indicated first in each cell, and payoffs for the column player P2 are second.

$P1 \backslash P2$	t_1	t_2
s_1	(3,6)	(1,1)
s_2	(1,2)	(2,4)

Find all mixed Nash equilibria of this game (including the pure ones).

Q3 Cunning Airlines

Recall the cunning airlines game from class. There are two players, say P1 and P2. Each player can report an integer in $\{2, 3, \dots, 99\}$. If P1 reports s and P2 reports t , their rewards are determined as follows.

- If $s = t$, both players receive s .
- If $s < t$, P1 receives $s + 2$ and P2 receives $s - 2$.
- If $t < s$, P1 receives $t - 2$ and P2 receives $t + 2$.

Find a Nash equilibrium of this game, and prove that it is the unique Nash equilibrium (that is, there are no other mixed Nash equilibria).

Q4 Guess $3/4^{\text{th}}$ of the average

Recall the game we played in class called “guess $2/3^{\text{rd}}$ of the average”. Consider a variant of this game played among three players.

Each of Alice, Bob, and Charlie picks a number in the set $\{1, 2, 3, 4\}$. The player whose number is closest to $3/4^{\text{th}}$ of the average of the three numbers wins the game (i.e., gets reward 1) and the other players lose the game (i.e., get reward 0). If multiple players are tied for being the closest to $3/4^{\text{th}}$ of the average, they share the reward equally (i.e., if two players are tied, each of them gets reward $1/2$, and if all three players are tied, each of them gets reward $1/3$), and the remaining player (if any) gets reward 0.

Find all Nash equilibria of this game.

Q5 Russell Crowe was wrong!

Watch the scene from the movie *A Beautiful Mind* that purports to explain what a Nash equilibrium is (it’s easy to find this on YouTube). This can be modeled as a game with four players (the men) and each player having five actions (corresponding to the women). Explain why the solution proposed by Russell Crowe (playing John Nash) is *not* a Nash equilibrium.