CSC304
Algorithmic Game Theory & Mechanism Design

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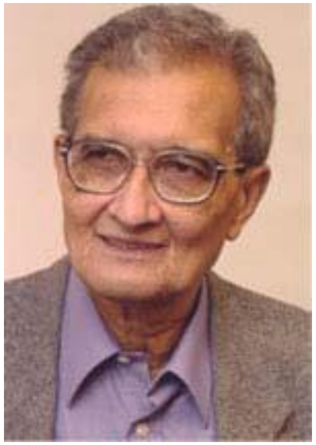
Voting

Social Choice Theory

- Mathematical theory for aggregating individual preferences into collective decisions

Social Choice Theory

- Originated in ancient Greece
- Formal foundations
- 18th Century (Condorcet and Borda)
- 19th Century: Charles Dodgson (a.k.a. Lewis Carroll)
- 20th Century: Nobel prizes to Arrow and Sen



Social Choice Theory

- Want to select a collective outcome based on (possibly different) individual preferences
  - Presidential election, restaurant/movie selection for group activity, committee selection, facility location, …

- How is it different from allocating goods?
  - One outcome that applies to all agents
  - Technically, we can think of allocations as "outcomes"
    - Very restricted case with lots of ties
    - An agent is indifferent as long as her allocation is the same
  - We want to study the more general case

Social Choice Theory

- Set of voters $N = \{1, \dots, n\}$
- Set of alternatives A, $|A| = m$
- Voter i has a preference ranking $\succ_i$ over the alternatives
- Preference profile $\vec{\succ}$ is the collection of all voters' rankings

1	2	3
a	c	b
b	a	a
c	b	c

Social Choice Theory

- Social choice function f
  - Takes as input a preference profile $\vec{\succ}$
  - Returns an alternative $a \in A$

- Social welfare function f
  - Takes as input a preference profile $\vec{\succ}$
  - Returns a societal preference $\succ^*$

- For now, voting rule = social choice function

1	2	3
a	c	b
b	a	a
c	b	c

Voting Rules

- Plurality
  - Each voter awards one point to her top alternative
  - Alternative with the most point wins
  - Most frequently used voting rule
  - Almost all political elections use plurality

  - Is this intuitively a good outcome?

1	2	3	4	5
a	a	a	b	b
b	b	b	c	c
c	c	c	d	d
d	d	d	e	e
e	e	e	a	a

Winner
a

Voting Rules

- Borda Count
  - Each voter awards $m - k$ points to alternative at rank k
  - Alternative with the most points wins
  - Proposed in the 18^{th} century by chevalier de Borda
  - Used for elections to the national assembly of Slovenia

1	2	3
a (2)	c (2)	b (2)
b (1)	a (1)	a (1)
c (0)	b (0)	c (0)

Total
a: 2+1+1 = 4
b: 1+0+2 = 3
c: 0+2+0 = 2

Winner
a

Current uses [edit]

Political uses [edit]

The Borda count is used for certain political elections in at least three countries, Slovenia and the tiny Micronesian nations of Kiribati and Nauru. In Slovenia, the Borda count is used to elect two of the ninety members of the National Assembly: one member represents a constituency of ethnic Italians, the other a constituency of the Hungarian minority. As noted above, members of the Parliament of Nauru are elected based on a variant of the Borda count that involves two departures from the normal practice: (1) multi-seat constituencies, of either two or four seats, and (2) a point-allocation formula that involves increasingly small fractions of points for each ranking, rather than whole points. In Kiribati, the president (or *Beretitenti*) is elected by the plurality system, but a variant of the Borda count is used to select either three or four candidates to stand in the election. The constituency consists of members of the legislature (*Maneaba*). Voters in the legislature rank only four candidates, with all other candidates receiving zero points. Since at least 1991, tactical voting has been an important feature of the nominating process.

The Republic of Nauru became independent from Australia in 1968. Before independence, and for three years afterwards, Nauru used instant-runoff voting, importing the system from Australia, but since 1971, a variant of the Borda count has been used.

The modified Borda count has been used by the Green Party of Ireland to elect its chairperson.[6][7]

The Borda count has been used for non-governmental purposes at certain peace conferences in Northern Ireland, where it has been used to help achieve consensus between participants including members of Sinn Féin, the Ulster Unionists, and the political wing of the UDA.

