

(PRINT) Name: _____ Student No.: _____

Signature: _____ Marks: _____ / 55

CSC 304 — Algorithmic Game Theory and Mechanism Design

Midterm 1, October 19, 2018

This test has six (6) pages. Make sure that you have a complete paper.

Answer all questions on this paper. You are allowed to use one, two-sided 8.5" x 11" sheet of *handwritten* notes. No other materials or aids of any type are permitted.

Time: 50 minutes; Total Marks: 55

Q1. Nash Equilibria (25 pts)

Consider the following normal-form game involving two players. The row player (Alice) and the column player (Bob) have four actions each. In each cell of the payoff matrix, the first number indicates the payoff to the row player (Alice) and the second number indicates the payoff to the column player (Bob). Note that the payoffs are negative, so the players incur costs.

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
<i>W</i>	-4, -7	-6, -5	-5, -6	-6, -5
<i>X</i>	-5, -6	-1, -5	-2, -1	-5, -5
<i>Y</i>	-6, -6	-4, -5	-3, -3	-1, -2
<i>Z</i>	-7, -1	-3, -3	-6, -5	-7, -7

- (a) [2.5 points] If Alice plays *Z*, what is Bob's best response? You do not need to justify your answer.
- (b) [2.5 points] If Bob plays *C*, what is Alice's best response? You do not need to justify your answer.
- (c) [5 points] It is possible to iteratively eliminate strictly dominated actions and reduce the game to a 2×2 game in which only two actions of each player remain. State which actions you would remove and in which order. You do not need to justify your answer.
- (d) [5 points] In the reduced 2×2 game produced in part (c), find all (if any) pure Nash equilibria. You only need to identify the equilibria; no explanation is needed.
- (e) [10 points] In the reduced 2×2 game produced in part (c), find all mixed Nash equilibria, apart from those already found in part (d), using the indifference principle.

Use this page to continue answering any question or as a scrap paper.

Q2. Pure Nash Equilibria (10 pts)

Consider a two-player game. A mixed strategy profile (s_1, s_2) consists of a mixed strategy s_1 of player 1 and a mixed strategy s_2 of player 2. Given the profile (s_1, s_2) , the payoffs of players 1 and 2 are denoted $u_1(s_1, s_2)$ and $u_2(s_1, s_2)$, respectively. Suppose (s_1^*, s_2^*) is a mixed Nash equilibrium of the game such that

$$\begin{aligned} u_1(s_1^*, s_2^*) &> u_1(s_1, s_2^*), \forall \text{ mixed strategies } s_1 \neq s_1^* \text{ of player 1,} \\ u_2(s_1^*, s_2^*) &> u_2(s_1^*, s_2), \forall \text{ mixed strategies } s_2 \neq s_2^* \text{ of player 2.} \end{aligned}$$

Either prove that (s_1^*, s_2^*) is necessarily a pure Nash equilibrium, or provide an example in which the above conditions are satisfied but (s_1^*, s_2^*) is not a pure Nash equilibrium.

NOTE: Recall that a pure strategy is also a mixed strategy by definition, and similarly, a pure Nash equilibrium is also a mixed Nash equilibrium by definition.

Use this page to continue answering any question or as a scrap paper.

Q3. Stackelberg Equilibria (20 pts)

Consider the following game between two players. Again, the row player is Alice, the column player is Bob, the first number in each cell denotes the payoff to Alice, and the second number in each cell denotes the payoff to Bob.

	A	B
X	5, 4	0, 8
Y	0, 1	3, 2

(a) [5 points] Suppose both players have to choose their strategies simultaneously. Identify the unique Nash equilibrium of the game by iteratively eliminating strictly dominated actions. You only need to write down the actions removed, the order in which they are removed, and the unique Nash equilibrium; no further explanation is needed.

(b) [5 points] If Bob can commit to a *pure* strategy and let Alice know of his strategy, what is the best pure strategy for him to commit to? What would be his reward in the resulting outcome? Explain your answer.

(c) [10 points] If Bob can commit to a *mixed* strategy and let Alice know of his strategy, what is the best mixed strategy for him to commit to? What would be his reward in the resulting outcome? Explain your answer. If Alice is indifferent between her two pure strategies, assume that she plays the pure strategy that gives Bob a higher payoff.

HINT: You do not need to solve a complicated linear program for this question. Just think about which action Bob would want Alice to respond with.

Use this page to continue answering any question or as a scrap paper.