Other uses [edit]

The Borda count is used in elections by some educational institutions in the United States.

- University of Michigan
  - Central Student Government
  - Student Government of the College of Literature, Science and the Arts (LSASG)
- University of Missouri: officers of the Graduate-Professional Council
- University of California Los Angeles: officers of the Graduate Student Association
- Harvard University: officers of the Civil Liberties Union
- Southern Illinois University at Carbondale: officers of the Faculty Senate,
- Arizona State University: officers of the Department of Mathematics and Statistics assembly.
- Wheaton College, Massachusetts: faculty members of committees.
- College of William and Mary: members of the faculty personnel committee of the School of Business Administration (tie-breaker).

The Borda count is used in elections by some professional and technical societies.

- International Society for Cryobiology: Board of Governors.
- Tempo sustainable design network: management committee.
- U.S. Wheat and Barley Scab Initiative: members of Research Area Committees.
- X.Org Foundation: Board of Directors.

The OpenGL Architecture Review Board uses the Borda count as one of the feature-selection methods.

The Borda count is used to determine winners for Toastmasters International speech contests. Judges offer a ranking of their top three speakers, awarding them three points, two points, and one point, respectively. All unranked candidates receive zero points.

The modified Borda count is used to elect the President for the United States member committee of AIESEC.

The Borda count, and points-based systems similar to it, are often used to determine awards in competitions.

The Borda count is a popular method for granting sports awards in the United States. Uses include:

- MLB Most Valuable Player Award (baseball)
- Heisman Trophy (college football)[8]
- Ranking of NCAA college teams

The Eurovision Song Contest uses a positional voting method similar to the Borda count, with a different distribution of points: only the top ten entries are considered in each ballot, the favorite entry receiving 12 points, the second-placed entry receiving 10 points, and the other eight entries getting points from 8 to 1. Although designed to favor a clear winner, it has produced very close races and even a tie.

The People's Remix Competition uses a Borda variant where each voter ranks only the top three contestants.

The Borda count is used for wine trophy judging by the Australian Society of Viticulture and Oenology, and by the RoboCup autonomous robot soccer competition at the Center for Computing Technologies, in the University of Bremen in Germany.

The Finnish Associations Act lists three different modifications of the Borda count for holding a proportional election. All the modifications use fractions, as in Nauru. A Finnish association may choose to use other methods of election, as well.[9]

Borda count in real life

Voting Rules

- Positional Scoring Rules
  - Defined by a score vector $\vec{s} = (s_1, \ldots, s_m)$
  - Each voter gives s_k points to alternative at rank k

- A family containing many important rules
  - Plurality = $(1,0, \ldots, 0)$
  - Borda = $(m-1, m-2, \ldots, 0)$
  - k-approval = $(1, \ldots, 1,0, \ldots, 0)$ $\leftarrow$ top k get 1 point each
  - Veto = $(0, \ldots, 0, -1)$
  - …

Voting Rules

- Plurality with runoff
  - First round: two alternatives with the highest plurality scores survive
  - Second round: between these two alternatives, select the one that majority of voters prefer

- Similar to the French presidential election system
  - Problem: vote division
  - Happened in the 2002 French presidential election

Voting Rules

- Single Transferable Vote (STV)
  - $m - 1$ rounds
  - In each round, the alternative with the least plurality votes is eliminated
  - Alternative left standing is the winner
  - Used in Ireland, Malta, Australia, New Zealand, …

- STV has been strongly advocated for due to various reasons

STV Example

2 voters	2 voters	1 voter
a	b	c
b	a	d
c	d	b
d	c	a

→

2 voters	2 voters	1 voter
a	b	c
b	a	b
c	c	a

↓

2 voters	2 voters	1 voter
a	b	b
b	a	a

←

2 voters	2 voters	1 voter
b	b	b

Voting Rules

- Kemeny's Rule
  - Social welfare function (selects a ranking)
  - Let $n_{a \succ b}$ be the number of voters who prefer a to b
  - Select a ranking σ of alternatives = for every pair (a, b) where $a \succ_\sigma b$, we make $n_{b \succ a}$ voters unhappy
  - Total unhappiness $K(\sigma) = \sum_{(a,b): a \succ_\sigma b} n_{b \succ a}$
  - Select ranking σ^* with the minimum total unhappiness
- Social choice function
  - Choose the top alternative in Kemeny ranking

Condorcet Winner

- Definition: Alternative x beats y in a pairwise election if a strict majority of voters prefer x to y
  - We say that the majority preference prefers x to y
- Condorcet winner beats every other alternative in pairwise election
- Condorcet paradox: when the majority preference is cyclic

1	2	3
a	b	c
b	c	a
c	a	b

Majority Preference

$a \succ b$

$b \succ c$

$c \succ a$

Condorcet Consistency

- Condorcet winner is unique, if one exists

- A voting rule is Condorcet consistent if it always selects the Condorcet winner if one exists

- Among rules we just saw:
  - *NOT* Condorcet consistent: all positional scoring rules (plurality, Borda, …), plurality with runoff, STV
  - Condorcet consistent: Kemeny (WHY?)

Majority Consistency

- Majority consistency: If a majority of voters rank alternative x first, x should be the winner.

- Question: What is the relation between majority consistency and Condorcet consistency?

  1. Majority consistency $\Rightarrow$ Condorcet consistency
  2. Condorcet consistency $\Rightarrow$ Majority consistency
  3. Equivalent
  4. Incomparable

Condorcet Consistency

- Copeland
  - Score(x) = # alternatives x beats in pairwise elections
  - Select x^* with the maximum score
  - Condorcet consistent (WHY?)

- Maximin
  - Score(x) = $\min_{y} n_{x \succ y}$
  - Select x^* with the maximum score
  - Also Condorcet consistent (WHY?)

Which rule to use?

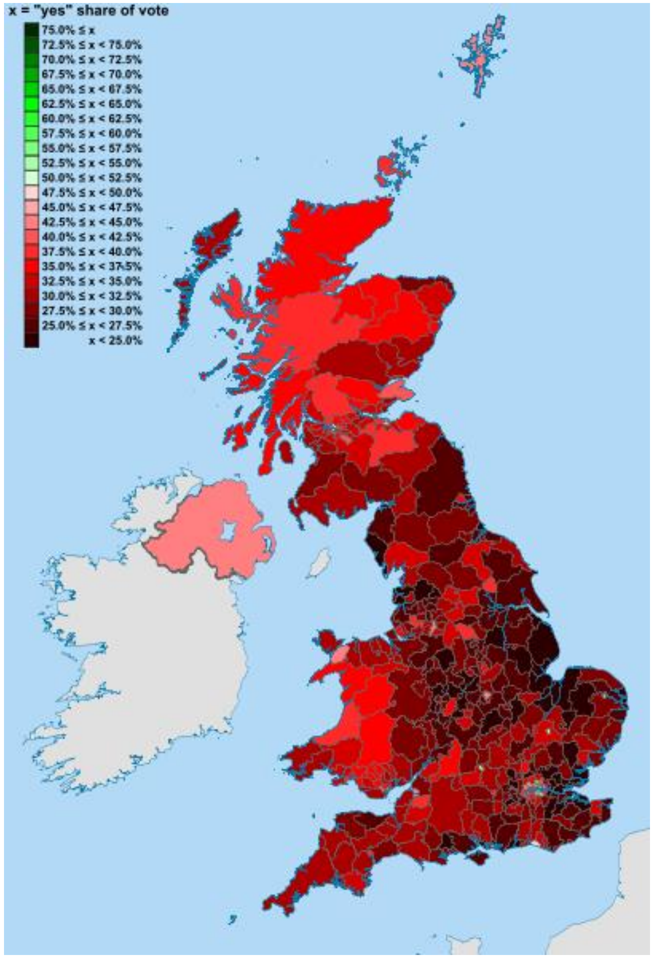
- We just introduced infinitely many rules
  - (Recall positional scoring rules…)

- How do we know which is the “right” rule to use?
  - Various approaches
  - Axiomatic, statistical, utilitarian, …

- How do we ensure good incentives without using money?
  - Bad luck! [Gibbard-Satterthwaite, next lecture]

Is Social Choice Practical?

- UK referendum: Choose between plurality and STV for electing MPs
- Academics agreed STV is better...
- ...but STV seen as beneficial to the hated Nick Clegg
- Hard to change political elections!



Gibbard-Satterthwaite Theorem

Strategyproofness

- Would any of these rules incentivize voters to report their preferences truthfully?

- A voting rule f is strategyproof if for every
  - preference profiles $\vec{\succ}$,
  - voter i, and
  - preference profile $\vec{\succ}'$ such that $\succ'_j = \succ_j$ for all $j \neq i$

  - it is not the case that $f(\vec{\succ}') \succ_i f(\vec{\succ})$

Strategyproofness

- None of the rules we saw are strategyproof!

- Example: Borda Count
  - In the true profile, b wins
  - Voter 3 can make a win by pushing b to the end

Winner
b

1	2	3
b	b	a
a	a	b
c	c	c
d	d	d

→

1	2	3
b	b	a
a	a	c
c	c	d
d	d	b

Winner
a

Borda's Response to Critics

My scheme is intended only for honest men!

Random 18th century French dude

Strategyproofness

- Are there any strategyproof rules?
  - Sure
- Dictatorial voting rule
  - The winner is always the most preferred alternative of voter i
- Constant voting rule
  - The winner is always the same
- Not satisfactory (for most cases)

Dictatorship

Constant function

Three Requirements

- Strategyproof: Already defined. No voter has an incentive to misreport.
- Onto: Every alternative can win under some preference profile.
- Nondictatorial: There is no voter i such that $f(\overrightarrow{\succ})$ is always the top alternative for voter i.

Gibbard-Satterthwaite

- Theorem: For $m \geq 3$, no deterministic social choice function can be strategyproof, onto, and nondictatorial simultaneously ☹

- Proof: We will prove this for $n = 2$ voters.
  - Step 1: Show that SP is equivalent to "strong monotonicity"
  - Strong Monotonicity (SM): If $f(\overrightarrow{\succ}) = a$, and $\overrightarrow{\succ}'$ is such that $\forall i \in N, x \in A$: $a \succ_i x \Rightarrow a \succ_i' x$, then $f(\overrightarrow{\succ}') = a$.
    - If a is winning, and the votes change so that in each vote, a still defeats each alternative it defeated before, then a should still win.

Gibbard-Satterthwaite

- Theorem: For $m \geq 3$, no deterministic social choice function can be strategyproof, onto, and nondictatorial simultaneously ☹

- Proof: We will prove this for $n = 2$ voters.
  - Step 2: Show that SP+onto implies "Pareto optimality"
  - Pareto Optimality (PO): If $a \succ_i b$ for all $i \in N$, then $f(\overrightarrow{\succ}) \neq b$.
    - If there is a different alternative that *everyone* prefers, your choice is not Pareto optimal (PO).

Gibbard-Satterthwaite

- Proof for n=2: Consider a problem instance $I(a, b)$

$I(a, b)$ → (points to the row "b | a")

$\succ_1$	$\succ_2$
a	b
b	a
Arbitrary	Arbitrary

Say $f(\succ_1, \succ_2) = a$

- PO: $f(\succ_1, \succ_2) \in \{a, b\}$

$\succ_1$	$\succ_2'$
a	b
b	Same as before
Same as before	a

$f(\succ_1, \succ_2') = a$

- PO: $f(\succ_1, \succ_2') \in \{\mathrm{a}, \mathrm{b}\}$
- SP: $f(\succ_1, \succ_2') \neq b$

$\succ_1''$	$\succ_2''$
a	A N Y
A N Y	

$f(\succ'') = a$

- Due to strong monotonicity

Gibbard-Satterthwaite

- Proof for n=2:
  - If f outputs a on instance $I(a, b)$, voter 1 can get a elected whenever she puts a first.
    - In other words, voter 1 becomes dictatorial for a.
    - Denote this by $D(1, a)$.
  - If f outputs b on $I(a, b)$
    - Voter 2 becomes dictatorial for b, i.e., we have $D(2, b)$.

- For every pair of alternatives (a, b), at least one of $D(1, a)$ and $D(2, b)$ holds.

Gibbard-Satterthwaite

- Proof for n=2:
  - Take a pair (a^*, b^*)
  - Suppose wlog that $D(1, a^*)$ holds
  - Then, we show that voter 1 is a dictator, i.e., $D(1, x)$ holds for every other x as well
  - Take $x \neq a^*$
  - Because $|A| \geq 3$, there exists $y \in A \backslash \{a^*, x\}$.
  - For (x, y), at least one of $D(1, x)$ and $D(2, y)$ holds
  - But $D(2, y)$ is incompatible with $D(1, a^*)$
    - Who wins if voter 1 puts a^* first and voter 2 puts y first?
  - Thus, we have $D(1, x)$, as required. ■

Circumventing G-S

- Randomization
  - Gibbard characterized all randomized strategyproof rules
  - Somewhat better, but still too restrictive
- Restricted preferences
  - Median for facility location (more generally, for single-peaked preferences on a line)
  - Will see other such settings later
- Money
  - E.g., VCG is nondictatorial, onto, and strategyproof, but charges payments to agents

Circumventing G-S

- Equilibrium analysis
  - Maybe good alternatives still win under Nash equilibria?
- Lack of information
  - Maybe voters don’t know how other voters will vote?

Circumventing G-S

- Computational complexity (Bartholdi et al.)
  - Maybe the rule is manipulable, but it is NP-hard to find a successful manipulation?
  - Groundbreaking idea! NP-hardness can be good!!

- Not NP-hard: plurality, Borda, veto, Copeland, maximin, …
- NP-hard: Copeland with a peculiar tie-breaking, STV, ranked pairs, …

Circumventing G-S

- Computational complexity
  - Unfortunately, NP-hardness just says it is hard for *some worst-case instances.*
  - What if it is actually easy for most practical instances?
  - Many rules admit polynomial time manipulation algorithms for fixed #alternatives (m)
  - Many rules admit polynomial time algorithms that find a successful manipulation on almost all profiles (the fraction of profiles converges to 1)

- Interesting open problems regarding the design of voting rules that are hard to manipulate on average

Social Choice

- Let’s forget incentives for now.
- Even if voters reveal their preferences truthfully, we do not have a “right” way to choose the winner.

- Who is the right winner?
  - On profiles where the prominent voting rules have different outputs, all answers seem reasonable [HW3].

Axiomatic Approach

Axiomatic Approach

- Goal: Define a set of reasonable desiderata, and find voting rules satisfying them
  - Ultimate hope: a unique voting rule satisfies the axioms we are interested in!
- Sadly, it's often the opposite case.
  - Many combinations of reasonable axioms cannot be satisfied by any voting rule.
  - GS theorem: nondictatorship + ontoness + strategyproofness = $\emptyset$
  - Arrow's theorem: we'll see
  - …

Axiomatic Approach

- Unanimity: If all voters have the same top choice, that alternative is the winner.

$$\left(top(\succ_i) = a \;\; \forall i \in N\right) \Rightarrow f\left(\overrightarrow{\succ}\right) = a$$

  - I used $top(\succ_i) = a$ to denote $a \succ_i b \; \forall b \neq a$

- Pareto optimality: If all voters prefer a to b, then b is not the winner.

$$\left(a \succ_i b \; \forall i \in N\right) \Rightarrow f\left(\overrightarrow{\succ}\right) \neq b$$

- Q: *What is the relation between these axioms?*
  - *Pareto optimality* $\Rightarrow$ *Unanimity*

Axiomatic Approach

- Anonymity: Permuting votes does not change the winner (i.e., voter identities don't matter).
  - E.g., these two profiles must have the same winner:
    {voter 1: $a \succ b \succ c$, voter 2: $b \succ c \succ a$}
    {voter 1: $b \succ c \succ a$, voter 2: $a \succ b \succ c$}

- Neutrality: Permuting the alternative names permutes the winner accordingly.
  - E.g., say a wins on {voter 1: $a \succ b \succ c$, voter 2: $b \succ c \succ a$}
  - We permute all names: $a \rightarrow b$, $b \rightarrow c$, and $c \rightarrow a$
  - New profile: {voter 1: $b \succ c \succ a$, voter 2: $c \succ a \succ b$}
  - Then, the new winner must be b.

Axiomatic Approach

- Neutrality is tricky
  - As we defined it, it is inconsistent with anonymity!
    - Imagine {voter 1: $a \succ b$, voter 2: $b \succ a$}
    - Without loss of generality, say a wins
    - Imagine a different profile: {voter 1: $b \succ a$, voter 2: $a \succ b$}
      - Neutrality: We just exchanged $a \leftrightarrow b$, so winner is b.
      - Anonymity: We just exchanged the votes, so winner stays a.
  - Typically, we only require neutrality for…
    - Randomized rules: E.g., a rule could satisfy both by choosing a and b as the winner with probability ½ each, on both profiles
    - Deterministic rules allowed to return ties: E.g., a rule could return $\{a, b\}$ as tied winners on both profiles.

Axiomatic Approach

- Majority consistency: If a majority of voters have the same top choice, that alternative wins.

$$\left(|\{i: top(\succ_i) = a\}| > \frac{n}{2} \right) \Rightarrow f(\vec{\succ}) = a$$

  - Satisfied by plurality, but not by Borda count

- Condorcet consistency: If a defeats every other alternative in a pairwise election, a wins.

$$\left(|\{i: a \succ_i b\}| > \frac{n}{2}, \forall b \neq a \right) \Rightarrow f(\vec{\succ}) = a$$

  - Condorcet consistency $\Rightarrow$ Majority consistency
  - Violated by both plurality and Borda count

Axiomatic Approach

- Is even the weaker axiom majority consistency a reasonable one to expect?

1	2	3	4	5
a	a	a	b	b
b	b	b		
			a	a

Axiomatic Approach

- Consistency: If a is the winner on two profiles, it must be the winner on their union.

$$f(\overrightarrow{\succ}_1) = a \wedge f(\overrightarrow{\succ}_2) = a \Rightarrow f(\overrightarrow{\succ}_1 + \overrightarrow{\succ}_2) = a$$

  - Example: $\overrightarrow{\succ}_1 = \{a \succ b \succ c\}$, $\overrightarrow{\succ}_2 = \{a \succ c \succ b, b \succ c \succ a\}$
  - Then, $\overrightarrow{\succ}_1 + \overrightarrow{\succ}_2 = \{a \succ b \succ c, a \succ c \succ b, b \succ c \succ a\}$

- Is this reasonable?
  - Young [1975] showed that subject to mild requirements, a voting rule is consistent if and only if it is a positional scoring rule!
  - Thus, plurality with runoff, STV, Kemeny, Copeland, Maximin, etc are *not* consistent.

Axiomatic Approach

- Weak monotonicity: If a is the winner, and a is "pushed up" in some votes, a remains the winner.
  - $f(\overrightarrow{\succ}) = a \Rightarrow f(\overrightarrow{\succ}') = a$ if
    1. $b \succ_i c \Leftrightarrow b \succ_i' c, \forall i \in N,\ b, c \in A \backslash \{a\}$
       "Order among other alternatives preserved in all votes"
    2. $a \succ_i b \Rightarrow a \succ_i' b, \forall i \in N,\ b \in A \backslash \{a\}$ (a only improves)
       "In every vote, a still defeats all the alternatives it defeated"
- Contrast: strong monotonicity requires $f(\overrightarrow{\succ}') = a$ even if $\overrightarrow{\succ}'$ only satisfies the 2nd condition
  - It is thus too strong. Equivalent to strategyproofness!
  - Only satisfied by dictatorial/non-onto rules [GS theorem]

Axiomatic Approach

- Weak monotonicity: If a is the winner, and a is "pushed up" in some votes, a remains the winner.
  - $f(\overrightarrow{\succ}) = a \Rightarrow f(\overrightarrow{\succ}') = a$, where
    - $b \succ_i c \Leftrightarrow b \succ_i' c, \forall i \in N,\ b, c \in A \backslash \{a\}$ (Order of others preserved)
    - $a \succ_i b \Rightarrow a \succ_i' b, \forall i \in N,\ b \in A \backslash \{a\}$ (a only improves)

- Weak monotonicity is satisfied by most voting rules
  - Only exceptions (among rules we saw): STV and plurality with runoff
  - But this helps STV be hard to manipulate
    - [Conitzer & Sandholm 2006]: "Every weakly monotonic voting rule is easy to manipulate on average."

Axiomatic Approach

- STV violates weak monotonicity

7 voters	5 voters	2 voters	6 voters
a	b	b	c
b	c	c	a
c	a	a	b

7 voters	5 voters	2 voters	6 voters
a	b	a	c
b	c	b	a
c	a	c	b

- First c, then b eliminated
- Winner: a

- First b, then a eliminated
- Winner: c

NOT IN SYLLABUS

Utilitarian Approach
(Only if time permits)

Utilitarian Approach

- Each voter i still submits a ranking $\succ_i$
  - But the voter has "implicit" numerical utilities $\{v_i(a) \geq 0\}$

$$\Sigma_a \, v_i(a) = 1$$
$$a \succ_i b \Rightarrow v_i(a) \geq v_i(b)$$

- Goal:
  - Select a^* with the maximum social welfare $\sum_i v_i(a^*)$
    - Cannot always find this given only rankings from voters

  - Refined goal: Select a^* that gives the best worst-case approximation of welfare

Distortion

- The distortion of a voting rule f is its approximation ratio of social welfare, on the worst preference profile.

$$dist(f) = \sup_{valid\ \{v_i\}} \frac{\max_b \sum_i v_i(b)}{\sum_i v_i\left(f(\vec{\succ})\right)}$$

  - where each v_i is valid if $\sum_a v_i(a) = 1$
  - $\vec{\succ} = (\succ_1, \dots, \succ_n)$ where $\succ_i$ represents the ranking of alternatives according to v_i

Example

- Suppose there are 2 voters and 3 alternatives
- Suppose our f returns c on this profile

Rankings

1	2
a	c
b	a
c	b

Utilities

1	2
a : 1.0	c : 0.5
b : 0.0	a : 0.5
c : 0.0	b : 0.0

Social welfare
a = 1.5 (optimal)
c = 0.5
$dist(f) \geq 3$

Utilities

1	2
a : 0.4	c : 0.7
b : 0.3	a : 0.2
c : 0.3	b : 0.1

Social welfare
c = 1.0 (optimal)

$dist(f) \geq 1$

…

$dist(f)$ is the largest such number you can find by constructing consistent utility profiles

Optimal Deterministic Rules

- Theorem [Caragiannis et al. '17]:
  Plurality achieves $O(m^2)$ distortion.
- Proof:
  - The winner is the top choice of at least n/m voters.
  - Each voter must have utility at least $1/m$ for her top choice. (WHY?)
  - Plurality achieves social welfare at least $\frac{n}{m} \cdot \frac{1}{m} = \frac{n}{m^2}$
  - No alternative can achieve social welfare more than n (WHY?)
  - QED!

Optimal Deterministic Rules

- Theorem [Caragiannis et al. '17]:
  Every deterministic voting rule has $\Omega(m^2)$ distortion.
- Proof:
  - n voters divided into $m-1$ blocks of equal size
  - Preference profile:
    - voters in block i put a_i first, a_m next, and the rest arbitrarily
  - If output = $a_m \Rightarrow \infty$ distortion (WHY?)
  - If output $\in \{a_1, \dots, a_{m-1}\} \Rightarrow \Omega(m^2)$ distortion
    - Derivation on the board!

$$n/(m-1) \text{ times } \left\{ \begin{array}{l} a_1 \succ a_m \succ \cdots \\ a_2 \succ a_m \succ \cdots \\ a_3 \succ a_m \succ \cdots \\ \vdots \\ a_{m-1} \succ a_m \succ \cdots \end{array} \right.$$

Optimal Randomized Rules

- Theorem [Boutilier et al. '15]:
  There is a randomized rule with $O\left(\sqrt{m \cdot \log m}\right)$ distortion.

Optimal Randomized Rules

- Theorem [Boutilier et al. ‘15]:
  No randomized rule has distortion better than $\sqrt{m}/3$.
- Theorem [Ebadian et al. ‘22]:
  There is a randomized rule with $O(\sqrt{m})$ distortion.

Utilitarian Approach

- Pros: Uses minimal assumptions and yields a uniquely optimal voting rule
- Cons: The optimal rule is difficult to compute and unintuitive to humans

- This approach is currently deployed on RoboVote.org
  - It has been extended to select a set of alternatives, select a ranking, select public projects subject to a budget constraint, etc.